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S-DUALITY AND COMPACTIFICATION OF TYPE IIB SUPERSTRING ACTION

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ABSTRACT

The $\mathbf{SL(2,R)}$ invaraint ten dimensional type IIB superstring effective action is compactified on a torus to D spacetime dimensions. The transformation properties of scalar, vector and tensor fields, appearing after the dimensional reduction, are obtained in order to maintain the $\mathbf{SL(2,R)}$ invariance of the reduced effective action. The symmetry of the action enables one to generate new string vacua from known configurations. As illustrative examples, new black hole solutions are obtained in five and four dimensions from a given set of solutions of the equations of motion.

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It is recognised that dualities play a central role in our understanding of the dyanamics of string theory[1]. The intimate connections between various superstring theories and the nonperturbative features of these theories in diverse dimensions are unravelled by the web of duality relations[2,3]. The S-duality transformation relates strong and weak coupling phases of a given theory in some cases, whereas in some other situation strong and weak coupling regimes of two different theories are connected. One familiar example is the heterotic string toroidally compactified from ten to four dimension and for such a theory S-duality is the generalization of the familiar electric-magnetic duality. Another situation arises in six spacetime dimensions; when the ten dimensional hetetoric string is compactified on T^4. The S-duality, on this occasion, relates the six dimensional heterotic string to type IIA theory compactified on K_3.

It was conjectured that type IIB theory in ten dimensions is endowed with $\mathbf{SL(2,Z)}$ symmetry [4,5]. There is mounting evidence for this symmetry and it has played a very important role in providing deeper insight into the nonperturbative attributes of type IIB theory [6,7]. Furthermore, there is an intimate connection between type IIB theory compactified on a circle and the M-theory compactified on T^2 leading to a host of interesting results [8]. We recall that the bosonic massless excitations of type IIB theory consist of graviton, dilaton and antisymmetric tensor in the NS-NS sector; denoted by $\hat{G}_{MN}, \hat{\phi}$ and $\hat{B}^{(1)}_{MN}$, respectively. The R-R counterparts are 'axion', $\hat{\chi}$, another antisymmetric tensor field, $\hat{B}^{(2)}_{MN}$ and a four index antisymmetric potential, $\hat{C}_{MNPR}$, with self-dual field strength. The Lorentz indices in ten dimensions are denoted by M,N,P,... and the field are defined with a hat. The complex moduli, $\hat{\lambda} = \hat{\chi} + ie^{-\hat{\phi}}$ is known to transform nontrivially under $\mathbf{SL(2,R)}$ and same is the case for the two second rank tensor fields $\hat{B}^(1)$ and $B^{(2)}$ (see below for the exact transformation rules). The $\mathbf{SL(2,R)}$ eventually breaks to the robust discrete symmetry $\mathbf{SL(2,Z)}$.

The purpose of this investigation is to study toroidal compactification of the type IIB theory and implications of $\mathbf{SL(2,R)}$ symmetry for the reduced action. Furthermore, we would like know, what are the types of interaction terms involving the moduli, dilaton and axion, are permitted when we impose $\mathbf{SL(2,R)}$ symmetry. It will be shown that a manifestly $\mathbf{SL(2,R)}$ invariant reduced action can be written down by defining the transformation properties for the scalar, gauge and tensor fields, which appear as a consequence of toroidal compactification of the ten dimensional theory. We shall show that the interactions involving the complex moduli, expressed in terms of the dilaton and the axion, can be restricted by demanding the $\mathbf{SL(2,R)}$ invariance of the effective action.

The compactifications of type IIA and type IIB theories as we go from ten to nine dimensions have been studied by Bergshoeff, Hull and Otin [9] and they have explored implications of various dualities for this compactification; more recently, Andrianopoli and collaborators [10] have studied compactification of type II theories and M-theory in various dimensions. It is well known that type IIA and type IIB theories are related by T-duality below ten dimensions [11]. Furthermore, in lower dimensions the S-duality combines with the T-duality leading to U-duality; for example in 8-dimensions, the U-duality group is $\mathbf{SL(3,Z)}\times\mathbf{SL(2,Z)}$ and various branes belong to representations [12] of this larger group. Recently, the five dimensional string effective action, obtained by toroidal compactification of type IIB superstring action, has attracted considerable of attention in establishing Beckenstein-Hawking area-entropy relations for extremal black holes and the near extremal ones [13-18]. Therefore, it is of interest to obtain type IIB effective action, through dimensional reduction, in lower dimensional spacetime and explore the implications of $\mathbf{SL(2,R)}$ duality transformations.

Let us consider the ten dimensional action for the type IIB theory:

$$\begin{aligned}\hat{S} \;=\; & \frac{1}{2\kappa^2}\int d^{10}x\sqrt{-\hat{G}}\Big\{e^{-2\hat{\phi}}\left(\hat{R}+4(\partial\hat{\phi})^2-\frac{1}{12}\hat{H}^{(1)}_{MNP}\hat{H}^{(1)MNP}\right)-\frac{1}{2}(\partial\hat{\chi})^2\\ & -\frac{1}{12}\hat{\chi}^2\hat{H}^{(1)}_{MNP}\hat{H}^{(1)MNP}-\frac{1}{6}\hat{\chi}\hat{H}^{(1)}_{MNP}\hat{H}^{(2)MNP}-\frac{1}{12}\hat{H}^{(2)}_{MNP}\hat{H}^{(2)MNP}\Big\} \qquad (1)\end{aligned}$$

Here $\hat{G}_{MN}$ is the ten dimensional metric in the string frame and $\hat{H}^{(1)}$ and $\hat{H}^{(2)}$ are the field strengths associated with the potentials $\hat{B}^{(1)}$ and $\hat{B}^{(2)}$ respectively. It is well known that in ten dimensions, it is not possible to construct a covariant action [19] for $\hat{C}_{MNPR}$ with self-dual field strength and therefore, we have set this field to zero throughout this paper; however, one can dimensionally reduce this field while carrying out compactification; we set it to zero for convenience. In order to express the action in a manifestly $\mathbf{SL(2,R)}$ invariant form [20,7], recall that the axion and the dilaton parametrize [21] the coset $\frac{\mathbf{SL(2,R)}}{SO(2)}$. We over to the Einstein frame through the conformal transformation $\hat{g}_{MN}=e^{-\frac{1}{2}\hat{\phi}}\hat{G}_{MN}$ and the action (1) takes the form

$$\hat{S}_E=\frac{1}{2\kappa^2}\int d^{10}x\sqrt{-\hat{g}}\Big\{\hat{R}_{\hat{g}}+\frac{1}{4}Tr(\partial_N\hat{\mathcal{M}}\partial^N\hat{\mathcal{M}}^{-1})-\frac{1}{12}\hat{\mathbf{H}}^T_{MNP}\hat{\mathcal{M}}\hat{\mathbf{H}}^{MNP}\Big\} \qquad (2)$$

Here $\hat{R}_{\hat{g}}$ is the scalar curvature computed from the Einstein metric. The matrix $\hat{\mathcal{M}}$ is defined as:

$$\hat{\mathcal{M}} = \begin{pmatrix} \hat{\chi}^2 e^{\hat{\phi}} + e^{-\hat{\phi}} & \hat{\chi} e^{\hat{\phi}} \\ \hat{\chi} e^{\hat{\phi}} & e^{\hat{\phi}} \end{pmatrix}, \qquad \hat{\mathbf{H}} = \begin{pmatrix} \hat{H}^{(1)} \\ \hat{H}^{(2)} \end{pmatrix} \tag{3}$$

Note that $det\hat{\mathcal{M}}$ is unity. The action is invariant under following transformations,

$$\hat{\mathcal{M}} \to \Lambda \hat{\mathcal{M}} \Lambda^T, \qquad H \to (\Lambda^T)^{-1} H, \qquad \hat{g}_{MN} \to \hat{g}_{MN}, \tag{4}$$

where $\Lambda \in \mathbf{SL(2,R)}$. Let us introduce a 2×2 matrix, Σ and consider a generic form of Λ

$$\Lambda = \begin{pmatrix} a & b \\ c & d \end{pmatrix}, \qquad \Sigma = \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix} \tag{5}$$

with $ad - bc = 1$. It is easy to check that,

$$\Lambda \Sigma \Lambda^T = \Sigma, \qquad \Sigma \Lambda \Sigma = \Lambda^{-1} \tag{6}$$

and

$$\hat{\mathcal{M}} \Sigma \hat{\mathcal{M}} = \Sigma, \qquad \Sigma \hat{\mathcal{M}} \Sigma = \hat{\mathcal{M}}^{-1} \tag{7}$$

Thus Σ plays the role of $\mathbf{SL(2,R)}$ metric and the symmetric matrix $\hat{\mathcal{M}} \in \mathbf{SL(2,R)}$.

It is evident that the second term of eq(2) can be written as

$$\frac{1}{4} Tr[\partial_N \hat{\mathcal{M}} \Sigma \partial^N \hat{\mathcal{M}} \Sigma] \tag{8}$$

The Einstein equation can be derived by varying the action with respect to the metric and the equation of motion associated with the antisymmetric tensor fields can be obtained in a straight forward manner. The $\hat{\mathcal{M}}$ -equation of motion follows from the variation of the action if we keep in mind that $\hat{\mathcal{M}}$ is a symmetric $\mathbf{SL(2,R)}$ matrix satisfying the properties mentioned above. Thus, if we consider an infinitesimal transformation, we arrive at following relations.

$$\Lambda = \mathbf{1} + \epsilon, \qquad \Lambda \in \mathbf{SL(2,R)} \tag{9}$$

$$\epsilon\Sigma + \Sigma\epsilon^T = 0, \qquad \hat{\mathcal{M}} \to \epsilon\hat{\mathcal{M}} + \hat{\mathcal{M}}\epsilon^T + \mathcal{M} \tag{10}$$

Now the desired equation of motion, derived from the above action, is

$$\partial_M(\sqrt{-\hat{g}}\hat{g}^{MN}\hat{\mathcal{M}}\Sigma\partial_N\hat{\mathcal{M}}\Sigma) - \frac{1}{6}\hat{\mathbf{H}}^T\hat{\mathcal{M}}\hat{\mathbf{H}} = 0 \tag{11}$$

Note that this is a matrix equation of motion and we have suppressed the indices for notational conveniences. It is worthwhile, at this stage to point out some similarities with the the global $O(d,d)$ symmetry that arises when one considers toroidal compactifications to lower spacetime dimensiona. The metric Σ is analogous to the metric, η, associated with the $O(d,d)$ transformations and the $\hat{\mathcal{M}}$ equation of motion bears resemblance with the corresponding M-matrix of the $O(d,d)$ case [22,23].

Let us consider compactification of the action of the type IIB theory on a d-dimensional torus to obtain the dimensionally reduced effective action [24-27]. Let the D-dimensional spacetime coordinates be denoted by $x^\mu, \mu = 1, 2, ..D-1$ and the internal coordinates be labelled by y^α and α takes d values so that $D + d = 10$. The following choice of 10-dimensional vielbein [24,25] is convenient to derive the reduced action:

$$\hat{e}^A_M = \begin{pmatrix} e^r_\mu & \mathcal{A}^\beta_\mu E^a_\beta \\ 0 & E^a_\alpha \end{pmatrix} \tag{12}$$

The D-dimensional spacetime metric is $g_{\mu\nu} = e^r_\mu e^s_\nu \eta_{rs}$, η_{rs} being D-dimensional Lorentzian signature flat metric and the internal metric is $\mathcal{G}_{\alpha\beta} = E^a_\alpha E^b_\beta \delta_{ab}$. In our notation above, A, r and a denote the local Lorentz indices of $\hat{e}^A_M, e^r_\mu$ and E^a_α respectively and M, μ and α are the corresponding global indices. Let us assume that the backgrounds are independent of the set of internal coordinates y^α and derive the reduced effective action. Note that, with choice of our vielbein, $\sqrt{-\hat{g}} = \sqrt{-g}\sqrt{\mathcal{G}}$. The 10-dimensional Einstein-Hilbert Lagrangian density is decomposed, when we go down to lower dimensions, as sum of three terms consiting of $\sqrt{-g}R$, kinetic energy term for the scalars, $\mathcal{G}_{\alpha\beta}$ and the kinetic energy term for the Abelian gauge fields $\mathcal{A}^\alpha_\mu$. The matrix $\hat{\mathcal{M}}$ defined in terms of the dilaton and axion becomes a matrix which carries x-dependence only and we denote it by $\mathcal{M}$ from now on.

The term $\hat{\mathbf{H}}^T\hat{\mathcal{M}}\hat{\mathbf{H}}$ is expressed as sum of three terms (each of the term is a scalar with no free index): a term with one Lorentz index and two internal indices, another term which has two Lorentz indices and one internal index and a term with three Lorentz indices. The term with all internal indices, $H_{\alpha\beta\gamma}$, vanishes, since we assume y-indepence of backgrounds and $\mathbf{H}$ always involves derivatives. When we want to obtain a D-dimenional tensor from a given 10-dimensional one, we first convert the global indices of the 10-dimensional tensor to local indices by multiplying suitable numbers of $\hat{e}$'s and $\hat{e}^{-1}$'s in ten dimensions, then we multiply with e's and e^{-1} of D-dimensions. For the case at hand,

$$H^{(i)}_{\mu\alpha\beta} = \partial_\mu B^{(i)}_{\alpha\beta} \tag{13}$$

$$H^{(i)}_{\mu\nu\alpha} = F^{(i)}_{\mu\nu\alpha} - B^{(i)}_{\alpha\beta}\mathcal{F}^{\beta}_{\mu\nu} \tag{14}$$

Where, the upper index $i = 1, 2$, $\mathcal{F}^{\alpha}_{\mu\nu} = \partial_\mu \mathcal{A}^{\alpha}_{\nu} - \partial_\nu \mathcal{A}^{\alpha}_{\mu}$ and $F^{(i)}_{\mu\nu\alpha} = \partial_\mu A^{(i)}_{\nu\alpha} - \partial_\nu A^{(i)}_{\mu\alpha}$ and the gauge potential $A^{(i)}_{\mu\alpha} = \hat{B}^{(i)}_{\mu\alpha} + B^{(i)}_{\alpha\beta}\mathcal{A}^{\beta}_{\mu}$. The antisymmetric field strength in D-dimensions takes the following form.

$$H^{(i)}_{\mu\nu\rho} = \partial_\mu B^{(i)}_{\nu\rho} - \frac{1}{2}[\mathcal{A}^{\alpha}_{\mu} F^{(i)}_{\nu\rho\alpha} + A^{(i)}_{\mu\alpha}\mathcal{F}^{\alpha}_{\nu\rho}] + cycl.perm \tag{15}$$

We mention in passing, the presence of Abelian Chern-Simons term in the expression for the field strength [25], resulting from the dimensional reduction. The two form potential is defined as

$$B^{(i)}_{\mu\nu} = \hat{B}^{(i)}_{\mu\nu} + \frac{1}{2}\mathcal{A}^{\alpha}_{\mu} A^{(i)}_{\nu\alpha} - \frac{1}{2}\mathcal{A}^{\alpha}_{\nu} A^{(i)}_{\mu\alpha} - \mathcal{A}^{\alpha}_{\mu} B^{(i)}_{\alpha\beta}\mathcal{A}^{\beta}_{\nu} \tag{16}$$

Notice that $\mathbf{H}_{\mu\nu\rho}$ is antisymmetric in all its tensor indices as should be the case.

The 10-dimensional effective action is invariant under general coordinate transformations as well as the gauge transformations associated with the two antisymmetric tensor fields. When we examine the local symmetries of the theory in D-dimensions after dimensional reduction, we find that there is general coordinate transformation invariance in D-dimensions. The Abelian gauge transformation, associated with $\mathcal{A}^{\alpha}_{\mu}$, has its origin in 10-dimensional general coordinate transformations. The field strength $H^{(i)}_{\mu\nu\alpha}$ is invariant under a suitable gauge transformation once we define the gauge transformation for $F^{(i)}_{\mu\nu\alpha}$ since $\mathcal{F}^{\alpha}_{\mu\nu}$ is gauge invariant under the gauge transformation of $\mathcal{A}$-gauge fields. Finally, the tensor field strength $H^{(i)}_{\mu\nu\rho}$, defined above, can be shown to be gauge invariant by defining appropriate gauge transformations for $B^{(i)}_{\mu\nu}$; $\delta B^{(i)}_{\mu\nu} = \partial_\mu \xi^{(i)}_{\nu} - \partial_\nu \xi^{(i)}_{\mu}$.

The D-dimensional effective action takes the following form

$$S_E = \int d^D x\sqrt{-g}\sqrt{\mathcal{G}}\Big\{R + \frac{1}{4}[\partial_\mu \mathcal{G}_{\alpha\beta}\partial^\mu \mathcal{G}^{\alpha\beta} + g^{\mu\nu}\partial_\mu ln\mathcal{G}\partial_\nu ln\mathcal{G} - g^{\mu\lambda}g^{\nu\rho}\mathcal{G}_{\alpha\beta}\mathcal{F}^\alpha_{\mu\nu}\mathcal{F}^\beta_{\lambda\rho}]$$
$$-\frac{1}{4}\mathcal{G}^{\alpha\beta}\mathcal{G}^{\gamma\delta}\partial_\mu B^{(i)}_{\alpha\gamma}\mathcal{M}_{ij}\partial^\mu B^{(j)}_{\beta\delta} - \frac{1}{4}\mathcal{G}^{\alpha\beta}g^{\mu\lambda}g^{\nu\rho}H^{(i)}_{\mu\nu\alpha}\mathcal{M}_{ij}H^{(j)}_{\lambda\rho\beta}$$
$$-\frac{1}{12}H^{(i)}_{\mu\nu\rho}\mathcal{M}_{ij}H^{(j)\mu\nu\rho} + \frac{1}{4}Tr(\partial_\mu\mathcal{M}\Sigma\partial^\mu\mathcal{M}\Sigma)\Big\} \tag{17}$$

The above action is expressed in the Einstein frame, $\mathcal{G}$ being determinant of $\mathcal{G}_{\alpha\beta}$. If we demand $\mathbf{SL(2,R)}$ invariance of the above action, then the backgrounds are required to satisfy following transformation properties:

$$\mathcal{M} \to \Lambda\mathcal{M}\Lambda^T, \qquad H^{(i)}_{\mu\nu\rho} \to (\Lambda^T)^{-1}{}_{ij}H^{(j)}_{\mu\nu\rho} \tag{18}$$

$$A^{(i)}_{\mu\alpha} \to (\Lambda^T)^{-1}{}_{ij}A^{(j)}_{\mu\alpha}, \qquad B^{(i)}_{\alpha\beta} \to (\Lambda^T)^{-1}{}_{ij}B^{(j)}_{\alpha\beta} \tag{19}$$

and

$$g_{\mu\nu} \to g_{\mu\nu}, \qquad \mathcal{A}^\alpha_\mu \to \mathcal{A}^\alpha_\mu, \qquad \mathcal{G}_{\alpha\beta} \to \mathcal{G}_{\alpha\beta} \tag{20}$$

and $\Lambda \in \mathbf{SL(2,R)}$.

It is evident from the D-dimensional action that dilaton and axion interact with antisymmetric tensor fields, gauge fields and the scalars due to the presence of $\mathcal{M}$ matrix in various terms and these interaction terms respect the $\mathbf{SL(2,R)}$ symmetry. It is important know what type of dilatonic potential is admissible in the above action which respects the S-duality symmetry. The only permissible interaction terms, preserving the symmetry, are of the form

$$Tr[\mathcal{M}\Sigma]^n, \qquad n \in \mathbf{Z} \tag{21}$$

It is easy to check using the properties of Σ and $\mathcal{M}$ matrices; such as $Tr(\mathcal{M}\Sigma) = 0$ and $Tr(\mathcal{M}\Sigma\mathcal{M}\Sigma) = 2$, that

$$Tr[\mathcal{M}\Sigma]^n = 0, \qquad and, \qquad Tr[\mathcal{M}\Sigma]^n = 2, \tag{22}$$

For odd $n \in \mathbf{Z}$ and even $n \in \mathbf{Z}$ respectively. Therefore, we reach a surprizing conclusion that the presence of interaction terms of the form in eq.(21)only adds constant term which amounts to adding cosmological constant term to the reduced action. Note that the Einstein metric is $\mathbf{SL(2,R)}$ invariant and one can add terms involving higher powers of curvature (higher derivatives of metric) to the action and maintain the symmetry. However, we are considering the case when the gravitational part of the action has the Einstein-Hilbert term only.

Now we proceed to present illustrative examples which demonstrate the application of solution generating technique to derive new backgroundis by implementing $\mathbf{SL(2,R)}$ transformations on an initial set which satisfy the equation of motion. The first example is a five dimensional effective action [28] which has the following form

$$\int d^5x\sqrt{-g}\Big\{R+\frac{1}{4}Tr(\partial_\mu\mathcal{M}\Sigma\partial^\mu\mathcal{M}\Sigma)-\frac{1}{12}e^{-\phi}H^{(1)}_{\mu\nu\rho}H^{(1)\mu\nu\rho}-\frac{1}{4}e^{\phi}F_{\mu\nu}F^{\mu\nu}\Big\} \tag{23}$$

This action corresponds to the following choice of backgrounds

$$H^{(2)}_{\mu\nu\rho}=0,\qquad \mathcal{A}^{\alpha}_{\mu}=0,\qquad H^{(1)}_{\mu\nu\alpha}=0,\qquad \chi=0,\qquad B^{(i)}_{\alpha\beta}=0,\qquad \mathcal{G}=1 \tag{24}$$

Of the five field strengths, $F_{\mu\nu\alpha}$, $\alpha=5,6,7,8,9$, coming from compactification of $H^{(2)}_{MNP}$, we choose only one of them to be nonvanishing and set rest to zero. Moreover, $\mathcal{M}=$ diag ($e^{-\phi}$, e^{ϕ}), since $\chi=0$. This five dimensional action is quite similar to the one considered by Strominger and Vafa [13] in their seminal paper in which they derived the Beckenstein-Hawking area-entropy relation for a class of five dimensional extremal black holes. It is easy to see that there will be conserved charges Q_H and Q_F proportional to

$$\int_{S^3}*\,e^{-\phi}H^{(1)},\qquad \int_{S^3}*\,e^{\phi}F \tag{25}$$

respectively. The dilaton equation takes the form

$$(\nabla\phi)^2+\frac{1}{12}e^{-\phi}H^{(1)}_{\mu\nu\rho}H^{(1)\mu\nu\rho}-\frac{1}{4}e^{\phi}F_{\mu\nu}F^{\mu\nu}=0 \tag{26}$$

Thus if we have constant dilaton configuration ϕ_c, then e^{ϕ_c} will be proporational to $(\frac{Q_F}{Q_H})^2$. Since the action is invariant under $\mathbf{SL(2,R)}$ transformation, we can generate new backgrounds with nonzero $H^{(2)}$, $F^{(1)}$ and χ, by suitable implementaion of the symmetry transformation [29,30]; note that the F appearing in the action is $F^{(2)}$ according to our choice. The simplest form of $\Lambda=\begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix}$ takes e^{ϕ} to $e^{-\phi}$ and is just the strong-weak duality transformation. A more interesting transformation is when

$$\Lambda=\frac{1}{\sqrt{2}}\begin{pmatrix}cosh\theta-sinh\theta & cosh\psi+sinh\psi\\ sinh\psi-cosh\psi & cosh\theta+cosh\theta\end{pmatrix} \tag{27}$$

Here θ and ψ are two 'boost' parameters. It is easy to construct the new matrix $\mathcal{M}'$ from eq.(18) and we find

$$e^{\phi'} = \frac{1}{2}[e^{-\phi}(cosh\psi + sinh\psi)^2 + e^{\phi}(cosh\theta + sinh\theta)^2] \tag{28}$$

and

$$\begin{aligned} \chi' e^{\phi'} &= \frac{1}{2}[e^{-\phi}(cosh\theta - sinh\theta)(cosh\psi + sinh\psi) \\ &+ e^{\phi}(cosh\theta + sinh\theta)(sinh\psi - cosh\psi)] \end{aligned} \tag{29}$$

Now we have both the gauge field strengths and they are given by

$$F^{(1)}_{\mu\nu} = -\frac{1}{\sqrt{2}}(cosh\psi + sinh\psi)F_{\mu\nu} \tag{30}$$

$$F^{(2)}_{\mu\nu} = \frac{1}{\sqrt{2}}(cosh\theta - sinh\theta)F_{\mu\nu} \tag{31}$$

where $F_{\mu\nu}$ appearing in the right hand side of the above equation is the one that was introduced in the five dimensional action, eq.(23). Furthermore, the new antisymmetric tensor field strengths are,

$$H^{(1)'}_{\mu\nu\rho} = \frac{1}{\sqrt{2}}(cosh\theta + sinh\theta)H^{(1)}_{\mu\nu\rho} \tag{32}$$

$$H^{(2)'}_{\mu\nu\rho} = \frac{1}{\sqrt{2}}(cosh\theta - sinh\theta)H^{(1)}_{\mu\nu\rho} \tag{33}$$

We note that the Einstein metric is invariant under these transformations and therefore, the spacetime geometry remains unchanged and we expect that the Hawking temperature, T_H to be the same for the family of black holes obtained through this procedure. Indeed, they will carry charges with respect to the two field strengths as well as have axionic charges. Note that these charges will be characterised by the pair of parameters θ and ψ. Of course, the residual unbroken symmetry is $\mathbf{SL(2,Z)}$ and then the transformation matrix Λ will have integer elements, satisfying the requisite constraints.

Our next example is the four dimensional black hole solution discussed by Shapere, Trivedi and Wilczek [31]. Let us recall that the charged black hole solution was obtained from an effective action which had metric, dilaton and a gauge field. Next, these authors obtained solutions in the presence of the axion. The axion field appears after one implements Poincare duality transformation on the field strength of the antisymmetric tensor (in their case it came from the NS-NS sector).

If we start with an action with graviton, dilaton and a gauge field, we can generate new solutions in the following ways.

(i) We can envisage the gauge field arising from the compactification of the metric, say one of the $\mathcal{A}^{\alpha}_{\mu}$ fields. Then, under the $\mathbf{SL(2,R}$ transformations $\mathcal{A}$ remains invariant according to our rules; however, we shall generate nontrivial χ field which comes from the RR sector and the transformed dilaton will be different from the one we started with as we demonstrated in the previous example.

(ii) On the other hand, if our gauge field arises from compactification of the antisymmetric fields,we can have either $A^{(1)}_{\mu}$ or $A^{(2)}_{\mu}$ as we like, then both the dilaton and the gauge field will transform to generate new background configurations.

It is important to note that the action considered in [31] has axion, dilaton, gauge field in addition to graviton. This action is not invariant under *their* $\mathbf{SL(2,R)}$ transformations ($\mathbf{E}^2-\mathbf{B}^2 \to \mathbf{B}^2-\mathbf{E}^2$), under the duality transformation), however the equations of motion are duality invariant. The action considered by us has an axion from the RR sector and the antisymmetric tensors $H^{(i)}_{\mu\nu\rho}$ and gauge fields $A^{(i)}_{\mu\alpha}$ transform nontrivially under the S-duality transformation eq.(19). Furthermore, the action itself is invariant under the symmetry transformation.

Let us consider the following four dimensional effective action

$$S_4 = \int d^4x\sqrt{-g}[R - \frac{1}{2}(\partial\phi)^2 - \frac{1}{4}e^{-\phi}F^{(1)}_{\mu\nu}F^{(1)\mu\nu}] \tag{34}$$

The action of reference 31 can be obtained from the above one by scaling the dilaton by a factor of two and removing the factor of $\frac{1}{4}$ from the gauge field strength squared term. We keep the superscript 1 to remind that this gauge field strength came from compactification of $H^{(1)}_{MNP}$. In this case, $\mathcal{M}$ is also diagonal in absence of RR axion field, χ. If we implement an $\mathbf{SL(2,R)}$ transformation, the new dilaton and RR axion will be given by the same expression as eqns.(28) and (29). However, now we shall generate new gauge field configurations

$$A_\mu^{(1)'} = \frac{1}{\sqrt{2}}(cosh\theta + sinh\theta)A_\mu^{(1)}, \tag{35}$$

$$A_\mu^{(2)'} = \frac{1}{\sqrt{2}}(cosh\psi - sinh\psi)A_\mu^{(1)} \tag{36}$$

Recall that initially $A_\mu^{(2)} = 0$. We mention in passing that if the four dimensional action had terms corresponding to $H_{\mu\nu\rho}^{(1)}$ and $H_{\mu\nu\rho}^{(2)}$ squares with $\mathcal{M}$ matrix, then a Poincare duality transformation on these two field strengths would have given rise to two additional 'axions'. We are considering a different scenario; however, the action in eq.(34) does admit charged black hole solutions.

To summarize, the ten dimensional type IIB superstring action can be expressed in $\mathbf{SL(2,R)}$ invariant form with the introduction of the metric Σ. The compactified theory on a d-dimensional torus respects the symmetry when we specify the transformation properties of the resulting scalar and vector fields. It is shown that the $\mathbf{SL(2,R)}$ invariant interactions terms involving only $\mathcal{M}$-matrix results in adding a cosmological constant term even when we construct general form of invariants such as trace of the product $\Sigma\mathcal{M}$. Since the action is invariant under the symmetry, we can construct new backgrounds from a set of background which satisfies equations of motion. We presented two illustrative examples: one in the case of five dimensional black hole solution of Strominger and Vafa[13] and the other for the four dimensional black hole solutions of Shapere, Trivedi and Wilczek [31].

It is evident that $\mathbf{SL(2,R)}$ transformations together with the T-dualities will enable us to generate a large family of gauge inequivalent background confugurations when we implement them on type IIB vacuum solutions in diverse dimensions.

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