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**E791**

**A Search for  $D^0 - \bar{D}^0$  Mixing and Doubly-Cabibbo-Suppressed  
Decays of the  $D^0$  in Hadronic Final States**

E.M. Aitala et al.

The E791 Collaboration

*Fermi National Accelerator Laboratory  
P.O. Box 500, Batavia, Illinois 60510*

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**A search for  $D^0 - \overline{D}^0$  mixing  
and doubly-Cabibbo-suppressed decays of the  $D^0$   
in hadronic final states**

E. M. Aitala,<sup>9</sup> S. Amato,<sup>1</sup> J. C. Anjos,<sup>1</sup> J. A. Appel,<sup>5</sup> D. Ashery,<sup>15</sup> S. Banerjee,<sup>5</sup>  
I. Bediaga,<sup>1</sup> G. Blaylock,<sup>8</sup> S. B. Bracker,<sup>16</sup> P. R. Burchat,<sup>14</sup> R. A. Burnstein,<sup>6</sup> T. Carter,<sup>5</sup>  
H. S. Carvalho,<sup>1</sup> N. K. Coptý,<sup>13</sup> L. M. Cremaldi,<sup>9</sup> C. Darling,<sup>19</sup> A. Fernandez,<sup>12</sup>  
P. Gagnon,<sup>2</sup> C. Gobel,<sup>1</sup> K. Gounder,<sup>9</sup> A. M. Halling,<sup>5</sup> G. Herrera,<sup>4</sup> G. Hurvits,<sup>15</sup> C. James,<sup>5</sup>  
P. A. Kasper,<sup>6</sup> S. Kwan,<sup>5</sup> D. C. Langs,<sup>11</sup> J. Leslie,<sup>2</sup> B. Lundberg,<sup>5</sup> S. MayTal-Beck,<sup>15</sup>  
B. Meadows,<sup>3</sup> R. H. Milburn,<sup>17</sup> A. Napier,<sup>17</sup> A. Nguyen,<sup>7</sup> A. B. d'Oliveira,<sup>3,12</sup>  
K. O'Shaughnessy,<sup>2</sup> K. C. Peng,<sup>6</sup> L. P. Perera,<sup>3</sup> M. V. Purohit,<sup>13</sup> B. Quinn,<sup>9</sup>  
S. Radeztsky,<sup>18</sup> A. Rafatian,<sup>9</sup> N. W. Reay,<sup>7</sup> J. J. Reidy,<sup>9</sup> A. C. dos Reis,<sup>1</sup> H. A. Rubin,<sup>6</sup>  
A. K. S. Santha,<sup>3</sup> A. F. S. Santoro,<sup>1</sup> A. J. Schwartz,<sup>11</sup> M. Sheaff,<sup>18</sup> R. A. Sidwell,<sup>7</sup>  
A. J. Slaughter,<sup>19</sup> M. D. Sokoloff,<sup>3</sup> N. R. Stanton,<sup>7</sup> K. Stenson,<sup>18</sup> D. J. Summers,<sup>9</sup>  
S. Takach,<sup>19</sup> K. Thorne,<sup>5</sup> A. K. Tripathi,<sup>10</sup> S. Watanabe,<sup>18</sup> R. Weiss-Babai,<sup>15</sup> J. Wiener,<sup>11</sup>  
N. Witchey,<sup>7</sup> E. Wolin,<sup>19</sup> D. Yi,<sup>9</sup> S. Yoshida,<sup>7</sup> R. Zaliznyak,<sup>14</sup> and C. Zhang<sup>7</sup>

(Fermilab E791 Collaboration)

<sup>1</sup> *Centro Brasileiro de Pesquisas Físicas, Rio de Janeiro, Brazil*

<sup>2</sup> *University of California, Santa Cruz, California 95064*

<sup>3</sup> *University of Cincinnati, Cincinnati, Ohio 45221*

<sup>4</sup> *CINVESTAV, Mexico*

<sup>5</sup> *Fermilab, Batavia, Illinois 60510*

<sup>6</sup> *Illinois Institute of Technology, Chicago, Illinois 60616*

<sup>7</sup> *Kansas State University, Manhattan, Kansas 66506*

<sup>8</sup> *University of Massachusetts, Amherst, Massachusetts 01003*

<sup>9</sup> *University of Mississippi, University, Mississippi 38677*

<sup>10</sup> *The Ohio State University, Columbus, Ohio 43210*

<sup>11</sup> *Princeton University, Princeton, New Jersey 08544*

<sup>12</sup> *Universidad Autonoma de Puebla, Mexico*

<sup>13</sup> *University of South Carolina, Columbia, South Carolina 29208*

<sup>14</sup> *Stanford University, Stanford, California 94305*

<sup>15</sup> *Tel Aviv University, Tel Aviv, Israel*

<sup>16</sup> *317 Belsize Drive, Toronto, Canada*

<sup>17</sup> *Tufts University, Medford, Massachusetts 02155*

<sup>18</sup> *University of Wisconsin, Madison, Wisconsin 53706*

<sup>19</sup> *Yale University, New Haven, Connecticut 06511*

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## Abstract

We present results of a search for  $D^0 - \bar{D}^0$  mixing and doubly-Cabibbo-suppressed decays of the  $D^0$  in Fermilab experiment E791, a fixed-target charm hadroproduction experiment. We look for evidence of mixing in the decay chain  $D^* \rightarrow \pi D \rightarrow \pi(K\pi \text{ or } K\pi\pi\pi)$ . If the charge of the pion from the  $D^*$  decay is the same as the charge of the kaon from the  $D$  decay (a “wrong-sign” event), mixing may have occurred. Mixing can be distinguished from other sources of wrong-sign events (such as doubly-Cabibbo-suppressed decays) by analyzing the distribution of decay times. We see no evidence of mixing. Allowing for possible CP violation, our results for  $r_{mix}$ , the ratio of mixed to unmixed decays, are  $r_{mix}(\bar{D}^0 \rightarrow D^0) = (0.18^{+0.43}_{-0.39} \pm 0.17)\%$  and  $r_{mix}(D^0 \rightarrow \bar{D}^0) = (0.70^{+0.58}_{-0.53} \pm 0.18)\%$ . The sensitivity of these results is comparable to that of previous measurements, but the assumptions made in fitting the data are notably more general. We present results from many fits to our data under various assumptions. If we assume  $r_{mix} = 0$ , we find that the ratios of doubly Cabibbo-suppressed decays to Cabibbo-favored decays

are  $r_{ds}(K\pi) = (0.68^{+0.34}_{-0.33} \pm 0.07)\%$  and  $r_{ds}(K\pi\pi\pi) = (0.25^{+0.36}_{-0.34} \pm 0.03)\%$ .

12.15.Ff, 13.25.Ft, 14.40.Lb

## I. INTRODUCTION

The Standard Model predicts a rate for  $D^0 - \bar{D}^0$  mixing which is many orders of magnitude below the reach of present experiments. Typical calculations [1] give  $r_{mix}$ , the ratio of mixed to unmixed decay rates, in the range  $10^{-10}$  to  $10^{-7}$ . In contrast, various extensions to the Standard Model [2] allow a mixing rate close to the current experimental sensitivity of  $10^{-3}$  to  $10^{-2}$ . Consequently, a discovery of  $D^0 - \bar{D}^0$  mixing at currently measurable levels would be inconsistent with the Standard Model, and would provide a clear signal for new physics.

Experimentally, mixing is identified by a change in the charm quantum number of the neutral  $D$  meson between its production and decay. In the analysis presented in this paper, the charm of the produced  $D$  is determined from the decay  $D^{*+} \rightarrow D^0 \pi^+$  (or  $D^{*-} \rightarrow \bar{D}^0 \pi^-$ ), where the charge of the pion indicates whether a  $D^0$  or a  $\bar{D}^0$  was produced.  $D$  decays are reconstructed in four all-charged hadronic decay modes  $D \rightarrow K^- \pi^+$ ,  $D \rightarrow K^+ \pi^-$ ,  $D \rightarrow K^- \pi^- \pi^+ \pi^+$  or  $D \rightarrow K^+ \pi^+ \pi^- \pi^-$ . (Hereafter, we will omit the charge superscripts from the final states where context allows.) Possible evidence for mixing comes from the detection of a meson produced as a  $D^0$  ( $\bar{D}^0$ ) decaying to a “wrong-sign” final state which contains a  $K^+$  ( $K^-$ ), with the kaon charge opposite to that expected for unmixed decays.

Fermilab experiment E691 [3] has previously used this technique to set what is currently the strictest upper limit on mixing,  $r_{mix} < 0.37\%$ , albeit with specific assumptions which we will address in this paper. Fermilab experiment E615 obtained a limit of  $r_{mix} < 0.56\%$  by looking for same-sign muon pairs in  $\pi$ -tungsten interactions, based on a specific model for charm production [4]. Evidence for wrong-sign decays has been presented by the CLEO collaboration [6], which measures the ratio of wrong-sign to right-sign decays to be  $(0.77 \pm 0.25 \pm 0.25)\%$  for the  $K\pi$  final state. However, the CLEO experiment was unable to distinguish between mixing and doubly-Cabibbo-suppressed decays, which also produce wrong-sign events. Recently we have reported on a search for mixing using semileptonic decays of the  $D^0$  ( $\bar{D}^0$ ) which do not have a doubly-Cabibbo-suppressed background

[5]. We found that  $r_{mix} < 0.50\%$ .

It is possible to distinguish doubly-Cabibbo-suppressed (DCS) and mixing contributions to the wrong-sign rate by studying the distribution of  $D$  decay times. In the limit of small mixing, the rate for wrong-sign decays takes the form

$$, [D^0(t) \rightarrow f] = \frac{e^{-\Gamma t}}{4} |\langle f|H|\overline{D}^0\rangle_{CF}|^2 \left| \frac{q}{p} \right|^2 \times \left[ 4|\lambda|^2 + \left( (\Delta M)^2 + \frac{(\Delta\Gamma)^2}{4} \right) t^2 + (2Re(\lambda)\Delta\Gamma + 4Im(\lambda)\Delta M)t \right], \quad (1)$$

where

$$\lambda \equiv \frac{p}{q} \frac{\langle f|H|D^0\rangle_{DCS}}{\langle f|H|\overline{D}^0\rangle_{CF}}, \quad (2)$$

and  $p$  and  $q$  describe the relationship between the charm eigenstates  $|D^0\rangle$  and  $|\overline{D}^0\rangle$  and the mass eigenstates  $|D_{1,2}\rangle$ :

$$\begin{aligned} |D_1\rangle &= p|D^0\rangle + q|\overline{D}^0\rangle, \\ |D_2\rangle &= p|D^0\rangle - q|\overline{D}^0\rangle. \end{aligned} \quad (3)$$

The amplitude  $\langle f|H|D^0\rangle_{DCS}$  represents the DCS decay of the  $D^0$  while  $\langle f|H|\overline{D}^0\rangle_{CF}$  is the Cabibbo-favored counterpart for the decay of  $\overline{D}^0$ . The parameters  $\Delta M$  and  $\Delta\Gamma$  describe the differences in mass and width of the two physical states. The term proportional to  $|\lambda|^2$  in Eq. (1) describes the contribution from DCS amplitudes, the term proportional to  $t^2$  describes the lowest-order contribution from mixing, and the term proportional to  $t$  represents the interference between mixing and DCS amplitudes. We can apply this formula to the measured time distribution of wrong-sign decays to determine the separate contributions from DCS and mixing amplitudes.

## II. EFFECTS OF CP VIOLATION

Equation (1) describes the rate for  $D^0$  to decay to a wrong-sign final state  $f$ . Within the context of some new physics models, it is possible that the rate for  $\overline{D}^0$  to decay to  $\overline{f}$  is

not the same, and that CP is violated to a significant extent. (For recent discussions of the role of CP violation in  $D^0 - \bar{D}^0$  mixing see [7], [8].) Formally, the conjugate equation is

$$, [\bar{D}^0(t) \rightarrow \bar{f}] = \frac{e^{-\Gamma t}}{4} |\langle \bar{f} | H | D^0 \rangle_{CF}|^2 \left| \frac{p}{q} \right|^2 \\ \times \left[ 4|\bar{\lambda}|^2 + \left( (\Delta M)^2 + \frac{(\Delta, )^2}{4} \right) t^2 + (2Re(\bar{\lambda})\Delta, + 4Im(\bar{\lambda})\Delta M)t \right], \quad (4)$$

with

$$\bar{\lambda} \equiv \frac{q}{p} \frac{\langle \bar{f} | H | \bar{D}^0 \rangle_{DCS}}{\langle \bar{f} | H | D^0 \rangle_{CF}}. \quad (5)$$

In principle, CP violation can arise through a difference between Equations (1) and (4) in any one of the three terms. Any term in (1) can differ from its charge conjugate in (4) as a result of the interference of two or more contributing amplitudes which have non-zero relative phases of both the CP-conserving (strong) and CP-violating (weak) type.

Inequality of the two constant terms (*i.e.*,  $\left| \frac{p}{q} \right|^2 |\lambda|^2 \neq \left| \frac{q}{p} \right|^2 |\bar{\lambda}|^2$ , but see comment [9]) is referred to as *direct* CP violation. This could be significant if two or more comparable DCS amplitudes contribute with different CP-conserving and CP-violating phases. However, the Standard Model contribution (which is expected to dominate) provides only one weak, CP-violating phase. Direct CP violation is therefore likely to be small. Similarly, the two charge conjugate terms proportional to  $t^2$  will be the same unless there are two or more mixing amplitudes with relative weak and strong phases. On the contrary, most models suggest that if mixing occurs at all, it is likely to be dominated by a single CP-violating phase. The most likely source of CP violation, if it occurs, will be in the interference terms proportional to  $t$ , due to the relative phases between DCS and mixing amplitudes. We will explore some of these possibilities in the study of our data which follows.

### III. DESCRIPTION OF E791

We report the results of a search for  $D^0 - \bar{D}^0$  mixing and DCS decays using hadronic decays found in data from our experiment, Fermilab E791. We collected approximately



20 billion hadronic interactions in the 1991-2 fixed-target run using the TPL spectrometer [10] with a 500 GeV/ $c$   $\pi^-$  beam. There were five foil targets: one 0.5-mm thick Pt foil followed by four 1.6-mm thick diamond foils with 15 mm center-to-center separations. This arrangement allowed us to greatly reduce secondary interaction backgrounds by selecting only charm candidates which decayed in air.

The target region was preceded by six planes of silicon microstrip detectors and 8 proportional wire chambers (PWC's) used for beam tracking and was followed by 17 additional planes of silicon microstrip detectors for measuring tracks produced at and downstream of the primary vertex. The track momenta and slopes were also measured in the downstream spectrometer which had two magnets, 35 planes of drift chambers, and two PWC's. Two threshold Čerenkov counters provided  $\pi/K$  separation in the 6 - 60 GeV/ $c$  momentum range [11].

The mixing analysis in this paper relies heavily on track reconstruction, which begins by using hits in the silicon detector and folds in additional information from the downstream devices. The tracking efficiency is approximately 80% for particles with momenta greater than 30 GeV/ $c$  and drops to around 60% for particle momenta of 10 GeV/ $c$ . The mean number of reconstructed tracks used to fit the primary vertex is seven. After reconstruction, events with evidence of multiple vertices were kept for further analysis. The list of reconstructed vertices is used in the selection criteria described below.

We determined our production (primary) and decay vertex resolutions by comparing reconstructed and true vertex positions using our Monte Carlo detector simulation. The transverse resolutions quoted below are one-dimensional values. Longitudinal and transverse position resolutions for the primary vertex are 350 and 6  $\mu m$ , respectively. For the mean  $D^0$  momentum of 65 GeV/ $c$  the longitudinal resolutions for  $K\pi$  and  $K\pi\pi\pi$  vertices are 320 and 395  $\mu m$ , respectively, and increase by 33 and 36  $\mu m$ , respectively, for every 10 GeV/ $c$   $D^0$  momentum. Similarly, for the mean momentum, 65 GeV/ $c$ , of the observed  $D^0$ 's, the transverse resolutions for  $K\pi$  and  $K\pi\pi\pi$  vertices are 10 and 12  $\mu m$  respectively and decrease by about 0.5  $\mu m$  for every 10 GeV/ $c$  increase in  $D^0$  momentum.

The kaon and pion identification efficiencies and misidentification probabilities vary with momentum and with the signatures we require in the Čerenkov detector. For typical particle momenta in the range 20 GeV/ $c$  to 40 GeV/ $c$ , the Čerenkov identification efficiency of a kaon is around 58% when the probability for a pion to be misidentified as a kaon is 4%. In the same momentum range the Čerenkov identification efficiency of a pion is around 93% when the probability for a kaon to be misidentified as a pion is 35%.

#### IV. SELECTION OF DATA SAMPLE

A search for the rare wrong-sign mixing and DCS decays requires selection criteria that emphasize background reduction. We achieve this goal in two stages: initially reconstructing displaced secondary vertices and using a few loose criteria for selecting  $D^0$  decays to reduce the data sample, and then optimizing the data selection with artificial neural networks.

Initial reduction of the large E791 data set to a manageable size was achieved with the aid of a few simple criteria. Here, we describe the cuts made in these initial stages for the  $D^0 \rightarrow K\pi$  mode. (When we refer to  $K\pi$  or  $K\pi\pi\pi$  in this paper, without any explicit signs, we include the charge conjugate states. Otherwise, we indicate a specific final state by explicitly specifying at least the kaon charge or specifying right-sign (RS) or wrong-sign (WS) decays.) Two-prong vertices were used to start the search for  $D^0$  decays. The invariant mass of the two-prong  $D^0$  candidate, assumed to be  $K\pi$ , was required to be in the range 1.7 to 2.0 GeV/ $c^2$ . The kaon candidate was chosen as the one with the higher probability of being a kaon based on Čerenkov information. To further reduce the contributions from misidentified  $D^0$  decays,  $K\pi$  candidates were rejected if the reverse hypothesis ( $\pi K$ ) fell within  $2\sigma$  of the  $D^0$  mass, where  $\sigma$  is the measurement resolution for the  $D^0$  mass. Similarly, to reduce contamination from  $D^0$  decays to  $K^+K^-$  and  $\pi^+\pi^-$ ,  $K\pi$  candidates were rejected if the  $K^+K^-$  or  $\pi^+\pi^-$  mass hypotheses fell within  $2\sigma$  of the  $D^0$  mass. To help ensure that the reconstructed secondary vertex was a true decay vertex, we required that it be separated from the primary vertex by at least 8 standard deviations ( $\sigma_{\Delta z}$ ) in the beam direction (*i.e.*,

$\Delta z/\sigma_{\Delta z} > 8$ ). Two further requirements ensured that the reconstructed  $D^0$  was consistent with originating at the primary vertex. First, the impact parameter of the reconstructed  $D$  momentum with respect to the primary vertex,  $b_p$ , was required to be less than  $60 \mu\text{m}$ . Second, the component of the reconstructed  $D$  momentum perpendicular to the  $D$  line-of-flight (as determined from the primary and secondary vertex positions),  $p_T^D$ , was required to be less than  $0.35 \text{ GeV}/c$ . The  $K$  and  $\pi$  decay tracks were required to be well-reconstructed in the silicon detectors and drift chambers. Finally, the momentum asymmetry of the  $K$  and  $\pi$  as measured in the laboratory frame (*i.e.*,  $p_{asy} \equiv |\vec{p}_K - \vec{p}_\pi|/|\vec{p}_K + \vec{p}_\pi|$ ) was required to be less than 0.65. This reduced the contribution from random track combinations, which tended to be asymmetric.

The cuts for the  $D^0 \rightarrow K\pi\pi\pi$  case were similar in the initial stages. We used candidates arising from both 4-prong vertices and 3-prong vertices (with an added track). The two vertex samples contributed roughly equal amounts to the signal. We required  $\Delta z/\sigma_{\Delta z} > 8$ ,  $b_p < 60 \mu\text{m}$ ,  $p_T^D < 0.5 \text{ GeV}/c$ , and that the decay vertex was outside the target foils. The  $D^0$  candidate mass was required to be in the range  $1.7 - 2.0 \text{ GeV}/c^2$ . Candidate  $K\pi\pi\pi$  decays were rejected if either hypothesis with opposite kaon charge was within  $2\sigma$  of the  $D^0$  mass. Tracks were required to have momenta greater than  $0.5 \text{ GeV}/c$  and to have greater than  $4\sigma$  of transverse separation from the primary vertex where  $\sigma$  is the measurement resolution on the separation. Finally, the decay tracks were required to form a vertex that was no more than  $2.5 \text{ cm}$  downstream of the final target. Beyond that point the silicon detectors and other material in the beam path provided large numbers of secondary interactions.

In the final stage of analysis, we used two-layer feed-forward neural networks to optimize the signal selection [12,13]. Specifically, we chose selection criteria that maximize  $S/\sqrt{B}$  where  $S$  and  $B$  were the signal and the background under the signal for the right-sign decays. A vector, whose components are variables such as the ones just mentioned, was fed into each neural net as the input layer. Each node in the next (hidden) layer computed the sigmoid of the sum of an offset and the inner product of the input vector with a weight vector. The results from this layer in turn formed the input for a single node in the final

(output) layer. Thus, the networks effectively combined information from each variable we would otherwise have “cut” on and provided a single output value in the range 0 to 1. This output was monotonically related to the probability that a given candidate was signal and not background.

Since our two major sources of background were false  $D^0$  candidates and real  $D^0$  candidates combined with random pions to produce fake  $D^*$  candidates, we used separate neural networks to classify the  $D^0$  and  $D^*$  candidates. Although there are only two modes of  $D^0$  decay examined in this work, three  $D^0$  samples were used to train the neural nets: one for the  $D^0 \rightarrow K\pi$  mode and two for the  $D^0 \rightarrow K\pi\pi\pi$  mode. The two separate  $D^0 \rightarrow K\pi\pi\pi$  samples contained candidates from vertices that had either all four or only three of the four tracks.

Separate neural nets were trained for each of the three samples; only  $D^0$  candidates not combining with pions to give a  $D^*$  candidate were used. Thus, the three neural nets used in  $D^0$  optimization were all trained with samples not used in this analysis and were thus statistically unbiased. A fourth neural net was trained using part of the right-sign  $D^{*+} \rightarrow D^0\pi^+$  sample to classify  $D^{*+}$  candidates. Every net was trained using events in the peak region as “signal” and the remaining events as “background”. We selected only those events for which the product of the  $D^0$  and  $D^*$  net outputs was greater than a certain value rather than making individual “cuts” on many variables.

In the  $D^0 \rightarrow K^-\pi^+$  mode, the net was presented with twelve input variables: the  $p_T$  of the  $D^0$  relative to the incident pion beam direction, the separation between the secondary and primary vertices ( $\Delta z$ ),  $p_T^D$ ,  $b_p$ , the  $\chi^2$  per degree of freedom for the secondary vertex fit, the Čerenkov -based probability for the kaon to be a kaon, the momentum asymmetry ( $p_{asy}$ ), the consistency probability for the secondary vertex to be in a target foil, the track fit  $\chi^2$  per degree of freedom for the two tracks and the number of tracking systems traversed by each of the two tracks. The two nets for the  $D^0 \rightarrow K^-\pi^+\pi^+\pi^-$  mode used seven variables, of which the first six variables were the same as the first six just listed for the  $D^0 \rightarrow K^-\pi^+$  mode. The seventh variable was the smallest contribution to the  $\chi^2$  of the fit to the primary

vertex from any of the four decay tracks. Finally, the  $D^*$  neural net was constructed using only five variables: the  $\chi^2$  per degree of freedom for the track fit for the pion from the  $D^{*+}$  decay (referred to hereafter as the “bachelor pion”), the number of tracking stations in which the pion is detected, the probability that it is not a fictitious track, its momentum and the  $\chi^2$  per degree of freedom for the vertex fit of the  $D^0$  and the bachelor pion. There were three nodes in the hidden layer for the  $D^0 \rightarrow K^-\pi^+$  and  $D^*$  nets and four nodes in the hidden layers of each of the  $D^0 \rightarrow K^-\pi^+\pi^+\pi^-$  nets.

Although we considered many variables, we pruned the list down to the variables listed above and also pruned some of the connections to the hidden layer nodes when their contributions to the output were deemed unimportant using a technique called subset reduction, implemented as follows. Nodes in a given layer were viewed as a linear array, one row for each event. The matrix thus formed was subjected to singular value decomposition using QRcp factorization [14]. The “energy content” of the nodes was determined by the resulting eigenvalues [15]. Nodes with an “energy content”  $< 1\%$  were deleted.

We also tried other techniques for selecting events, including the more common method of using independent “cuts” in each variable and a Binary Decision Tree (BDT) technique [16]. The sensitivity of the neural net technique was about 10% higher than the BDT in the  $D^0 \rightarrow K\pi$  mode and about 30% higher than the BDT in the  $D^0 \rightarrow K\pi\pi\pi$  mode; in turn the BDT was better than the commonly used “cuts” technique. One further advantage of the neural net technique was that the output could be used to choose the best candidate in an event, should there be more than one (a rare occurrence). This simplified the statistical analysis in the fits to our data.

The results of our neural net optimizations are shown in Figure 1 for right-sign and wrong-sign  $K\pi$  and  $K\pi\pi\pi$  final states. In this figure, we plot the candidate  $D^0$  mass ( $m(K\pi)$  or  $m(K\pi\pi\pi)$ ) versus  $Q$ , defined as  $Q \equiv m(K\pi\pi) - m(K\pi) - m(\pi)$  or  $Q \equiv m(K\pi\pi\pi) - m(K\pi\pi\pi) - m(\pi)$ . For real  $D^*$  decays,  $Q$  has a value of about 5.8 MeV. In the right-sign plots (top of Figure 1), clear signals are apparent over small backgrounds. The bands of events at  $m(K\pi)$  or  $m(K\pi\pi\pi) \approx 1.865$  GeV/ $c^2$  are due to real  $D^0$  decays combining with

random pions in the event to give a false  $D^*$  candidate. These bands are more readily seen in the wrong-sign plots (bottom of Figure 1) where the vertical scale is expanded by a factor of 20. This background, which we will refer to as “random pion” background, is the dominant one in our analysis. We will call the remaining broad background visible in the plots the “false  $D^0$ ” background.

In the right-sign plot for the  $K\pi$  mode, there is a signal of 5643 events above a background of 235 events. In the right-sign plot for the  $K\pi\pi\pi$  mode, there is a signal of 3469 events above a background of 146 events. The signals and backgrounds are estimated in a region spanning  $\pm 1.75\sigma$  around the peak in  $Q$ , for  $1.77 < m_{D^0} < 1.97 \text{ GeV}/c^2$ . The precise region used to estimate  $S$  and  $B$  is not important for the optimization. The resulting sensitivities ( $S/\sqrt{B}$ ) for the two modes are 368 ( $K\pi$ ) and 287 ( $K\pi\pi\pi$ ).

## V. FIT TECHNIQUE

To estimate the contributions to the data samples from the various sources, we perform an unbinned maximum likelihood fit, using the following form for the  $\ln$  of the likelihood:

$$\ln \mathcal{L} = \sum_i \ln \mathcal{L}_i - \sum_f \left( \frac{1}{2} \ln [2\pi N_{pred}^f] + \frac{[N_{pred}^f - N_{obs}^f]^2}{2N_{pred}^f} \right), \quad (6)$$

where the first sum is over all the  $D^*$  candidates, the second sum is over the eight decays used in the analysis,  $\mathcal{L}_i$  represents the likelihood for each candidate, and  $N_{obs}^f$  is the observed number of candidates for each final state. The argument of the second sum is the logarithm of a normalized Gaussian, and serves to constrain the number of candidates predicted by the fit,  $N_{pred}^f$ . There are four contributions to each  $\mathcal{L}_i$ : signal, misidentified  $D^0 \rightarrow KK$  and  $D^0 \rightarrow \pi\pi$  decays, random pions with real  $D^0$ 's, and random pions with false  $D^0$ 's.

$$\mathcal{L}_i = S(m_i, Q_i, t_i) + M(m_i, Q_i, t_i) + P(m_i, Q_i, t_i) + F(m_i, Q_i, t_i), \quad (7)$$

where  $m_i$ ,  $Q_i$ , and  $t_i$  are the  $D$  mass,  $Q$  value and proper decay time of each candidate. A wrong-sign signal event is described by simple Gaussian terms in  $m_i$  and  $Q_i$ , multiplied by

a sum of the three different decay time distributions that represent the DCS, mixing and interference contributions (see Eq. (1)):

$$S(m_i, Q_i, t_i) = \frac{1}{N_{pred}} \frac{1}{\sqrt{2\pi}\sigma_D} e^{-(m_{D^0}-m_i)^2/2\sigma_D^2} \times \frac{1}{\sqrt{2\pi}\sigma_Q} e^{-(Q_{D^*}-Q_i)^2/2\sigma_Q^2} \times \{A_{dcs} B_{exp}(t_i) + A_{mix} B_{mix}(t_i) + A_{int} B_{int}(t_i)\}, \quad (8)$$

where

$$\begin{aligned} B_{exp}(t_i) &= B_{exp}^0 \epsilon(t_i) \int dt e^{-(t-t_i)^2/2\sigma_0^2} e^{-\Gamma t}, \\ B_{mix}(t_i) &= B_{mix}^0 \epsilon(t_i) \int dt e^{-(t-t_i)^2/2\sigma_0^2} t^2 e^{-\Gamma t}, \\ B_{int}(t_i) &= B_{int}^0 \epsilon(t_i) \int dt e^{-(t-t_i)^2/2\sigma_0^2} t e^{-\Gamma t}, \end{aligned} \quad (9)$$

$\epsilon(t_i)$  is the reconstruction efficiency and  $\sigma_0$  is the decay time resolution. Each  $B(t_i)$  is normalized to unit integral so that  $A_{dcs}$ ,  $A_{mix}$ , and  $A_{int}$  can be interpreted as the number of *observed* candidates of each type. The Gaussian smearing integrals are performed analytically with a smearing width  $\sigma_0 = 0.05$  ps.

The reconstruction efficiency  $\epsilon(t)$  is the first of three functions that must be modeled for the fit. It is desirable to measure this function using real data rather than using Monte Carlo simulation. Fortunately, this can be accomplished with a sample of right-sign events. Since there is no mixing contribution to the right-sign decay rate, the true decay time distribution for right-sign decays is proportional to  $e^{-\Gamma t}$  with  $\Gamma = (0.415 \text{ ps})^{-1}$  [17]. The reconstructed distribution is proportional to  $\epsilon(t) \int dt e^{-(t-t_i)^2/2\sigma_0^2} e^{-\Gamma t}$ . Therefore, dividing the measured right-sign distribution (corrected for non- $D^0$  background using sideband subtraction) by the known smeared exponential gives a distribution proportional to the efficiency [18]. Figure 2 shows the results of that measurement for both the  $K\pi$  and  $K\pi\pi\pi$  final states, which we will use to represent  $\epsilon(t)$  in the fit.

Despite the explicit mass cuts designed to reduce backgrounds from  $D^0 \rightarrow KK$  and  $D^0 \rightarrow \pi\pi$  decays described in Section IV, some contamination remains. The misidentified  $D^0 \rightarrow KK$  and  $D^0 \rightarrow \pi\pi$  events are described by

$$M(m_i, Q_i, t_i) = \frac{1}{N_{pred}} A_{\pm} U(m_i) V(Q_i) B_{exp}(t_i). \quad (10)$$

where the functions  $U(m_i)$  and  $V(Q_i)$  are obtained from Monte Carlo simulations of  $KK$  and  $\pi\pi$  reflections remaining after all cuts, including explicit mass cuts designed to minimize these reflections. The parameters  $A_+$  and  $A_-$  describe the number of events in the wrong-sign  $K^+\pi^-$  and  $K^-\pi^+$  samples, respectively. Similar backgrounds for the  $K\pi\pi\pi$  mode are not significant (see Section VII for further discussion).

The random pion background is described by

$$P(m_i, Q_i, t_i) = \frac{1}{N_{pred}} A_{\pi} \frac{1}{\sqrt{2\pi} \sigma_D} e^{-(m_{D^0} - m_i)^2 / 2\sigma_D^2} R(Q_i) B_{exp}(t_i). \quad (11)$$

The background shape in  $Q$ , represented by  $R(Q)$ , is independent of the candidate  $D$  mass. We model this shape by combining a  $D^0$  candidate from one event with a  $\pi$  from another event. As long as the  $D^0$  is not strongly correlated with other tracks in the event, and the selection cuts are not dependent on the spatial relation between the  $D^0$  and other tracks in the event, this technique should provide an accurate model of background. Monte Carlo studies confirm the validity of this method. The resulting distribution is compared to the wrong-sign  $D^0 \rightarrow K\pi$  and  $D^0 \rightarrow K\pi\pi\pi$  data samples in Figure 3.

The false  $D^0$  background is adequately described by a linear function in  $m_i$ :

$$F(m_i, Q_i, t_i) = \frac{1}{N_{pred}} \frac{A_0(1 + A_1(m_i - m_0))}{\Delta m} R(Q_i) B_{false}(t_i), \quad (12)$$

where  $m_0$  is an arbitrary reference chosen to be  $1.87 \text{ GeV}/c^2$  and  $\Delta m$  is the  $D^0$  mass interval ( $0.2 \text{ GeV}/c^2$ ). The function  $R(Q)$  is observed to be the same as in the case of the random pion and real  $D^0$  background described by Equation (11) above. The function  $B_{false}(t_i)$  describes the time distribution of the false  $D^0$  background. We model this distribution using candidates from the  $D$  mass sidebands of the right-sign event sample (Figure 4). Since this background is very small, we do not need to model it with great precision, and the statistics of Figure 4 are adequate.

The likelihood function for right-sign decays is constructed similarly. Since right-sign decays were used to model  $\epsilon(t)$  and  $B_{false}(t)$ , we do not use the lifetime information for



these events in the fit. Moreover, right-sign decays are not subject to mixing or interference, so the fit functions for these events are given by the simplified formulae

$$\begin{aligned}
S(m_i, Q_i) &= \frac{1}{N_{pred}} \frac{1}{\sqrt{2\pi} \sigma_D} e^{-(m_{D^0} - m_i)^2 / 2\sigma_D^2} \times \frac{1}{\sqrt{2\pi} \sigma_Q} e^{-(Q_{D^*} - Q_i)^2 / 2\sigma_Q^2} \times A_{rs}, \\
P(m_i, Q_i) &= \frac{1}{N_{pred}} A_\pi \frac{1}{\sqrt{2\pi} \sigma_D} e^{-(m_{D^0} - m_i)^2 / 2\sigma_D^2} R(Q_i), \\
F(m_i, Q_i) &= \frac{1}{N_{pred}} \frac{A_0(1 + A_1(m_i - m_0))}{\Delta m} R(Q_i).
\end{aligned} \tag{13}$$

We fit all the data, both right-sign and wrong-sign  $D^0 \rightarrow K\pi$  and  $D^0 \rightarrow K\pi\pi\pi$ , simultaneously in one fit. Separate terms for charge conjugate final states are provided to allow for the most general possible form for CP violation. Under these conditions, we have four signal parameters ( $A_{dcs}$ ,  $A_{mix}$ ,  $A_{int}$  and  $A_\pm$ ) and three background parameters ( $A_\pi$ ,  $A_0$  and  $A_1$ ) for the two wrong-sign decay modes  $D^0 \rightarrow K^+\pi^-$  and  $\bar{D}^0 \rightarrow K^-\pi^+$ . We have three signal parameters ( $A_{dcs}$ ,  $A_{mix}$  and  $A_{int}$ ) and three background parameters ( $A_\pi$ ,  $A_0$  and  $A_1$ ) for the two wrong-sign decay modes  $D^0 \rightarrow K^+\pi\pi\pi$  and  $\bar{D}^0 \rightarrow K^-\pi\pi\pi$ . For each right-sign mode ( $\bar{D}^0 \rightarrow K^+\pi^-$ ,  $\bar{D}^0 \rightarrow K^+\pi\pi\pi$ ,  $D^0 \rightarrow K^-\pi^+$ , and  $D^0 \rightarrow K^-\pi\pi\pi$ ) we have one signal parameter ( $A_{rs}$ ) and three background parameters ( $A_\pi$ ,  $A_0$  and  $A_1$ ). Additionally, we have five mass parameters ( $m_{D^0}$ ,  $\sigma_{K\pi}$ ,  $\sigma_{K\pi\pi\pi}$ ,  $Q_{D^*}$ ,  $\sigma_Q$ ) to describe the signal Gaussian functions. Separate  $D^0$  mass resolutions are used for the  $K\pi$  and  $K\pi\pi\pi$  final states. The resolution in  $Q$  is dominated by the bachelor pion, and is therefore the same for the two final states.

With this list we have 47 parameters for a complete description of the data. However, we expect that the false  $D^0$  backgrounds for right-sign and wrong-sign decays to the same  $D^0$  final state should have the same slope parameter ( $A_1$ ), although the level ( $A_0$ ) may differ since they are combined with pions of different charge to form the  $D^*$  candidates. This observation reduces the number of parameters to 43.

It is convenient to express the wrong-sign signal parameters  $A_{dcs}$ ,  $A_{mix}$  and  $A_{int}$  in terms of the ratios of produced wrong-sign events to produced right-sign events, since these are the parameters of primary physics interest. For the wrong-sign  $K^-\pi^+$  final state

$$r_{dcs}(K^-\pi^+) = \frac{A_{dcs}(\bar{D}^0 \rightarrow K^-\pi^+)}{A_{rs}(\bar{D}^0 \rightarrow K^+\pi^-)},$$

$$\begin{aligned}
r_{mix}(K^-\pi^+) &= \frac{A_{mix}(\overline{D}^0 \rightarrow K^-\pi^+)}{A_{rs}(\overline{D}^0 \rightarrow K^+\pi^-)} \times c_{mix}, \\
r_{int}(K^-\pi^+) &= \frac{A_{int}(\overline{D}^0 \rightarrow K^-\pi^+)}{A_{rs}(\overline{D}^0 \rightarrow K^+\pi^-)} \times c_{int},
\end{aligned} \tag{14}$$

and similarly for  $K^+\pi^-$ ,  $K^-\pi\pi\pi$  and  $K^+\pi\pi\pi$ . The  $c$ 's in the expressions for  $r_{mix}$  and  $r_{int}$  are given by

$$\begin{aligned}
c_{mix} &= \frac{\int dt_i, \epsilon(t_i) \int dt e^{-\Gamma t} e^{-(t-t_i)^2/2\sigma_0^2}}{\int dt_i, \frac{1}{2}, {}^3 \epsilon(t_i) \int dt t^2 e^{-\Gamma t} e^{-(t-t_i)^2/2\sigma_0^2}}, \\
c_{int} &= \frac{\int dt_i, \epsilon(t_i) \int dt e^{-\Gamma t} e^{-(t-t_i)^2/2\sigma_0^2}}{\int dt_i, {}^2 \epsilon(t_i) \int dt t e^{-\Gamma t} e^{-(t-t_i)^2/2\sigma_0^2}}.
\end{aligned} \tag{15}$$

These terms correct for the different integrated efficiencies for reconstructing wrong-sign DCS, mixing and interference events. Table I shows these correction factors for both the  $K\pi$  and  $K\pi\pi\pi$  final states.

Although the production characteristics of  $D^0$  and  $\overline{D}^0$  are different in our experiment, the ratios in Equation (14) are designed to cancel this effect. In constructing these ratios we implicitly assume that the Cabibbo-favored amplitudes  $\langle \overline{f} | H | D^0 \rangle$  and  $\langle f | H | \overline{D}^0 \rangle$  are equal in magnitude, as mentioned previously [9]. With this assumption,  $r_{mix}$  of Equation (14) can be interpreted according to convention as

$$r_{mix} = \frac{1}{2, {}^2} \left| \frac{q}{p} \right|^2 \left( (\Delta M)^2 + \frac{(\Delta, )^2}{4} \right). \tag{16}$$

Finally, we note that the values  $r_{mix}(D^0 \rightarrow \overline{D}^0)$  and  $r_{mix}(\overline{D}^0 \rightarrow D^0)$  should be independent of the  $D$  decay final state. Thus, without loss of generality, we can assume that  $r_{mix}(D^0 \rightarrow K^+\pi^-) = r_{mix}(D^0 \rightarrow K^+\pi\pi\pi)$  and  $r_{mix}(\overline{D}^0 \rightarrow K^-\pi^+) = r_{mix}(\overline{D}^0 \rightarrow K^-\pi\pi\pi)$ . This eliminates two more parameters from our fit [19], leaving us with a total of 41 independent parameters to describe the full data set.

## VI. FIT RESULTS

We fit the data over the range 1.77 to 1.97 GeV/ $c^2$  in  $m_D$ , 0.0 to 0.020 GeV/ $c^2$  in  $Q$  and 0.0 to 4.0 ps in  $t$ . Tables II and III show the results for the 41-parameter fit just

described. The wrong-sign ratios are all consistent with zero, indicative of small DCS to Cabibbo-favored ratios and very little mixing. There is also no evidence for CP violation. Figures 5 and 6 show the fit results overlaid on the data distributions for  $m_D$ ,  $t$ , and  $Q$ . Good agreement is evident in every distribution.

In performing the fit, we discovered that the  $r_{dcs}$  and  $r_{mix}$  terms are strongly anticorrelated with the  $r_{int}$  terms, and strongly correlated with each other. Figure 7 demonstrates how these correlations come about in a hypothetical case where the interference contribution approximately cancels the contribution from pure mixing. This plot demonstrates that even when the full time evolution deviates only slightly from the pure exponential form of DCS decays, a large contribution from mixing can be present if it is offset by a destructive interference contribution. This implies that the fitted values for the interference contribution and the mixing contribution are strongly anticorrelated.

Figure 8 illustrates the correlations in our particular fit by showing the likelihood contour plots for representative pairs of parameters for the  $D^0 \rightarrow K^+\pi^-$  mode. These strong correlations account for much of the uncertainty in the wrong-sign ratios. Table IV gives the correlation coefficients for the different wrong-sign ratios. The correlations of these ratios with all other parameters of the fit are negligible. We note that the correlations would be slightly reduced in an experiment with better efficiency at short decay times where there is good discrimination between  $r_{dcs}$  and the other terms.

If we assume, in addition to the assumptions of Section V, that there is no significant CP violation in either the DCS or mixing terms (see the discussion in Section II), we can eliminate three more parameters from the fit. Table V shows the results of that 38-parameter fit for the wrong-sign ratios. As expected, the central values for  $r_{dcs}$  and  $r_{mix}$  are between the corresponding separate terms in Table II, and all the fit errors are reduced.

We note that the earlier measurement by the E691 collaboration [3] assumed that the interference terms  $r_{int}$  were negligible. Recently, there has been lively discussion concerning the validity of this assumption [7,8,20]. Although some authors suggest that the phase between DCS and mixing amplitudes may be small, and therefore that the interference terms

$r_{int}$  should also be small, we prefer to quote our results without this constraint. Nonetheless, to compare our measurements with the previous results from E691, we show in Table VI the results for  $r_{dcs}$  and  $r_{mix}$  when we set the interference terms to zero. The reduction of the fit errors from Table V is indicative of the strong correlations with the  $r_{int}$  parameters, which are now fixed at zero. E691 also touched on this point by considering several different fixed values of the interference term (only one interference term was allowed in their model). Their results showed behavior similar to what we see in our data: strong correlation between wrong-sign ratios, and reduced fit errors when fixing the interference term to zero.

Finally, we explore the possibility that mixing is completely negligible, as one would expect from purely Standard Model contributions. In this case, we fit only for the DCS terms, obtaining  $r_{dcs}(K\pi) = (0.68_{-0.33}^{+0.34} \pm 0.07)\%$  and  $r_{dcs}(K\pi\pi\pi) = (0.25_{-0.34}^{+0.36} \pm 0.03)\%$ . The result for  $D^0 \rightarrow K\pi$  is consistent with the CLEO measurement [6] for the total wrong-sign rate:  $r_{ws}(K\pi) = (0.77 \pm 0.25 \pm 0.25)\%$ .

## VII. SYSTEMATIC UNCERTAINTIES

Systematic uncertainties in the fit arise primarily from our modeling of the three functions  $\epsilon(t)$ ,  $R(Q)$ , and  $B_{false}(t)$ . By using the right-sign data samples to estimate these functions, we have minimized our dependence on Monte Carlo models, but some uncertainty remains. The results of our studies of systematic uncertainties are summarized in Tables VIII, VII, IX and X. Below we describe the entries in Table VIII which are the systematic errors obtained from studies on the constrained fit of Table V. The entries in Tables VII, IX, and X are obtained similarly. We find that the systematic uncertainties in the analysis are small compared with the statistical errors from the fit.

The uncertainties in the first row arise from our estimates of the size of the reflection in  $K\pi$  from misidentified  $D^0 \rightarrow K^-K^+$  and  $D^0 \rightarrow \pi^-\pi^+$  events. Most of this uncertainty arises from our knowledge of the branching ratios for these modes. We have estimated this error as 20% of the difference between the results of fits with and without terms describing

these reflections.

The uncertainties listed in the third row (“statistics of effy & bkgd distrs”) result from the uncertainties in our estimates of  $\epsilon(t)$ ,  $R(Q)$ , and  $B_{false}(t)$  due to the finite size of our right-sign data sample. The error bars on the corresponding histograms in Figures 2, 3, and 4 show the level of uncertainty involved. Statistical uncertainties on the model of the function  $R(Q)$  (Figure 3) have been greatly reduced by combining each  $D^0$  candidate with pions from many different events, so that the uncertainties in this histogram are negligible in comparison with the uncertainties of Figures 2 and 4. In order to propagate these statistical uncertainties to uncertainties on the fitted parameters, we perform many fits to the data, modifying each bin in each of the histograms of  $\epsilon(t)$ ,  $R(Q)$  and  $B_{false}(t)$  by a Gaussian fluctuation with resolution given by the error bars. The rms spreads of the fitted parameters from 25 such fits are given in Table VIII.

The uncertainties listed in the fourth row of Table VIII (“binning of effy & bkgd distrs”) are due to the fact that we have represented the  $\epsilon(t)$ ,  $R(Q)$ , and  $B_{false}(t)$  functions by binned histograms rather than smooth functions. We have, of course, tried to choose bin sizes small enough so that binning effects are not significant. In order to verify this claim, we replaced the histograms in the fit with smoothed functions which were derived from the histogram data, and repeated the fit. The differences between the parameter values with the smoothed functions and the parameter values with the histograms are quoted in Table VIII as the uncertainties due to binning. We expect this method to give an overestimate of the binning effect, since it also includes the effect of some statistical fluctuations in the measured histograms which are adjusted by the smoothing function. As anticipated, the binning effects are small.

The uncertainties listed in the fifth row of the table (“time resolution”) are due to the resolution on the measured decay time. Since the smearing is small, a good assumption is that  $\epsilon(t)$  is almost the same with and without smearing. For this analysis, the most likely resolution on the decay time is about 0.03 ps, with some measurements having a resolution as large as 0.08 ps. In order to quantify the error due to smearing, we replace the functions

of Equation (9) by functions convoluted with a fixed Gaussian resolution:

$$\begin{aligned}
B_{exp}(t_i) &= B_{exp}^0 \int dt e^{-(t-t_i)^2/2\sigma_0^2} e^{-\Gamma t} \epsilon(t), \\
B_{mix}(t_i) &= B_{mix}^0 \int dt e^{-(t-t_i)^2/2\sigma_0^2} t^2 e^{-\Gamma t} \epsilon(t), \\
B_{int}(t_i) &= B_{int}^0 \int dt e^{-(t-t_i)^2/2\sigma_0^2} t e^{-\Gamma t} \epsilon(t),
\end{aligned} \tag{17}$$

where  $\epsilon(t)$  is obtained as in Section V and the integrals are obtained numerically. We then perform the fit with three different values for  $\sigma_0$ : 0.02 ps, 0.05 ps and 0.08 ps. We quote the average of the central value differences (from the fits for 0.02 ps and 0.05 ps and from the fits for 0.08 ps and 0.05 ps) as variations in Table VIII, line 5. When the exponential lifetime is modified by our detector acceptance which is poor at low lifetimes, there is a “peak” at  $\approx 0.5$  ps. We observe that the time smearing affects the likelihood most near this “peak”, while mixed events are most likely around higher values of decay time. Consequently, the DCS ratios exhibit the largest variation, while the mixing ratio is relatively stable.

The uncertainties in the sixth line of the table (“mass resolution”) come from the assumption of a constant Gaussian resolution in  $m(K\pi)$  and  $m(K\pi\pi\pi)$ . In truth, the mass resolution should depend on the  $D^0$  momentum and on the kinematics of the decay. We have studied the dependence on momentum and verified a noticeable correlation between resolution and momentum. For the  $K\pi$  decay mode, about 90% of the events have a mass resolution between 12 and 16 MeV/ $c^2$ , with a tail reaching out to about 25 MeV/ $c^2$  at high momentum. For the  $K\pi\pi\pi$  mode, the variation is much smaller, with all events exhibiting a mass resolution in the range 8.5 to 11 MeV/ $c^2$ . To quantify this effect, we have varied the mass resolutions  $\pm 2$  MeV/ $c^2$  in the fit and recorded the maximum variations in wrong-sign ratios in Table VIII. The fit results are quite insensitive to variation of the resolution on  $m(K\pi\pi\pi)$ , but change slightly with the resolution on  $m(K\pi)$ . A change in  $m(K\pi)$  resolution primarily affects  $r_{dcs}(K\pi)$ , but because of the correlations in the fit, it will also alter  $r_{mix}$  and  $r_{dcs}(K\pi\pi\pi)$ , as shown.

Some other assumptions and biases in our fit model bear further comment. First of all, we have assumed that the efficiency function  $\epsilon(t)$  for reconstructing a  $D^0$  from a  $D^*$  decay (signal

terms from Eq. (8)) is the same as the efficiency function for reconstructing a primary  $D^0$  (random pion term from Eq. (11)). Since our reconstruction and selection criteria are only weakly dependent on the  $D^0$  production kinematics, we expect to find very little difference in efficiency for these two sources of  $D$  mesons. Studies of reconstructed  $D^0$  decays which are *not* associated with a bachelor pion appear to confirm that the difference is negligible. Secondly, the  $K\pi\pi\pi$  final state may result from different resonant substructures in the Cabibbo-favored and DCS amplitudes. This can, in principle, lead to different efficiencies for the DCS and interference terms in the fit. (The two-body  $K\pi$  mode is, of course, immune to this problem.) Once again however, the fact that our reconstruction depends only slightly on the decay kinematics leads to effects at the level of only 1%. The very similar time dependence of the efficiency functions for the  $K\pi$  and  $K\pi\pi\pi$  final states (Fig. 2) demonstrates how little  $\epsilon(t)$  depends on the  $D$  decay.

We are also aware that training the  $D^*$  neural net on a sample of right-sign  $D^*$  decays can, in principle, produce a small bias in that sample (but not the wrong-sign samples which were not used for training the net). Careful selection of input variables for the neural net that are not correlated with  $Q$  (our variables depend only on parameters that describe the bachelor pion) should prevent any significant bias. We have investigated this effect by subdividing the training sample into 10 subsamples, training a neural net on one subsample, and applying the resulting net to the remaining samples as a test of the bias. We then repeat the process on each subsample to get a better statistical average. By comparing the sensitivity (measured as  $S/\sqrt{B}$ ) of the training samples with the sensitivity of the test samples, we determine the level of bias. We find that the number of right-sign signal events could be biased upwards by about 1%. This is a negligible effect compared to our statistical error.

Finally, we have investigated the effects of various charm backgrounds that might feed into our wrong-sign data samples. The largest source comes from doubly-misidentified decays of  $D^0 \rightarrow K\pi$  or  $K\pi\pi\pi$ , in which the  $K$  and a  $\pi$  of opposite charge are both misidentified (as  $\pi$  and  $K$ ) by the Čerenkov detector. Although we explicitly cut against these misidentified

decays in our data selection, a small fraction is expected to pass our cuts. Our Čerenkov measurements allow  $D^0$  decays to be doubly misidentified around 1.3% of the time. However, only about 15% of these candidates make it into the signal region of the misidentified mass. The selection cut on the reflected mass of each candidate further reduces this background by about a factor of 20. Furthermore, since the background is very broadly distributed in mass, rather than peaked in the signal region, we expect the fit to respond only weakly by changing the wrong-sign ratios, probably at the level of a few times  $10^{-4}$  or lower. Since this background has an exponential decay time distribution, it will be interpreted as a signal for  $r_{dcs}$ , and will not affect the measurement of  $r_{mix}$  or  $r_{int}$  at all.

There are other potential sources of charm background from  $D^* \rightarrow \pi D$  decays with the  $D$  decaying to a mode other than  $K\pi$  or  $K\pi\pi\pi$ . We have especially examined decays that might be misidentified as  $D^0 \rightarrow K\pi$ , including  $D^0 \rightarrow K^-\pi^+\pi^0$  and  $K^-\mu^+\nu$  (doubly-misidentified), and  $D^0 \rightarrow K^+K^-$ ,  $\pi^+\pi^-$  and  $\pi^+\pi^-\pi^0$  (singly-misidentified). As a general rule, the misidentification rates for these modes are similar to what was observed for the double misidentification above (all misidentification is dominated by the Čerenkov selection criteria for the kaon candidate), while the misidentified masses (in some cases after losing a neutral particle) are well outside the signal region. The  $D^0 \rightarrow K^+K^-$  and  $D^0 \rightarrow \pi^+\pi^-$  were the most significant, and were handled as described previously. None of the other decays were seen to contribute a significant background to our data samples.

The last line in Table VIII shows the contribution from all the systematic errors added in quadrature. These totals are about half the size of the statistical errors in Table V, and about one quarter to one third the size of the statistical errors of Table II.

### VIII. DISCUSSION

At the current level of sensitivity, mixing searches begin to constrain some models [21]. There also are other search methods that are promising. Using the same  $D^*$  decay chain to identify the produced  $D$  meson, but looking at semileptonic decays of the  $D$ , is one



possibility. Although semileptonic decays are harder to reconstruct due to missing neutrinos, they are not subject to contributions from DCS amplitudes, and therefore do not suffer from the main limitations discussed in this paper. In a separate publication we describe such a search [5] whose result is  $r_{mix} < 0.50\%$  at the 90% CL. The possibility exists for even higher statistics searches in future experiments. Alternatively, it may be possible to detect mixing via the lifetime difference between the two physical eigenstates by comparing the measured lifetimes for different CP final states [22]. Of course, this approach will only detect mixing if it is associated with a substantial lifetime difference as opposed to mixing that only results from a mass difference. We are investigating this method as well. Finally, the cleanest signal for mixing might be found at a  $\tau$ -charm factory which produces  $D^0 - \bar{D}^0$  pairs on resonance. As has been discussed previously [23], certain hadronic final states from these  $D^0 - \bar{D}^0$  pairs can only be produced by mixing and not by DCS amplitudes. We remain hopeful that one of these techniques may be used to detect  $D^0 - \bar{D}^0$  mixing, and thus provide information about the existence of new physics.

## IX. SUMMARY

We have searched for evidence of  $D^0 - \bar{D}^0$  mixing and DCS decays by looking for wrong-sign decays in the decay chain  $D^* \rightarrow \pi D$  with  $D \rightarrow K\pi$  or  $D \rightarrow K\pi\pi\pi$ . Our results are summarized in Table XI.

We have seen no evidence for mixing in either  $D^0$  decay mode. The full results of a maximum likelihood fit to the data are given in Table II. The possibility of additional sources of wrong-sign decays from DCS amplitudes limits our sensitivity for detecting mixing alone. Using the criterion  $\Delta \ln \mathcal{L} = 0.82$  [24], we calculate the one-sided, 90% C.L. upper limits to be  $r_{mix}(\bar{D}^0 \rightarrow D^0) < 0.74\%$  and  $r_{mix}(D^0 \rightarrow \bar{D}^0) < 1.45\%$ . If we assume no CP violation in either the mixing or DCS terms of the fit (as in Table V), but allow for CP violation in  $r_{int}$ , we calculate the upper limit for the common value of  $r_{mix}$  to be 0.85%.

Our quoted sensitivity to mixing is similar to that of E691, but our analysis is no-

tably more general in its assumptions concerning DCS-mixing interference and CP violation. Assuming no DCS-mixing interference constrains the mixing and DCS contributions much more severely, but we do not feel this assumption is justifiable in general. Nevertheless, for comparison we note that our limit in this case is  $r_{mix} < 0.33\%$ . All our results for the  $K\pi$  final state are also consistent with the CLEO measurement of  $r_{ws}(K\pi) = (0.77 \pm 0.25 \pm 0.25)\%$  [6] for wrong-sign decays. In particular, if the mixing amplitude is zero, we find  $r_{dcs}(K\pi) = (0.68^{+0.34}_{-0.33} \pm 0.07)\%$  and  $r_{dcs}(K\pi\pi\pi) = (0.25^{+0.36}_{-0.34} \pm 0.03)\%$ .

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the same for all three time distributions of Eq. (9). Of course, the efficiency may more correctly be described as a function of the true decay time instead and should therefore be included inside the integrals of Eq. (9). Since the smearing is small, we do not do so and discuss the associated systematic error in Section VII below.

- [19] Instead of constraining  $r_{mix}$  to be the same for both  $K\pi$  and  $K\pi\pi\pi$  modes, we could of course fit the modes separately and combine the results to obtain an estimate of the common value of  $r_{mix}$ . However, because of the strong correlations present, it is easier to let the fit perform the constraint than to propagate the full correlation matrices in a weighted mean of the separate modes. The results are essentially the same in either case.
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1. There is an ambiguity about what functions to choose for the a priori probability density functions of the fit parameters.
2. There is a practical difficulty in doing a numerical integration over a space of 41 parameters.

## TABLES

TABLE I. Correction factors for the difference in integrated reconstruction efficiencies associated with the decay time distributions of the mixing and interference terms. See Eq. (15) in the text.

$c_{mix}(K\pi)$	0.388
$c_{int}(K\pi)$	0.499
$c_{mix}(K\pi\pi\pi)$	0.359
$c_{int}(K\pi\pi\pi)$	0.473

TABLE II. Signal and background parameters for the most general fit as described in Section V.

	$D^0 \rightarrow K^- \pi^+$	$D^0 \rightarrow K^- \pi \pi \pi$	$\overline{D}^0 \rightarrow K^+ \pi^-$	$\overline{D}^0 \rightarrow K^+ \pi \pi \pi$
$A_{rs}$	$2269^{+51}_{-50}$	$1314^{+38}_{-37}$	$2967^{+58}_{-57}$	$1677^{+43}_{-42}$
$A_\pi$	$746^{+35}_{-34}$	$311^{+23}_{-22}$	$797^{+36}_{-36}$	$368^{+25}_{-24}$
$A_0$	$338^{+26}_{-24}$	$278^{+20}_{-19}$	$422^{+28}_{-27}$	$356^{+23}_{-22}$
	$\overline{D}^0 \rightarrow K^- \pi^+$	$\overline{D}^0 \rightarrow K^- \pi \pi \pi$	$D^0 \rightarrow K^+ \pi^-$	$D^0 \rightarrow K^+ \pi \pi \pi$
$r_{dcs}(\%)$	$0.80^{+1.46}_{-1.37} \pm 0.47$	$-0.67^{+1.44}_{-1.35} \pm 0.41$	$1.26^{+1.94}_{-1.79} \pm 0.49$	$0.33^{+1.91}_{-1.70} \pm 0.32$
$r_{mix}(\%)$	$0.18^{+0.43}_{-0.39} \pm 0.17$		$0.70^{+0.58}_{-0.53} \pm 0.18$	
$r_{int}(\%)$	$-0.11^{+1.08}_{-1.16} \pm 0.43$	$0.22^{+1.12}_{-1.18} \pm 0.41$	$-1.46^{+1.49}_{-1.59} \pm 0.47$	$-0.89^{+1.35}_{-1.46} \pm 0.35$
$A_\pi$	$735^{+33}_{-32}$	$323^{+21}_{-21}$	$752^{+33}_{-32}$	$315^{+21}_{-21}$
$A_0$	$244^{+25}_{-24}$	$214^{+17}_{-16}$	$332^{+28}_{-27}$	$238^{+18}_{-17}$
$A_\pm$	$59^{+15}_{-14}$		$49^{+15}_{-14}$	
	$K^- \pi^+$	$K^- \pi \pi \pi$	$K^+ \pi^-$	$K^+ \pi \pi \pi$
$A_1$	$-3.29^{+0.82}_{-0.80}$	$-2.86^{+0.77}_{-0.75}$	$-3.73^{+0.69}_{-0.67}$	$-2.65^{+0.71}_{-0.69}$

TABLE III. Mass parameters for the fit described in Section V. All values are in  $\text{MeV}/c^2$ . There are systematic uncertainties on these parameters that are bigger than the statistical errors shown here, but they have inconsequential effects on the parameters of Table II.

$m_{D^0}$	$1865.8^{+0.1}_{-0.1}$
$\sigma_{K\pi}$	$15.16^{+0.17}_{-0.17}$
$\sigma_{K\pi\pi\pi}$	$10.76^{+0.15}_{-0.15}$
$Q_{D^*}$	$5.92^{+0.01}_{-0.01}$
$\sigma_Q$	$0.76^{+0.01}_{-0.01}$



TABLE IV. Two matrices summarizing the correlation coefficients for the wrong-sign ratios from the fit of Table II. Only the lower halves of the symmetric matrices are shown. Correlations with the fit parameters that are not shown are negligible.

	$r_{dcs}(K^-\pi^+)$	$r_{int}(K^-\pi^+)$	$r_{mix}(K^-\pi^+ \& K^-\pi\pi\pi)$	$r_{dcs}(K^-\pi\pi\pi)$	$r_{int}(K^-\pi\pi\pi)$
$r_{dcs}(K^-\pi^+)$	1.00				
$r_{int}(K^-\pi^+)$	-0.39	1.00			
$r_{mix}(K^-\pi^+ \& K^-\pi\pi\pi)$	-0.45	-0.36	1.00		
$r_{dcs}(K^-\pi\pi\pi)$	-0.19	-0.19	0.48	1.00	
$r_{int}(K^-\pi\pi\pi)$	0.32	0.26	-0.73	-0.87	1.00

	$r_{dcs}(K^+\pi^-)$	$r_{int}(K^+\pi^-)$	$r_{mix}(K^+\pi^- \& K^+\pi\pi\pi)$	$r_{dcs}(K^+\pi\pi\pi)$	$r_{int}(K^+\pi\pi\pi)$
$r_{dcs}(K^+\pi^-)$	1.00				
$r_{int}(K^+\pi^-)$	-0.93	1.00			
$r_{mix}(K^+\pi^- \& K^+\pi\pi\pi)$	0.80	-0.94	1.00		
$r_{dcs}(K^+\pi\pi\pi)$	0.55	-0.63	0.67	1.00	
$r_{int}(K^+\pi\pi\pi)$	-0.72	0.84	-0.89	-0.90	1.00

TABLE V. Fit results for the wrong-sign ratios assuming no CP violation in either the DCS or mixing terms of the fit. Thus, we have used the constraints  $r_{dcs}(D^0 \rightarrow K^+\pi^-) = r_{dcs}(\bar{D}^0 \rightarrow K^-\pi^+)$ ,  $r_{dcs}(D^0 \rightarrow K^+\pi\pi\pi) = r_{dcs}(\bar{D}^0 \rightarrow K^-\pi\pi\pi)$ , and  $r_{mix}(D^0 \rightarrow \bar{D}^0) = r_{mix}(\bar{D}^0 \rightarrow D^0)$ .

	$D^0 \rightarrow K^+\pi^-$	$\bar{D}^0 \rightarrow K^-\pi^+$	$D^0 \rightarrow K^+\pi\pi\pi$	$\bar{D}^0 \rightarrow K^-\pi\pi\pi$
$r_{dcs}(\%)$	$0.90^{+1.20}_{-1.09} \pm 0.44$		$-0.20^{+1.17}_{-1.06} \pm 0.35$	
$r_{mix}(\%)$		$0.39^{+0.36}_{-0.32} \pm 0.16$		
$r_{int}(\%)$	$-0.84^{+0.92}_{-1.00} \pm 0.43$	$-0.46^{+0.89}_{-0.97} \pm 0.41$	$-0.25^{+0.87}_{-0.94} \pm 0.36$	$-0.29^{+0.89}_{-0.96} \pm 0.37$

TABLE VI. Fit results for the wrong-sign ratios assuming no CP violation and no interference between DCS and mixing amplitudes. We feel the latter assumption is debatable, but refer to the discussion in references [7,8,20].

	$D^0 \rightarrow K^+ \pi^-$	$\overline{D}^0 \rightarrow K^- \pi^+$	$D^0 \rightarrow K^+ \pi \pi \pi$	$\overline{D}^0 \rightarrow K^- \pi \pi \pi$
$r_{dcs}(\%)$	$0.17^{+0.39}_{-0.38} \pm 0.08$		$-0.32^{+0.42}_{-0.39} \pm 0.06$	
$r_{mix}(\%)$	$0.21^{+0.09}_{-0.09} \pm 0.02$			

TABLE VII. Systematic uncertainties for representative parameters of the general fit of Table II. Entries are explained in the text.

	$r_{dcs}(K^- \pi) (\%)$	$r_{dcs}(K^- \pi \pi \pi) (\%)$	$r_{mix}(\overline{D}^0 \rightarrow D^0) (\%)$
fit value	$0.80^{+1.46}_{-1.37}$	$-0.67^{+1.44}_{-1.35}$	$0.18^{+0.43}_{-0.39}$
$K^+ K^-$ , $\pi^+ \pi^-$ reflections	$\pm 0.02$	$\pm 0.02$	$\pm 0.01$
statistics of effy & bkgd distrs	$\pm 0.35$	$\pm 0.24$	$\pm 0.12$
binning of effy & bkgd distrs	$\pm 0.15$	$\pm 0.18$	$\pm 0.08$
time resolution	$\pm 0.19$	$\pm 0.26$	$\pm 0.07$
mass resolution	$\pm 0.21$	$\pm 0.11$	$\pm 0.04$
total systematic uncertainty	$\pm 0.47$	$\pm 0.41$	$\pm 0.17$

TABLE VIII. Systematic uncertainties for representative parameters in the fit of Table V. This fit describes the most likely case in the Standard model where we have allowed for CP violation only in the interference term. Entries are explained in the text.

	$r_{dcs}(K^-\pi)$ (%)	$r_{dcs}(K^-\pi\pi\pi)$ (%)	$r_{mix}(\overline{D}^0 \rightarrow D^0)$ (%)
fit value	$0.80^{+1.46}_{-1.37}$	$-0.67^{+1.44}_{-1.35}$	$0.18^{+0.43}_{-0.39}$
$K^+K^-$ , $\pi^+\pi^-$ reflections	$\pm 0.02$	$\pm 0.02$	$\pm 0.01$
statistics of effy & bkgd distrs	$\pm 0.35$	$\pm 0.24$	$\pm 0.12$
binning of effy & bkgd distrs	$\pm 0.15$	$\pm 0.18$	$\pm 0.08$
time resolution	$\pm 0.19$	$\pm 0.26$	$\pm 0.07$
mass resolution	$\pm 0.21$	$\pm 0.11$	$\pm 0.04$
total systematic uncertainty	$\pm 0.47$	$\pm 0.41$	$\pm 0.17$

TABLE IX. Systematic uncertainties in the no-CP-violation, no-interference fit of Table VI. Entries are explained in the text.

	$r_{dcs}(K\pi)$ (%)	$r_{dcs}(K\pi\pi\pi)$ (%)	$r_{mix}$ (%)
fit value	$0.17^{+0.39}_{-0.38}$	$-0.32^{+0.42}_{-0.39}$	$0.21^{+0.09}_{-0.09}$
$K^+K^-$ , $\pi^+\pi^-$ reflections	$\pm 0.04$	$\pm 0.00$	$\pm 0.00$
statistics of effy & bkgd distrs	$\pm 0.03$	$\pm 0.04$	$\pm 0.01$
binning of effy & bkgd distrs	$\pm 0.04$	$\pm 0.04$	$\pm 0.01$
time resolution	$\pm 0.00$	$\pm 0.00$	$\pm 0.00$
mass resolution	$\pm 0.04$	$\pm 0.02$	$\pm 0.00$
total systematic uncertainty	$\pm 0.08$	$\pm 0.06$	$\pm 0.02$

TABLE X. Systematic uncertainties in the no-mixing fit. Entries are explained in the text.

	$r_{dcs}(K\pi)$ (%)	$r_{dcs}(K\pi\pi\pi)$ (%)
fit value	$0.68^{+0.34}_{-0.33}$	$0.25^{+0.36}_{-0.34}$
$K^+K^-$ , $\pi^+\pi^-$ reflections	$\pm 0.04$	$\pm 0.00$
statistics of effy & bkgd distrs	$\pm 0.02$	$\pm 0.02$
binning of effy & bkgd distrs	$\pm 0.01$	$\pm 0.00$
time resolution	$\pm 0.01$	$\pm 0.01$
mass resolution	$\pm 0.04$	$\pm 0.02$
total systematic uncertainty	$\pm 0.07$	$\pm 0.03$

TABLE XI. A summary of values from our four fits. Results from other experiments are also listed for comparison.

Fit type	This Result	Other Comparable Result
Most general, no CP assumptions	$r_{mix}(\overline{D}^0 \rightarrow D^0) = (0.18^{+0.43}_{-0.39} \pm 0.17)\%$ $r_{mix}(D^0 \rightarrow \overline{D}^0) = (0.70^{+0.58}_{-0.53} \pm 0.18)\%$	
CP violation only in interference term	$r_{mix} = (0.39^{+0.36}_{-0.32} \pm 0.16)\%$	$r_{mix} = (0.11^{+0.30}_{-0.27})\%$ (E791 [5]) (Semileptonic decays)
No CP violation, no interference	$r_{mix} = (0.21^{+0.09}_{-0.09} \pm 0.02)\%$	$r_{mix} = (0.05 \pm 0.20)\%$ (E691 [3])
No mixing	$r_{dcs}(K\pi) = (0.68^{+0.34}_{-0.33} \pm 0.07)\%$ $r_{dcs}(K\pi\pi\pi) = (0.25^{+0.36}_{-0.34} \pm 0.03)\%$	$r_{ws}(K\pi) = (0.77 \pm 0.25 \pm 0.25)\%$ (CLEO [6])

# FIGURES

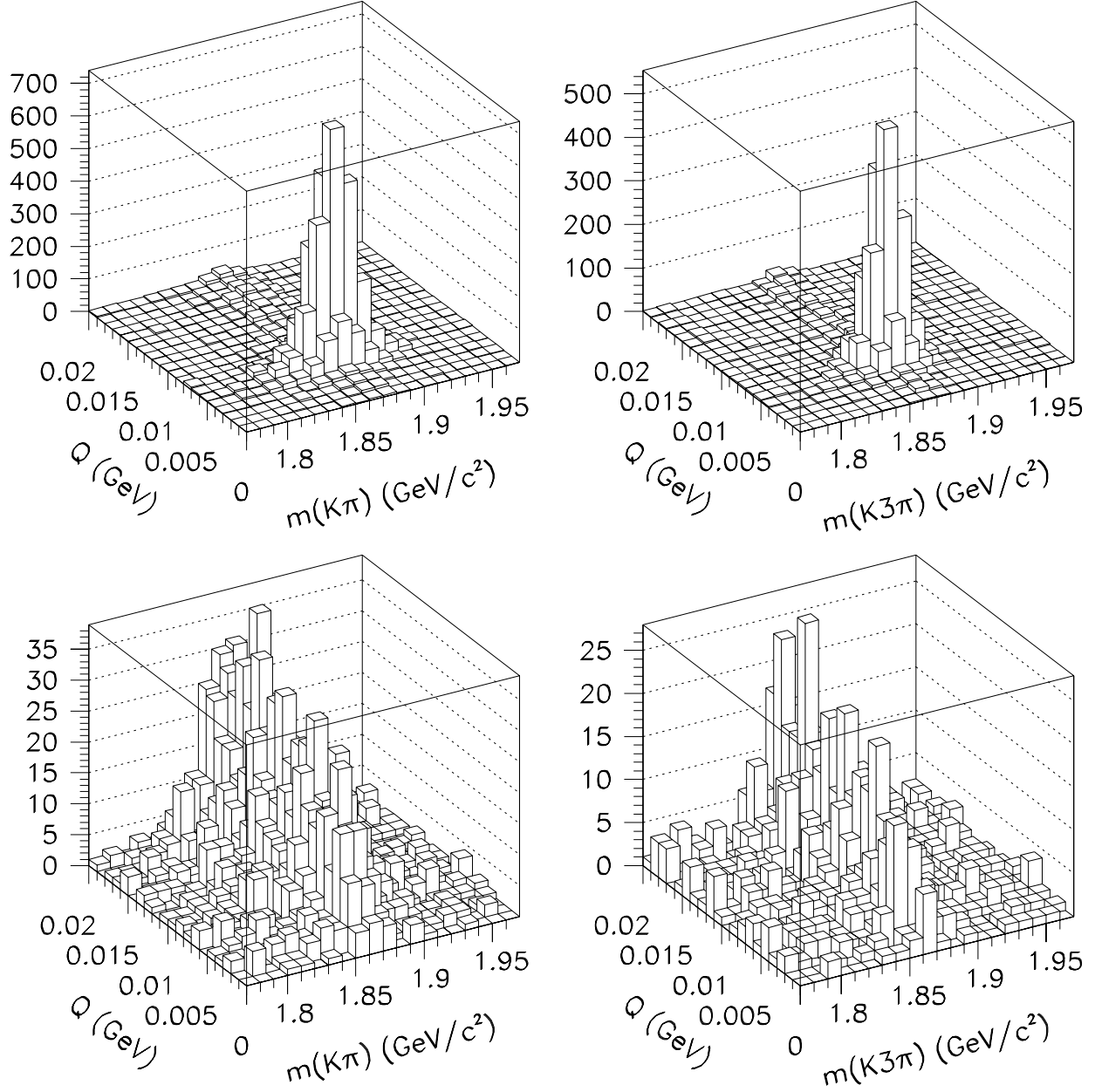


FIG. 1. Plots of  $Q$  (defined in the text) versus the candidate  $D$  mass for right-sign  $D \rightarrow K\pi$  (top-left), right-sign  $D \rightarrow K\pi\pi\pi$  (top-right), wrong-sign  $D \rightarrow K\pi$  (bottom-left), and wrong-sign  $D \rightarrow K\pi\pi\pi$  (bottom-right). Clean signals are apparent in both right-sign plots. In all four plots, the bands of events at  $m(K\pi)$ ,  $m(K\pi\pi\pi) \approx 1.87 \text{ GeV}/c^2$  are due to real  $D$  decays combining with random pions to give false  $D^*$  candidates.

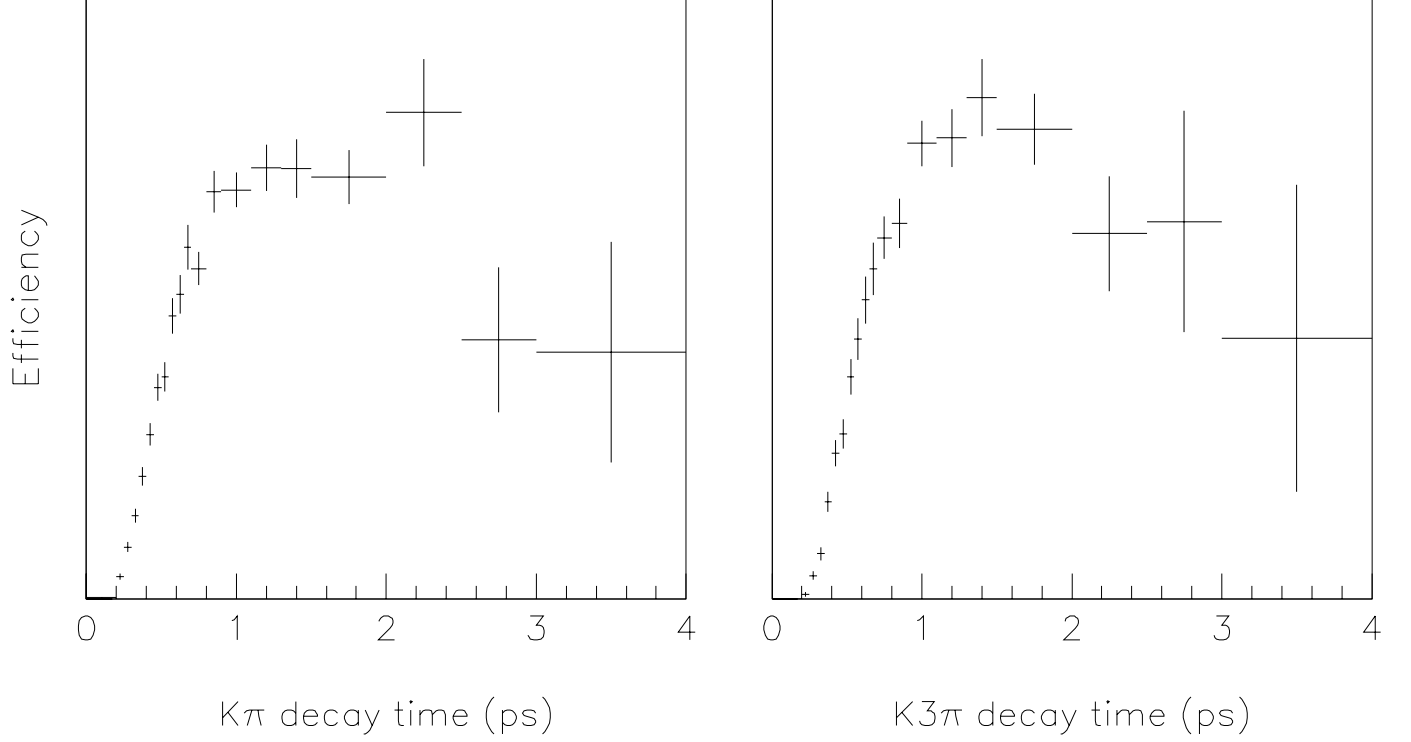


FIG. 2. Reconstruction efficiencies for  $D^0 \rightarrow K\pi$  (left) and  $D^0 \rightarrow K\pi\pi\pi$  (right), as measured from the right-sign data samples. The low efficiency at short decay times is typical of fixed-target experiments which identify charm decays by a secondary decay vertex. The drop in efficiency at long decay times is due to our selection criteria which remove decays occurring in downstream target foils. Efficiencies for charge conjugate final states (e.g.,  $\epsilon_{K-\pi^+}$  and  $\epsilon_{K+\pi^-}$ ) are observed to be the same within errors, and have been combined in the above plots. The vertical scales are arbitrary.

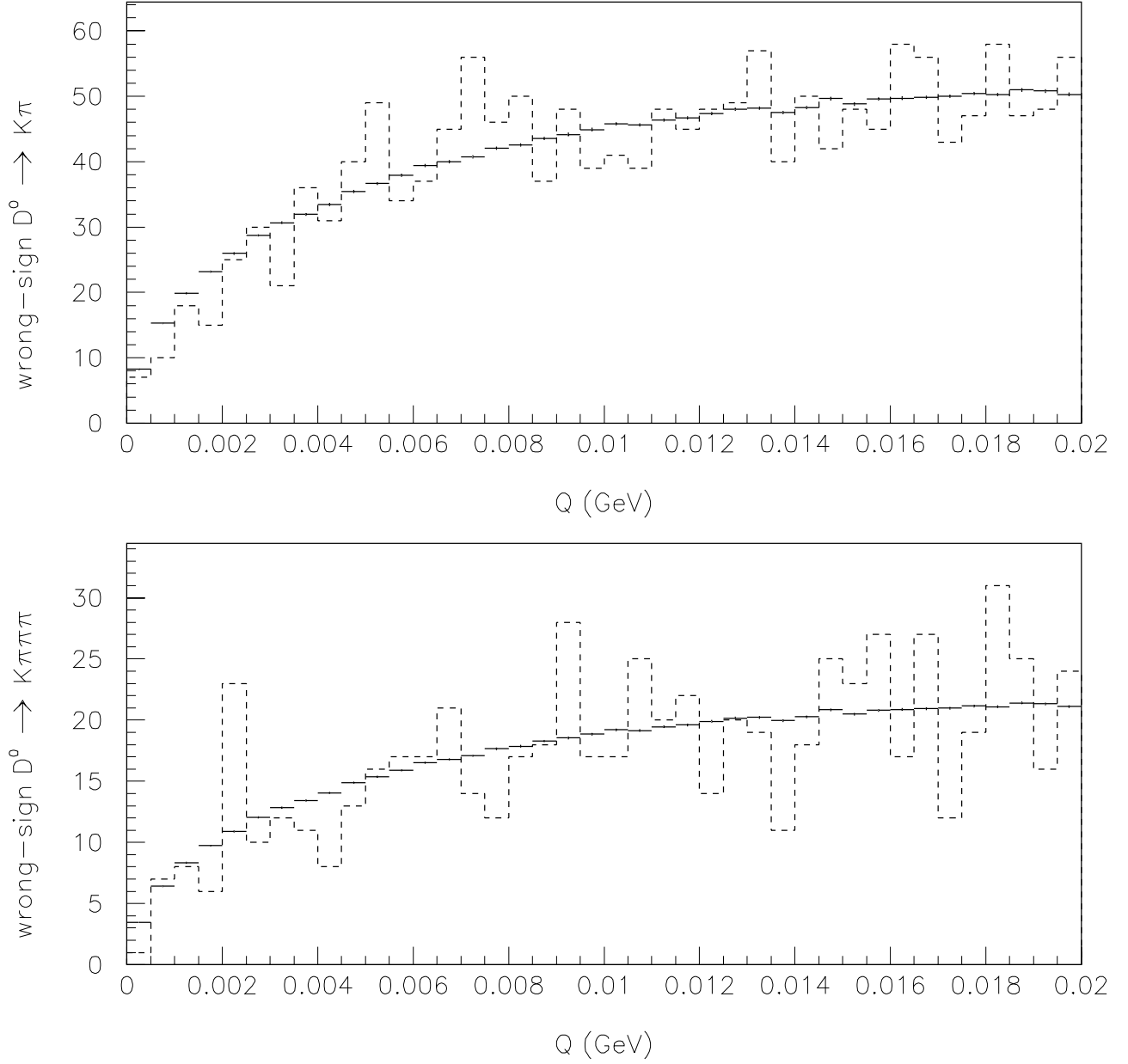


FIG. 3. Histograms show distributions of  $Q \equiv m(K\pi\pi) - m(K\pi) - m(\pi)$  (top) and  $Q \equiv m(K\pi\pi\pi\pi) - m(K\pi\pi\pi) - m(\pi)$  (bottom) for wrong-sign  $D$  candidates in the mass range 1.835 to 1.895  $\text{GeV}/c^2$ . The points with error bars show the distributions from combining  $D^0$  candidates and  $\pi$ 's from separate events, normalized to the histograms. These distributions are used to represent  $R(Q)$  in the likelihood fits.

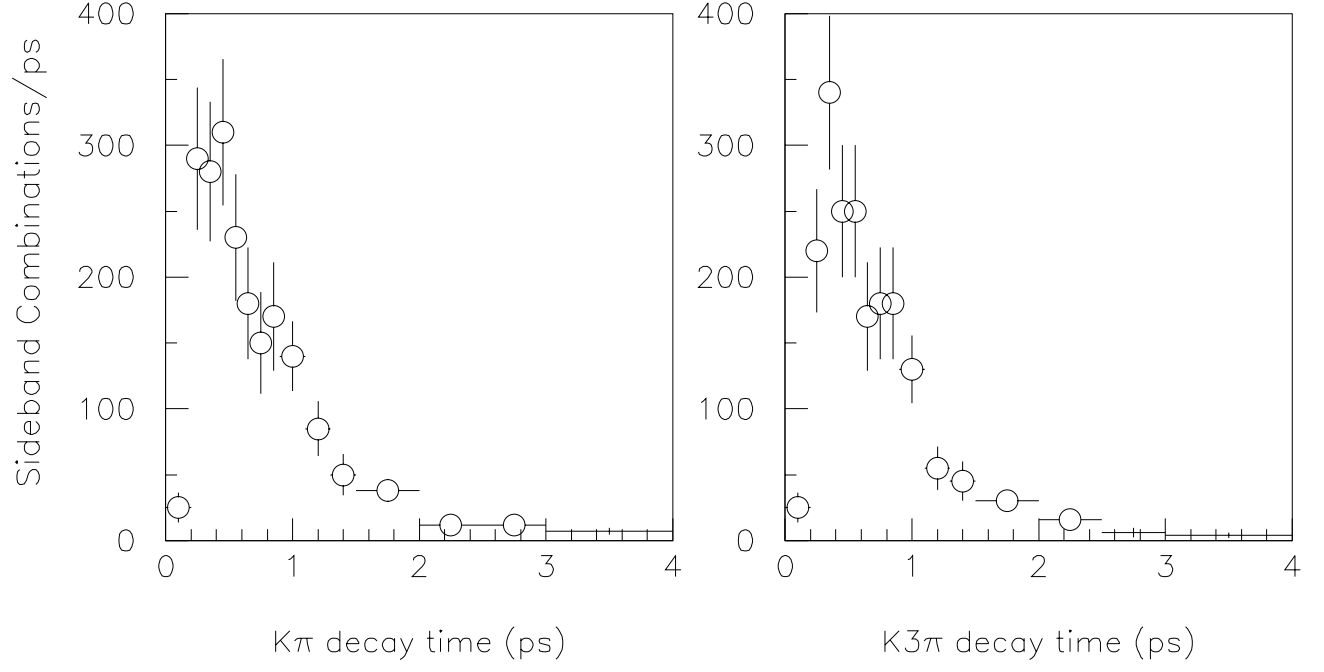


FIG. 4. The measured decay time distribution for  $D^0 \rightarrow K\pi$  candidates (left) and  $D^0 \rightarrow K\pi\pi\pi$  candidates (right), taken from the sidebands  $1.77 < m_D < 1.81 \text{ GeV}/c^2$  and  $1.93 < m_D < 1.97 \text{ GeV}/c^2$ . These distributions are used to represent  $B_{false}(t)$  in the likelihood fit.



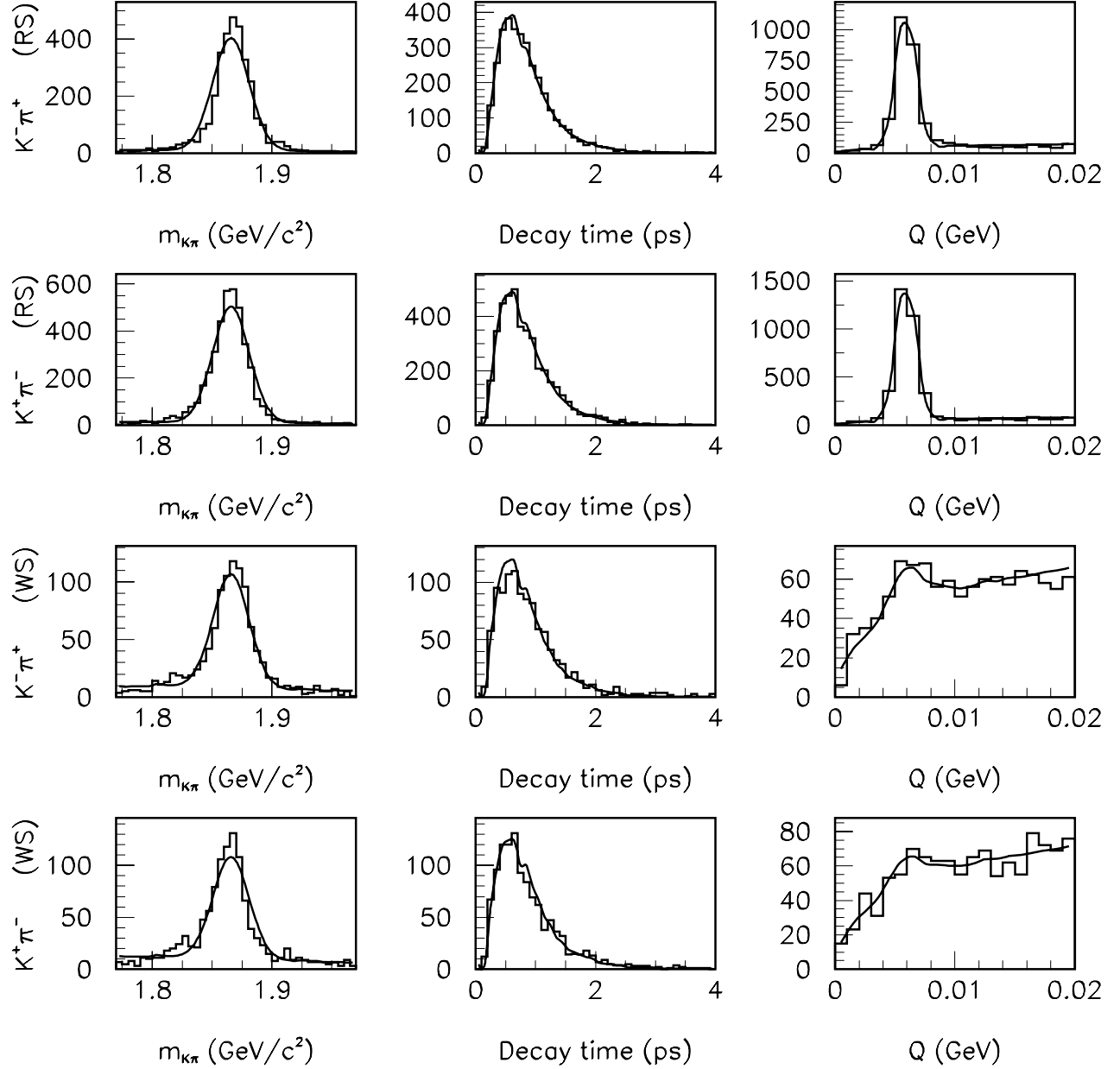


FIG. 5. Projections of the four  $D \rightarrow K\pi$  data samples onto each of three distributions  $m_{K\pi}$ ,  $t$  and  $Q$ . Data are from the  $D^0$  mass range ( $1.770 < m(K\pi) < 1.970$ )  $\text{GeV}/c^2$ . Solid curves show the projections of the most general global fit whose results are summarized in Table II. The broad component of the peaks in the WS  $Q$  plots is described well by reflections of  $K^+K^-$  and  $\pi^+\pi^-$  signals.

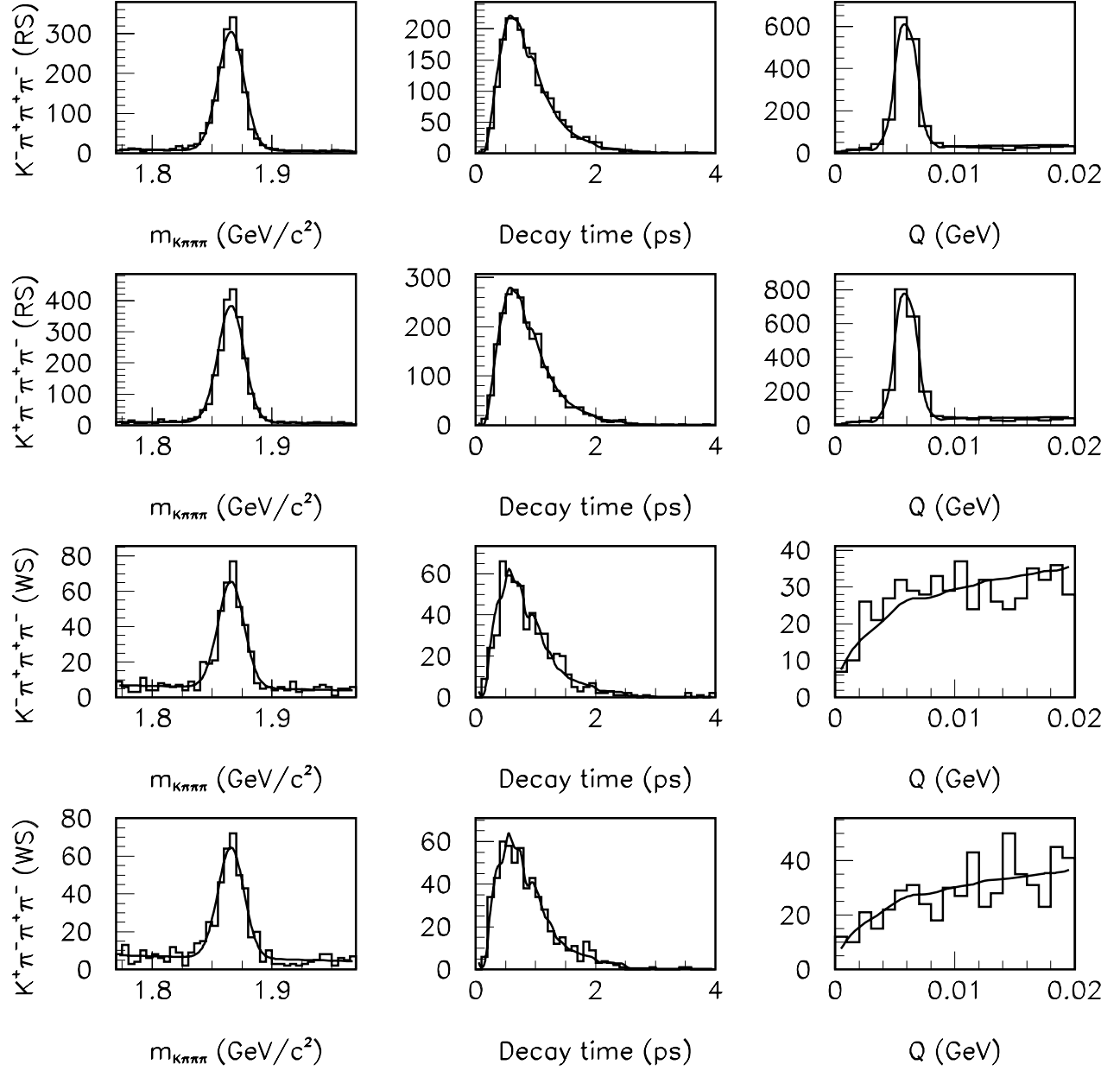


FIG. 6. Projections of the four  $D \rightarrow K\pi\pi\pi$  data samples onto each of three distributions  $m_{K\pi\pi\pi}$ ,  $t$ , and  $Q$ . Data are from the  $D^0$  mass range ( $1.770 < m(K\pi\pi\pi) < 1.970$ )  $\text{GeV}/c^2$ . Solid curves show the projections of the most general global fit whose results are summarized in Table II.

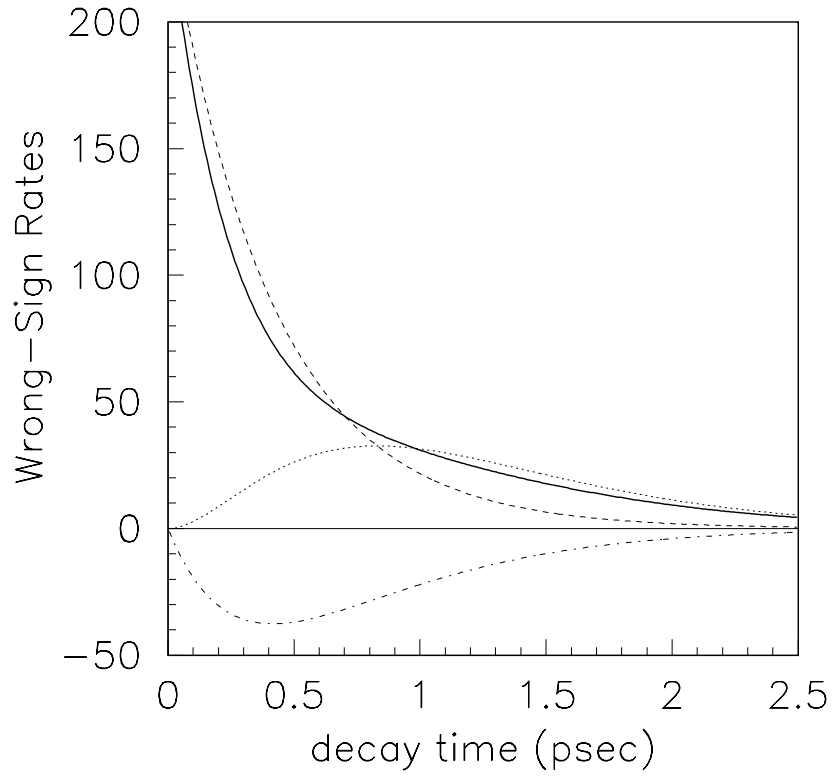


FIG. 7. A hypothetical plot of the time dependence of wrong-sign decays taken from [7]. The dashed line represents the DCS contribution. The dotted line shows the contribution due to mixing. The dash-dot line shows the contribution from destructive interference of DCS and mixing amplitudes when the interference is 30% of its maximum. The solid line is the sum of all three contributions. The vertical scale is arbitrary.

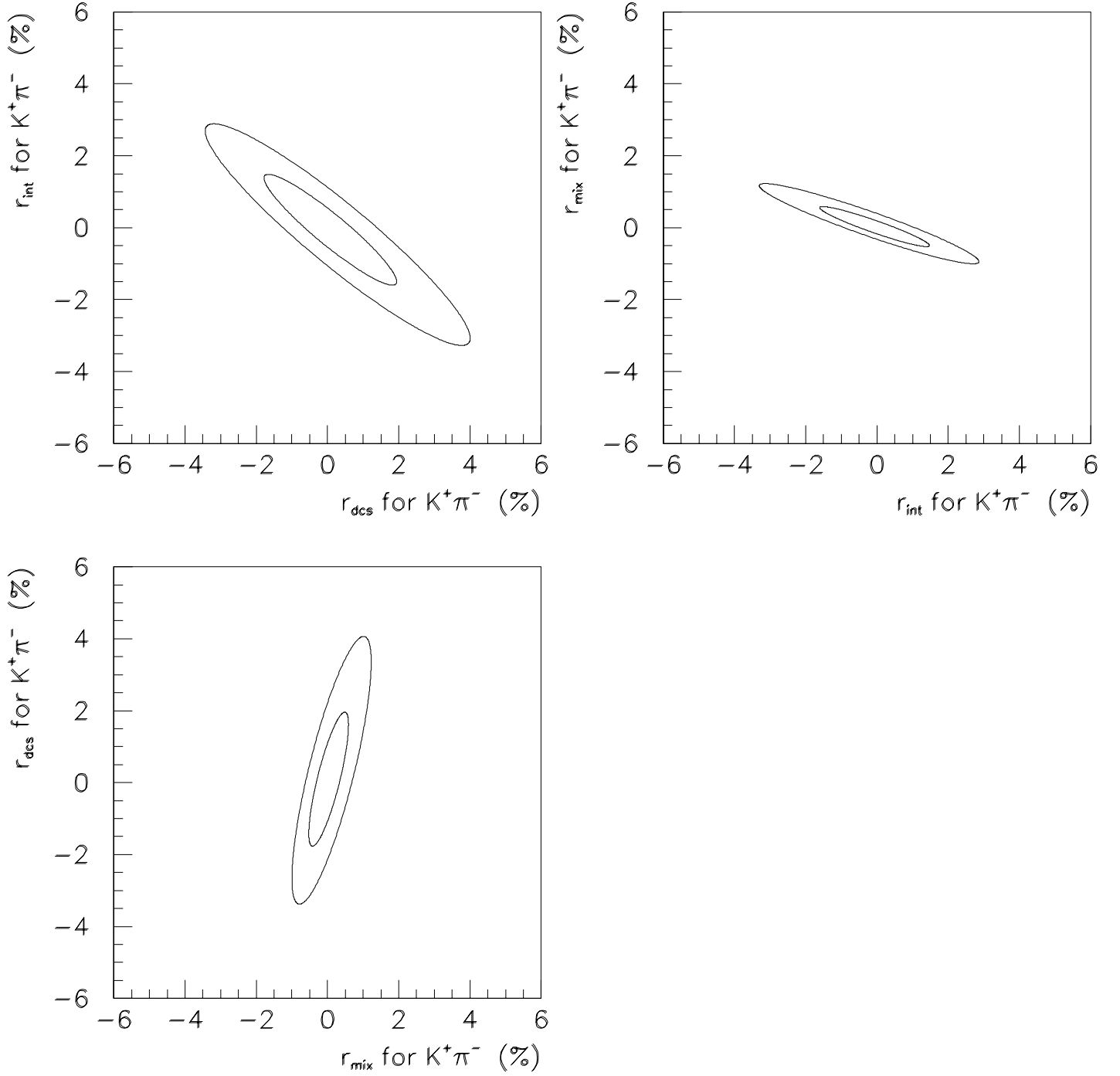


FIG. 8. Likelihood contours corresponding to  $\Delta \ln \mathcal{L} = 0.5$  and 2.0 for the fit of Table II, illustrating the correlations among the three parameters  $r_{\text{dcs}}(K^+\pi^-)$ ,  $r_{\text{mix}}(K^+\pi^-)$  and  $r_{\text{int}}(K^+\pi^-)$ . Strong correlations among these parameters are apparent. The correlations among other wrong-sign ratios are similar.