The Light Quark Masses with an $O(a)$-Improved Action

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We present the recent Fermilab calculations of the masses of the light quarks, using tadpole-improved Sheikholeslami-Wohlert (SW) quarks. Various sources of systematic errors are studied. Our final result for the average light quark mass in the quenched approximation evaluated in the $\overline{MS}$ scheme is $\overline{m}_q(\mu = 2\,\mathrm{GeV}; n_f = 0) = (m_u + m_d)/2 = 3.6 \pm 0.6$ MeV.

1. Introduction

We present recent results on the light quark mass determination using the SW action [1], which are updates of the last year's results [2]. For results from wilson and staggered fermions see [3][4][5].

The basic procedure is to extract the pseudoscalar masses (m_{PS}) numerically for a range of quark masses and determine the linear coefficient in the chiral extrapolation,

$$(m_{PS}\,a)^2 = A\,\tilde{m}_q^{lat}\,a \qquad (1)$$

where $\tilde{m}_q^{lat} = \ln(1 + 1/2\tilde{\kappa} - 1/2\tilde{\kappa}_c)$, with $\tilde{\kappa} \equiv \kappa u_0$ and $u_0 \equiv \sqrt[4]{<U_P>_{MC}}$ [6].

Using the experimentally measured pion mass as an input, we obtain the light quark bare mass $\tilde{m}_q^{lat}$, which is the average of the up and down quark masses. We convert it to the light quark mass $\overline{m}_q$ in the $\overline{MS}$ scheme by perturbation theory.

Table 1 shows the lattices used for the simulation. We use the SW fermion action. For $\beta = 5.5, 5.7$ and 5.9 the clover coefficient c is the

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Table 1
Lattice details ($n_f = 0$)

β	5.5	5.7	5.9	6.1
κ's	4	4	4	4
configs	40	300	100	100
a^{-1} (GeV)	0.79	1.16	1.80	2.55
L^3 ($T = 2L$)	8^3	12^3	16^3	24^3
L^{phys} (fm)	2.0	2.1	1.8	1.9
c	1.69	1.57	1.50	1.40

tadpole improved tree-level value $1/u_0^3$. However, for $\beta = 6.1$, we use $c = 1.40$ instead of $1/u_0^3 = 1.46$. All calculations are done in the quenched approximation. The lattice spacing a is determined from the 1P-1S charmonium splitting.

2. Systematic Errors

We use the multi-state smearing method [7] to suppress excited state contamination. The smearing sources are fits to the measured wavefunctions of the pseudoscalar ground and excited
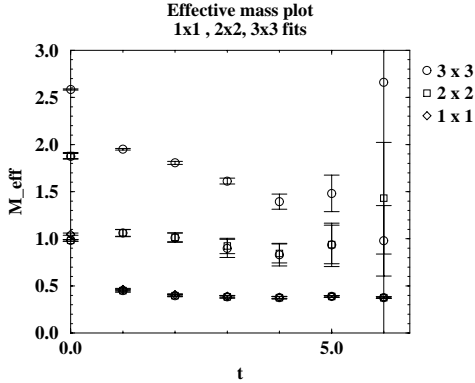
Effective mass plot
1x1 , 2x2, 3x3 fits

Figure 1. The pseudoscalar effective mass plot for $\beta = 5.9$, $\kappa = 0.1385$. The 1×1 one-state (diamond), 2×2 two-state (square), 3×3 three-state (circle) fits are shown.

Chiral extrapolation
4 points v.s. 3 points

Figure 2. The Chiral extrapolation of m_{PS}^2 for $\beta = 5.9$. The four κ fit (solid line) and the three κ fit (dashed line) are almost indistinguishable.

states with the following forms,

$$f_{1S}(\mathrm{r}) = \exp(-\mu_{1S}\mathrm{r}), \quad (2)$$

$$f_{2S}(\mathrm{r}) = (1 - \nu_{2S})\exp(-\mu_{2S}\mathrm{r}). \quad (3)$$

For $\beta = 5.5$, 5.7 and 5.9, we use 1S, 2S and local sources, while for $\beta = 6.1$, only 1S and local sources are used. We choose 2×2 two-state fits as our best fits. In order to estimate the systematic error of excited state contamination, we compare our best fits with the results from 1×1 one-state and 3×3 three-state fits. We find that the difference is less than 1% for $\beta = 5.7$, 5.9 and about 1-1.5% for $\beta = 6.1$. (See Figure 1.)

As the chiral extrapolation error, we take the difference in the chiral extrapolation with three κ's and four κ's. The results are again less than 1% for $\beta = 5.7$ and 5.9, and about 3% for $\beta = 6.1$. (See Figure 2.)

The one loop the renormalization factor which connects the lattice bare mass with $\overline{MS}$ mass is,

$$\overline{m}_q(\mu) = \widetilde{m}_q^{lat} \left[1 + \alpha_{\mathrm{V}}(q^*)(\gamma_0(\ln \widetilde{C}_m - \ln(a\mu)))\right] . (4)$$

The mean-field improved bare mass $\widetilde{m}_q^{lat}$ is given by $\widetilde{m}_q^{lat} = m_0/(1 - (\pi/3)\alpha_{\mathrm{V}} + \dots)$ in perturbation theory [6]. $\gamma_0 = 2/\pi$ is the leading quark mass
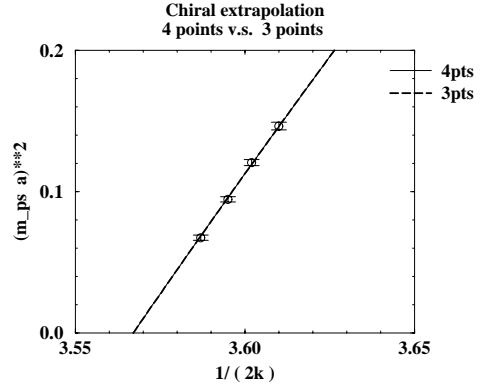
Table 2
Our results for A, the coefficient of the linear fit in the m_π^2 chiral extrapolation, $\widetilde{m}_q^{lat}$ (in MeV), the tadpole improved lattice bare mass, and $\overline{m}_q(2\,\mathrm{GeV})$ (in MeV), in the $\overline{MS}$ scheme, renormalized at 2 GeV ($q^* = \pi/a, 1/a$.) .

β	5.5	5.7	5.9	6.1
A	5.3(7)	4.1(5)	3.05(7)	2.24(10)
$\widetilde{m}_q^{lat}$	4.34(17)	3.9(1)	3.3(1)	3.2(1)
$\overline{m}_q(\pi/a)$	4.75(19)	4.41(12)	3.90(13)	3.84(18)
$\overline{m}_q(1/a)$	6.20(25)	4.89(13)	4.19(14)	4.05(19)

anomalous dimension. $\widetilde{C}_m$ for SW-improved light quarks is 4.72 [8].

Using Eq.(4), we first convert the lattice quark mass to the $\overline{MS}$ mass at $\mu = \pi/a$ or $1/a$, then run it to the common scale of 2 GeV. In Eq.(4), there is another scale q^*, which is the scale for the gauge coupling constant. Since we do not know the two-loop correction, it is not obvious which scale we should take for q^*. We estimate the size of unknown higher order corrections to Eq.(4) by varying q^* between $1/a$ and π/a. This procedure is consistent with assuming a coefficient of order unity for the α_{V}^2 term. Our estimates are 30%, 13%, 7%, 5% for $\beta = 5.5, 5.7, 5.9, 6.1$.

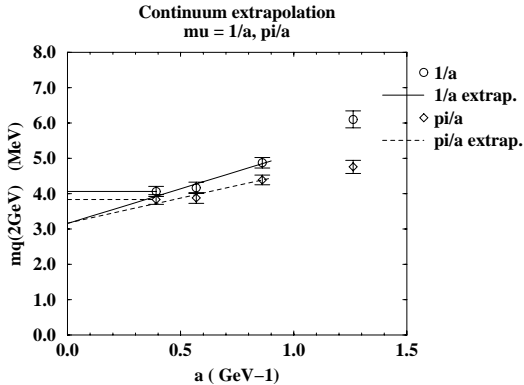**Continuum extrapolation**
mu = 1/a, pi/a

Figure 3. The continuum extrapolation of $\overline{m}_q$. The upper value is the $\beta = 6.1$ result, and the lower value is the naive linear extrapolation of the $\beta = 5.7, 5.9, 6.1$ data. The data for $\mu = q^* = 1/a$ (circle, solid line), π/a (diamond, dashed line) are presented in the same graph.

There are both $O(\alpha a)$ and $O(a^2)$ corrections to the action, and the continuum extrapolation could change depending on the relative size of these subleading terms. All we can say is that there is a systematic downward trend as we approach to the continuum. Without a theoretical argument to tell us about the a-dependence, we take the $\beta = 6.1$ result as an upper value and take the linearly extrapolated value using $\beta = 5.7, 5.9,$ 6.1 as a lower value. Our estimate of the continuum extrapolation error is 11%. (See Figure 3.)

3. Summary

In summary, our error estimates are,

excited states	< 1.5%
chiral extrapolation	~ 3%
perturbative	5%
continuum extrapolation	11%
combined	17%

The perturbative and a dependent errors are intertwined. We combine them linearly in the following way. As we saw earlier, the scale of the coupling constant q^* is arbitrary. When we discuss the continuum limit, we therefore perform the extrapolation of the data for both $q^* = 1/a$ and π/a (Figure 3). The outer points so obtained are taken as the limits of the combined error bar. The remaining errors are much smaller and combined in quadrature. Our final result for the light quark mass in the $\overline{MS}$ scheme in the quenched approximation is

$$\overline{m}_q(\mu = 2\,\text{GeV}; n_f = 0) = 3.6 \pm 0.6\,\text{MeV}. \qquad (5)$$

4. Acknowledgments

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