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New Models of Gauge and Gravity Mediated Supersymmetry Breaking

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Abstract

We show that supersymmetry breaking in a class of theories with $SU(N)\times SU(N-2)$ gauge symmetry can be studied in a calculable sigma model. We use the sigma model to show that the supersymmetry breaking vacuum in these theories leaves a large subgroup of flavor symmetries intact, and to calculate the masses of the low-lying states. By embedding the Standard Model gauge groups in the unbroken flavor symmetry group we construct a class of models in which supersymmetry breaking is communicated by both gravitational and gauge interactions. One distinguishing feature of these models is that the messenger fields, responsible for the gauge mediated communication of supersymmetry breaking, are an integral part of the supersymmetry breaking sector. We also show how, by lowering the scale that suppresses the nonrenormalizable operators, a class of purely gauge mediated models with a combined supersymmetry breaking-cum-messenger sector can be built. We briefly discuss the phenomenological features of the models we construct.

1 Introduction.

In order to be relevant to nature, supersymmetry must be spontaneously broken. An attractive idea in this regard is that the breaking occurs nonperturbatively [1] in a strongly coupled sector of the theory and is then communicated to the Standard Model fields by some “messenger” interaction. One possibility is that the role of the messenger is played by gravity—giving rise to the so-called hidden sector models (for a review, see [2]). Another possibility [3], [2], which has received considerable attention recently [4]-[8] is that the supersymmetry breaking is communicated by gauge interactions—the gauge mediated models.

The past few years have seen some remarkable progress in the understanding of non-perturbative supersymmetric gauge theories [9], [10]. This progress has made a more thorough investigation of supersymmetry breaking possible [11]. We begin this paper by extending the study of supersymmetry breaking in a class of theories with $SU(N) \times SU(N-2)$ gauge symmetry. These theories were first considered in ref. [12]. We use some elegant observations by Y. Shirman [13] to show that the low-energy dynamics of these theories can be studied in terms of a calculable low-energy sigma model. We use the sigma model to show that the supersymmetry breaking vacuum in these theories preserves a large group of flavor symmetries, and to calculate the spectrum of low-energy excitations.

We then turn to model building. The models we construct have two sectors: a supersymmetry breaking sector—consisting of an $SU(N) \times SU(N-2)$ theory mentioned above—and the usual Standard Model sector. The basic idea is to embed the Standard Model gauge groups in the unbroken flavor symmetries of the supersymmetry breaking sector. As a result, in these models the breaking of supersymmetry can be communicated directly by the Standard Model gauge groups. This is to be contrasted with models of gauge mediated supersymmetry breaking constructed elsewhere [4], in which a fairly elaborate messenger sector is needed to accomplish the feed-down of supersymmetry breaking.

In the models under consideration here, the scale of supersymmetry breaking turns out to be high, of order the intermediate scale, i.e., 10^{10} GeV. As a result, the gravity mediated effects are comparable to the gauge mediated ones. The resulting phenomenology in these “hybrid” models is different from both the gravity and gauge mediated cases. Scalars acquire both universal soft masses due to gravity and non-universal masses due to gauge interactions, while gauginos receive masses only due to gauge interactions. Since the scale of supersymmetry breaking is large, the gravitino has an electroweak scale mass. Finally, there is new physics in this theory, at about 10 TeV, at which scale all light degrees of freedom of the supersymmetry breaking sector, including those carrying Standard Model quantum numbers, can be probed.

The biggest drawback of these models is the following. Since the scale of supersymmetry breaking is so high, one cannot, at least in the absence of any information regarding the higher dimensional operators in the Kähler potential, rule out the presence of flavor changing neutral currents. In this respect these models are no better than the usual hidden sector models.

The high scale of supersymmetry breaking arises as follows. Within the context of the $SU(N) \times SU(N-2)$ models, in order to embed the Standard Model gauge groups in the unbroken flavor symmetries, one is lead to consider large values of N, namely $N \geq 11$. In these theories supersymmetry breaking occurs only in the presence of non-renormalizable operators in the superpotential and the dimension of these operators grows as N grows. On suppressing the effects of these operators by the Planck scale, one is lead to a large supersymmetry breaking scale.

If one lowers the scale that suppresses the non-renormalizable operators, the supersymmetry breaking scale is lowered as well. We use the resulting theories to construct purely gauge mediated models with a combined supersymmetry breaking and messenger sector. The lower scale suppressing the non-renormalizable operators could arise due to new nonperturbative dynamics. It could also arise if the Standard Model gauge groups are dual to an underlying microscopic theory. We will not explicitly discuss how this lower scale arises here. A brief study of the phenomenology of the purely gauge mediated models we construct reveals some features which should be more generally true in models of this type. We hope to return to a detailed phenomenological study of these models in the future.

A few more comments are worth making with respect to the models considered here. First, from the perspective of a hidden sector theory, the hybrid models are examples of theories without any fundamental gauge singlets in which gauginos obtain adequately big soft masses.[1]

Second, one concern about constructing models in which the supersymmetry breaking sector carries Standard Model charges is that this typically leads to a loss of asymptotic freedom for the Standard Model gauge groups and the existence of Landau poles at fairly low energies. One interesting idea on how to deal with this problem involves dualizing [10] the theory and regarding the resulting dual theory—which is usually better behaved in the ultraviolet—as the underlying microscopic theory. In the "hybrid" models discussed here, one finds that the Landau poles are pushed beyond an energy scale of order 10^{16} GeV. This is a sufficiently high energy scale that even without appealing to duality their presence might not be a big concern. For example, new GUT scale physics (or conceivably even string theory related physics) could enter at this scale. The non-renormalizable operators, mentioned above, which are responsible for the large scale of supersymmetry breaking in the "hybrid models" are also responsible for pushing up the scale at which Landau poles appear; to this extent

[1] For an example of a hidden sector theory, in which supersymmetry breaking involves a global supersymmetric theory and a singlet, and yields reasonable gaugino masses, see ref. [14].

their presence is an attractive feature which one might want to retain.

Finally, we would like to comment on the low-energy effective theory used to study the breaking of supersymmetry in the $SU(N) \times SU(N-2)$ theories. This effective theory arises as follows. First, at very high energies, the $SU(N-2)$ group is broken, giving rise to an effective theory consisting of some moduli fields and a pure $SU(N)$ theory coupled to a dilaton. The $SU(N)$ theory then confines at an intermediate energy scale giving rise to a low-energy theory involving just the dilaton and the moduli. Gaugino condensation in the $SU(N)$ theory gives rise to a term in the superpotential of this low-energy theory and as a result, the superpotential has a runaway behavior characteristic of a theory containing a dilaton. However, one finds that this runaway behavior is stabilized due to a non-trivial Kähler potential involving the dilaton. It has been suggested that a similar phenomenon might be responsible for stabilizing the runaway behavior of the dilaton in string theory [15]. In the globally supersymmetric models considered here the stabilization occurs due to a calculable non-trivial Kähler potential in the effective theory linking the dilaton with the other moduli.

2 The Supersymmetry Breaking Sector.

2.1 The $\mathbf{SU(N) \times SU(N-2)}$ Models.

In this section we will briefly review the models, introduced in [12], that will play the role of a supersymmetry breaking sector. They have an $SU(N) \times SU(N-2)$ gauge group, with *odd* N, and matter content consisting of a single field, $Q_{\alpha\dot{\alpha}}$, that transforms as $(\square, \square)$ under the gauge groups, $N-2$ fields, $\bar{L}_I^\alpha$, transforming as $(\bar{\square}, \mathbf{1})$, and N fields, $\bar{R}_A^{\dot{\alpha}}$, that transform as $(\mathbf{1}, \bar{\square})$. Here, as in the subsequent discussion, we denote the gauge indices of $SU(N)$ and $SU(N-2)$ by α and $\dot{\alpha}$, respectively, while $I = 1 \ldots N-2$ and $A = 1 \ldots N$ are flavor indices. We note that these theories are chiral—no mass terms can be added for any of the matter fields.

We begin by considering the classical moduli space. It is described by the gauge invariant mesons and baryons:

$$\begin{aligned} Y_{IA} &= \bar{L}_I \cdot Q \cdot \bar{R}_A \ , \\ b^{AB} &= \frac{1}{(N-2)!} \, \varepsilon^{ABA_1 \cdots A_{N-2}} \, \varepsilon_{\dot{\alpha}_1 \cdots \dot{\alpha}_{N-2}} \, \bar{R}_{A_1}^{\dot{\alpha}_1} \cdots \bar{R}_{A_{N-2}}^{\dot{\alpha}_{N-2}} \ , \end{aligned} \tag{2.1}$$

and $\bar{\mathcal{B}} = Q^{N-2} \cdot \bar{L}^{N-2}$. These invariants are not independent but subject to classical constraints [12].

We will consider the theory with the tree-level superpotential

$$W_{tree} = \lambda^{IA} \, Y_{IA} + \frac{1}{M^{N-5}} \, \alpha_{AB} \, b^{AB} \ . \tag{2.2}$$

The superpotential W_{tree} lifts all classical flat directions, provided that λ_{IA} has maximal rank, $N-2$, the matrix α_{AB} also has maximal rank $(N-1)$, and its cokernel contains the cokernel of λ^{IA} (rank $\lambda = N-2$). With this choice of couplings, W_{tree} also preserves a nonanomalous, flavor dependent R symmetry. To see this, choose for example $\alpha^{AN} = 0, \lambda^{IN} = \lambda^{I(N-1)} = 0$ (to lift the classical flat directions). Then one sees that the field $\bar{R}_N$ appears in each of the baryonic terms of the superpotential (2.2), while it does not appear in any of the Yukawa terms. Assigning different R charges to the four types of fields, $\bar{R}_N$, $\bar{R}_{A<N}$, Q, and $\bar{L}_I$, one has to satisfy four conditions: two conditions ensuring that the superpotential (2.2) has R charge 2, and two conditions that the gauge anomalies of this R symmetry vanish. It is easy to see that there is a unique solution to these four conditions.

The couplings in the superpotential will be chosen to preserve a maximal global symmetry[2]. We will take the nonvanishing components of the Yukawa matrix to be $\lambda^{IA} = \delta^{IA}\lambda$, for $A = 1, ..., N-2$. The antisymmetric matrix α_{AB} will have the following nonvanishing elements: $\alpha_{AB} = aJ_{AB}$, for $A, B < N-2$ and $\alpha_{AB} = J_{AB}$, for $A, B = N-1, N-2$. This choice of couplings preserves an $SP(N-3)$ global nonanomalous symmetry.[3]

The dynamics of these models was discussed in [12], where it was shown that when the superpotential (2.2) is added, the ground state dynamically breaks supersymmetry. In the next section we will study supersymmetry breaking in these theories in more detail.

2.2 The Low-Energy Nonlinear Sigma Model.

2.2.1 The Essential Ideas.

We show in this section that for a region of parameter space the breaking of supersymmetry in the $SU(N) \times SU(N-2)$ theories can be conveniently studied in a low-energy effective theory. We identify the degrees of freedom, which appear in this supersymmetric nonlinear sigma model, and show that both the superpotential and the Kähler potential in the sigma model can be reliably calculated in the region of moduli space where the vacuum is expected to occur. This is interesting since the underlying theory that gives rise to the sigma model is not weakly coupled. In the following section, we then explicitly construct and minimize the potential responsible for supersymmetry breaking, thereby deducing the unbroken flavor symmetries and the spectrum of the low-energy excitations.

It is convenient to begin by considering the limit $M \to \infty$. In this limit, the baryonic

[2]This choice of couplings, which preserves the maximal global symmetries, has been made for simplicity. For the discussion of model building that follows, it is enough to preserve an $SU(3)\times SU(2)\times U(1)$ symmetry. Doing so introduces extra parameters in the superpotential eq. (2.2) but does not alter the subsequent discussion in any significant way.

[3]In our notation $SP(2k)$ is the rank k unitary symplectic group with $2k$ dimensional fundamental representation. J_{AB} is the $SP(2k)$ invariant tensor; we take $J_{12} = 1$ and $J^{AB}J_{BC} = -\delta^A_C$.

flat directions, described by the gauge invariant fields b^{AB}, are not lifted and the model has runaway directions along which the energy goes to zero asymptotically [13]. As was mentioned above, we take λ^{IA} of eq. (2.2) to be $\lambda^{IA} = \delta^{IA}\lambda$, for $A = 1, ..., N-2$. The runaway directions are specified by the condition that $b^{NN-1} \to \infty$. The other baryons b^{AB} can in addition be non-zero along these directions. We will see that once one is sufficiently far along these directions the low-energy dynamics can be described by a calculable effective theory.

Let us first consider the simplest runaway direction, $b^{NN-1} \to \infty$, with all the other $b^{AB} = 0$. Along this direction the $\bar{R}$ fields have vacuum expectation values given by $\bar{R}^{\dot{\alpha}}_A = v\delta^{\dot{\alpha}}_A$ with $v \to \infty$. Since the $SU(N-2)$ symmetry is completely broken at a scale v, its gauge bosons get heavy and can be intergated out. In the process, several components of the $\bar{R}_A$ fields get heavy or eaten and can be removed from the low-energy theory as well. In addition, on account of the Yukawa coupling in (2.2) all $N-2$ flavors of $SU(N)$ quarks become massive, with mass λv, and can be integrated out. Thus one is left with an intermediate scale effective theory containing the light components of the $\bar{R}_A$ fields and the pure $SU(N)$ gauge theory. There is one slightly novel feature about the $SU(N)$ group in this effective theory: its strong coupling scale Λ_{1L} is field dependent. On integrating out the Q and L fields one finds that

$$\Lambda_{1L}^{3N} = \Lambda_1^{2N+2}\ \lambda^{N-2} b^{N\ N-1}, \tag{2.3}$$

with Λ_1 being the scale of the ultraviolet $SU(N)$ theory. Thus the field $b^{N\ N-1}$ acts as a dilaton for the $SU(N)$ group in the low-energy theory. Going further down in energy one finds that the $SU(N)$ group confines at a scale Λ_{1L}, leaving the dilaton, b^{NN-1}, and the other light components of $\bar{R}_A$ as the excitations in the final low-energy theory. Gaugino condensation in the $SU(N)$ theory gives rise to a superpotential [9] of the form:

$$W = \lambda^{\frac{N-2}{N}}\ \Lambda_1^{\frac{2N+2}{N}}\ \left(b^{N-1\ N}\right)^{\frac{1}{N}} \tag{2.4}$$

in this low-energy theory.

So far we have considered the simplest runaway direction, $b^{NN-1} \to \infty$, with all the other $b^{AB} = 0$. There are other runaway directions, along which some of the other baryons go to infinity as well, at a rate comparable or faster than b^{NN-1}. In these cases the underlying dynamics giving rise to the effective theory can be sometimes different from that described above. However, one can show that the effective theory, consisting of the light components of $\bar{R}_A$, with the non-perturbative superpotential (2.4), describes the low-energy dynamics along these directions as well.

It is not surprising that the exact superpotential can be calculated in this effective theory. What is more remarkable is that, as has been argued in [13], the corrections to the classical Kähler potential are small along these runaway directions and thus the Kähler potential can be calculated in the effective theory as well. Thus, as promised above, the effective theory is completely calculable. Let us briefly summarize Shirman's argument here. Since the $SU(N-2)$ theory is broken at a high scale, the corrections to the Kähler potential one is worried about must involve the effects of the strongly coupled $SU(N)$ group[4] with a strong coupling scale Λ_{1L}, eq. (2.3). These corrections are of the form $\bar{R}^\dagger \bar{R} f(t)$, with $t = \Lambda_{1L}^\dagger \Lambda_{1L}/(\bar{R}^\dagger \bar{R}) \sim (\Lambda_1^\dagger\ \Lambda_1)^{\frac{2N+2}{3N}}/(\bar{R}^\dagger \bar{R})^{1-(N-2)/(3N)}$. We are interested in the behavior of $f(t)$ when $R \to \infty$, i.e., $t \to 0$. Now, it is easy to see that this limit can also be obtained when $\Lambda_1 \to 0$. In this case it is clear that the strong coupling effects due to the $SU(N)$ group must go to zero and thus the corrections to the Kähler potential for $\bar{R}$ must be small. Hereafter, we will take the Kähler potential to be classical. The discussion above shows that this is a good approximation as long as $\Lambda_{1L} \ll v$, where v denotes the vacuum expectation value of the $\bar{R}$ fields.

Let us now briefly summarize what has been learned about the theory when $M \to \infty$. We found that the theory had runaway directions. The low-energy dynamics along these directions can be described by an effective theory consisting of the light components of the fields $\bar{R}_A$. Finally, both the superpotential and the Kähler potential in this effective theory can be calculated.

Armed with this knowledge of the $M \to \infty$ limit we ask what happens when we consider M to be large but not infinite. It was shown in [12] that once the last term in (2.2) is turned on, the theory does not have any runaway directions and breaks supersymmetry. However, and this is the crucial argument, for a large enough value of M the resulting vacuum must lie along the runaway directions discussed above (since the runaway behavior is ultimately stopped by the $1/M^{N-5}$ terms in (2.2)), and therefore the breaking of supersymmetry can be analyzed in terms of the low-energy theory discussed above.

2.2.2 The Explicit Construction.

We now turn to explicitly constructing the low-energy effective theory. The light degrees of freedom of the $\bar{R}$ fields can be described either in terms of the appropriate components of $\bar{R}_A$ or the gauge invariant baryons b^{AB}. The use of the baryons is more convenient [16], since it automatically takes care of integrating out the heavy $SU(N-2)$ vector fields and their superpartners at tree level (see also [17], [18]), and provides an explicitly gauge invariant description of the low-energy physics.

The Kähler potential for the light fields is $K = \bar{R}^\dagger e^V \bar{R}|_{V=V(\bar{R}^\dagger,\bar{R})}$, where the heavy vector

[4] As mentioned above along some of the runaway directions the underlying dynamics is somewhat different. Correspondingly the strongly coupled effects do not always involve the full $SU(N)$ group. However, an analogous argument shows that the corrections to the classical Kähler potential are small along these directions as well.

superfield V is integrated out by solving its classical equation of motion. In terms of the baryons, this Kähler potential can be calculated, as in [16]:

$$K = c_N \left(b^\dagger_{AB}\, b^{AB} \right)^{\frac{1}{N-2}} , \tag{2.5}$$

where $c_N = (N-2)\, 2^{-\frac{1}{N-2}}$. The baryons b^{AB} are not independent, but obey the constraints:

$$b^{A\ B}\, b^{N-1\ N} = b^{N-1\ A}\, b^{N\ B} - b^{N-1\ B}\, b^{N\ A} , \tag{2.6}$$

which follow from their definition (2.1) and Bose symmetry. We can now use these constraints to solve for the redundant baryons in terms of an appropriately chosen set thereby obtaining the required Kähler potential. Counting the number of eaten degrees of freedom and comparing with the analysis in terms of the fields $\bar{R}_A$ along the D-flat directions, it is easy to see that $b^{N-1\ N}$, $b^{N-1\ A}$, and $b^{N\ B}$, with $A, B = 1, ..., N-2$, are good coordinates[5] (in a vacuum where $b^{N\ N-1} \neq 0$) and we consequently use them as the independent fields.

For notational convenience, we introduce the fields S and $P^{\alpha A}$, (hereafter $A, B = 1, ..., N-2; \alpha = 1, 2$) via the definitions: $S = b^{N-1\ N}$, $P^{1A} = b^{N-1 A}$ and $P^{2A} = b^{NA}$. The Kähler potential (2.5) and superpotential of the effective theory, after using the constraint (2.6) to solve for the redundant degrees of freedom, become:

$$K = (N-2) \left(S^\dagger\ S + P^\dagger_{\alpha A}\ P^{\alpha A} + \frac{P^{\alpha A}\ P^B_\alpha\ P^\dagger_{\beta A}\ P^{\dagger\beta}_B}{2\ S^\dagger\ S} \right)^{\frac{1}{N-2}} , \tag{2.7}$$

and

$$W = \lambda^{\frac{N-2}{N}}\ \Lambda_1^{\frac{2N+2}{N}}\ S^{\frac{1}{N}} - \frac{2}{M^{N-5}}\ P^{1\ N-2} + \frac{2\ a}{M^{N-5}} \sum_{A,B=1}^{N-3} \frac{J_{AB}\ P^{1A}\ P^{2B}}{S} , \tag{2.8}$$

respectively. The superpotential above was obtained by adding the last term of (2.2)—with the matrix α_{AB} chosen to preserve $SP(N-3)$, as described in Section 2.1—to the nonperturbatively generated superpotential, eq. (2.4).

We will see, in the following sections, that the sigma model has a stable supersymmetry breaking vacuum. As discussed above, the field S is a dilaton for the $SU(N)$ gauge group. The first term in the superpotential (2.8) could have lead to runaway behavior. This runaway behavior is, however, stopped by the Kähler potential (2.7), which links the dilaton to the other moduli.

[5]For example, along the flat direction $\bar{R}^{\dot\alpha}_A = v\delta^{\dot\alpha}_A$, discussed in Section 2.2.1, the components of $\bar{R}_A$ that remain light are $\bar{R}^{\dot\alpha}_N$, $\bar{R}^{\dot\alpha}_{N-1}$, and v (which describes fluctuations corresponding to motion along the flat direction). Using the definitions of the baryons (2.1) one can see that fluctuations of $b^{N-1\ A}$, with $A < N-1$, around these expectation values correspond to the field $\bar{R}^{\dot\alpha}_N$, while fluctuations of $b^{N\ A}$ ($A < N-1$) and $b^{N-1\ N}$ correspond to $\bar{R}^{\dot\alpha}_{N-1}$ and v, respectively.

2.3 Mass Scales and Spectrum.

2.3.1 Mass Scales.

With the sigma model in hand we can now write down the the potential—it is given in terms of the Kähler potential and the superpotential as $V = W_i K^{-1\ ij^*} W^*_{j^*}$ [19]. The explicit minimization of the potential in our case needs to be done numerically but several features about the resulting ground state can be deduced in a straightforward way.

Notice first, that the superpotential has two scales Λ_1 and M. These will determine the various scales which appear in this problem. The scale of the vacuum expectation values v can be obtained by balancing the first two terms in the superpotential (2.8) and is given by

$$v \equiv M \left[\frac{\lambda^{\frac{N-2}{2N+2}} \Lambda_1}{M} \right]^{\frac{2N+2}{(N-1)\ (N-2)}} . \tag{2.9}$$

In order for our approximations to be justified v needs to be large enough. Quantitatively, we need $\Lambda_{1L}/v \ll 1$, where Λ_{1L} is the strong coupling scale of the intermediate scale $SU(N)$ theory. Since the first term in the superpotential (2.8) is of order Λ_{1L}^3, we need the condition

$$\frac{\Lambda_{1L}}{v} \sim \left(\frac{v}{M}\right)^{\frac{N-5}{3}} \ll 1 \tag{2.10}$$

to be valid. Eq. (2.10) can be met, for $N > 5$, if $v \ll M$ [6].

Hereafter it will be convenient to use v and M as the two independent energy scales. The scale of the typical F components that give rise to supersymmetry breaking is $\sim W/v$, i.e. of order F where

$$F \equiv M^2 \left(\frac{v}{M}\right)^{N-3} , \tag{2.11}$$

while the masses of the fields in the sigma model are $\sim W/v^2$, i.e. of order m, where

$$m \equiv M \left(\frac{v}{M}\right)^{N-4} . \tag{2.12}$$

Note that (for $N > 5$) the scale of supersymmetry breaking, $F^{1/2}$, eq. (2.11), is much higher than the scale of the masses, eq. (2.12), if $M \gg v$.

We turn now to the global symmetries. As is clear from eq. (2.8), the superpotential has an $SP(N-3)$ symmetry under which P^{1A} and P^{2A} transform as fundamentals. First we note that although there might exist vacua that break the $SP(N-3)$ global symmetry, an $SP(N-7)$ global symmetry is always preserved, since the light spectrum only has two fundamentals of the $SP(N-3)$ global symmetry. Second, intuitively it is clear that when the parameter a that appears in the third term of the superpotential (2.8) is large, the ground

[6] For $N = 5$, this condition can be met by making a dimensionless Yukawa coupling small.

Table 1: Vacuum expectation values x, z, eq. (2.13), vacuum energy ε, and mass matrix parameters $\alpha, \beta, \gamma, \delta$, eq. (2.14,2.15), in the $SU(N) \times SU(N-2)$ models, for $5 \leq N \leq 27$, N-odd.

N	x	z	α	β	γ	δ	ε
5	.0299	.0429	.873	-.437	.748	.250	.201
7	.0298	.0357	.197	-.0493	.317	.0947	.0774
9	.0283	.0317	.0893	-.0149	.209	.0496	.0443
11	.0262	.0284	.0520	-.00650	.159	.0308	.0293
13	.0241	.0256	.0343	-.00343	.129	.0212	.0211
15	.0222	.0233	.0245	-.00204	.109	.0154	.0159
17	.0205	.0214	.0183	-.00131	.0946	.0118	.0125
19	.0190	.0197	.0143	-.000892	.0835	.00928	.0100
21	.0177	.0183	.0114	-.000635	.0748	.00751	.00828
23	.0165	.0170	.00936	-.000468	.0677	.00619	.00694
25	.0155	.0159	.00780	-.000355	.0619	.00520	.00590
27	.0146	.0150	.00661	-.000275	.0570	.00442	.00508

state of this theory should preserve the global $SP(N-3)$ symmetry. In the limit of large a, the fields that transform under the $SP(N-3)$ symmetry can be integrated out if the field S has an expectation value. The resulting theory of the light fields (the fields S and $P^{\alpha N-2}$) is expected to have a stable vacuum at nonvanishing value of S since the potential is singular for both zero and infinite field values. In fact, the numerical minimization of the potential shows that an $SP(N-3)$ symmetric stable vacuum exists for a wide range of values of a (not necessarily $\gg 1$).

2.3.2 Mass Spectrum.

With this background in mind we turn to the numerical minimization. We will study the vacuum that preserves the maximal global symmetry and will in particular be interested in the masses of the $SP(N-3)$ fundamentals $P^{\alpha A}$, $A < N-2$, since they will play the role of messenger fields in the subsequent discussion of model building. The numerical investigation shows that an extremum exists where the only nonvanishing vacuum expectation values are those of the fields S and P^{1N-2}. In particular the field P^{2N-2} does not acquire an expectation value.[7]

[7]There may exist other extrema of the potential where also the field $P^{2N-2} \neq 0$. We have not studied these in any detail.

The expectation values of the fields S and P^{1N-2} are:

$$\begin{aligned} S &= x\, v^{N-2} , \\ P^{1N-2} &= z\, v^{N-2} . \end{aligned} \tag{2.13}$$

All components of the S and $P^{\alpha N-2}$ supermultiplets have mass of order m, except the R axion—which becomes massive due to higher dimensional operators [17], necessary e.g. to cancel the cosmological constant—and the goldstino, which is a linear combination of the S and P^{1N-2} fermions. The fermionic components of the $SP(N-3)$ fundamentals $P^{\alpha A}$, $A = 1, ..., N-3$ have a Dirac mass term, which can be directly read off eq. (2.8) (the Kähler connection [19] does not contribute to the masses of the $SP(N-3)$ multiplets in the vacuum (2.13))[8]

$$\gamma\ a\ m \sum_{A,B=1}^{N-3} P^{1A}\ P^{2A}\ J_{AB}, \tag{2.14}$$

while the quadratic terms in their scalar components are:

$$m^2 \sum_{A,B=1}^{N-3} (P^{1A}\ P^{\dagger}_{2B}) \begin{pmatrix} (\alpha + \gamma^2\ a^2)\ \delta_A^C & \delta\ a\ J_{AD} \\ \delta\ a\ J^{BC} & (\beta + \gamma^2\ a^2)\ \delta_D^B \end{pmatrix} \begin{pmatrix} P^{\dagger}_{1C} \\ P^{2D} \end{pmatrix} . \tag{2.15}$$

The numerical values of the vacuum expectation values x, z (2.13) and the mass matrix parameters α, β, γ, and δ, as well as the vacuum energy ε (defined by $V = M_{SUSY}^4 = \varepsilon\ F^2$) are given in Table 1 for a range of values of N.

A few comments are now in order:

First, it is useful to consider the messenger fields' spectrum, (2.14) and (2.15), in the $a \gg 1$ limit. The fermion mass squared and the diagonal components of the scalar mass matrix become equal in this limit. Furthermore, the fermion mass squared is equal to the average of the squared masses of the scalar mass eigenstates, and the splitting in the supermultiplet (proportional to $\sqrt{a}$) is much smaller than the supersymmetric mass (proportional to a). The spectrum of the messenger fields in this limit is very similar to that obtained in the models of ref. [4], where gauge singlet fields are responsible for generating both the supersymmetric and supersymmetry breaking masses. This is because in the $a \gg 1$ limit, the masses of the $SP(N-3)$ fundamentals mainly arise due to the last term in the superpotential in eq. (2.8), which has the form of the singlet—messenger fields coupling in the models of ref. [4].

Second, it is very likely—at least in the $a \gg 1$ limit—that the vacuum we have explored here is in fact the global minimum of the theory. This is to be contrasted with the models of ref. [4], which contain a more elaborate messenger sector. In these models, the required vacuum—with an F term expectation value for the singlet, which couples to the messenger

[8] In eqs. (2.14) and (2.15) all kinetic terms have been brought to canonical form.

quarks—is only local. Usually there is a deeper minimum, in which the singlet F term expectation value vanishes, while the messenger quarks have expectation values, breaking the Standard Model gauge group at an unacceptably high scale (avoiding this problem requires an even more complicated messenger sector, as shown in ref. [6]).

In addition to the fields in the sigma model, when discussing the communication of supersymmetry breaking to the Standard Model sector, we will need some information on the spectrum of heavy fields in the $SU(N) \times SU(N-2)$ theory. The vacuum expectation values for the fileds S and P^{1N-2} (2.13) correspond to the expectation values of $\bar{R}_{1\ldots N-2}$ and $\bar{R}_N$ of order v. Correspondingly, due to the first term in (2.2) the fields Q and $\bar{L}^I$ get (supersymmetric) masses of order λv. Since the F components of the $\bar{R}$ fields also have expectation values, the fields Q and $\bar{L}^I$ also obtain a supersymmetry breaking mass squared splitting of order λF. For the discussion in the following section it is relevant to note that the ratio of the supersymmetry breaking mass squared splitting to the supersymmetric mass of the heavy fields Q and $\bar{L}^I$ is of order F/v—the same as the corresponding ratio for the light fields in the sigma model.

The components of the $\bar{R}$ fields which get eaten by the $SU(N-2)$ gauge bosons and their heavy superpartners (with mass of order $g_2 v$) also obtain supersymmetric mass splitting. The leading effect is that the scalar components in the heavy vector supermultiplets obtain supersymmetry breaking contributions to their masses of order $m \simeq F/v$. These contributions arise because of a shift of the expectation values of the heavy fields in response to the F-type vacuum expectation values of the light fields (a similar effect of the heavy tadpole is discussed in ref. [17]; see also [18]).

Having understood the supersymmetry breaking vacuum and the spectrum in some detail we now turn to using these theories for model building.

3 Communicating Supersymmetry Breaking.

3.1 Basic Ideas.

The basic idea is to construct a model containing two sectors: the usual Standard Model sector, consisting of the supersymmetric Standard Model and a supersymmetry breaking sector consisting of an $SU(N) \times SU(N-2)$ theory studied above. We saw above that the latter theories have an $SP(N-3)$ global symmetry which is left unbroken in the supersymmetry breaking vacuum. A subgroup of $SP(N-3)$ can be identified with the Standard Model gauge symmetries. The minimal $SP(2k)$ group in which one can embed $SU(3) \times SU(2) \times U(1)$ is $SP(8)$—this corresponds to taking $N = 11$. Alternatively, we can consider an embedding consistent with Grand Unification. For this purpose one can embed $SU(5)$ in $SP(10)$—using the $SU(13) \times SU(11)$ models.

The soft parameters—the Standard Model gaugino masses and soft scalar masses—receive contributions from several different energy scales. As discussed in the previous section, all heavy fields in the $SU(N)\times SU(N-2)$ theory that transform under the Standard Model gauge group obtain supersymmetry breaking mass splittings. The Q and $\bar{L}$ heavy fields transform as fundamentals under the Standard Model gauge group, whereas the eaten (and superpartners) components of the fields $\bar{R}$ transform as two fundamentals, a symmetric tensor (adjoint), and an antisymmetric tensor representation of $SP(N-3)$.

In this section we will present a brief discussion of the generation of the soft parameters. As in [4] gaugino masses arise at one loop, while soft scalar masses arise at two loops. The corresponding calculations are somewhat more involved than the ones from [4], [7]; more details will be presented in a subsequent paper [22].

We first consider the effects of the heavy Q and $\bar{L}$ fields. The contribution of these fields is analogous to that of the messenger fields in the models of [4]. Consequently their contribution to the gaugino masses is:

$$\delta_H m_{gaugino} \sim N_f \, \frac{g^2}{16\pi^2} \, \frac{F}{v} \sim N_f \, \frac{g^2}{16\pi^2} \, M \, \left(\frac{v}{M}\right)^{N-4} , \tag{3.16}$$

while their contribution to the soft scalar masses is :

$$\delta_H m_a^2 \sim N_f \, \frac{g^4}{128\pi^4} \, C_a \, S_Q \, \left(\frac{F}{v}\right)^2 . \tag{3.17}$$

In the equations above g denotes the appropriate Standard Model gauge coupling, C_a is the quadratic Casimir ($(N^2-1)/2N$ for an $SU(N)$ fundamental; for $U(1)_Y$ the corresponding coefficient is $3Y^2/5$, with Y the messenger hypercharge), S_Q is the Dynkin index of the messenger representation ($1/2$ for the fundamental of $SU(N)$). Finally, in eqs. (3.16) and (3.17) N_f denotes the number of messenger flavors for the appropriate Standard Model groups—in particular it is important to note that N_f is proportional to N so that these contributions increase in magnitude as the size of the $SU(N) \times SU(N-2)$ group increases.

Next we consider the contributions of the light fields (described by the sigma model). These effects will be described in more detail elsewhere [22], here we restrict ourselves to providing some rough order of magnitude estimates. Their contribution to the gaugino masses is of order:

$$\delta_L m_{gaugino} \sim \frac{g^2}{16\pi^2} \, m \sim \frac{g^2}{16\pi^2} \, M \, \left(\frac{v}{M}\right)^{N-4} , \tag{3.18}$$

where m denotes the typical mass scale in the sigma model, eq. (2.12). Since the supertrace of the light messenger mass matrix squared is nonvanishing (as can be inferred from eqs. (2.14), (2.15), and Table 1), their contribution to the soft scalar masses turns out to be logarithmically divergent [22]. The divergent piece is:

$$\delta_L m_a^2 = -\frac{g_a^4}{128\pi^4}\, C_a\, S_Q\, \mathrm{Str} M_{mess}^2\, \mathrm{Log}\frac{\Lambda^2}{m_f^2}, \tag{3.19}$$

where Λ is the ultraviolet cutoff and m_f is the Dirac mass of the messenger fermion, $m_f \sim F/v \sim m$. This logarithm is cutoff by the contributions of the heavy eaten components of the fields[9] $\bar{R}$, therefore the scale Λ in eq. (3.19) should be replaced by their mass, $\sim g_2 v$ [10]. Note also that in (3.19) $\mathrm{Str}M_{mess}^2 \sim m^2$, and that there is no large flavor factor N_f, since there are only two fundamentals of $SP(N-3)$ light messengers. In addition to the logarithmically divergent contribution (3.19), there are finite contributions analogous to those of the heavy fields (3.17), which are proportional to $\Delta m_{mess}^2/m_{mess} \sim F/v \sim m$ (the exact formula will be given in [22]).[11]

We can now use the above estimates for the Standard Model soft masses, eqs. (3.16), (3.17), (3.18), and (3.19), to obtain an estimate of the scales in the $SU(N) \times SU(N-2)$ theory. In section 2.3.1, we found that the scale of the messenger masses (2.12) is given by $m = M(v/M)^{(N-4)} = F/v$, while the scale of supersymmetry breaking (2.11) is $\sqrt{F} = M(v/M)^{(N-3)/2}$. Demanding, e.g. that $m_{gaugino} \sim (10^2 - 10^3)$ GeV, we obtain

$$\frac{v}{M} \sim \left(\frac{(10^4 - 10^5)\ \mathrm{GeV}}{M}\right)^{\frac{1}{N-4}} . \tag{3.20}$$

The scale of supersymmetry breaking (2.11) then becomes

$$\sqrt{F} \sim M\ \left(\frac{(10^4 - 10^5)\ \mathrm{GeV}}{M}\right)^{\frac{N-3}{2(N-4)}} . \tag{3.21}$$

3.2 Hybrid Models.

Since M suppresses the non-renormalizable operators in eq. (2.2) one natural value it can take is M_{Planck}. We consider this case in some detail here. On setting $M = M_{Planck} \simeq 2 \cdot 10^{18}$ GeV in the formula above gives $\sqrt{F} \sim 10^{18}(10^{-14} - 10^{-13})^{\frac{N-3}{2(N-4)}}$ GeV. As discused above, the smallest value of N for which the Standard Model groups can be embedded in the flavor group

[9] As far as the soft Standard Model parameters are concerned, this is the main effect of the supersymmetry breaking mass splittings in the eaten components of the $\bar{R}$ fields.

[10] In the full theory, the $\mathrm{Str}M^2$, appropriately weighted by the messengers' Dynkin indices vanishes. One can see this by noting that a nonvanishing supertrace would imply the existence of a counterterm for the Standard Model soft scalar masses. This counterterm would have to be nonpolynomial in the fields (for example, of the form $\Phi^\dagger\Phi\mathrm{Log}\bar{R}^\dagger\bar{R}$) and thus can not appear.

[11] For completeness, we note that with the general messenger scalar mass matrix (2.15), one loop contributions to the hypercharge D-term are generated. These can be avoided if the messengers fall in complete $SU(5)$ representations, or, alternatively, the parameter a is sufficiently large (for a not sufficiently large, however, there are two loop contributions to the $U(1)_Y$ D term, even in the complete $SU(5)$ representations case).

is $N = 11$. This corresponds to $\sqrt{F} \sim 10^{10}$ GeV, i.e. the supersymmetry breaking scale is of order the intermediate scale. It also follows from eq. (3.21) that on increasing N, the scale of supersymmetry breaking increases very slowly. For example, with $N = 13$—the smallest value consistent with Grand Unification—$\sqrt{F} \sim 10^{10} - 10^{11}$ GeV, still of order the intermediate scale. One consequence of the supersymmetry breaking scale being of order the intermediate scale is that the squark and slepton masses due to supergravity, of order F/M_{Planck}, will be comparable to the masses induced by the gauge interactions. These models can therefore be thought of as "hybrid models" in which scalar masses arise due to both supergravity and gauge interactions, while gaugino masses arise solely from the gauge interactions.

It is also illustrative to work out the other energy scales in the supersymmetry breaking sector. For concreteness we focus on the $N = 11$ theory. From eq. (2.9) we find that $v \sim 10^{16}$ GeV while from eq. (2.10), it follows that $\Lambda_{1L} \sim 10^{12}$ GeV. Notice in particular that $\Lambda_{1L} \ll v$ so that the requirement in eq. (2.10) is met and the approximations leading to the sigma model are valid. The underlying physics giving rise to supersymmetry breaking in this model can be described as follows. One starts with a $SU(11) \times SU(9)$ theory at very high energies. At $v \sim 10^{16}$ GeV, the $SU(9)$ symmetry is broken giving rise to a theory consisting of some moduli and a pure $SU(11)$ group coupled to a dilaton. The $SU(11)$ group confines at $\Lambda_{1L} \sim 10^{12}$ GeV, giving rise to a sigma model consisting of the moduli and the dilaton. Finally, supersymmetry breaks at 10^{10} GeV giving rise to masses for messenger quarks of order 10 TeV. It is worth noting that this large hierarchy of scales is generated dynamically. We also note that this hybrid model does not exhibit Landau poles (below scales, higher than $v \sim 10^{16}$ GeV) of the Standard Model gauge groups: between the messenger scale and the scale v, in addition to the usual quark, lepton and Higgs supermultiplets only two vectorlike $SU(3)$ flavors and two $SU(2)$ fundamentals contribute to the running of the gauge couplings. Above the scale 10^{16} GeV, new physics is expected to take over, as discussed in the Introduction.

The high scale of supersymmetry breaking in these models poses a problem and constitutes their most serious drawback. It implies that one cannot generically rule out the presence of large flavor changing neutral current effects. Such effects could arise due to higher dimensional operators in the Kähler potential. For these models to be viable, physics at the Planck scale would have to prevent such operators from appearing. In this respect these models are no better than the usual hidden sector models.

It is worth emphasizing the key features of the $SU(N) \times SU(N-2)$ theories that are ultimately responsible for the high scale of supersymmetry breaking. The requirement that the flavor group is big enough forces one to large values of N in these theories[12]. Furthermore,

[12]For smaller values of N, $N \leq 7$, the scale of supersymmetry breaking $\sqrt{F} \leq 10^9$ GeV, and the problem of flavor changing effects may be alleviated. However, in this case, we can not embed the whole Standard Model

supersymmetry breaking occurs only in the presence of nonrenormalizable operators whose dimension grows with N. Suppressing these operators by the Planck scale leads to the high scale of supersymmetry breaking.

3.3 Purely Gauge Mediated Models.

One would like to find other theories in which the requirement for a big enough flavor symmetry can be met without leading to such a high supersymmetry breaking scale. We discuss two possibilities in this context.

3.3.1 Lowering the scale M.

One possible way in which the supersymmetry breaking scale can be lowered is by making $M < M_{Planck}$. The $SU(N) \times SU(N-2)$ theory of Section 2.1 itself would in this case be an effective theory, which would arise from some underlying dynamics at scale M. However, to suppress the flavor changing neutral currents one would have to forbid D terms of the form

$$\frac{\bar{R}^\dagger \ \bar{R} \ \Phi^\dagger \ \Phi}{M^2} \ , \tag{3.22}$$

where Φ denote Standard Model fields, in the effective theory. Such terms, if present in a flavor non-universal form, would be problematic (at least for N sufficiently large to accommodate the whole Standard Model gauge group). It is possible that they might be absent in a theory where the last two terms in eq. (2.8) arose due to non-perturbative dynamics that only couples to the $\bar{R}$ fields but not to the Standard Model.

Once the supersymmetry breaking scale is lowered these theories can be used to construct purely gauge mediated models of supersymmetry breaking. The feeddown of supersymmetry breaking to the Standard Model in these models proceeds as described in Section 3.1. Both gaugino and scalar soft masses receive contributions from the heavy, eqs. (3.16), (3.17), and light, eqs. (3.18), (3.19), messengers. As follows from eq. (3.19) and the sigma model spectrum of Table 1 (note that $\mathrm{Str} M^2_{mess} \sim \alpha + \beta$), the logarithmically enhanced contribution of the light fields to the scalar masses is in fact negative. Consequently, obtaining positive soft scalar mass squares poses a significant constraint on the models. These masses can be positive if the additional finite contributions of the heavy and light messengers overcome the negative logarithmically enhanced contribution of the light messengers. This can happen in two ways. First the logarithmic contribution can be reduced in magnitude by lowering the scale $g_2 v$, which cuts off the logarithm, and bringing it sufficiently close to the scale m. For example,

gauge group in the unbroken $SP(N-3 \leq 4)$ global symmetry (in particular, the gluinos would have to be massless in this framework).

with $N = 11$, using Table 1, one can conclude that positive mass squares are obtained with a scale v two orders of magnitude larger than the scale $m \sim 10^4 - 10^5$ GeV. Note that lowering the scale $g_2 v$ amounts to lowering the scale M, eq. (2.2), at which new physics must enter. Second, we note that the positive finite contributions, eq. (3.17), of the heavy fields Q and $\bar{L}$, are enhanced by a factor of $N_f \sim N$. In addition, as is clear from the numerical results of Table 1, with increasing N the ratio of the supertrace (proportional to $\alpha + \beta$) to the finite contribution (proportional to $(\delta/\gamma)^2$) decreases. Consequently, models with N sufficiently large will yield positive mass squares, without requiring the scale M to be too close to the scale of the light messengers[13]. Having the scale M be as large as the GUT scale pushes the Landau poles up, which is an attractive feature of the models that one might want to retain. We leave a detailed analysis of this issue for future work [22]. We only mention here the phenomenologically interesting possibility that the two competing effects, (3.17), and (3.19) might yield squarks that are lighter than the gauginos.

We conclude this section by raising the possibility that the scale M could be less than M_{Planck} if the Standard Model gauge groups are dual to some underlying theory. In order to illustrate this, we return to our starting point, the $SU(N) \times SU(N-2)$ theory, with, as discussed above, the Standard Model groups embedded in the $SP(N-3)$ global symmetry. As a result of the additional degrees of freedom the Standard Model groups are severely non-asymptotically free, once all the underlying degrees in the $SU(N) \times SU(N-2)$ theory come into play. Consequently, it is appealing to dualize the theory and to regard the dual, which is better behaved in the ultraviolet, as the underlying microscopic theory. We see below that this could also lead to lowering the scale M, eq. (2.2), in the electric theory.

For purposes of illustration we work with the $N = 11$ case and consider dualizing the Standard Model $SU(3)$ and $SU(2)$ groups. In the process we need to re-express the baryonic operators in eq. (2.2) in terms of gauge invariants of the two groups and then use the duality transformation of SQCD, [10], to map these operators to the dual theory. Doing so shows that the baryonic operators can be expressed as a product involving some fields neutral under the Standard Model groups and mesons of the $SU(3)$ and $SU(2)$ groups. But the mesons map to fields which are singlets in the dual theory. Consequently, the resulting terms in the superpotential of the dual theory have smaller canonical dimensions and are therefore suppressed by fewer powers of M_{Pl}. For example, the operator $b^{N-1 N-2}$ can be written as a product involving the field $\bar{R}_N$, three mesons of $SU(3)$, and a meson of $SU(2)$; as a result in the dual it has dimension 5 and is suppressed by two powers of M_{Pl}. The deficit in terms of dimensions is made up by the scales μ_3 and μ_2 which enter the scale matching relations for

[13]Similar observations have been made recently in ref. [23]. We thank J. March-Russell for discussions in this regard.

the $SU(3)$ and $SU(2)$ theories, respectively [10], leading to a relation:

$$M = M_{Pl} \left(\frac{\mu_3^3\, \mu_2}{M_{Pl}^4}\right)^{\frac{1}{6}}. \tag{3.23}$$

For μ_3 and μ_2 much less than M_{Pl} we see that M is much lower than M_{Pl}.

While the above discussion is suggestive, several concerns need to be met before it can be made more concrete. First, as was mentioned above, one needs to argue that terms of the form (3.22) are suppressed adequately. We cannot at present, conclusively, settle this matter since the map for non-chiral operators under duality is not known. However, since the scales involved in the duality transformation are much smaller than the Planck scale, it is quite plausible that if an operator of the form eq. (3.22), suppressed by the Planck scale, is present in the dual theory it will be mapped to an operator in the electric theory that is adequately suppressed. Second, in the example discussed above, the Standard Model $U(1)_Y$ group continues to be non-asymptotically free. This can be avoided by considering theories in which the Standard Model groups are embedded in a GUT group. The simplest such example is the $N = 13$ theory with a GUT group $SU(5)$. The $SU(5)$ group has matter in the fundamental, antisymmetric and adjoint representations. Unfortunately, no compelling dual for this theory is known at present. [14] Finally, the above attempt at lowering M relied on taking the parameter(s) μ to be smaller than M_{Pl}. This might be unnatural in the dual theory. For example, in the dual theory considered here, the Yukawa coupling $\lambda^{IA}Y_{IA}$, eq. (2.2), turns into a mass term with a μ dependent coefficient. Naturalness, in this case suggests that μ is of order M_{Pl}. A detailed discussion of these issues is left for the future, hopefully, within the context of more compelling models and their duals.

3.3.2 Other Sigma Models.

We saw in our discussion of the hybrid models above that a large hierarchy of scales separates the microscopic theory from the sigma model. In view of this, one can ask if at least a sigma model can be constructed as an effective theory that yields a low enough supersymmetry breaking scale, while the nonrenormalizable operators are still suppressed by the Planck scale. The answer, it is easy to see, is yes. For example, we can take the dimensions of the fields in the effective lagrangian (2.7), (2.8) to be equal to, say, D—being thus different from their dimension, $N-2$, dictated by the underlying $SU(N) \times SU(N-2)$ theory—and change correspondingly the power of the $1/M$-factors, the powers in the Kähler potential and the power of S in the nonperturbative term in the superpotential, eq. (2.8). We should emphasise

[14] This theory can be dualized by following the methods of [20], [21] and unbinding each antisymmetric tensor by introducing an extra $SU(2)$ group. However, the resulting dual is quite complicated and contrived.

that we are not aware of any underlying microscopic theory which gives rise to such a sigma model. However, they do provide an adequate description of supersymmetry breaking. An analysis similar to the one above shows that these sigma models break supersymmetry, while leaving an $SP(N-3)$ flavor subgroup intact. The mass spectrum of low lying excitations in these theories is also qualitatively of the form in eq. (2.15) and eq. (2.14). Following then the same arguments that lead to eq. (3.21) for the supersymmetry breaking scale, we find that the exponent in eq. (3.21) changes to $(D-1)/(2(D-2))$ instead. Consequently, for $D=4$ or 5 (even with $M = M_{Planck}$), the scale of supersymmetry breaking is sufficiently low for supergravity effects to be unimportant.

It is illustrative to compare the energy scales obtained in such a model with those obtained in the "hybrid" models above. We consider the $D=4$ case for concreteness. The supersymmetry breaking scale in this case is of order 10^7 GeV, well below the intermediate scale, while the scale of the vacuum expectation values is $\sim 10^{11}$ GeV. Therefore the the sigma model breaks down at an energy scale well above the scale of supersymmetry breaking.[15]

Once the supersymmetry breaking scale is sufficiently lowered one can use these sigma models to construct purely gauge mediated models of supersymmetry breaking. We note, however, that we can not compute the Standard Model soft masses from the effective theory alone—we saw in Section 3.1 that the contribution of the heavy states not included in the sigma model can be as important as the ones from the light fields.

4 Phenomenological Implications.

In this section, we discuss the phenomenological implications of the "hybrid" models of dynamical supersymmetry breaking, introduced above. Towards the end we will briefly comment on some expected features of purely gauge mediated models with a combined supersymmetry breaking and messenger sector. In our discussion of hybrid models we will, where necessary, focus on the $SU(11) \times SU(9)$ model, in which the $SU(3) \times SU(2) \times U(1)$ groups are embedded in the $SP(8)$ global symmetry group.

We begin with two observations. First, since the supersymmetry breaking scale is high in these models, the gravitino has a weak scale mass and is not (for non-astrophysical purposes at any rate) the LSP. Second, since the supersymmetry breaking sector is coupled quite directly to the Standard Model sector, the masses of the (light) fields in the supersymmetry breaking sector are of order 10 TeV. Consequently, at this scale one can probe all the fields that play

[15]The scale at which the effective theory breaks down could be smaller than the perturbative estimate coming from the sigma model, $\sim 4\pi v$, would indicate. For example, if we had retained the corrections to the Kähler potential of order Λ_L/v, discussed in Section 2.2.1, we would have found that the model breaks down at a scale Λ_L, which is lower than v, but still higher than M_{SUSY}.

an essential role in the breaking of supersymmetry.

We now turn to the scalar soft masses. As noted in the previous section, scalars in these models receive contributions due to both gauge and gravitational effects. Gravitational effects give rise to universal soft masses of order $F/M_{Planck} \sim 10^2 - 10^3$ GeV at the Planck scale[16]. In addition, as described in Section 3.1, Standard Model gauge interactions induce non-universal contributions (3.19), (3.17). Since the soft masses receive contribution at various energy scales, the renormalization group running in the hybrid models is quite different from the running in supergravity hidden sector models and from that in gauge mediated, low-energy supersymmetry breaking models. We leave the detailed study of the renormalization group effects for future work.

Getting a big enough μ term in these models is a problem. Since the model is "hybrid", one could attempt to use $1/M^2_{Planck}$–suppressed couplings, such as $\int d^4\theta H_1 H_2 \bar{R}^\dagger \bar{R}$ or $\int d^2\theta H_1 H_2 (W^\alpha W_\alpha)_{SU(N)}$, to generate the desired μ and $B\mu$ terms. However, it is easy to see that while $B\mu \sim F^2/M^2_{Planck}$ is generally of the right order of magnitude, the resulting μ-parameter is $\mu \sim (v/M_{Planck})\sqrt{B\mu} \sim 10^{-2}\sqrt{B\mu}$ and is therefore too small. A similar conclusion results from considering, e.g. the F term bH_1H_2/M^{N-3}, with b being an $SP(N-3)$-singlet baryon, which can be used to generate a reasonable $B\mu$ term and a negligible μ term (to see this, we use $\langle b/M^{N-3}\rangle \sim M(v/M)^{N-2} + \theta^2 M^2 (v/M)^{2(N-3)}$, with $M \sim 10^{18}$ GeV, $v \sim 10^{16}$ GeV, and $N = 11$).

To avoid this small-μ problem, one could use the approach of ref. [4] and introduce a special sector of the theory, constrained by some discrete symmetry, which will be responsible for generating the μ term. For example, this could be achieved by requiring an appropriate $SP(N-3)$-singlet baryon, b/M^{N-3}, to play the role of the singlet field S of ref. [4] (see Section 4 of last paper in [4]) and the introduction of an additional singlet T with appropriate couplings in the superpotential. From the point of view of low-energy phenomenology, this approach implies that when analyzing the low-energy predictions of the model, μ and $B\mu$ should be treated as free parameters.

A few more comments are in order.

First, electroweak symmetry breaking will occur radiatively in these models, with the large top Yukawa driving the mass square of one Higgs field negative. Second, these models do not suffer from a supersymmetric CP problem. This can be seen immediately in the sigma model superpotential eq. (2.8), where all phases can be rotated away.[17] Finally, we note that the hybrid models are likely to inherit some of the cosmological problems of hidden sector models.

[16] We are assuming as usual here that the Kähler metric is flat.

[17] It can also be seen in the underlying $SU(N) \times SU(N-2)$ theory where all phases except for the θ angle of $SU(N-2)$ can be rotated away. Since the $SU(N-2)$ group is broken at a very high scale, its instantons are highly suppressed.

For example, the R axion, whose mass in this model can be seen to be of order the electroweak scale [17], is very weakly interacting, $f_{axion} \sim v \sim 10^{16}$ GeV, and may suffer the usual Polonyi problem. This problem could be solved, for example, by invoking weak scale inflation.

We end with a few comments about the phenomenological implications of purely gauge mediated models with a supersymmetry breaking-cum-messenger sector. As was mentioned in Section 3, such models can be constructed by lowering the scale M. A few key features emerge from considering such purely gauge mediated models, which are likely to be generally true in models of this kind. First, as we have seen above, the scale of supersymmetry breaking which governs the mass and interaction strength of the gravitino, is a parameter which can take values ranging from 10 TeV to 10^{10} GeV and can therefore be very different from the value of the messenger field masses. It should therefore be treated as an independent parameter in considering the phenomenology of these models. Second, one consequence of having a combined supersymmetry breaking and messenger sector is that several degrees of freedom responsible for the communication and the breaking of supersymmetry breaking can be probed at an energy of about 10 TeV. Finally, the form of the mass matrix of the messenger fields can be different from that in the models of ref. [4], as is clear from eqs. (2.15) and (2.14). In particular, the sum rule relating the fermion and boson masses is not respected in general. We expect this to be a general feature of such models. As discussed in Section 3.1, the nonvanishing supertrace for the light messenger fields gives a logarithmically enhanced contribution to the soft scalar masses. In the models discussed here, the supertrace is positive and the corresponding contribution to the soft scalar masses squared is negative. This poses a constraint on model building. The negative contribution can be controlled by lowering the scale M, or considering models with large N. This could lead to scalar soft masses that are lighter than the gaugino masses[18]. A detailed analysis of the spectrum and the resulting phenomenology is left for the future [22].

5 Summary.

In conclusion we summarize the main results of this paper and indicate some possible areas for future study:

- We began this paper by studying a class of supersymmetry breaking theories with an $SU(N) \times SU(N-2)$ gauge group. We showed how the breaking of supersymmetry in these theories can be studied in a *calculable* low-energy sigma model. The sigma model was used to show that a large subgroup of the global symmetries is left unbroken in these theories, and to calculate the low-energy mass spectrum after supersymmetry breaking.

[18] We acknowledge discussions with G. Anderson on this point.

- We then turned to using these theories for model building. The models we constructed had two sectors: a supersymmetry breaking sector, consisting of the above mentioned $SU(N) \times SU(N-2)$ theories, and the supersymmetric Standard Model. The essential idea was to identify a subgroup of the global symmetries of the supersymmetry breaking sector with the Standard Model gauge group. In order to embed the full Standard Model gauge group in this way, we were lead to consider large values of N, i.e. $N \geq 11$, and as a consequence of this large value of N, the supersymmetry breaking scale was driven up to be of order the intermediate scale, i.e. 10^{10} GeV. Hence, these models are of a "hybrid" kind—supersymmetry breaking is communicated to the Standard Model both gravitationally and radiatively through the Standard Model gauge groups in them.

- We briefly discussed the phenomenology of these models. The main consequence of the messenger fields being an integral part of the supersymmetry breaking sector is that several degrees of freedom responsible for both communicating and breaking supersymmetry can be probed at an energy of order 10 TeV. In the hybrid models gauginos acquire mass due to gauge mediated effects, while scalars acquire mass due to both gauge and gravitational effects. We leave a more detailed investigation of the resulting mass spectrum, including the effects of renormalization group running for further study.

- It is worth mentioning that in these models there is a large hierarchy of scales that is generated dynamically. For example, even though the scale of supersymmetry breaking is high, of order 10^{10} GeV, the masses of the messenger fields—the lightest fields in the supersymmetry breaking sector that carry Standard Model charges—are of order 10 TeV. Furthermore, the sigma model used for studying the low-energy dynamics breaks down at a scale 10^{12} GeV—well above the scale of supersymmetry breaking.

- Purely gauge mediated models can be constructed by lowering the scale M that suppresses the nonrenormalizable term in the superpotential. These purely gauge mediated models reveal the following features that should be generally true in models with supersymmetry breaking-cum-messenger sector that have an effective low-energy weakly coupled description. First, the supersymmetry breaking scale can in general be quite different from the scale of the messenger field masses—it can range from 10 TeV to 10^{10} GeV, while the messenger field masses are of order 10 TeV. Second, as in the hybrid models, several degrees of freedom that are responsible for communicating and breaking supersymmetry can be probed at an energy scale or order 10 TeV. Third, the Standard Model soft masses receive contributions at various energy scales. Because of a tradeoff between positive and negative contributions, the soft scalar masses can be lighter than the corresponding gaugino masses. A detailed investigation of the phenomenology of such models, incorporating these features, needs to be carried out. We leave such an investigation for the future.

- Finally, we hope to return to the construction of purely gauge mediated models of supersymmetry breaking with a combined supersymmetry breaking and messenger sector. One would like to construct a consistent microscopic theory which could give rise to an adequate supersymmetry breaking sector. A minimal model of this kind would serve to further guide phenomenology. It would also prompt an investigation of more theoretical questions—like those associated with the loss of asymptotic freedom for the Standard Model gauge groups.

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