



Some Examples of Chiral Moduli Spaces and Dynamical Supersymmetry Breaking

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Abstract

We investigate the low-energy dynamics of $SU(N)$ gauge theories with one antisymmetric tensor field, $N - 4 + N_f$ antifundamentals and N_f fundamentals, for $N_f \leq 3$. For $N_f = 3$ we construct the quantum moduli space, and for $N_f < 3$ we find the exact quantum superpotentials. We find two large classes of models with dynamical supersymmetry breaking. The odd N theories break supersymmetry once appropriate mass terms are added in the superpotential. The even N theories break supersymmetry after gauging an extra chiral $U(1)$ symmetry.



1 Introduction

There are two motivations for studying non-perturbative supersymmetry (SUSY) breaking. Firstly, it could explain why the electroweak scale (M_W) is so much smaller than the GUT or Planck scale (M_{Pl}). This could happen if the supersymmetry breaking scale is tied to the electroweak breaking scale [1]. The non-perturbative breaking would then relate M_W/M_{Pl} to the logarithmic running of dimensionless coupling constants. Supersymmetry would thus provide an explanation for both the fine tuning and naturalness problems associated with the ratio M_W/M_{Pl} . Secondly, since only chiral gauge theories can undergo dynamical supersymmetry breaking, its study could shed some light on the behaviour of non-perturbative chiral gauge theories - a subject of interest from other points of view as well.

The past year has seen some spectacular progress in our understanding of the non-perturbative behaviour of SUSY gauge theories (for a review see [2] and references therein). Most of it has been in vector-like theories. In this paper we extend these ideas to some chiral gauge theories as well, uncovering in the process several examples of dynamical supersymmetry breaking. We will present some of our important results along with a few details here. More results and details will follow in a subsequent paper.

Our general strategy is as follows. We will restrict ourselves to theories in which the scale of supersymmetry breaking is lower than the strong coupling scale (Λ) of the gauge theory¹. In such theories the heavy degrees of freedom can be integrated out at the scale Λ and a supersymmetric effective theory can be constructed in terms of the light fields. This effective theory can then be used to study the breaking of supersymmetry. If one is interested in explicitly calculating the vacuum energy and the spectrum, knowledge of both the Kähler potential and superpotential are necessary. However the Kähler potential is not needed in detail if one only wants to show that supersymmetry is broken. For this purpose it is enough to ensure that the Kähler potential has no singularities in terms of the light fields, i.e. the moduli. Supersymmetry breaking can then be established by analyzing the superpotential. Fortunately, a great deal can be said about the superpotential non-perturbatively, while the Kähler potential in $N = 1$ SUSY gauge theories remains poorly understood. Guided by these observations we first identify the correct moduli fields in the low-energy effective lagrangian and argue that the Kähler potential does not contain any singularities (strictly speaking, we will only establish the absence of singularities for finite values of the moduli). Then we turn our attention to the superpotential and investigate the question of supersymmetry breaking. The strategy discussed above is very similar to that of Intriligator, Seiberg and Shenker [3].

In this paper, we consider some simple chiral gauge theories [4] - those with gauge group

¹In practice we will achieve this by adding appropriately small terms in the superpotential.

$SU(N)$ and one antisymmetric tensor field, N_f fields in the fundamental and $N - 4 + N_f$ fields in the antifundamental representations. While the analysis is quite different for N odd and even, we find that in both cases for $N_f = 3$ these theories have a smooth moduli space in terms of appropriately identified variables. The classical singularities (present for example at the point where the gauge symmetry is restored) get smoothed out quantum mechanically. Our results in this regard are analogous to those obtained by Seiberg for SUSY QCD with $N_f = N_c$ [5].

Having established the quantum moduli spaces for these cases, we then add various terms to the superpotential and study the behaviour of these theories. In particular, by adding mass terms we find the quantum superpotentials for theories with $N_f \leq 3$. In fact we find that for appropriate mass terms the $SU(2k + 1)$ theories break supersymmetry. This yields a large class of models which exhibit supersymmetry breaking, some of which (with $N_f = 2$) are calculable. For the $SU(2k)$ case we have not found any examples of SUSY breaking in this manner. However by starting with the $SU(2k + 1)$ theories and breaking the gauge symmetry down to $SU(2k) \times U(1)$ one arrives at a closely related set of theories. An investigation shows that with appropriate Yukawa terms they dynamically break supersymmetry.

This letter is organized as follows. In sections 2, 3 and 4 we consider the $SU(2k + 1)$, $SU(2k)$ and $SU(2k) \times U(1)$ theories respectively. We end in section 5 with conclusions and some comments.

2 $SU(2k+1)$

In this section we consider the nonperturbative low-energy dynamics of models based on the gauge group $SU(2k + 1)$, with an antisymmetric tensor $A_{\alpha\beta}$, $2k - 3 + N_f$ antifundamentals \bar{Q}_i^a , ($i = 1, \dots, 2k - 3 + N_f$), and N_f fundamentals Q_a^a , ($a = 1, \dots, N_f$). In this paper we restrict ourselves to $N_f \leq 3$. In the absence of any superpotential the classical $SU(N)$ theory has a global $SU(N_f)_L \times SU(N - 4 + N_f)_R \times U(1)_Q \times U(1)_{\bar{Q}} \times U(1)_A \times U(1)_R$ symmetry. The charges of the fundamental fields and the coefficients of the anomalies (i.e. the "charges" of the strong coupling scale Λ^{b_0} , with $b_0 = 2N + 3 - N_f$ the first coefficient of the beta function) of the $U(1)$ -symmetries are:

	$U(1)_Q$	$U(1)_{\bar{Q}}$	$U(1)_A$	$U(1)_R$	
Q	1	0	0	0	
\bar{Q}	0	1	0	0	
A	0	0	1	0	
Λ^{b_0}	N_f	$N - 4 + N_f$	$N - 2$	$6 - 2N_f$	(2.1)

We start with the $N_f = 3$ case. Subsequently, we will add mass terms and flow to theories with a fewer number of flavors. The classical $N_f = 3$ theory has flat directions and hence

infinitely many inequivalent ground states. These flat directions can be described by the following gauge invariant operators²:

$$\begin{aligned}
M_i^a &= \bar{Q}_i^\alpha Q_\alpha^a \\
X_{ij} &= A_{\alpha\beta} \bar{Q}_i^\alpha \bar{Q}_j^\beta \\
Y^a &= Q_{\alpha_{2k+1}}^a \epsilon^{\alpha_1 \dots \alpha_{2k+1}} A_{\alpha_1 \alpha_2 \dots \alpha_{2k-1} \alpha_{2k}} \\
Z &= \epsilon^{\alpha_1 \dots \alpha_{2k+1}} A_{\alpha_1 \alpha_2 \dots \alpha_{2k-1} \alpha_{2k-2}} Q_{\alpha_{2k-1}}^a Q_{\alpha_{2k}}^b Q_{\alpha_{2k+1}}^c \epsilon_{abc} .
\end{aligned} \tag{2.2}$$

The inequivalent ground states correspond to different expectation values of these moduli. Not all the fields in (2.2) are independent. There is one constraint relating them, which follows from the Bose symmetry of the superfields. It is given by³:

$$Y \cdot M^2 \cdot X^{k-1} = c Z \text{ Pf} X , \tag{2.3}$$

where $\text{Pf} X \equiv \epsilon^{i_1 \dots i_{2k}} X_{i_1 i_2} \dots X_{i_{2k-1} i_{2k}}$, $Y \cdot M^2 \cdot X^{k-1} \equiv \epsilon_{abc} Y^a M_{i_1}^b M_{i_2}^c \epsilon^{i_1 \dots i_{2k}} X_{i_3 i_4} \dots X_{i_{2k-1} i_{2k}}$ and $c = k/3$.

Symmetry arguments show that for $N_f = 3$ no superpotential can be generated dynamically. Thus the vacuum degeneracy must persist and the quantum theory must have a moduli space of ground states. Considerations similar to those in supersymmetric QCD with $N_f = N_c$ [5] suggest that the classical constraint (2.3) is modified by non-perturbative effects, and becomes

$$Y \cdot M^2 \cdot X^{k-1} - c Z \text{ Pf} X = \Lambda^{4k+2} , \tag{2.4}$$

with Λ being the strong coupling scale of the theory.

As in the case of supersymmetric QCD this quantum modification meets several non-trivial tests. For example the fields on the quantum-deformed moduli space saturate the 't Hooft conditions for the unbroken global symmetries at various points of enhanced symmetry (the maximal enhanced symmetry is $SU(3)_L \times SP(2k)_R \times U(1)_R \times U(1)$). Furthermore, as we show below, on integrating out one of the quark flavors the instanton generated superpotential for the $N_f = 2$ case is correctly reproduced. We regard these tests as fairly persuasive and so will assume that the modified constraint (2.4) is correct. This constraint can be implemented by adding a term in the superpotential of the form:

$$W_{N_f=3} = L (Y \cdot M^2 \cdot X^{k-1} - c Z \text{ Pf} X - \Lambda^{4k+2}) , \tag{2.5}$$

with L being a Lagrange multiplier.

The low-energy effective Lagrangian can then be described in terms of the moduli fields (2.2) subject to the constraint (2.5). The quantum modification to the constraint (2.5) results

²It can be shown that all other invariants in this theory are products of the ones given in (2.2).

³This constraint is also needed to correctly account for the total number of degrees of freedom.

in smoothing out the singularities present classically at points of partially enhanced gauge symmetry. Therefore no fields other than the moduli become massless in any finite region of the quantum moduli space. Since singularities of the Kähler potential are due to the appearance of extra massless states, we are lead to conclude that the Kähler potential in terms of fields (2.2) is not singular for any finite values of the moduli.

One caveat needs to be added to the discussion of the previous paragraph. Strictly speaking, the points of partially enhanced gauge symmetry are removed from the quantum moduli space (2.4) only for finite moduli vevs. But these points can still be reached when some moduli become infinite (while others go to zero, in a way consistent with the quantum-modified constraint)⁴. In this limit some subgroup H of the gauge group is restored, with both the H gauge coupling and the H -breaking vevs tending to zero. The massless, weakly coupled gauge bosons of the restored gauge group descend into the low-energy theory, causing a singularity in the Kähler potential⁵. The correct low-energy degrees of freedom are then given by the weakly coupled quark and vector superfields. We will need to worry about these singularities at infinity in our discussion of supersymmetry breaking below.

Having understood the quantum moduli space and identified the appropriate moduli fields we now turn to perturbing this theory by adding various terms to the superpotential. By adding a mass perturbation for the third flavor and integrating out the heavy fields, we find the superpotential for the $N_f = 2$ theory :

$$W_{N_f=2} = \frac{\Lambda_{(2)}^{4k+3}}{\epsilon_{ac} Y^a M_{j_1}^c \epsilon^{j_1 \dots j_{2k-1}} X_{j_2 j_3} \dots X_{j_{2k-2} j_{2k-1}}} \quad (2.6)$$

where we have absorbed a numerical coefficient in the definition of the low-energy Λ ($\Lambda_{(2)}^{4k+3} = m\Lambda^{4k+2}$). The fields appearing in W are the $N_f = 2$ analogues of the fields appearing in (2.2). This superpotential has the simple physical interpretation of being induced by a one instanton term in the gauge theory.

Let us now perturb the $N_f = 2$ superpotential (2.6) by adding mass and Yukawa terms

$$\delta W = m_c^i M_i^c + \lambda^{ij} X_{ij}. \quad (2.7)$$

If SUSY is to remain unbroken, the superpotential must be an extremum with respect to all the fields. On extremising with respect to the mesons M_i^c we find that :

$$m_c^i = \frac{\Lambda_2^{4k+3}}{(Y \cdot M \cdot X^{(2k-2)/2})^2} Y_c \epsilon^{ij_2 \dots j_{2k-1}} X_{j_2 j_3} \dots X_{j_{2k-2} j_{2k-1}}. \quad (2.8)$$

⁴We thank M. Dine and Y. Shirman for a related discussion.

⁵This weak-coupling singularity can be explicitly seen in some $N = 2$ models, where the Kähler potential is known [6].

But these equations cannot be satisfied for a rank 2 mass matrix. To see this consider starting with a diagonal mass matrix (with the index i in (2.7) taking only two values, $2k - 2$ and $2k - 1$ respectively). Then (2.8) can be satisfied only if two contradictory conditions are met:

$$\frac{Y_1}{Y_2} = 0, \quad \text{and} \quad \frac{Y_2}{Y_1} = 0. \quad (2.9)$$

Clearly this is impossible for any values of Y_1 and Y_2 . Since we have already argued that the Kähler potential has no singularities for finite values of the moduli ⁶, we conclude that there is no SUSY preserving minimum in this region of moduli space. However, to establish SUSY breaking we need to also rule out the possibility that SUSY is restored when some of the moduli go to infinity. As was discussed above, sometimes the Kähler potential may be singular along such runaway directions. Thus, even though the superpotential cannot be extremized, the vacuum energy may vanish and SUSY may be restored. However, since these theories with the superpotential (2.7) have no classical flat directions, the appearance of such runaway directions is extremely improbable. We thus expect that these theories break supersymmetry.

We close this section with a few comments.

Firstly, for small $m \ll \Lambda$, $\lambda \ll 1$, the minimum of the scalar potential of the $N_f = 2$ theory is expected to be in the semiclassical region and the resulting models are therefore calculable. Readers primarily interested in examples of SUSY breaking may note that these models can be understood simply without recourse to the preceding discussion of moduli space etc. In this case the gauge symmetry is completely broken and the non-perturbative superpotential can be understood as simply arising from a single instanton effect.

Secondly, it has long been suggested [4], [7], that the $SU(2k + 1)$ theories with $2k - 3$ antifundamental fields break supersymmetry. These theories can be thought of as the $m \gg \Lambda$ limit of the theories discussed above. In this limit we cannot strictly speaking make any definite statements, nevertheless our results indicate that supersymmetry breaking persists in this case as well⁷.

3 $SU(2k)$

In this section we investigate the even- N theories with antisymmetric tensors. We keep the discussion brief since it closely parallels that in the previous section. Once again we restrict

⁶Although this was strictly shown for the theory with $N_f = 3$ and without any mass terms in the superpotential, we expect the conclusion to be true even after the mass terms are added. After giving mass to some fields one expects some of the moduli fields to get mass rather than extra massless states to appear. In fact, arguments similar to the ones following eq.(2.8) hold for the three-flavor case as well.

⁷Murayama[8] had suggested that the $SU(5)$ model in this class can be analysed by adding an extra flavor with a mass term. Our analysis above is very close in spirit to his. We have chosen to elaborate on the $N_f = 2$ case since this yields calculable models.

ourselves to $N_f \leq 3$. We find a quantum moduli space for $N_f = 3$, and dynamically generated superpotentials for $N_f < 3$. From the point of view of constructing models for SUSY breaking the results of this section will be primarily interesting as a stepping stone for constructing the $SU(2k) \times U(1)$ models discussed in the next section.

The fundamental fields of the $SU(2k)$ theory are the antisymmetric tensor $A_{\alpha\beta}$, $2k-4+N_f$ antifundamentals \bar{Q}_i^α , ($i = 1, \dots, 2k-4+N_f$), and N_f fundamentals Q_a^α , ($a = 1, \dots, N_f$). The classical moduli space for $N_f = 3$ is described by the following gauge invariant fields⁸:

$$\begin{aligned} M_i^a &= \bar{Q}_i^\alpha Q_\alpha^a \\ X_{ij} &= A_{\alpha\beta} \bar{Q}_i^\alpha \bar{Q}_j^\beta \\ Y_a &= \epsilon_{abc} Q_{\alpha_1}^b Q_{\alpha_2}^c \epsilon^{\alpha_1 \dots \alpha_{2k}} A_{\alpha_3 \alpha_4} \dots A_{\alpha_{2k-1} \alpha_{2k}} \\ \text{Pf} A &= \epsilon^{\alpha_1 \dots \alpha_{2k}} A_{\alpha_1 \alpha_2} \dots A_{\alpha_{2k-1} \alpha_{2k}} \end{aligned} \quad (3.1)$$

These invariants are subject to a constraint, which is modified by nonperturbative effects and becomes:

$$X^{k-1} \cdot M \cdot Y - b M^3 \cdot X^{k-2} \text{Pf} A = \Lambda^{4k} \quad (3.2)$$

Here $M^3 \cdot X^{k-2} \equiv \epsilon_{abc} M_{i_1}^a M_{i_2}^b M_{i_3}^c \epsilon^{i_1 \dots i_{2k-1}} X_{i_4 i_5} \dots X_{i_{2k-2} i_{2k-1}}$, $X^{k-1} \cdot M \cdot Y \equiv Y_a M_{i_1}^a X_{i_2 i_3} \dots X_{i_{2k-2} i_{2k-1}} \epsilon^{i_1 \dots i_{2k-1}}$ and $b = (k-1)/(3k)$. 't Hooft's anomaly matching conditions are saturated by the moduli fields subject to the constraint (3.2) (the maximal enhanced symmetry in this case is $SU(3)_V \times SP(2k-4) \times U(1)_R$).

The dynamical superpotentials for $N_f < 3$ ⁹ can be found by integrating out the extra flavors. For $N_f = 2$ we find the instanton induced superpotential

$$W_{N_f=2} = \frac{\Lambda_{(2)}^{4k+1}}{Y \text{Pf} X - 3b \epsilon_{ac} M_{i_1}^a M_{i_2}^c \epsilon^{i_1 \dots i_{2k-2}} X_{i_3 i_4} \dots X_{i_{2k-3} i_{2k-2}} \text{Pf} A} \quad (3.3)$$

The singularity at $Y \text{Pf} X = M^2 \cdot X^{k-2} \text{Pf} A$ reflects the existence of points on the moduli space with an unbroken $SU(2)$ gauge symmetry. For $N_f = 1$ the superpotential is due to gaugino condensation in the unbroken $SU(2)$ gauge group:

$$W_{N_f=1} = \frac{\Lambda_{(1)}^{2k+1}}{\left[M_{i_1} X_{i_2 i_3} \dots X_{i_{2k-4} i_{2k-3}} \epsilon^{i_1 \dots i_{2k-3}} \text{Pf} A \right]^{1/2}} \quad (3.4)$$

Finally, the $N_f = 0$ superpotential is induced by $SP(4)$ gaugino condensation [4]:

$$W_{N_f=0} = \frac{\Lambda_{(0)}^{(4k+3)/3}}{[\text{Pf} A \text{Pf} X]^{1/3}} \quad (3.5)$$

⁸As in the odd case, one can show that all other invariants are products of (3.1).

⁹The moduli fields appearing in the superpotentials for $N_f < 3$ are simply the restrictions of the $N_f = 3$ invariants (3.1) to a smaller number of flavors.

We have investigated these theories by adding various terms to the superpotentials, (3.3), (3.4) and (3.5), but have not found any examples of SUSY breaking. For example on adding a perturbation $\delta W = \text{Pf}A + \lambda^{ij}X_{ij}$ to (3.5) we can see that the $N_f = 0$ theory has a supersymmetric ground state.

4 $SU(2k) \times U(1)$

In this section we investigate the behaviour of theories with gauge group $SU(2k) \times U(1)$ and find a large class of models that do break SUSY. These theories can be obtained by starting with the $SU(2k+1)$ theories of section 2, and breaking the gauge symmetry down to $SU(2k) \times U(1)$. The symmetry breaking can be accomplished, for example, by adding an additional heavy field in the adjoint of $SU(2k+1)$. Since the $SU(2k+1)$ theory is expected to break SUSY the resulting $SU(2k) \times U(1)$ theory is a natural candidate for SUSY breaking. As we show below SUSY breaking does indeed occur in these models.

Our starting point will be the $SU(2k+1)$ models of section 2 with $N_f = 0$. The $SU(2k) \times U(1)$ theory resulting from symmetry breaking is then given by the $SU(2k)$ theory with $N_f = 1$ of section 3 with additional $2k - 3$ $SU(2k)$ singlets, S_i , ($i = 1, \dots, 2k - 3$). The $U(1)$ charges of the fields are $\bar{Q} \sim -1$, $S \sim 2k$, $A \sim 2$, $Q \sim 1 - 2k$.

We will only consider theories where the $U(1)$ gauge coupling is weak at the scale at which the $SU(2k)$ coupling gets strong¹⁰. In this case the low energy lagrangian can be simply constructed in two steps. First one can neglect the $U(1)$ interaction and construct the effective lagrangian of the $SU(2k)$ theory. Then one can gauge the $U(1)$ symmetry in this lagrangian, integrate out the resulting heavy particles and construct the final low energy lagrangian. Since the first step was already carried out in section 3 (the additional $SU(2k)$ singlets clearly do not pose any problems) we can directly turn to gauging the $U(1)$ symmetry in the lagrangian containing the $SU(2k)$ moduli fields, (3.1) and the singlets, S_i .

If the $U(1)$ symmetry is broken the relevant degrees of freedom in the low energy lagrangian are the $U(1)$ invariants built out of the $SU(2k)$ -invariant moduli. The $SU(2k)$ invariant moduli fields have charges $\text{Pf}A \sim 2k$, $M_i \sim -2k$, $X_{ij} \sim 0$, and $S_i \sim 2k$ under the $U(1)$ symmetry. Out of them we can build three types of $SU(2k) \times U(1)$ invariants

$$A_i = M_i \text{Pf}A, \quad B_{ij} = S_i M_j, \quad \text{and} \quad X_{ij} . \quad (4.1)$$

These fields are not all independent but obey the constraints¹¹

$$B_{ij}A_k - B_{ik}A_j = 0. \quad (4.2)$$

¹⁰Since the $U(1)$ coupling is irrelevant in the infrared it only gets weaker at lower energies.

¹¹These constraints are not all independent. However, adding a redundant set of constraints only amounts to redefining the Lagrange multipliers for the independent constraints and does not alter any of the conclusions.

Unlike the cases encountered previously, these constraints are not modified quantum - mechanically. This is expected due to the non-asymptotically free nature of the $U(1)$ gauge interaction and can also be seen explicitly by symmetry considerations. The moduli fields in the final low energy theory are thus given by X_{ij} , A_i and B_{ij} subject to (4.2). The Kähler potential in terms of these fields has singularities which occur on the submanifold where the $U(1)$ symmetry is restored. There extra massless particles (e.g. the $U(1)$ gauge boson) descend into the low energy theory resulting in the singular Kähler potential. In our analysis of SUSY breaking we will have to consider this submanifold separately.

The dynamical superpotential (3.4) can be written in terms of the $U(1)$ invariant fields as:

$$W_{dyn} = \frac{\Lambda_{(1)}^{2k+1}}{\left[A_{i_1} X_{i_2 i_3} \dots X_{i_{2k-4} i_{2k-3}} \epsilon^{i_1 \dots i_{2k-3}} \right]^{1/2}} . \quad (4.3)$$

Let us add to this the Yukawa couplings

$$W_{tree} = \gamma^{ij} X_{ij} + \lambda^{ij} B_{ij}, \quad (4.4)$$

and implement the constraints (4.2) via a Lagrange multiplier,

$$W_{constr} = L_{l_1 \dots l_{2k-3}}^i \epsilon^{l_1 \dots l_{2k-3}} B_{il_{2k-4}} A_{l_{2k-3}} .$$

We are now ready to show that the $SU(2k) \times U(1)$ model breaks supersymmetry. For simplicity, we first take $k = 3$. In this case $W_{constr} = L_j^i \epsilon^{jkl} B_{ik} A_l$ and the equations of motion for B_{ij} are $L_k^i \epsilon^{klj} A_l = \lambda^{ij}$. Solving the $i = 2, 3$ equations for L_k^2 and L_k^3 , and substituting back into the $i = 1$ equation we find three conditions; $\lambda^{11} + \lambda^{l2}(A_2/A_1) + \lambda^{l3}(A_3/A_1) = 0$, for $l = 1, 2, 3$. Clearly these cannot be satisfied when λ^{ij} has rank three (to see this consider going to a basis where λ^{ij} is diagonal). This argument can now be easily generalized for $k > 3$. Solving the equations of motion for the Lagrange multipliers one obtains a similar consistency condition $\lambda^{i1} + \sum_{j=2}^{2k-3} \lambda^{ij}(A_j/A_1) = 0$, $i = 1, \dots, 2k - 3$, which again cannot be satisfied by a non-degenerate Yukawa matrix. Thus, in all these models with nondegenerate Yukawa couplings we expect that SUSY is broken. We should add though, that, as in the $SU(2k + 1)$ case we cannot strictly rule out the possibility of runaway directions. Such directions might arise since the Kähler potential can become singular when some of the moduli vevs go to infinity. The vacuum energy then may go to zero, even though the superpotential is not extremized. However, as in the $SU(2k + 1)$ case, the $SU(2k) \times U(1)$ theories with superpotential (4.4) have no classical flat directions and we do not expect such runaway directions to be induced quantum-mechanically. We thus expect that supersymmetry is broken.

As mentioned earlier, we need to consider the submanifold on which the $U(1)$ symmetry is restored separately (on this submanifold the vevs of all $U(1)$ -charged $SU(2k)$ moduli, M_i , S_i

and PfA, go to zero). The correct degrees of freedom around any point in this submanifold are the $SU(2k)$ moduli fields and the $U(1)$ gauge field. By varying the full superpotential, which is the sum of (4.3) and (4.4), with respect to the $SU(2k)$ moduli it is easy to see, that there is no way in which S_i, M_i and PfA can tend to zero while preserving supersymmetry¹². Thus supersymmetry cannot be restored on this submanifold and these theories do indeed break SUSY.

The case $k = 2$ corresponds to the simplest model in this class. It is worth discussing in some more detail. The theory has a gauge group $SU(4) \times U(1)$, and only three $SU(4)$ moduli fields, denoted by M , PfA and S respectively¹³. The full superpotential (corresponding to a sum of the terms (4.3) and (4.4)) is given by:

$$W = \frac{\Lambda_{(1)}^5}{\sqrt{M \text{ Pf } A}} + \lambda S M . \quad (4.5)$$

The $U(1)$ invariants correspond precisely to the two combinations $M \text{ Pf } A$ and $S M$ and no constraints are needed in this case. Extremising the superpotential with respect to these fields clearly shows that SUSY is broken¹⁴. This model is among the simplest examples of SUSY breaking we know of. The superpotential (4.5) preserves an R-symmetry. On adding another term, $M \text{ Pf } A$, one finds that SUSY breaking persists even though the R-symmetry is now broken. This is another example of supersymmetry breaking without R-symmetry [9].

We end this section by commenting on the importance of correctly incorporating constraints (for example (4.2)) into the analysis when testing for SUSY breaking, especially with respect to runaway directions. As a toy model, consider a theory with the nonsingular Kähler potential $K = X^*X + Y^*Y$ and superpotential $W = X + L(XY - 1)$. If we first solve the constraint for Y , $Y = 1/X$, and then minimize the superpotential with respect to X , we would conclude that the theory breaks supersymmetry since $dW/dX = 1$. However, by solving the constraint we introduced a singularity in the Kähler metric, $K_{XX^*} \sim 1/(X^*X)^2$ at $X \rightarrow 0$, or $Y \rightarrow \infty$. Therefore the inverse Kähler metric has a zero eigenvalue and the model has a runaway direction. Had we kept the constraint, this behaviour would follow solely from the superpotential.

5 Conclusion

In this letter we studied the low-energy dynamics of chiral $SU(N)$ gauge theories with one antisymmetric tensor and $N - 4 + N_f$ antifundamental and N_f fundamental fields. We found

¹²This is true even if one allows for runaway directions.

¹³In this case the field X_{ij} (3.1) is absent.

¹⁴We do need to consider the points of restored $U(1)$ symmetry separately but as above they do not change the conclusion.

the quantum moduli spaces and exact superpotentials for the models with $N_f \leq 3$. We also found two large classes of models that broke supersymmetry dynamically. For the odd- N models this breaking occurred when suitable mass terms were added to the superpotential. For the even- N models the supersymmetry breaking occurred after gauging an additional chiral $U(1)$ symmetry. These results suggest that, perhaps, the set of theories which undergo dynamical supersymmetry breaking is quite large and might even be a fairly large subclass of all chiral SUSY gauge theories.

Clearly, much more needs to be done to further these investigations. In the short run it would be interesting to extend this analysis to a larger number of flavors, $N_f > 4$,¹⁵ hopefully in the recently proposed framework of duality [10], [11], [12]. From a phenomenological point of view it would be interesting to incorporate these theories into visible sector SUSY extensions of the standard model [13]. In the longer run one would like to understand better the essential ingredients required for supersymmetry breaking and attempt a more systematic construction of the possibly large class of theories that exhibit this phenomenon.

While completing this paper, we became aware of the recent preprint [14] where some of our results were obtained.

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¹⁵The four extra flavor case is analogous to the $N_f = N_c + 1$ case of SUSY QCD. The low-energy theory is a theory of the massless composites (the analog of the invariants (2.2, 3.1) for $N_f = 4$). At the origin of moduli space the theory is confining, with unbroken chiral symmetry. The anomaly matching conditions are saturated by the massless composites. We do not consider details here, since the $N_f = 3$ case is sufficient to establish nonsingularity of the Kähler potential.

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