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ABSTRACT

On-going projects to discover Type Ia supernovae at redshifts $z \sim 0.3 - 1$, coupled with improved techniques to narrow the dispersion in SN Ia peak magnitudes, have renewed the prospects for determining the cosmic deceleration parameter q_0. We estimate the expected uncertainty in the Hubble diagram determination of q_0 due to weak lensing by structure in the universe, which stochastically shifts the apparent brightness of distant standard candles. Although the results are sensitive to the density power spectrum on small scales, the induced flux dispersion $\sigma_m \lesssim 0.04\Omega_{m,0}^{1/2}$ mag for sources at $z \leq 0.5$, well below the "intrinsic" spread of nearby SN Ia magnitudes, $\sigma_M \simeq 0.2$ mag. Thus, density inhomogeneities do not significantly impact the current program to measure q_0, in contrast to a recent claim. If, however, light-curve shape and other calibrators can reduce the effective intrinsic spread to 0.1 mag at high z, then weak lensing could increase the observed spread by 30 % in an $\Omega_{m,0} = 1$ universe for SNe at $z \gtrsim 1$.

Subject headings: cosmology: large-scale structure of the universe, supernovae

1. Introduction

The determination of the cosmological parameters via the "classical" cosmological tests, such as the redshift-magnitude relation (Hubble diagram), remains a holy grail for observational astronomy. Past attempts to measure the deceleration parameter q_0 using galaxies as standard candles have foundered on the uncertainty in galaxy luminosity evolution (Tinsley

1972, Peebles 1993). Recently, there has been renewed hope that q_0 can be determined, thereby constraining the mean mass density and the cosmological constant, by using distant Type Ia supernovae (e.g., Goobar & Perlmutter 1995).

Progress has come on two fronts. First, there is growing evidence that samples of SNe Ia, when suitably culled to excise peculiar lightcurves or spectra and cases of significant host-galaxy extinction, may provide reasonably good standard candles, with a dispersion $\sigma_M \simeq 0.3$ in peak absolute magnitude. Moreover, using the observed correlation between lightcurve shape and peak luminosity (Hamuy etal. 1995, Riess, Press, & Kirshner 1995) as well as spectroscopic features that have been found to correlate with luminosity (Branch, Nugent, & Fisher 1996, Nugent, etal. 1995), the effective dispersion of these 'standardized candles' can apparently be reduced to $\sigma_M \simeq 0.1 - 0.2$ mag. Second, several groups have begun observing campaigns to discover a large number of high-redshift Ia supernovae, with coordinated follow-up programs to measure the lightcurves of SN candidates; more than 20 candidates at $z \sim 0.4 - 0.6$ have been found so far (e.g., Perlmutter, etal. 1995, Schmidt, etal. 1995, Perlmutter, etal. 1996).

The case for SNe Ia as a population of 'standard' candles is also on reasonable physical footing. While the details of the explosion mechanism remain poorly understood, there is general consensus that SNe Ia are thermonuclear explosions of accreting Carbon-Oxygen white dwarfs. On the other hand, it is not yet clear whether the white dwarf progenitors must be near the Chandrasekhar mass (e.g., Hoeflich etal. 1996) or not (Livne & Arnett 1995), and thus the theoretical intrinsic spread in SN Ia luminosity is still a matter of some debate. In either case, there is optimism that, based on the correlations above, SNe Ia can be used to measure extragalactic distances and thereby to determine the cosmological parameters.

Recently, however, Kantowski, Vaughan, & Branch (1995) have argued that large-scale structure provides a stumbling block to the accurate determination of q_0 from standard candles. A bundle of light rays from a distant source is sheared and focused due to deflection by intervening density inhomogeneities. Since gravitational lensing conserves surface brightness, the change in the cross-section of the bundle implies that the image of the source appears magnified (or demagnified) relative to a homogeneous universe. Using the "swiss-cheese" model of large-scale structure, Kantowski etal. find that the resulting change in received flux (apparent magnitude) could lead to a systematic underestimate of q_0 if one interprets the observations assuming a homogeneous universe. For example, for a source at $z = 0.458$ (the redshift of SN 1992bi (Perlmutter, etal. 1995)), they find that the resulting error in q_0 can be as large as $\delta q_0 \simeq -0.33 q_0$. This claim is consonant with other studies which found that large-scale structure can significantly change the proportionality between angular-size distance and redshift, and therefore lead to difficulties, e.g., for the determination of H_0 from gravitational lens time delays (Kantowski 1969, Dyer & Roeder 1972, Alcock & Anderson 1985, 1986, Watanabe etal. 1992, Sasaki 1993).

In this Comment, we reevaluate this issue by estimating the rms fluctuation in the amplification of distant sources in a perturbed Friedmann-Robertson-Walker (FRW) universe. We find that the expected effects are smaller than those found by Kantowski, etal., of order several percent at most for sources at $z \lesssim 0.5$, and subdominant in comparison to the ~ 20 % intrinsic spread of nearby SN Ia magnitudes. Moreover, flux conservation implies that the magnification shift is random, with zero mean over all lines of sight (Weinberg 1976), not a systematic offset. As a result, the amplification due to large-scale structure will not seriously impact the accuracy of q_0 measurements in current surveys.

The primary reason for this different conclusion is that the swiss-cheese model does not conform to our current understanding of the large-scale mass distribution of the universe. In particular, it does not accurately reflect the observational information gained from galaxy redshift and peculiar velocity surveys and the cosmic microwave background (CMB) anisotropy. This point has been made recently by Seljak (1994) and Bar-kana (1995) in the context of the determination of H_0 from QSO lens time delays. The argument that these effects should be small has also been made qualititatively by Peebles (1993). The issue was first laid out clearly by Gunn (1967), who showed that the rms fluctuation in the apparent brightness of a distant source can be expressed as a radial integral of the two-point correlation function of the mass distribution. Gunn's argument was updated to reflect the substantial advances in our understanding of large-scale structure in more recent numerical (Jaroszynski, etal. 1990) and analytic (Babul & Lee 1991) studies (unbeknownst to the present author until this work was completed). In fact, Babul & Lee's 'DP' model is close to the model for the power spectrum that we adopt below, although there are some important quantitative differences which we outline later.

From the smallness of the CMB anisotropy on large scales to observations of galaxies and galaxy clusters on small scales, we have strong indications that the spacetime metric of the universe is well-described by a weakly perturbed FRW model. In the longitudinal gauge, the line element can be written

$$ds^2 = a^2(\tau)\left[-(1+2\phi)d\tau^2 + (1-2\phi)[d\chi^2 + F^2(\chi)(d\theta^2 + \sin^2\theta d\phi^2)]\right] \qquad (1)$$

where $a(\tau)$ is the cosmic scale factor, χ denotes the comoving radial coordinate, and $\tau = \int dt/a$ is the conformal time, with τ_0 denoting the present. The function $F(\chi)$ depends on the spatial curvature K: $F(\chi) = K^{-1/2}\sin K^{1/2}\chi$ for $K > 0$, $F = \chi$ for $K = 0$, and

$F = (-K)^{-1/2}\sinh(-K)^{1/2}\chi$ for $K < 0$. The curvature can be expressed in terms of the present density parameter and the Hubble parameter $H_0 = H(\tau_0)$, $K = (\Omega_0 - 1)H_0^2 a_0^2$, where $\Omega = \Omega_m + \Omega_\Lambda$ includes both non-relativistic matter (m) and vacuum energy density (the cosmological constant Λ) and $\Omega_m = \bar{\rho}/\rho_c = 8\pi G\bar{\rho}/3H^2$, with $\bar{\rho}(\tau)$ the mean density of matter. The metric perturbation variable ϕ is the relativistic analog of the Newtonian gravitational potential; over scales less than the Hubble length H^{-1} it obeys the Poisson equation,

$$\nabla^2\phi = \frac{3}{2}\Omega_m H^2 a^2 \delta \tag{2}$$

where the density contrast $\delta(\mathbf{x},\tau) \equiv \rho(\mathbf{x},\tau)/\bar{\rho} - 1$. Observationally, the fluctuations in spatial geometry correspond to $\phi \lesssim 10^{-4} - 10^{-5}$ from galaxy scales to the Hubble radius. We will assume $\phi \ll 1$ but do not place any restrictions on the amplitude of δ.

For a photon with direction $\hat{\mathbf{n}}$, the null geodesic equation in the metric (1) determines the rate of change in propagation direction due to inhomogeneities along the light path; to lowest order in ϕ, the geodesic can be parametrized by the radial coordinate χ, yielding $d\hat{\mathbf{n}}/d\chi = -2\nabla_\perp\phi$, where the derivative is transverse to the line of sight; we are implicitly assuming small deflection angles, so that the two-dimensional sphere can be approximated by a plane perpendicular to the unperturbed line of sight. Directions in this plane can be parameterized by the two-dimensional angle $\boldsymbol{\theta}$. A photon observed at direction $\boldsymbol{\theta}$ would have been seen at direction $\boldsymbol{\beta} = \boldsymbol{\theta} + \boldsymbol{\delta\theta}$ in the absence of weak lensing effects, i.e., $\boldsymbol{\delta\theta}$ is the net transverse deflection of the light rays due to density fluctuations. From the geodesic equation, for a source at comoving distance χ_s we have (e.g., Kaiser 1992, Pyne & Birkinshaw 1996, Villumsen 1995, Seljak 1996), defining $\boldsymbol{\chi} = -\hat{\mathbf{n}}\chi$,

$$\boldsymbol{\delta\theta} = -2\int_0^{\chi_s} \frac{F(\chi_s - \chi)}{F(\chi_s)} \boldsymbol{\nabla}_\perp \phi(\boldsymbol{\chi}, \tau = \tau_0 - \chi) d\chi \; . \tag{3}$$

Here we have implicitly replaced the perturbed by the unperturbed path in the integral; while this is not a good approximation (e.g., Frieman, Harari, & Surpi 1994), we will only make use of the weaker assumption that the statistical properties of the potential field ϕ are identical along the perturbed and unperturbed paths (e.g., Kaiser 1992).

The amplification of the observed image relative to the (unlensed) source is given by $A = 1/\det\, M$, where the 2×2 amplification matrix $M_{ij} = \partial\beta_i/\partial\theta_j = \delta_{ij} + \Phi_{ij}$, and $\Phi_{ij} = \partial\delta\theta_i/\partial\theta_j$. We can decompose the deformation tensor as $\Phi_{ij} = -\kappa\delta_{ij} + \gamma_{ij}$, where the trace ($\kappa$), the expansion, describes the uniform dilation or contraction of ray bundles, and the traceless part (γ_{ij}) describes their shear. In the limit of small deflection angles, we thus have

$A = 1/[(1-\kappa)^2 - \gamma^2] \simeq 1 + 2\kappa$, where $\pm\gamma$ are the eigenvalues of γ_{ij} and $\kappa = -(\Phi_{11} + \Phi_{22})/2$. The flux perturbation is therefore $\delta A = A - 1 = -\mathrm{Tr}\Phi$. Using eqns. (2, 3), we find

$$\delta A = \frac{3(H_0 a_0)^2 \Omega_{m,0}}{F(\chi_s)} \int_0^{\chi_s} d\chi F(\chi) F(\chi_s - \chi) \frac{a_0}{a(\tau_0 - \chi)} \delta(\boldsymbol{\chi}, \tau = \tau_0 - \chi) \tag{4}$$

where $\Omega_{m,0}$ is the present matter density parameter.

To find the rms flux amplification along a random line of sight, we assume $\delta(\mathbf{x}, \tau)$ can be described as a continuous, homogeneous random process; this ignores discreteness in the mass density, an excellent approximation since the dark matter very likely consists of objects of mass less than $10^6 M_\odot$. It is convenient to Fourier-transform the density field, $\delta(\boldsymbol{\chi}) = (2\pi)^{-3} \int d^3k\ \delta(\mathbf{k}) \exp(i\mathbf{k} \cdot \boldsymbol{\chi})$, where the flat-space transform suffices for the small angles we are considering. Defining the density power spectrum via $\langle \delta(\mathbf{k}) \delta^*(\mathbf{k}') \rangle = (2\pi)^3 P(\mathbf{k}) \delta_D^3(\mathbf{k} - \mathbf{k}')$, we have (defining $\sigma_A^2 = \langle (\delta A)^2 \rangle$)

$$\sigma_A^2 = 9\pi \Omega_{m,0}^2 (H_0 a_0)^4 \int_0^{\chi_s} d\chi F^2(\chi) \frac{F^2(\chi_s - \chi)}{F^2(\chi_s)} (1 + z(\tau))^2 \int \frac{\Delta^2(k, z)}{k} \frac{dk}{k} \tag{5}$$

where $\Delta^2(k, z) = k^3 P(k, z)/2\pi^2 = d\sigma_\delta^2/d\ln k$ is the contribution per logarithmic wavenumber interval to the variance of the density field. Here, $z(\tau)$ is the redshift at epoch $\tau = \tau_0 - \chi$, given implicitly by

$$\chi(z) = \frac{1}{H_0 a_0} \int_0^z \frac{dz'}{[\Omega_{m,0}(1 + z')^3 + (1 - \Omega_{m,0} - \Omega_\Lambda)(1 + z')^2 + \Omega_\Lambda]^{1/2}} \ . \tag{6}$$

For $\Omega_\Lambda = 0$, this has a well-known analytic solution for $z(\chi)$ (Mattig 1958). In (5), we have used a small-angle approximation, so that only waves nearly perpendicular to the line of sight contribute to the amplification (Blandford, etal. 1991, Kaiser 1992); in addition, we have assumed that the density correlation length is small compared to the Hubble radius, so we only include contributions to $\langle \delta(\mathbf{k}, \tau) \delta^*(\mathbf{k}', \tau') \rangle$ from equal times, $\tau = \tau'$.

Over a limited range of wavenumber k and redshift z, the power spectrum scales with time as $\Delta^2(k, z) = \Delta^2(k)(1 + z)^{-\epsilon}$, separating the integrals in (5). At small k, $\Delta \ll 1$, and it obeys linear perturbation theory, $\epsilon = 2$. At large k, where $\Delta \gg 1$, the clustering is highly non-linear and has approximately reached virial equilibrium, i.e., galaxies and groups of galaxies reach a fixed physical size; in this 'stable clustering' regime, if the two-point density correlation function obeys $\xi(r) \sim r^{-\gamma}$, then $\epsilon = 3 - \gamma$. Since the observed *galaxy* correlation function is a power-law with $\gamma \simeq 1.8$ at $r < 10h^{-1}$ Mpc, we expect $\epsilon \simeq 1.2$ on small scales, although with the caveat that the galaxy and mass distributions may differ significantly on these scales (see below). A third more radical possibility is that the observed clustering is merely "painted on" and expands with the Hubble flow; in this case, $\epsilon = 0$. For the numerical estimates below, we will thus consider the range $\epsilon = 0 - 2$. Since the rms amplification is dominated by structure in the non-linear regime, we expect $\epsilon \simeq 1.2$ to be the most accurate representation of the evolution. Recent N-body simulations confirm that the growth factor is intermediate between the stable clustering and linear regimes on small scales at recent epochs (Colin, Carlberg, & Couchman 1996), while it may be faster than linear ($\epsilon > 2$) on intermediate scales; in either case, the results presented below constitute an upper limit on the lensing effect.

To model the present power-spectrum on small scales, we could simply Fourier transform the galaxy correlation function, $\xi_g(r) = (r/r_0)^{-1.8}$, where the galaxy correlation length $r_{0,g} \simeq 5.4h^{-1}$ Mpc. However, this does not take into account the *bias* between the galaxy and mass distributions, i.e., the likelihood that light does not trace mass on these scales. To remedy this, we can incorporate dynamical information. From the (simplified version of the) cosmic virial theorem (Peebles 1980, 1993), the predicted pairwise velocity dispersion on small scales is roughly $\sigma_v^2(r) \sim [3/2(3-\gamma)]H_0^2\Omega_{m,0}r^2(r_0/r)^\gamma$. Assuming the matter correlation function has the same slope as that of the galaxies, this yields an estimate for the density correlation length, $r_0 \simeq 5.4h^{-1}$ Mpc$(0.1/\Omega_{m,0})^{0.55}(\sigma_v(1h^{-1}\ \mathrm{Mpc})/300\ \mathrm{km/sec})^{1.1}$. The standard estimates of the galaxy pairwise velocity dispersion at $r \simeq 1h^{-1}$ Mpc separation have been $\sigma_v \sim 300$ km/sec (Davis & Peebles 1983, Bean, etal. 1983). Recent redshift surveys, however, have yielded higher values, $\sigma_v \sim 500 - 700$ km/sec (Guzzo, etal. 1995, Marzke, etal. 1995). Moreover, Somerville, Davis, & Primack (1996) and Somerville, Primack, & Nolthenius (1996) have found that estimates of $\sigma_v(r)$ in both simulations and galaxy catalogs show large scatter. However, when the cores of rich clusters, which contain only a small fraction of the mass, are excluded, it appears that $\sigma_v \sim 300$ km/sec. In any case, to obtain an upper bound for the weak lensing effect, we will take $\sigma_v(1h^{-1}\ \mathrm{Mpc}) = 650$ km/sec, implying $\Delta^2(k) \simeq 8.7(k/h\ \mathrm{Mpc}^{-1})^{1.8}\Omega_{m,0}^{-1}(\sigma_v(1)/650)^2$. Note that the assumption made here that the galaxy and density correlation functions have the same small-scale slope is not well motivated on highly non-linear scales, and the results below should be interpreted with this caveat.

For this model of the power spectrum, and in general for $\gamma > 1$, the k-integral in (5) diverges in the ultraviolet and must be cut-off. Physically, the slope of the density correlation function must fall below unity below some length scale (even though the *galaxy* correlation function is an unbroken $\gamma \simeq 1.8$ power law down to $r \sim 10h^{-1}$ kpc). This flattening is expected to happen at least at the scale of individual galaxy halos (Kaiser 1992): below this scale, $\xi(r)$ is dominated by correlations within individual halos, yielding $\xi(r) \sim r^{-(2\nu-3)}$

for halos with density profile $\rho(r) \sim r^{-\nu}$ (Peebles 1974, McClelland & Silk 1977, Sheth & Jain 1996); for nearly isothermal halos, $\nu \simeq 2$, the corresponding slope is $\gamma \simeq 1$. Up to logarithmic corrections, we model this effect by imposing a cutoff in (5) at the approximate halo scale, $k_c = 1/(100h^{-1}$ kpc). Using this model in (5), the rms flux perturbation at fixed z_s scales approximately as $\Omega_{m,0}^{1/2}(\sigma_v(1)/650)(k_c/10h\ \mathrm{Mpc}^{-1})^{0.4}$.

Fig.1 shows the dispersion in apparent magnitude for distant standard candles as a function of redshift, $\sigma_m = 1.086\sigma_A$ mag, for cosmological models with $\Omega_{m,0} = 0.1$, 0.3, and 1. For $\Omega_{m,0} = 1$, we show results for $\epsilon = 0$, 1.2, and 2 to bracket the plausible range of evolution models. For the other cases, we show results for $\epsilon = 1.2$ only; since structure formation freezes out early in $\Omega_{m,0} < 1$ models (at $1 + z_f \simeq \Omega_{m,0}^{-1}$), stable clustering should be a reliable prescription here. For $\Omega_{m,0} = 0.3$, we show results for a spatially flat model with non-zero cosmological constant, $\Omega_\Lambda = 0.7$, and for an open model with $\Lambda = 0$.

Since the imposition of a sharp cutoff in the density power spectrum appears to be a rather crude approximation, as a check we have also explored models for the small-scale power spectrum based on the cold dark matter (CDM) model and CDM with non-zero Λ, with scale-invariant primordial fluctuations, extended into the non-linear regime using scaling formulae derived from N-body simulations (Hamilton etal. 1991, Peacock & Dodds 1994, Jain, Mo, & White 1995, Baugh & Gaztanaga 1996, Peacock & Dodds 1996). In these models, $P(k)$ scales as k at small wavenumber but turns over to k^{-3} at large k, yielding much better convergence for the flux dispersion σ_A. We normalize such models by requiring that they reproduce the observed galaxy cluster X-ray temperature distribution function according to the predictions of Press-Schechter theory (White, Efstathiou, & Frenk 1993); for CDM models, this corresponds approximately to imposing the constraint that the linear theory rms mass fluctuation in a sphere of radius 8 h^{-1} Mpc is $\sigma_8 = 0.6\Omega_{m,0}^{-C(\Omega_{m,0},\Lambda)}$, where $C = 0.36 + 0.31\Omega_{m,0} - 0.28\Omega_{m,0}^2$ for open models ($\Lambda = 0$) and $C = 0.59 - 0.16\Omega_{m,0} + 0.06\Omega_{m,0}^2$ for spatially flat (non-zero Λ) models (Viana & Liddle 1995). The results for the amplification dispersion in these cluster-normalized CDM models are shown in Fig. 2 for the same model parameters as in Fig. 1. The results for σ_A agree reasonably well with those of the power-law model, although they are somewhat higher for the models with $\Omega_{m,0} < 1$. This difference is due in part to the fact that the cluster normalization yields more small-scale power in these models than the cosmic virial theorem normalization for the corresponding power-law model. Thus, while the uncertainties in σ_v and in the accuracy of the cosmic virial theorem are still significant, this comparison suggests that our estimate for σ_A should be accurate to within a factor of two.

The implications of weak lensing for the determination of q_0 follow directly from Figs. 1 and 2. For sources at $z \leq 0.7$, $\sigma_m \leq 0.06$, well below the "intrinsic" $0.2 - 0.3$ mag spread in nearby SN Ia magnitudes. At $z \ll 1$, from the redshift-magnitude relation, the resulting '1σ' uncertainty in the deceleration parameter, $q_0 = \Omega_{m,0}/2 - \Omega_\Lambda$, for a single source at redshift z is $\sigma_{q_0} \simeq \sigma_A/z$. For example, for $z = 0.5$ and $\Lambda = 0$, we find $\sigma_{q_0} \simeq 0.1q_0^{1/2}$ ($\lesssim 0.07$ for $\Omega_{m,0} \leq 1$). In the future, if SN Ia searches discover sources at $z \gtrsim 1$, then weak lensing may be a significant factor. In particular, if light-curve shape and other correlators with peak magnitude can reduce the effective intrinsic spread to 0.1 mag (even at high z), then density fluctuations could increase the observed dispersion in an $\Omega_{m,0} = 1$ universe for sources at $z = 1$ by of order 30%. Since the amplification is caused by the foreground mass distribution, one could in principle use the angular correlation function of δA to probe the large-scale mass power spectrum; in practice, this will require thousands of well-measured SNe Ia at redshifts $z \gtrsim 0.5$ spread over hundreds of square degrees. This may be possible with the Next Generation Space Telescope.

The results shown here can be compared to those of Babul & Lee (1991). While they considered only the Einstein-de Sitter $\Omega_{m,0} = 1$ case, we have presented results for arbitrary Ω_m and Λ. Quantitatively, their 'DP' model for the power spectrum is quite similar to that used here, but it overestimated the rms amplification σ_A because it did not include the constraint from galaxy pairwise velocities on small scales. They also presented results for the $\Omega = 1$ CDM model with bias factor $b \simeq 2.5$; this normalization, which reflected earlier, lower estimates of the small-scale pairwise velocity dispersion, is now known to be unacceptably low and therefore substantially underestimates σ_A. Thus, while their results bracket those here, the estimates in Figs. 1 and 2 should be more accurate.

A final issue of concern for these surveys is amplification bias. A fraction of the SNe in a magnitude-limited survey are strongly amplified ($\delta A \gg \sigma_A$) by foreground mass concentrations very close to the line of sight; this would cause large systematic errors in the estimate of q_0. However, the lensing galaxy or cluster responsible could generally be seen and such events removed from the sample. Even if the lenses are too faint to be detected, the probability for such strong lensing events at moderate redshift, $z_s \lesssim 1$, is known to be very small, based on the low incidence of multiply imaged QSOs. While the optical depth τ for significant amplification by foreground galaxies (e.g., $\delta A \gtrsim 0.1$) is much higher than that for multiple imaging, it is still quite low. For example, for $\Omega_{m,0} = 1$ and assuming isothermal galaxy halos extend to $r \gtrsim 50h^{-1}$ kpc, integration over the galaxy luminosity function with the Faber-Jackson relation gives $\tau(\delta A > 0.1, z_s = 1) \simeq 6.5 \times 10^{-3}$ for amplification by individual foreground galaxies; for a non-zero cosmological constant satisfying $\Omega_\Lambda = 1 - \Omega_m < 0.8$, $\tau(> 0.1, 1) < 0.02$. The result of these near encounters is a very small non-Gaussian tail in the amplification distribution at large δA. On the other hand, if a substantial fraction of the cosmic density is in compact objects, the high amplification tail can be much more significant (Schneider & Wagoner 1987, Linder, Schneider, & Wagoner 1988, Rauch 1991). If high amplification ($\gtrsim 1$ mag) events are not soon found in the high-redshift SN Ia searches, one will be able to constrain the contribution to Ω from objects of mass $\gtrsim 10^{-3} M_{\odot}$.

For completeness, I note that weak lensing does not affect the shape of SN Ia lightcurves: for a source at redshift z_s, the fall-off from the peak is given by the usual time dilation factor, $\Delta t = \Delta t_i(1 + z_s)$, where Δt_i is the fall-off timescale in the SN rest frame.

Note added: After this work was accepted for publication, two recent numerical efforts have clarified the weak lensing effects of large-scale structure (Wambsganss, etal. 1996, D. E. Holz and R. M. Wald, to be published). Using ray-shooting techniques through N-body and phenomenologically constructed universes, it has been directly confirmed that the mean flux from a distant standard candle is that given by the standard FRW luminosity distance. However, the distribution of the amplification is not symmetric about this mean. The majority of lines of sight pass through regions underdense compared to the mean, leading to modest de-amplification, while a smaller number of rays pass near dense mass concentrations and suffer substantial amplification. Thus, for a survey with a finite number of sources, there can be a small bias in the results for q_0, but the effect is very small at the current redshifts ($z \lesssim 0.6$) probed by the SN Ia surveys.

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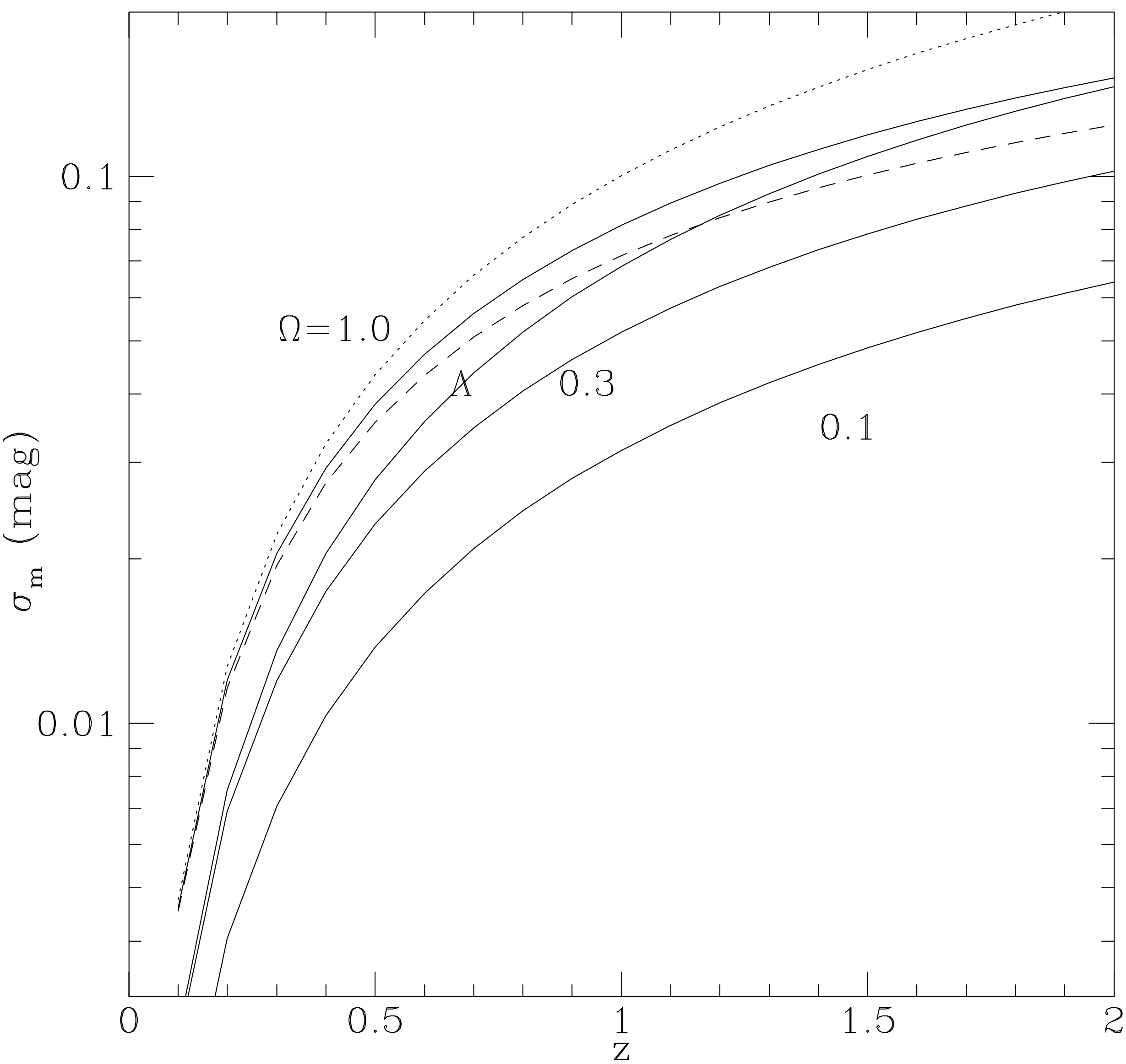

Fig. 1.— Fig. 1. Dispersion in flux (in magnitudes) for a source at redshift z, for $\Omega_{m,0} = 0.1$, 0.3, 1.0. Solid curves assume stable clustering, $\epsilon = 1.2$. For $\Omega_{m,0} = 1$, dashed curve corresponds to linear theory evolution, $\epsilon = 2$, and dotted curve corresponds to 'painted on' structure ($\epsilon = 0$). Lower curves marked $\Omega_{m,0} = 0.1, 0.3$ are open models; curve marked 'Λ' is a flat universe with $\Omega_\Lambda = 0.7$. The small-scale power spectrum has been cut off at $k_c = 1/(100h^{-1}$ kpc); the dispersion scales as $\sigma \propto k_c^{0.4}$.

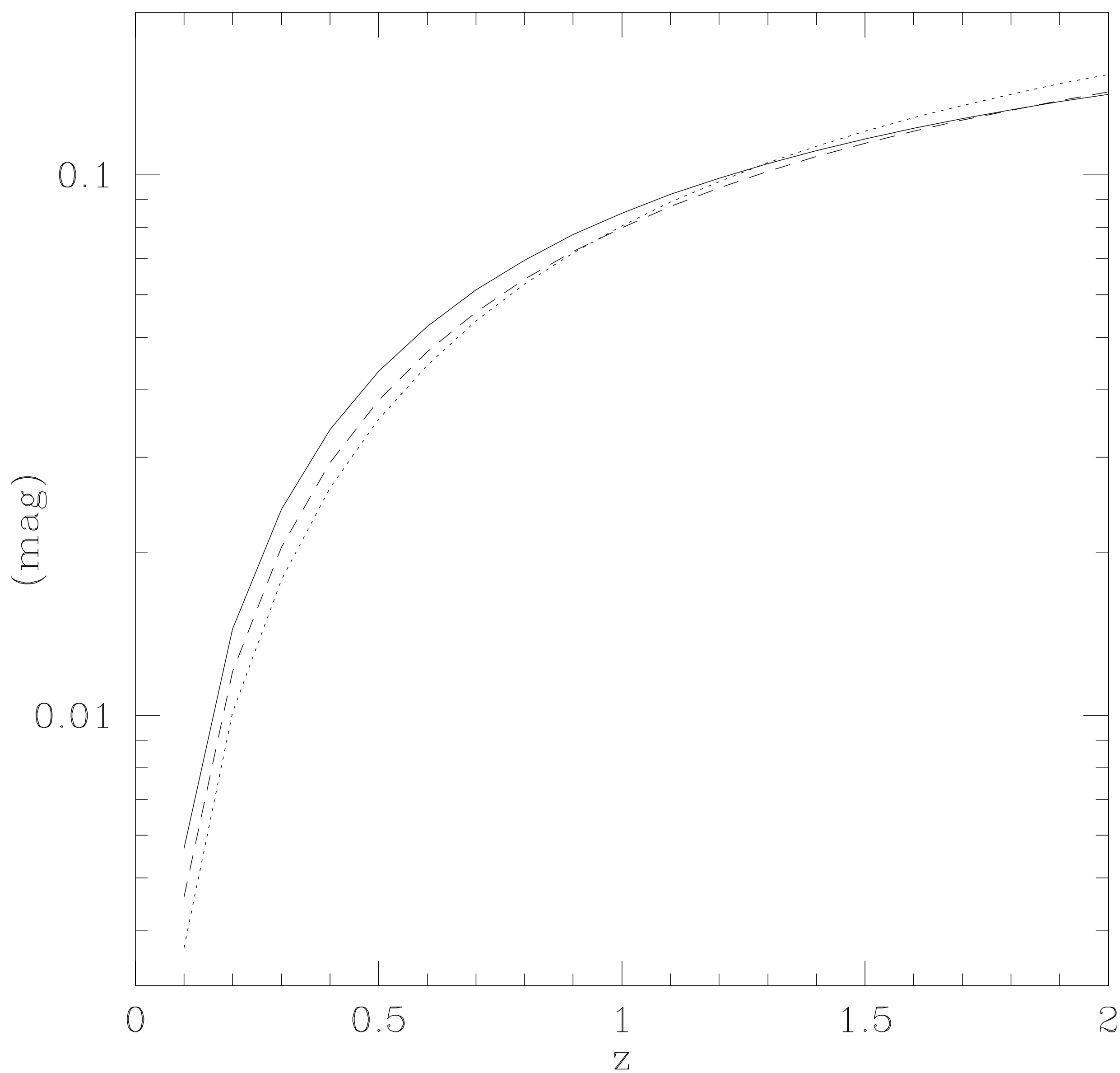

Fig. 2.— Fig. 2. Flux dispersion vs. redshift for models with the same cosmological parameters as in Fig. 1, but for the CDM model of structure formation instead of the phenomenological power-law model. The models are standard CDM $\Omega_{m,0} = 1$, $h = 0.5$ (solid), a flat ΛCDM model with $\Omega_\Lambda = 0.7$, $h = 0.7$ (dotted), and an open CDM model with $\Omega_{m,0} = 0.3$, $h = 0.7$ (dashed).