Electromagnetic Splittings and Light Quark Masses

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A method for computing electromagnetic properties of hadrons in lattice QCD is described. The electromagnetic field is introduced dynamically, using a noncompact formulation. Employing enhanced electric charges, the dependence of the pseudoscalar meson mass on the (anti)quark charges and masses can be accurately calculated. At $\beta = 5.7$ with Wilson action, the $\pi^+ - \pi^0$ splitting is found to be 4.9(3) MeV. Using the measured $K^0 - K^+$ splitting, we also find $m_u/m_d = .512(6)$. Systematic errors are discussed. Preliminary results for vector meson splittings are also presented.

1. Light Quark Masses

If a fundamental theory of quark masses ever emerges, it may be as important to resolve the theoretical uncertainty in the light quark masses as it is to accurately measure the top quark mass (e.g. in deciding whether nature avoids the strong CP problem via a massless up quark). The particle data tables [1] give wide ranges for the up ($2 < m_u < 8$ MeV) and down ($5 < m_d < 15$ MeV) quarks, while lowest order chiral perturbation theory [2] gives $m_u/m_d = 0.57 \pm 0.04$. Numerical lattice calculations provide, in principle, a very precise way of studying the dependence of hadron masses on the lagrangian quark mass parameters[3]. As the electromagnetic contribution to hadronic mass splittings within isomultiplets is comparable to the up-down quark mass difference, an accurate determination of the light quark masses requires inclusion of electromagnetic effects dressed by nonperturbative QCD dynamics. Here, we discuss a method for studying such electromagnetic effects[4]. In addition to the SU(3) color gauge field, we introduce a U(1) electromagnetic field on the lattice which is treated by quenched Monte Carlo methods. The resulting SU(3)×U(1) (Coulomb-gauge) configurations are then analyzed by standard hadron propagator techniques.

The small size of electromagnetic mass splittings makes their accurate determination by conventional lattice techniques difficult if the electromagnetic coupling is taken at its physical value. We have found that calculations done at larger values (roughly 2 to 6 times physical) of the quark electric charges lead to accurately measurable isosplittings in the light pseudoscalar meson spectrum, while still allowing perturbative extrapolation to physical values.

We proceed as follows: quark propagators are generated in the presence of background SU(3)×U(1) fields where the SU(3) component represents the usual gluonic gauge degrees of freedom, while the U(1) component incorporates an abelian photon field (with a noncompact gauge action) which interacts with quarks of specified electric charge. Quark propagators are calculated for a variety of electric charges and light quark mass values. The gauge configurations were generated at $\beta = 5.7$ on a $12^3 \times 24$ lattice. 200 configurations each separated by 1000 Monte Carlo sweeps were used. In the results reported here, we have used four different values of charge given by e_q =0, -0.4, +0.8, and -1.2 in units in which the electron charge is $e = \sqrt{4\pi/137} = .3028\ldots$. For each quark charge we calculate propagators for three light quark mass values in order to allow a chiral extrapolation. From the resulting

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12 quark propagators, 144 quark-antiquark combinations can be formed, leading 78 independent meson propagators and masses.

2. Expected Chiral Behaviour

Once the full set of meson masses is computed, the analysis proceeds by a combination of chiral and QED perturbation theory. In pure QCD it is known that, in the range of masses considered here, the square of the pseudoscalar meson mass is accurately fit by a linear function of the quark masses[5,6]. We have found that this linearity persists even in the presence of electromagnetism[4]. For each of the charge combinations studied, the dependence of the squared meson mass on the bare quark mass is well described by lowest order chiral perturbation theory. Thus we write the pseudoscalar mass squared as

$$m_P^2 = A(e_q, e_{\bar{q}}) + m_q B(e_q, e_{\bar{q}}) + m_{\bar{q}} B(e_{\bar{q}}, e_q) \quad (1)$$

where $e_q, e_{\bar{q}}$ are the quark and antiquark charges, and $m_q, m_{\bar{q}}$ are the bare quark masses, defined in terms of the Wilson hopping parameter by $(\kappa^{-1} - \kappa_c^{-1})/2a$. (Here a is the lattice spacing.) Because of the electromagnetic self-energy shift, the value of the critical hopping parameter must be determined independently for each quark charge. This is done by requiring that the mass of the neutral pseudoscalar meson vanish at $\kappa = \kappa_c$, as discussed below.

For the physical values of the quark charges, we expect that an expansion of the coefficients A and B in (1) to first order in e^2 should be quite accurate. For the larger values of QED coupling that we use in our numerical investigation, the accuracy of first order perturbation theory is less clear: in fact, a good fit to all our data requires small but nonzero terms of order e^4, corresponding to two-photon diagrams. Comparison of the order e^4 terms with those of order e^2 provides a quantitative check on the accuracy of QED perturbation theory. Only those e^4 terms which significantly reduce the χ^2 per degree of freedom have been kept.

According to Dashen's theorem, in the chiral limit the value of m_P^2 is proportional to the square of the total charge. Thus, we have also allowed the values of the critical hopping parameters for each of the quark charges to be fit parameters, requiring that the mass of the neutral mesons vanish in the chiral limit. Thus A takes the form $A^{(1)}(e_q + e_{\bar{q}})^2$ to order e^2. (Order e^4 terms here were not found necessary to fit the data.) The coefficient B in (1) which parametrizes the slope of m_P^2 may also be expanded in perturbation theory. Of the five possible e^4 terms in $B^{(2)}(e_q, e_{\bar{q}})$, only the e_q^4, $e_q^3 e_{\bar{q}}$ and $e_q^2 e_{\bar{q}}^2$ terms were found to improve the χ^2. The coefficients in A and B, along with the four values of κ_c for the four quark charges, constitute a 12-parameter fit to the meson mass values.

3. Lattice Formulation Including EM

We have chosen a noncompact abelian gauge action $S_{\rm em}$ to ensure that the theory is free in the absence of fermions, and is always in the nonconfining, massless phase. (Of course, lattice gauge invariance still requires a compact gauge-fermion coupling). An important aspect of a noncompact formalism is the necessity for a gauge choice. We use QCD lattice configurations which have all been converted to Coulomb gauge for previous studies of heavy-light mesons. Coulomb gauge turns out to be both practically and conceptually convenient in the QED sector as well.

For the electromagnetic action, we take

$$S_{\rm em} = \frac{1}{4e^2} \sum_{n\mu\nu} (\nabla_\mu A_{n\nu} - \nabla_\nu A_{n\mu})^2 \quad (2)$$

with e the bare electric coupling, n specifies a lattice site, ∇_μ the discrete lattice right-gradient in the μ direction and $A_{n\mu}$ takes on values between $-\infty$ and $+\infty$. Electromagnetic configurations were generated using (2) as a Boltzmann weight, subject to the linear Coulomb constraint $\bar{\nabla}_i A_{ni} = 0$ with $\bar{\nabla}$ a lattice left-gradient operator. The action is Gaussian-distributed so it is a trivial matter to generate a completely independent set in momentum space, recovering the real space Coulomb-gauge configuration by Fast Fourier transform. We fix the global gauge freedom remaining after the Coulomb gauge condition is imposed by setting the $p = 0$ mode equal to zero for the transverse modes, and the $\vec{p} = 0$ mode

Table 1
Coefficients of fitting function, Eq.(1). Terms consistent with zero were dropped from this fit. Numerical values are in GeV^2 and GeV for A and B terms respectively.

	Fit
A	$0.0143(10)(e_q + e_{\bar{q}})^2$
$B^{(0)}$	$1.594(11)$
$B^{(1)}$	$0.205(22)e_q^2 + 0.071(9)e_q e_{\bar{q}} + 0.050(7)e_{\bar{q}}^2$
$B^{(2)}$	$0.064(17)e_q^4 - 0.031(4)e_q^2 e_{\bar{q}}^2$
	$+0.033(6)e_q^3 e_{\bar{q}} - 0.031(4)e_q^2 e_{\bar{q}}^2$

to zero for the Coulomb modes on each time-slice. (This implies a specific treatment of finite volume effects which will be discussed below.) The resulting Coulomb gauge field $A_{n\mu}$ is then promoted to a compact link variable $U_{n\mu}^{\rm em} = e^{\pm iqA_{n\mu}}$ coupled to the quark field in order to describe a quark of electric charge $\pm qe$. Quark propagators are then computed in the combined SU(3)×U(1) gauge field.

4. Preliminary Results

For charge zero quarks, propagators were calculated at hopping parameter 0.161, 0.165, and 0.1667, corresponding to bare quark masses of 175, 83, and 53 MeV respectively. The gauge configurations are generated at $\beta = 5.7$, and we have taken the lattice spacing to be $a^{-1} = 1.15$ GeV. After shifting by the improved perturbative values, we select the same three hopping parameters for the nonzero charge quarks[4]. Because this shift turns out to be very close to the observed shift of κ_c, the quark masses for nonzero charge are nearly the same as those for zero charge. For all charge combinations, meson masses were extracted by a two-exponential fit (using both smeared and local sources) to the pseudoscalar propagator over the time range $t = 3$ to 11. Errors on each mass value are obtained by a single-elimination jackknife. The resulting data is fitted to the chiral/QED perturbative formula (1) by χ^2 minimization. The fitted parameters are given in Table 1. Errors were obtained by performing the fit on each jackknifed subensemble. Aside from very small corrections of order $(m_d - m_u)^2$, the $\pi^+ - \pi^0$ mass splitting is of purely electromagnetic origin, and thus should be directly calculable by our method. Because we have used the quenched approximation, $u\bar{u}$ and $d\bar{d}$ mesons do not mix. The squared neutral pion mass is obtained by averaging the squared masses of the $u\bar{u}$ and $d\bar{d}$ states. Thus, to zeroth order in e^2, the terms proportional to quark mass cancel in the difference $m_{\pi^+}^2 - m_{\pi^0}^2$. This difference is essentially given by the single term

$$m_{\pi^+}^2 - m_{\pi^0}^2 \approx A^{(1)} e^2 \qquad (3)$$

Using the coefficients listed in Table 1, and the experimental values of the π^0, K^0, and K^+ masses, we may directly solve the resulting three equations for the up, down, and strange masses. The $\pi^+ - \pi^0$ splitting may then be calculated, including the very small contributions from the order $e^2 m_q$ terms. We obtain

$$m_{\pi^+} - m_{\pi^0} = 4.9 \pm 0.3 \text{MeV} \qquad (4)$$

compared to the experimental value of 4.6 MeV. (The electromagnetic contribution to this splitting is estimated [7] to be 4.43 ± 0.03 MeV.) Our calculation can be compared to the value 4.4 MeV (for $\Lambda_{\rm QCD} = 0.3$ GeV and $m_s = 120$ MeV) obtained by Bardeen, Bijnens and Gerard[8] using large N methods. The values obtained for the bare quark masses are

$$m_u = 3.86(3), \quad m_d = 7.54(5), \quad m_s = 147(1) \qquad (5)$$

The errors quoted are statistical only, and are computed by a standard jackknife procedure. The small statistical errors reflect the accuracy of the pseudoscalar mass determinations, and should facilitate the future study of systematic errors (primarily finite volume, continuum extrapolation and quark loop effects)[3], which are expected to be considerably larger. The relationship between lattice bare quark masses and the familiar current quark masses in the $\overline{MS}$ continuum regularization is perturbatively calculable[3]. For mass ratios (which are independent of renormalization prescription) we obtain

$$\frac{m_d - m_u}{m_s} = .0249(3)\ , \quad \frac{m_u}{m_d} = .512(6) \qquad (6)$$

5. Finite Volume Corrections

The presence of massless, unconfined degrees of freedom implies that finite volume effects are potentially much larger than in pure QCD, falling as inverse powers of the lattice size, instead of exponentially. We have estimated the size of these corrections phenomenologically along the lines of Bardeen, et.al[8], who model the low-q^2 contribution to the $\pi^+ - \pi^0$ splitting in terms of π, ρ, and $A1$ intermediate states. This analysis gives

$$\delta m_\pi^2 = \frac{3e^2}{16\pi^2}\int_0^{M^2} \frac{m_A^2 m_\rho^2}{(q^2+m_\rho^2)(q^2+m_A^2)} dq^2 \qquad (7)$$

We may use this result to estimate the finite volume correction by casting the expression as a four-dimensional integral over d^4q and then constructing a finite volume version of it by replacing the integrals with discrete sums (excluding the $q = 0$ mode). For a $12^3 \times 24$ box with $a^{-1} = 1.15$ GeV, we find that the infinite volume value of 5.1 MeV is changed to $\delta m_\pi = 4.8$ MeV, indicating that the result we have obtained in our lattice calculation should be corrected upward by about 0.3 MeV, or about 6%. In upcoming studies this estimate will be checked directly on larger box sizes.

6. Vector Mesons

The same techniques can be applied to vector mesons and heavy-light mesons. For the light vector mesons the expected form of the mass matrix includes the following terms.

$$\begin{aligned} m_V &= A(e_q, e_{\bar q}) + m_q B(e_q, e_{\bar q}) + m_{\bar q} B(e_{\bar q}, e_q) \\ &\quad + (m_q^2 + m_{\bar q}^2) C(e_q, e_{\bar q}) + m_q m_{\bar q} D(e_q, e_{\bar q}) \end{aligned} \qquad (8)$$

(Including nonanalytic terms[6] (e.g. $O(m^{3/2})$) does not subtantially alter our final results.) Calculating all the vector mesons mass combinations and fitting to this form, we obtain the coefficients shown in Table 2. From these results we obtain the mass differences

$$m_{\rho^+} - m_{\rho^0} = -0.74 \pm 1.14(\text{stat}) \text{ MeV} \qquad (9)$$

$$m_{K^{*0}} - m_{K^{*+}} = 3.58 \pm 0.77(\text{stat}) \text{ MeV} \qquad (10)$$

Table 2
Coefficients (in lattice units) of fitting function, Eq.(8) for vector mesons.

	Fit
A	$0.567(1) + 0.0068(1)(e_q + e_{\bar q})^2$
B	$0.523(47) + 0.205(22)e_q^2 + 0.073(9)e_{\bar q}^2$ $+0.138(9)e_q e_{\bar q} - 0.027(1)e_q^2 e_{\bar q}^2$ $-0.017(1)e_q^4 - 0.013(1)e_{\bar q}^4$ $-0.009(1)e_q^3 e_{\bar q} - 0.013(1)e_q e_{\bar q}^3$
C	0.81(29)
D	0.38(50)

7. Conclusions

Here we have focused mainly on the pseudoscalar meson masses. This is the most precise way of determining the quark masses as well as providing an important test of the method in the $\pi^+ - \pi^0$ splitting [4]. Further calculations of electromagnetic splittings in the vector mesons and the baryons [9], as well as in heavy-light systems, are possible using the present method. This will provide an extensive opportunity to test the precision of the method and gain confidence in the results. Eventually reliable lattice calculations for all isospin breaking effects should be possible.

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REFERENCES

1. Review of Particle Properties, Phys. Rev. **D50** (1994).
2. J. Gasser and H. Leutwyler, Phys. Rep. 87 (1982) 77.
3. P. B. Mackenzie in these proceedings.
4. A. Duncan, E. Eichten and H. Thacker, Phys. Rev. Lett. **76** (1996) 3894.
5. H. Thacker in these proceedings.
6. S. R. Sharpe in these proceedings.
7. J. Gasser and H. Leutwyler, Nucl. Phys. **B250** (1985) 465.
8. W. A. Bardeen, J. Bijnens and J. M. Gerard, Phys. Rev. Lett. **62** (1989) 1343.
9. A. Duncan in these proceedings.