RENORMALIZATION OF HADRONIC DIFFRACTION AND THE STRUCTURE OF THE POMERON

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August 1, 1995

ABSTRACT

A phenomenological renormalization scheme for hadronic diffraction is proposed, which unitarizes the triple-pomeron Regge amplitude while preserving its M^2 and t dependence. Predictions for $pp/p\bar{p}$ single diffractive, double diffractive and double pomeron exchange cross sections are presented and compared with experimental results. A new interpretation of hard and deep inelastic diffractive data emerges in which the momentum sum rule is obeyed by the constituents of a pomeron described as a mixture of quark and gluon color singlets in a ratio dictated by asymptopia.

1 Introduction

It is well known that pomeron exchange in Regge theory accounts for the main features of high energy elastic, diffractive and total cross sections [1, 2]. In particular, for proton-(anti)proton interactions, it accounts for the rise of the total cross section and the shrinking of the forward elastic peak with energy, and also describes correctly the M^2 and t dependence of single diffraction dissociation (SD). Furthermore, the concept of factorization provides relationships between cross sections that pass successfully the test of experimental observation [1].

The early success of the simple Regge-pole model has been, however, tempered by the more recent measurements of the $p\bar{p}$ single diffraction (SD) dissociation cross section at the S$p\bar{p}$S Collider [3] and at the Tevatron [4, 5]. As seen in Fig. 1, the theoretical prediction for the SD cross section based on standard Regge theory (dashed curve) has a much steeper energy dependence than the data. Such a result was, of course, not unexpected, since it is well known that the SD cross section in Regge theory with a pomeron trajectory intercept $\alpha(0) \geq 1$ rises faster than the total cross section, and if this rise were to continue it would lead to violation of unitarity at the TeV energy scale. However, the need for *unitarizing* the simple Regge-pole description of cross sections was elevated to a crisis by the $p\bar{p}$-collider measurements, and several attempts have been made to implement unitarization, based generally on applying "screening corrections" to the diffractive amplitude (e.g. [6]). In this paper, we propose a simple phenomenological unitarization procedure, which consists in *renormalizing* the "pomeron flux factor" that appears in the triple-pomeron amplitude for SD so that its integral over the entire diffractive phase space, which denotes the total number of pomerons "carried" by the proton, is not allowed to exceed unity. This procedure preserves the shapes of the M^2 and t distributions in SD and predicts correctly (solid line in Fig. 1) the experimentally observed SD cross section at all energies.

Below, after a discussion of the standard Regge theory for proton-(anti)proton cross sections, we present our renormalization procedure and apply it to single diffraction, double diffraction and double pomeron exchange, and also to hard diffractive and deep inelastic scattering processes, where it leads to a pomeron whose hard constituents satisfy the momentum sum rule.

2 Standard Regge single diffraction

In standard Regge theory with a supercritical pomeron trajectory, $\alpha(t) = 1 + \epsilon + \alpha' t$, the $p\bar{p}$ total, elastic, and SD cross sections are given by (see Fig. 2)

$$\sigma_T = \beta_1(0)\beta_2(0)\left(\frac{s}{s_0}\right)^{\alpha(0)-1} = \sigma_0^{p\bar{p}}\left(\frac{s}{s_0}\right)^{\epsilon} \tag{1}$$

$$\frac{d\sigma_{el}}{dt} = \frac{\beta_1^2(t)\beta_2^2(t)}{16\pi}\left(\frac{s}{s_0}\right)^{2[\alpha(t)-1]} = \frac{\sigma_T^2}{16\pi}\left(\frac{s}{s_0}\right)^{2\alpha' t} F^4(t) \approx \frac{\sigma_T^2}{16\pi}\, e^{b_{el}(s)t} \tag{2}$$

$$F^4(t) \approx e^{b_{0,el}t} \quad \Rightarrow \quad b_{el}(s) = b_{0,el} + 2\alpha' ln\left(\frac{s}{s_0}\right) \tag{3}$$

$$\frac{d^2\sigma_{sd}}{dtd\xi} = \frac{\beta_1^2(t)}{16\pi}\,\xi^{1-2\alpha(t)}\left[\beta_2(0)\,g(t)\,\left(\frac{s'}{s'_0}\right)^{\alpha(0)-1}\right] = f_{\mathcal{P}/p}(\xi,t)\,\sigma_T^{\mathcal{P}\bar{p}}(s',t) \tag{4}$$

where $\beta_1(t)$ is the coupling of the pomeron ($\mathcal{P}$) to the proton, F(t) the (anti)proton form factor, $g(t)$ the triple-pomeron coupling, s' the s-value in the $\mathcal{P}-\bar{p}$ reference frame, $\xi = s'/s = M^2/s$ the fraction of the momentum of the proton carried by the pomeron (M is the diffractive mass), and s_0, s'_0 are constants.

In (4), in analogy with (1), the term in the square brackets is interpreted as the $\mathcal{P}-\bar{p}$

total cross section

$$\sigma_T^{\mathcal{P}\bar{p}}(s',t) = \beta_2(0)g(t)\left(\frac{s'}{s'_0}\right)^{\alpha(0)-1} = \sigma_0^{\mathcal{P}\bar{p}}\left(\frac{s'}{s'_0}\right)^{\epsilon} \tag{5}$$

where in writing $\beta_2(0)g(t) = \sigma_0^{\mathcal{P}\bar{p}}$ we have assumed that the triple-pomeron coupling constant, $g(t)$, is independent of t, as suggested by experiment [1]. This interpretation leads naturally to viewing SD as a process in which pomerons "carried" by the proton interact with the antiproton. The *pomeron flux factor* is given by

$$f_{\mathcal{P}/p}(\xi,t) \equiv \frac{d^2\sigma_{sd}/d\xi\, dt}{\sigma_T^{\mathcal{P}\bar{p}}(s',t)} = \frac{\beta_1^2(t)}{16\pi}\,\xi^{1-2\alpha(t)} = \frac{\sigma_0^{pp}}{16\pi}\,\xi^{1-2\alpha(t)}\,F^2(t) \tag{6}$$

The constants s_0 and s'_0, which represent energy scales in the pomeron propagator, are not specified by the theory. For a *universal* pomeron the energy scale should be process independent and hence $s'_0 = s_0$. Thus, since $\beta^2(0) = \sigma_0^{pp} = (\sigma_T\ s^{-\epsilon})\,s_0^{\epsilon}$, there are two free parameters in (4) which cannot be determined from the elastic and total cross sections, namely s_0 and $g(t) = g(0)$. From the SD cross section one can determine only the product $g(0)s_0^{\epsilon/2}$, so that *in the standard Regge theory* the normalization of the pomeron flux factor, which depends on s_0 and thereby on the value of $g(0)$, is arbitrary. However, the *pomeron flux renormalization* scheme that we propose provides an additional constraint, so that the constants s_0 and $g(0)$ can now be determined *independently*, resulting in an unambiguously normalized pomeron flux and a unique triple pomeron coupling constant.

For the numerical evaluation of $f_{\mathcal{P}/p}(\xi,t)$, we use the pomeron trajectory obtained from the recent CDF results [4]:

$$\alpha(t) = 1+\epsilon+\alpha'\,t = 1+(0.115\pm0.008)+[(0.26\pm0.02)\ \mathrm{GeV}^{-2}]\,t \tag{7}$$

The value of ϵ is the weighted average of three values: one obtained from the rise of the total cross section with energy, $\epsilon = 0.112\pm0.013$, and the other two from the ξ-dependence of the SD cross section at $\sqrt{s}$ =546 GeV, $\epsilon = 0.121\pm0.011$, and at 1800 GeV, $\epsilon = 0.103\pm0.017$. The value of α' is obtained [4] from a fit to the form of Eq. 3 of experimentally measured elastic scattering slope parameters at small-t by CDF and at lower energies at the ISR. The pomeron trajectory in (7) is somewhat different from that being used widely in the literature, $\alpha(t) = 1.08+0.25\,t$, which was derived [2] before the CDF data were available.

Anticipating the result $s_0 \approx 1$ GeV^2 that we will obtain by applying our renormalization procedure to SD, and using the CDF total cross section value of 80.03 ± 2.24 mb at 1800 GeV, we obtain $\sigma_0^{p\bar{p}} = \sigma_T^{p\bar{p}}s^{-\epsilon} = 14.3$ mb and $\sigma_0^{pp}/16\pi = 0.73$ GeV^{-2}. Note that at $\sqrt{s}$=1800 GeV the terms in the cross section that fall as $1/\sqrt{s}$ or faster are negligible and therefore Eq. 1 can be used to evaluate $\sigma_0^{p\bar{p}}$ directly from the measured σ_T.

The nucleon form factor, $F(t)$, is obtained from elastic scattering. In the small-t region, the t-dependence of elastic scattering is represented well by $F^4(t) \approx e^{b_{0,el}t}$. From the elastic slope parameter at $\sqrt{s}$ =1800 GeV, $b_{el} = 16.98 \pm 0.25$ GeV^{-2} [4], using $\alpha' = 0.26$ GeV^{-2} we obtain (Eq. 3) $b_{0,el} = b_{el} - 2\alpha' ln(s/s_0) = 9.2$ GeV^{-2} and hence $F^2(t) \approx e^{b_{0,sd}t} = e^{(1/2)b_{0,el}t} = e^{4.6t}$. The value 4.6 is consistent with $b_{0,sd} = 4.2 \pm 0.5$ GeV^{-2} measured in [4] at $\sqrt{s} = 1800$ GeV. However, this simple exponential expression underestimates the cross section at large t. Since the pomeron seems to couple to quarks like an isoscalar photon, it was proposed [7] that the appropriate form factor for elastic and diffractive scattering is the isoscalar form factor measured in electron-nucleon scattering

$$F_1(t) = \frac{4m^2 - 2.8t}{4m^2 - t} \left[\frac{1}{1 - \frac{t}{0.7}} \right]^2 \tag{8}$$

where m is the mass of the nucleon. This form factor describes well the t-dependence of elastic scattering over a broad range of t [7], but its validity in high-t diffraction has not been adequately checked due to the sparseness of relevant data. At small t, $F_1^2(t)$ can be approximated with an exponential whose slope parameter, $b(t) = \frac{d}{dt} ln F_1^2(t)$, is 4.6 GeV^{-2} at $t \approx 0.04$ GeV^2, consistent with the simple exponential expression we obtained above. Below, in deriving the energy dependence of the SD cross section, we will use $F^2(t) = e^{4.6t}$, since it corresponds closely to the expression used by experimenters to derive the integrated SD cross sections from data.

3 Renormalization of hadronic diffraction

Our unitarization procedure consists in *renormalizing* the pomeron flux factor by treating it as a probability density whose integral is not allowed to exceed unity, *i.e.*

$$f_N(\xi, t) = \frac{f_{\mathcal{P}/p}(\xi, t)}{N(\xi_{min})}; \quad N(\xi_{min}) = \begin{cases} A(\xi_{min}) \equiv \int_{\xi_{min}}^{0.1} d\xi \int_{t=0}^{\infty} f_{\mathcal{P}/p}(\xi, t)\, dt \\ 1, \qquad \text{if } A(\xi_{min}) < 1 \end{cases} \tag{9}$$

For $p\bar{p}$ soft SD, ξ_{min}=M_0^2/s with $M_0^2 = 1.5$ GeV^2 (effective threshold) and $\xi_{max} = 0.1$ (coherence limit) [1], *i.e.* the flux is integrated over the *entire* diffraction region. An approximate numerical expression for the flux integral is given by

$$N(\xi_{min}) \approx \left(\frac{\xi_0}{\xi_{min}} \right)^{2\epsilon} \left(\frac{s_0}{1 \text{ GeV}^2} \right)^{\epsilon} \Rightarrow 0.25\, s^{2\epsilon} \qquad [s \text{ in GeV}^2] \tag{10}$$

where $\xi_0 = 0.004$ is the ξ_{min} value for which the flux integral is unity, and in the last step we used $\xi_{min} = 1.5/s$ (valid for *soft* SD) and $s_0 = 1$. Because of the $1/\xi^{1+2\epsilon}$ dependece of the flux factor and the $1/s$ dependence of ξ_{min}, the flux integral is insensitive to the coherence limit, ξ_{max}, for which we use the conventional value of 0.1.

3.1 Single diffraction dissociation

The total proton-(anti)proton single diffraction cross section values from fixed target (pp at $\sqrt{s} = 14$ and 20 and $p\bar{p}$ at 14 GeV) [8], ISR [9], UA4 [3], E710 [5] and CDF [4] experiments are shown in Fig. 1. Dissociation of *both* nucleons is included, and the published cross sections were corrected to correspond to $\xi < 0.05$ in order to reduce possible non-pomeron contributions. The dashed curve in this figure shows the cross section calculated using Eqs. 4, 5 and 6 with the parameters discussed above and $\sigma_0^{\mathcal{P}p} = 2.6$ mb, chosen to fit the low energy data. As mentioned earlier, the calculated cross section rises much faster with energy than the observed and becomes comparable to the total cross section at $\sqrt{s} \approx 1.8$ TeV, in violation of unitarity.

The solid line in Fig. 1 shows the result obtained with the renormalized flux. The position of the 'knee' in this curve occurs at the $\sqrt{s}$-value at which the flux integral becomes unity, which depends on the parameter s_0. Therefore, s_0 *is determined from the position of this 'knee' in the data.* In Fig. 1, the 'knee' occurs at $\sqrt{s} = 22$ GeV for $s_0 = 1$ GeV^2. The uncertainty in s_0 is given by $\delta s_0/s_0 = -2\delta s/s = -4(\delta\sqrt{s})/\sqrt{s}$. Thus, a (reasonable) 10% uncertainty in the $\sqrt{s}$-position of the 'knee' results in a 40% uncertainty in s_0. However, the effect on the flux normalization below the 'knee', which varies as s_0^{ϵ}, is 4%, and the effect on the determination of the triple-pomeron coupling (see below), which is $\sim s_0^{-\epsilon/2}$, is only 2%. The flux normalization *above the 'knee'* is, of course, not affected.

The renormalized differential and total SD cross sections are given by ($s' = M^2 = s\xi$)

$$\frac{d^2\sigma_{sd,N}}{dtd\xi} = \sigma_0^{\mathcal{P}p}\left(\frac{s\xi}{s_0'}\right)^{\epsilon} f_N(\xi,t)$$

$$\sigma_{sd,N}(\xi_{min} < \xi < \xi_{max}) = \sigma_0^{\mathcal{P}p}\left(\frac{s}{s_0'}\right)^{\epsilon}\int_{\xi_{min}}^{\xi_{max}}\int_{t=0}^{\infty}\xi^{\epsilon} f_N(\xi,t)\,d\xi dt = \sigma_0^{\mathcal{P}p}\left(\frac{s}{s_0'}\right)^{\epsilon}\langle\xi^{\epsilon}\rangle \tag{11}$$

where, again, $s_0' = 1$ GeV^2 and $\xi_{min} = (1.5\text{ GeV}^2)/s$. Since ξ_{min} decreases with increasing s, $\langle\xi^{\epsilon}\rangle$ also decreases and therefore $\sigma_{sd,N}$ increases at a rate *slower* than s^{ϵ}, i.e. slower than the (dominant at high energies) pomeron exchange component of the total cross section, which increases as s^{ϵ}. Thus, as required by unitarity, the renormalized SD cross section remains safely below the total cross section at all energies.

Above $\sqrt{s} = 22$ GeV, where the pomeron flux factor integral becomes unity, the total renormalized SD cross section, calculated from Eq. 11 and multiplied by 2, has an approximately logarithmic s-dependence given by

$$\sigma_{sd}^{p\bar{p}}(s)_{\xi<0.05} = 4.3 + 0.3\ln s \quad [s\text{ in GeV}^2] \tag{12}$$

3.2 Pomeron-proton total cross section

The pomeron-proton total cross section is related intimately to the renormalized SD cross section through Eq. 11. Fitting the data with this equation not only yields the constant $\sigma_0^{\mathcal{P}p}$ but also verifies the assumed $\sim (M^2)^\epsilon$ energy dependence, where $M^2 = s'$ is the pomeron-proton center of mass energy. From this fit we therefore infer that

$$\sigma_T^{\mathcal{P}p} = 2.6\ (s')^\epsilon\ \text{mb} \quad [s' \text{ in GeV}^2] \tag{13}$$

Thus, the pomeron behaves like a hadron. The ratio of $\sigma_0^{\mathcal{P}p}$ to σ_0^{pp} is

$$\sigma_0^{\mathcal{P}p/pp} = \frac{\sigma_0^{\mathcal{P}p}}{\sigma_0^{pp}} = 0.18 \tag{14}$$

3.3 Triple-pomeron coupling constant

From $\sigma_0^{\mathcal{P}p}$ we obtain the value of the triple-pomeron coupling constant (see Eqs. 5 & 1), assuming that it is independent of t:

$$g(t) = \frac{g(t)\beta(0)}{\beta(0)} = \frac{\sigma_0^{\mathcal{P}p}}{(\sigma_0^{pp})^{\frac{1}{2}}} = 0.69\ \text{mb}^{\frac{1}{2}} = 1.1\ \text{GeV}^{-1} \tag{15}$$

This value of $g(t)$ is almost a factor of two higher than the value $g(t) = 0.364 \pm 0.025\ \text{mb}^{\frac{1}{2}}$ reported in Ref. [8]. This apparent discrepancy is due to the different parameterization ($\epsilon = 0$ and $\sigma_0^{pp} = \sigma_T^{pp}$) used in evaluating $g(t)$ from the data in [8].

If the pomeron couples to quarks [7], the pomeron-quark coupling constant, β_0, may be evaluated by equating the value of $\sigma_0^{p\bar{p}}$ of Eq. 1 with $[3\beta_0]^2$, which yields

$$\beta_0 = \frac{\sqrt{\sigma_0^{p\bar{p}}}}{3(\hbar c)} = 2.0\ \text{GeV}^{-1} \tag{16}$$

The ratio of the triple-pomeron to the pomeron-quark coupling, $g(t)$ to β_0, is given by

$$\frac{g(t)}{\beta_0} = 0.55 \tag{17}$$

3.4 Double diffraction dissociation

In double diffraction dissociation (DD) both nucleons dissociate, as shown in Fig. 3. Assuming pomeron exchange and factorization, the DD cross section may be obtained from the SD and elastic scattering cross sections using Eqs. 11 & 2,

$$\frac{d^3\sigma_{dd}}{dM_1^2\,dM_2^2\,dt} = \frac{1}{d\sigma_{el}/dt}\,\frac{d^2\sigma_1}{dM_1^2dt}\,\frac{d^2\sigma_2}{dM_2^2dt} = \left(\frac{\sigma_0^{\mathcal{P}p}}{4\sqrt{\pi}\hbar c}\right)^2\left(\frac{s^\epsilon}{N_s}\right)^2\frac{e^{b_{dd}t}}{(M_1^2M_2^2)^{1+\epsilon}} \tag{18}$$

where N_s is the integral of the pomeron flux factor at the s-value of the collision and b_{dd} the slope parameter for double-diffraction given by

$$b_{dd} = 2\alpha' ln\left[\frac{s\;s_0}{M_1^2M_2^2}\right] = 2\alpha'\,\Delta y \tag{19}$$

The nucleon form factors drop out in the division of the differential cross sections. In (19), Δy is the rapidity gap between the two diffractive clusters (see Fig. 3). If we now apply the requirement $\Delta y > 2.3$, which corresponds to the coherence requirement for single diffraction $\Delta y_{sd} = ln(s/M^2) > ln(1/0.1) = 2.3$, we obtain the *coherence condition for double diffraction*:

$$\frac{M_1^2\,M_2^2}{s\;s_0} < 0.1 \tag{20}$$

With this condition as a constraint, b_{dd} is positive for all mass combinations. If Δy were to become negative, which would correspond to mass clusters overlapping in rapidity, b_{dd} would also become negative and the cross section would increase with t. We therefore interpret Eq. 20 to mean that coherence breaks down for rapidity gaps smaller than 2.3 units, and integrate Eq. 18 subject to the coherence condition to obtain the total DD cross section (energies are in GeV):

$$\sigma_{dd} = K(s)\int_{M_1^2=1.5}^{0.1s/1.5}\int_{M_2^2=1.5}^{0.1s/M_1^2}\frac{dM_1^2\,dM_2^2}{(M_1^2\;M_2^2)^{1+\epsilon}\;ln(ss_0/M_1^2\,M_2^2)} \tag{21}$$

$$\text{where}\quad K(s) = \frac{1}{2\alpha'}\left(\frac{\sigma_0^{\mathcal{P}p}}{4\sqrt{\pi}\hbar c}\right)^2\left(\frac{s^\epsilon}{N_s}\right)^2$$

Table 1 lists cross sections at several energies calculated using this equation. The decrease of the cross section with energy is due to the faster increase of the elastic relative to the diffractive cross section.

A practical way of measuring the inclusive double diffractive cross section at hadron colliders is to look for events with a *fixed* rapidity gap centered at $y = 0$. Table 2 lists the cross sections expected at the Tevatron, $\sqrt{s} = 1800$ GeV, as a function of the width Δy of such a rapidity gap. These cross sections were calculated from Eq. 21 with $M_{1,max}^2 = M_{2,max}^2 = m_p\,\sqrt{s}\;e^{-\Delta y/2}$. As shown, the cross section decreases slowly as the rapidity gap width increases.

Using the rapidity gap technique, the UA5 collaboration measured the DD cross section at the CERN S$p\bar{p}$S collider and reported values of 3.5 ± 2.5 (4.0 ± 2.2) mb at

Table 1: Total double diffraction cross sections.

$\sqrt{s}$ $[GeV]$	σ_{dd}^{T} $[mb]$
30	3.1
200	2.3
630	1.7
900	1.6
1800	1.3
14000	0.75

Table 2: σ_{dd} versus Δy at the Tevatron.

Δy (central)	$\sigma_{dd}^{\Delta y}$ $[mb]$
2.3	0.59
2.5	0.57
3.0	0.52
3.5	0.47
4.0	0.42
4.5	0.39

$\sqrt{s} = 200$ (900) GeV, respectively [11]. These values are within $1\,\sigma$ of those in Table 1, but are systematically higher. The higher experimental values may be attributed to an underestimate of the detector acceptance for DD events, which was obtained with a Monte Carlo simulation where single diffractive clusters were generated on each side of the rapidity region and were allowed to reach independently and simultaneously mass values up to $M_{max}^2 = 0.05s$. This procedure allows overlapping diffractive clusters in violation of the coherence condition of Eq. 20, resulting in a lower acceptance for DD events and hence a larger cross section.

At the Tevatron, where the energy of $\sqrt{s}$=1800 GeV provides a rapidity range of 15 units, accurate measurements of DD cross sections as a function of rapidity gap width can be performed using minimum bias data triggered by the "beam-beam" counters. Such data are already available in the CDF and D0 experiments. The measurements can best be done by fitting the particle multiplicity distribution in a given region of $\Delta\eta$ centered at $\eta = 0$ and extracting from the fit the number of *excess* events in the zero multiplicity bin. The fraction of these *rapidity gap* events to the total number of events in the sample can then be compared directly with the values in Table 2 divided by the non-diffractive inelastic cross section of 50 mb.

3.5 Double pomeron exchange

In double pomeron exchange (DPE) two pomerons, one from each incoming hadron, interact to form a diffractive cluster of mass M centered at rapidity y_M (Fig. 3). The cross section for DPE is obtained from the SD and total cross sections using factorization (see [10]):

$$\frac{d^4\sigma}{d\xi_1 d\xi_2 dt_1 dt_2} = \frac{1}{\sigma_T^{p\bar{p}}} \frac{d^2\sigma_1}{d\xi_1 dt_1} \frac{d^2\sigma_2}{d\xi_2 dt_2} \tag{22}$$

The mass of the cluster and the rapidity of its centroid are related to the variables $\xi_{1,2}$:

$$M^2 = s\,\xi_1\xi_2 \tag{23}$$

$$y_M = \frac{1}{2}\, ln\frac{\xi_1}{\xi_2}$$

The condition $\xi_{1,2} < 0.1$ for SD translates to the condition

$$M^2 < 0.01s$$

Using Eqs. 1 and 11, and changing the variables from $\xi_{1,2}$ to M^2 and y, we obtain the expression

$$\frac{d^2\sigma}{dM^2 dy_M} = \sigma_0^{p\bar{p}} \left(\frac{\sigma_0^{\mathcal{P}p}}{16\pi(\hbar c)^2}\frac{s^\epsilon}{N_s}\right)^2 \left\{(M^2)^{1+\epsilon}\left[(b_0 + \alpha' ln\frac{s}{M^2})^2 - (2\alpha' y_M)^2\right]\right\}^{-1} \tag{24}$$

where $b_0 = 4.6$ GeV^{-2} is the slope parameter of the simple exponential proton form factor $F^2(t)$, which is used here for simplicity. For a given mass M, y_M varies within the range $\pm\frac{1}{2}ln\frac{M^2}{(0.1)^2 s}$, so that $|2y_M| < ln\frac{s}{M^2}$ and the term in the square brackets is a function decreasing logarithmically with increasing M^2. As a result, the DP cross section falls approximately as $1/M^2$. A numerical integration of this equation for the range 1 GeV$^2 < M^2 < (0.1)^2\, s$ yields the inclusive DPE cross sections of 61, 76, 69 and 50 μb at $\sqrt{s}=$ 50, 630, 1800 and 14000 GeV, respectively. The calculated value of 76 μb at 630 GeV is consistent with the experimental value of 30-150 μb reported by the UA8 experiment [12]. The DP cross section is approximately constant through the entire range from the ISR to the LHC collider energies. On a finer scale, it rises initially with energy and then starts falling as the $\alpha' lns$ term in the denominator becomes comparable to the slope b_0.

4 The structure of the pomeron

The structure of the pomeron has been investigated in $p\bar{p}$ colliders by UA8 [13], which observed diffractive dijets at $\sqrt{s} = 630$ GeV and $|t| \sim 1.5$ GeV^2, and by CDF [14], which searched for and placed upper limits for diffractive dijet and W production at $\sqrt{s} = 1800$ GeV and t close to zero (the CDF data are integrated over t). At HERA, where e^-p collisions occur at $\sqrt{s} \approx 300$ GeV, the quark content of the pomeron has been probed directly with virtual high-Q^2 photons in e^-p deep inelastic scattering (DIS) [15, 16], and the gluon content evaluated from diffractive DIS and photoproduction data[16]. Below, we apply our renormalization procedure to the $p\bar{p}$ collider and HERA results and obtain the pomeron structure function.

4.1 Hard diffraction at $p\bar{p}$ colliders

The study of hard diffractive processes at $p\bar{p}$ colliders was pioneered by the UA8 experiment, which observed high-P_T [13] diffractive dijets in $p + \bar{p} \to p + J_1 + J_2 + X$ for $0.04 < \xi < 0.1$ and $0.9 < |t| < 2.3$ GeV^2 at the CERN $Sp\bar{p}S$ collider at $\sqrt{s} = 630$ GeV. From the η-distribution of the jets, assumed to be due to collisions between the pomeron and antiproton constituents, UA8 estimated that the partonic structure of the pomeron is $\sim 57\%$ *hard* $[6\beta(1-\beta)]$, $\sim 30\%$ *superhard* $[\delta(1-\beta)]$, and $\sim 13\%$ *soft* $[6(1-\beta)^5]$. However, the measured dijet production rate turned out to be [17] much smaller than the rate calculated for a pomeron made of hard quark or gluon constituents obeying the momentum sum rule. The rate calculation was based on a standard pomeron flux in a model [18] which extends factorization to hard processes. The discrepancy between the measured and calculated rates was expressed in terms of a "discrepancy factor", D, by which the normalized hard structure function, $6\beta(1-\beta)$, has to be multiplied in the Monte Carlo calculation to yield the observed rate. This factor was found to be D=$0.46 \pm 0.08 \pm 0.24$ ($0.19 \pm 0.03 \pm 0.10$) for a hard-quark(gluon) dominated pomeron [17]. Clearly, this result shows that the momentum sum rule is not obeyed by the pomeron in this model. However, in the standard (un-renormalized) theory, this apparent discrepancy is not really meaningful, since the pomeron flux normalization is arbitrary due to lack of knowledge of the energy scale in the pomeron propagator, s_0. A more meaningful test of the existence of a unique structure function for the pomeron would be to compare the pomeron structure found by UA8 with that found at other energies or in other processes. As we shall see below, such a comparison of experimental data leads to inconsistencies in the standard flux D-factors, which are resolved by our flux renormalization procedure. By renormalizing the flux, the rates predicted for UA8 decrease by a factor of ~ 4.5, which is the integral of the flux at $\sqrt{s} = 630$ GeV, bringing the D-factors close to unity and therefore into agreement with the momentum sum rule.

At the Tevatron, the CDF Collaboration has used the rapidity gap technique to search for diffractive dijet and W production [14]. While dijet production is sensitive to both the quark and gluon component of the pomeron, W production probes mainly the quark component, since to leading order it occurs through $q\bar{q} \to W$. Since no hard diffraction was observed, the CDF (preliminary) results are expressed in terms of upper limits at 95% CL. In the dijet case, for a hard-gluon pomeron structure the limit on the discrepancy factor is $D < 0.14$ for the standard flux and $D < 1.2$ for the renormalized flux. The standard flux limit excludes the UA8 central value of 0.19, although it overlaps with its 1σ systematic error; the renormalized flux limit simply means that the observation of no signal is consistent with the prediction based on the momentum sum rule. In the W-case, no explicit upper limit is given in [14], but it is stated that the ratio of diffractive to non-diffractive events is $\sim 0\pm$ *a few* %, which for a full hard-quark pomeron structure should be compared with the predictions of $\sim 23\%$ for the standard flux, 17% for a calculation [19] using the flux of [18], and 2.8% for the renormalized flux.

The UA8 and preliminary CDF results are all consistent with the renormalized flux predictions based on a hard pomeron that obeys the momentum sum rule, while the standard flux D-factors are almost mutually inconsistent and are too small compared to the D-factors found at HERA, as will shall see below.

4.2 Deep inelastic diffraction at HERA

Both the H1 [15] and ZEUS [16] Collaborations have reported measurements of the diffractive structure function $F_2^D(Q^2, \xi, \beta)$ (integrated over t, which is not measured), where β (not to be confused with the pomeron-hadron coupling!) is the fraction of the pomeron's momentum carried by the quark being struck. Both experiments find that the ξ-dependence factorizes out and has the form $1/\xi^{1+2\epsilon}$, which is the same as the expression in the pomeron flux factor (see Eq. 6). Moreover, the fits yield $\epsilon \approx 0.1$, which is in agreement with the value measured in *soft* collisions. It therefore appears that *the same pomeron* is involved in hard as in soft collisions.

The quark structure function of the pomeron can be obtained by dividing the measured diffractive structure function by the pomeron flux and by the average charge of the quarks in the pomeron. Such an analysis has been done by both H1 and ZEUS using the standard flux factor. However, since flux renormalization alters the picture, we have reanalyzed the H1 data using the renormalized pomeron flux[1].

H1 integrates the diffractive form factor $F_2^D(Q^2, \xi, \beta)$ over ξ and provides values for the expression

$$\tilde{F}_2^D(Q^2, \beta) = \int_{0.0003}^{0.05} F_2^D(Q^2, \xi, \beta) d\xi \tag{25}$$

[1] *Note added in proof*: The ZEUS data, which are very similar to the H1 data, were not available at the time of the writing of the original manuscript of this paper.

The pomeron structure function is related to $\tilde{F}_2^D(Q^2,\beta)$ by factorization:

$$\tilde{F}_2^D(Q^2,\beta) = \left[\frac{\int_{0.0003}^{0.05} d\xi \int_0^\infty f_{\mathcal{P}/p}(\xi,t)\; dt}{N(s,Q^2,\beta)}\right] F_2^{\mathcal{P}}(Q^2,\beta) \tag{26}$$

The expression in the brackets is the renormalized flux factor. The integral in the numerator has the value 2.0, and the denominator shows explicitly how factorization breaks down due to flux renormalization. This break-down of factorization, which is a direct consequence of unitarization, *does not affect* the ξ distribution. For fixed Q^2 and β, $\xi_{min} = (Q^2/\beta s)$. Therefore, the flux integral is given by (see Eq. 10)

$$N(\xi_{min}) = N(s,Q^2,\beta) \approx \left(\frac{\beta s}{Q^2}\,\xi_0\right)^{2\epsilon} = 3.8\left(\frac{\beta}{Q^2}\right)^{0.23} \tag{27}$$

where we have used $\sqrt{s} = 300$ GeV and $\xi_0 = 0.004$, the value of ξ_{min} for which the flux integral is unity. Since ξ_0 is larger than ξ_{min} for all the (Q^2,β)-bins of the H1 data, flux renormalization must be applied to all the bins.

Assuming now that the pomeron structure function receives contributions from the four lightest quarks, whose average charge squared is 5/18, the quark content of the pomeron is given by

$$f_q^{\mathcal{P}}(Q^2,\beta) = \frac{18}{5}\,F_2^{\mathcal{P}}(Q^2,\beta) \tag{28}$$

The H1 results for $\tilde{F}_2^D(Q^2,\beta)$ are plotted in Fig. 4a as a function of β for four Q^2-bins: Q^2 =8.5, 12, 25 and 50 GeV^2. Using the standard flux, the quark component of the pomeron is $f_q^{\mathcal{P}}(Q^2,\beta) = 1.8\,\tilde{F}_2^D(Q^2,\beta)$ (right-hand axis in Fig. 4a), where the factor of 1.8 is 18/5 divided by 2.0 (the integral of the standard flux factor). The structure in Fig. 4a is rather flat in β, in contrast with the (mostly) hard structure found by UA8, and has a small but significant Q^2-dependence. The average value of the quark content of the pomeron is $\bar{f}_q \sim 1/3$.

Flux renormalization changes this picture significantly. The values of $f_q^{\mathcal{P}}(Q^2,\beta)$ obtained with the renormalized flux are shown in Fig. 4b. The renormalized points reveal a mostly hard structure (similar to that of UA8!) with no visible Q^2 dependence. We take this last fact as an indication that the pomeron *reigns in the kingdom of asymptopia* and compare the data points with the asymptotic momentum fractions expected for any quark-gluon construct by leading-order perturbative QCD, which for n_f quark flavors are

$$f_q = \frac{3n_f}{16+3n_f} \qquad\qquad f_g = \frac{16}{16+3n_f} \tag{29}$$

For $n_f = 4$, $f_q = 3/7$ and $f_g = 4/7$. The quark and gluon components of the pomeron structure are taken to be $f_{q,g}^{\mathcal{P}}(\beta) = f_{q,g}\;[6\beta(1-\beta)]$. The pomeron in this picture is a combination of valence quark and gluon color singlets and its complete structure function, which *obeys the momentum sum rule*, is given by [2]

$$f^{\mathcal{P}}(\beta) = \frac{3}{7}[6\beta(1-\beta)]_q + \frac{4}{7}[6\beta(1-\beta)]_g \tag{30}$$

The data in Fig. 4b are in reasonably good agreement with the quark-fraction of the structure function given by $f_q^{\mathcal{P}}(\beta) = (3/7)[6\beta(1-\beta)]$, except for a small excess at the low-β region. An excess at low-β is expected in this picture to arise from interactions of the photon with the gluonic part of the pomeron through gluon splitting into $q\bar{q}$ pairs. Such interactions, which are suppressed by an order of α_s, result in an *effective* quark β-distribution of the form $3(1-\beta)^2$. We therefore compare in Fig. 4b the data with the distribution

$$f_{q,eff}^{\mathcal{P}}(\beta) = (3/7)[6\beta(1-\beta)] + \alpha_s(4/7)[3(1-\beta)^2] \tag{31}$$

using $\alpha_s = 0.1$. Considering that this distribution involves *no free parameters*, the agreement with the data is remarkable!

5 Conclusion

Regge theory describes well the main features of available hadronic cross sections with one notable exception: the predicted single diffraction cross section rises much faster than the total, leading to violation of unitarity at present hadron collider energies. We have proposed a simple phenomenological unitarization procedure, which slows down the rise of the SD cross section to below that of the total and brings agreement between theoretical predictions and all available experimental data for $pp/p\bar{p}$ SD, DD and DPE cross sections. This procedure, which leaves intact the ξ and t dependence of single diffraction dissociation, consists of *renormalizing* the "pomeron flux factor" in the triple-pomeron amplitude by resetting its integral over all diffractive phase space to unity when it becomes larger than one. By applying our renormalization procedure to hard diffractive production at hadron colliders and to diffractive DIS at HERA, we obtain a consistent parameter-free picture of the complete pomeron partonic structure, in which the pomeron appears as a momentum sum-rule obeying combination of valence di-quark and di-gluon color-singlets in a ratio of $3 \div 4$, as suggested by asymptopia for four quark flavors.

[2] *Note added in proof*: From a flux independent measurement of the g/q ratio of the pomeron hard partonic content based on the combined results of diffractive DIS and photoproduction data, ZEUS reported [16] that between 30% and 80% of the momentum of the pomeron carried by partons is due to hard gluons, which is consistent with Eq. 30.

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Figure 1: Total $pp/p\bar{p}$ single diffraction cross section data (*both* sides) for $\xi < 0.05$ compared with predictions based on the standard and the renormalized pomeron flux.

Figure 2: Feynman diagrams for $p\bar{p}$ total, elastic, and single diffraction dissociation cross sections, including the triple-pomeron diagram for single diffraction.

Figure 3: Feynman diagrams, and rapidity regions occupied by the diffractive clusters, for double diffraction dissociation and for double pomeron exchange.

Figure 4: (a) The diffractive structure function measured by H1 at HERA; the right-hand y-axis gives the pomeron quark content obtained with the standard flux assuming 4 quark flavors. (b) The pomeron quark structure function obtained using the renormalized pomeron flux.