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Hadronization of $b \to c\bar{c}s$

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Abstract

The $b \to c\bar{c}s$ transition is usually believed to hadronize predominantly in $\overline{B} \to X_c D_s^{(*)-}$ with the $D_s^{(*)-}$ originating from the virtual W. We demonstrate in a variety of independent ways that other hadronization processes cannot be neglected. The invariant mass of $\bar{c}s$ has sizable phase-space beyond $m_D + m_K$. The rate for $\overline{B} \to D\overline{D}\,\overline{K}X$ could be significant and should not be ignored as was done in previous experimental analyses. We estimate the number of charmed hadrons per B-decay, n_c, to be ≈ 1.3 to higher accuracy than obtained in previous investigations. Even though n_c is currently measured to be about 1.1, observing a significant $\overline{B} \to D\overline{D}\,\overline{K}X$ would support $n_c \approx 1.3$. Many testable consequences result, some of which we discuss.

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At present, there appears to be a conflict between experiment and theory for fitting both the inclusive semileptonic branching ratio and the number of charmed hadrons per B decay [1–8],

$$n_c = 1 - B(b \to \text{no charm}) + B(b \to c\bar{c}s') \approx 1 + B(b \to c\bar{c}s') \ . \tag{1}$$

The prime indicates that the corresponding Cabibbo-suppressed mode is included. Experimentally the inclusive semileptonic BR has been measured accurately to be [9]

$$B(\overline{B} \to X\ell\nu) = (10.4 \pm 0.4)\% \ , \tag{2}$$

and n_c is measured as [9]

$$n_c = 1.10 \pm 0.06 \ . \tag{3}$$

A value of $B(b \to c\bar{c}s') \approx 0.1$, suggested by Eqs. (1) and (3), would lead to a theoretical prediction of $B(\overline{B} \to X\ell\nu)$ that is too large—i.e., inconsistent with its measured value (2). On the other hand, theory predicts $n_c \approx 1.3$ when the observed semileptonic BR is used as input, which is demonstrated below. Thus a conflict arises between (2) and (3) [2,3].

Recently, Bagan et al. and Voloshin made progress on the theoretical side [4–7]. Bagan et al. [4–6] performed a complete next-to-leading order analysis of inclusive B decays, which included important final state mass effects in the QCD corrections. The predicted $B(\overline{B} \to X\ell\nu)$ agrees with (2), within uncertainties that are dominated by renormalization scale-dependences in the perturbative calculation [4–6]. Simultaneously, an enhancement of $B(b \to c\bar{c}s')$ was found [4–7], albeit with considerable uncertainties. Table I summarizes these recent theoretical findings [6]. The main sources of the large errors in those studies are dependence on the renormalization-scale ($m_b/2 < \mu < 2\ m_b$), dependence on the renormalization-scheme ($\overline{MS}$ versus pole mass), and uncertainties in quark masses. Although this theoretical analysis hints that n_c may be larger than currently measured [5–7,2,3], it is difficult to draw firm conclusions from this direct calculation of $B(b \to c\bar{c}s')$ in view of the large uncertainties.

It should also be stressed at this point, that the experimental determination of $B(\overline{B} \to X\ell\nu)$ is reliable and accurate. In contrast, the measurement of n_c is a sum over the inclusive yields of many charmed hadron species in B decays. It is thus prone to large uncertainties, perhaps larger than currently realized.

Figure 1 displays the discrepancy graphically. We discuss now in some detail how the theoretical curve has been generated. Our objective is to draw the most accurate curve of n_c versus semi-electronic BR with presently available theoretical calculations. We do not use the prediction for $B(b \to c\bar{c}s')$ because it involves large errors, but rather proceed as follows. We start with

$$B(b \to c) = 1 - B(b \to \text{no charm}) \; , \tag{4}$$

where $B(b \to \text{no charm})$ is small, typically at the percent level. We take

$$r_{\not{c}} \equiv \Gamma(b \to \text{no charm})/\Gamma(b \to ce\nu) = 0.25 \pm 0.10, \tag{5}$$

to account for the small fraction of $b \to s + \text{no charm}$ [10] and charmless $b \to u$ transitions. Furthermore we use

$$r_\tau \equiv \frac{\Gamma(b \to c\tau\nu)}{\Gamma(b \to ce\nu)} = 0.25 \; , \tag{6}$$

which is in accordance with the result of Ref. [11] and also agrees with a recent ALEPH measurement [12],

$$B(b \to X\tau\nu) = (2.75 \pm 0.30 \pm 0.37)\% \; . \tag{7}$$

The last required ratio is $\Gamma(b \to c\bar{u}d')/\Gamma(b \to ce\nu)$ where the dominant uncertainties in $|V_{cb}|^2$ and in fermion masses cancel. Bagan et al. [4] have presented a complete computation of this quantity in next-to-leading logarithmic approximation taking all final-state charm quark mass effects into account. Based on this perturbative calculation and also including nonperturbative corrections up to $\mathcal{O}(1/m_b^2)$, the analysis of [4] yields,

$$r_{ud} \equiv \frac{\Gamma(b \to c\bar{u}d')}{\Gamma(b \to ce\nu)} = 4.0 \pm 0.4 \; . \tag{8}$$

Here the error comes almost entirely from the renormalization-scale uncertainty and represents a conservative estimate when working to order $\mathcal{O}(1/m_b^2)$. Because nonperturbative effects at $\mathcal{O}(1/m_b^3)$ could introduce rate-differences at the 10% level between B^- and $\overline{B}_d$ decays governed by $b \to c\bar{u}d$ [13], there is considerable room for additional studies.

Combining Eqs. (4), (5), (6), (8), the $b \to c\bar{c}s'$ branching fraction can be written as

$$\begin{aligned} B(b \to c\bar{c}s') &= 1 - (2 + r_\tau + r_{ud} + r_{\not{c}})\ B(\overline{B} \to X_c e\nu) \\ &= 1 - (6.50 \pm 0.40)\ B(\overline{B} \to X_c e\nu)\ . \end{aligned} \tag{9}$$

In this relation the very small contribution from $b \to u\bar{c}s'$ transitions has been neglected. Eqs. (1) and (9) yield the number of charms per B decay as

$$\begin{aligned} n_c &= 2 - (2 + r_\tau + r_{ud} + 2r_{\not{c}})\ B(\overline{B} \to X_c e\nu) \\ &= 2 - (6.75 \pm 0.40)\ B(\overline{B} \to X_c e\nu)\ , \end{aligned} \tag{10}$$

where we note that $B(b \to c\bar{c}s')$ drops out in the linear relation between n_c and $\overline{B} \to X_c e\nu$, and that the relation is largely free from uncertainties in masses of b and c quarks since the error is dominated by the uncertainty in r_{ud}. Figure 1 shows the discrepancy between theory given by Eq. (10) and experiment.

The precisely measured semileptonic BR together with Eqs. (9)-(10) gives

$$B(b \to c\bar{c}s') = 0.32 \pm 0.05\ , \tag{11}$$

$$n_c = 1.30 \pm 0.05. \tag{12}$$

This is our central result. Our predictions for $B(b \to c\bar{c}s')$ and for n_c agree with the central values obtained in previous theoretical investigations [5–7,2,3] but have smaller errors. As discussed in more detail below, such a large value of $B(b \to c\bar{c}s')$ requires a significant rate for $\overline{B} \to D\overline{D}\,\overline{K}X$. We predict the observation of (a) $\overline{B} \to D^{(*)}\overline{D}^{(*)}\overline{K}$ modes with significant BR's, (b) enhanced $\ell^+\overline{D}$ and ℓ^-D correlations where the primary lepton originates from one B and the charmed hadron from the other B in the event, and (c) enhanced DD and $\overline{D}\,\overline{D}$ correlations at the $\Upsilon(4S) \to B\overline{B}$.

If the predicted effects will be observed, then the $B(b \to c\bar{c}s')$ is larger than currently determined by experiment. The measured number of charm per B will not change by those observations, but the larger $B(b \to c\bar{c}s')$ would indicate that the current experimental value of n_c is underestimated. In that case, a careful re-evaluation of all errors involved in measuring n_c would be in order, including re-assessments of absolute BR's of the charmed hadrons some of which are poorly known. On the other hand, non-observation of our predictions would indicate an enhancement of the $b \to c\bar{u}d$ transition over the parton estimate [14] and/or a larger rate than anticipated for charmless $b \to s$ transitions [15,16].

Theory alone or experimental measurements alone have large inherent uncertainties for $B(b \to c\bar{c}s')$. We therefore adopted a hybrid approach which uses well measured quantities from experiment in conjunction with reliably calculated quantities from theory to determine $B(b \to c\bar{c}s')$ to higher accuracy [8].

One conventional way to determine $B(b \to c\bar{c}s)$ is to add the inclusive yield of D_s [9,17,15]

$$R_{D_s} \equiv B(\overline{B} \to D_s^- X) + B(\overline{B} \to D_s^+ X) \tag{13}$$

to the other observed final states governed by $b \to c\bar{c}s$ [18],

$$\begin{aligned} B(b \to c\bar{c}s) &= R_{D_s} + B(\overline{B} \to \Xi_c \overline{\Lambda}_c X) + B(\overline{B} \to (c\bar{c})X) = \\ &= 0.12 + 0.01 + 0.03 = 0.16 \pm 0.02 \ . \end{aligned} \tag{14}$$

$(c\bar{c})$ denotes charmonia not seen in $D\overline{D}X$ such as $J/\psi, \psi', \eta_c, \eta_c', \chi_c, h_c,\ {}^{1,3}\mathfrak{D}_2$. Within errors, this agrees with the experimental measurement of n_c

$$B(b \to c\bar{c}s') = n_c - 1 + B(b \to \text{no charm}) = 0.13 \pm 0.06 \ . \tag{15}$$

The agreement appears to support the low value of n_c.

Our determination of $B(b \to c\bar{c}s')$ suggests a different picture as to how $b \to c\bar{c}s$ hadronizes. A systematic classification shows that five classes of hadronization can occur, see Table II. Conventional wisdom [9,15,17] assumes that most of the inclusive D_s production in B decays originates from the virtual "W". Motivated by the observed inclusive momentum spectrum of D_s in B decays and by a simple theoretical argument given below, we predict instead that only about 70% of the inclusive D_s yield in B decays contribute to $\overline{B} \to DD_s^- X$ processes. The remaining D_s (about 30%) could occur in conjunction with $s\bar{s}$ fragmentation. We will return to this point below.

The branching ratio for class (a) is thus depleted and becomes about $0.7 R_{D_s}$. [This branching ratio can be at most R_{D_s}, which would soften our conclusion by a small amount only]. The branching ratios of the observed classes (a)-(c), do not add up to 30%. Thus class (d) must have a sizable branching fraction of about 20%,

$$B(\overline{B} \to D\overline{D}\,\overline{K}X) \sim 20\% \,. \tag{16}$$

There are several interesting experimental implications. Those modes can be studied at CLEO and at LEP. CLEO has higher statistics, whereas LEP has the ability to separate one B from the other b hadron. Thus far, however, they have not been seriously searched for. The low Q value in this process suggests that a significant portion will be three body [23],

$$\overline{B} \to D^{(*)}\overline{D}^{(*)}\overline{K} \,.$$

Because the responsible Hamiltonian is isospin zero, many isospin relations can be used to facilitate the observation of those modes [24].

Finally, the class (e) processes involve $s\bar{s}$ fragmentation. Their branching ratio could be non-negligible, at the few percent level [22]. A few exclusive final states would then carry the lion's share of the class (e) branching ratio, because of limited phase-space.

Before proposing a number of tests, we discuss briefly a few additional indications that support our hypothesis from

(a) a naive Dalitz plot analysis [25],

(b) measured inclusive kaon yields in B decays, and

(c) measured inclusive D momentum spectra in B decays.

Figure 2 shows the $b \to c\bar{c}s$ Dalitz plot resulting from the $(V-A) \times (V-A)$ matrix element, where the initial and final spins were averaged and summed. In this simple model, the $\bar{c}s$ system hadronizes as a $D_s^- X$ for invariant $\bar{c}s$ masses below $m_D + m_K$. In contrast, for

$$m_{\bar{c}s} > m_D + m_K$$

the $\bar{c}s$ is not seen as a $D_s^- X$ but rather as $\overline{D}\,\overline{K}X$. The Dalitz plot region contributing to D_s production in $b \to c\bar{c}s$ decay is $m_{\bar{c}s} < m_D + m_K$, and one obtains

$$\frac{\Gamma(b \to c + D_s^-)}{\Gamma(b \to c\bar{c}s)} \approx 0.35 \ .$$

This argument suggests that a large fraction of $b \to c\bar{c}s$ transitions has not been accounted for in previous investigations [9,17]. (See however the analyses of Refs. [25,26] which reach similar conclusions to ours.) Of course, the naive Dalitz plot argument is rather crude. It does not address issues of hadronization, resonance bands and their interferences, QCD-corrections, and interferences between penguin-amplitudes ($b \to s$) with the dominant spectator-amplitude ($b \to c\bar{c}s$). Nevertheless, the Dalitz plot conveys the important message that a significant fraction of $b \to c\bar{c}s$ processes could be seen in $D\overline{D}\,\overline{K}X$.

The surplus of the inclusive kaon yield in B decays beyond all the conventional sources again indicates a significant $B(\overline{B} \to D\overline{D}\,\overline{K}X)$ [22]. The indication is further strengthened by the large observed K-flavor correlation with its parent B-flavor at time of decay [27,28]. The flavor of the kaon in $\overline{B} \to D\overline{D}\,\overline{K}X$ is 100% correlated with its parent b-flavor. The momentum spectra of the inclusive D yields in B decays indicates an excess of low momentum D's over conventional sources [29]. A natural explanation can be found in $\overline{B} \to D\overline{D}\,\overline{K}X$.

We are now ready to suggest several tests. In addition to the "indirect" measurement using $B(b \to c\bar{c}s') \approx n_c - 1$ which involves large errors, we suggest to directly determine $B(b \to c\bar{c}s')$ by adding up the "wrong-sign" charm in tagged B decays [3,8],

$$B(b \to c\bar{c}s') \approx B(b \to \bar{c}) = B(\overline{B} \to D_s^- X) + B(\overline{B} \to \overline{D}X) + B(\overline{B} \to \overline{\Lambda}_c X) + \\ + B(\overline{B} \to \overline{\Xi}_c X) + B(\overline{B} \to (c\bar{c})X) \ . \quad (17)$$

The traditional lepton and $K^\pm$ tags could be supplemented by other tags, such as K^* and jet charge techniques. Further, the number of DD and DD_s events per $\Upsilon(4S) \to B\overline{B}$ decay can be combined with the single, inclusive D and D_s yields in untagged B decay to determine $B(\overline{B} \to \overline{D}X)$ and $B(\overline{B} \to D_s^- X)$ [3]. Of course, $B^0 - \overline{B}^0$ mixing effects must be corrected for [28]. No tagging is required to measure $B(\overline{B} \to (c\bar{c})X)$.

A sizable $B(\overline{B} \to D\overline{D}\ \overline{K}X)$ would show up as a $D^{(*)}\overline{K}$ (from cs) enhancement. The background at the $\Upsilon(4S)$ is much reduced because

$$\begin{array}{l}\Upsilon(4S) \to \overline{B}B \to \overline{D} \to K \\ \qquad\qquad\ \ \hookrightarrow D\ ,\end{array} \tag{18}$$

which naturally yields a DK correlation, while its $D\overline{K}$ correlation is suppressed. The Dalitz plot allows to enhance the $D^{(*)}\overline{K}$ signal correlation further by assuming

$$\frac{d\Gamma}{dm^2_{D^{(*)}\overline{K}}} \approx \frac{d\Gamma}{dm^2_{cs}}\ .$$

The invariant mass spectrum of the $b \to c\bar{c}s$ transition indicates that $D^{(*)}\overline{K}$ (from cs) tends to have a large invariant mass, see Fig. 2.

The inclusive D_s yield in B decays, R_{D_s}, has two roughly equal contributions. Figure 3 shows the measured momentum spectrum [19]. Whereas the high peak is dominated by the exclusive two-body modes $\overline{B} \to D^{(*)}D_s^{(*)-}$, the underlying dynamics of the remainder had been unclear. The factorization assumption is successful in predicting ratios of rates for the two-body modes $\overline{B} \to D^{(*)}D_s^{(*)-}$ [19]. Thus we assume factorization and predict that $b \to c + D_s^{(*)-}$ is dominated by the exclusive two-body decays $\overline{B} \to D^{(*)}D_s^{(*)-}$ in analogy to semileptonic decay of B mesons. We calculate that

$$\frac{\Gamma(\overline{B} \to D^{(*)}D_s^{(*)-})}{\Gamma(b \to c + D_s^{(*)-})} = 0.7 \pm 0.2\ , \tag{19}$$

where the quoted error refers to a variation in the b-quark mass, $4.4 \leq m_b \leq 5.2\ GeV$, and in the slope of the Isgur-Wise function [30], $\rho^2 = 0.84 \pm 0.15$. The numerator is the sum over the four exclusive two-body rates obtained [31,32] by using the heavy quark limit [33]. The denominator is the sum of two rates $b \to c + D_s$ and $b \to c + D_s^*$. It treats the $b \to c$ transition as if it were that of free quarks [15]. The decay constant f_{D_s}, the CKM elements and the factorization parameter [34] a_1 all cancel in the ratio. The prediction Eq. (19) can currently be tested since the ratio $\Gamma(\overline{B} \to D^{(*)} D_s^{(*)-})/\Gamma(b \to c + D_s^{(*)-})$ is an observable [35] in which the uncertainty due to $B(D_s \to \phi\pi)$ cancels. The prediction Eq. (19) together with the measured ratio [19]

$$\frac{B(\overline{B} \to D^{(*)} D_s^{(*)-})}{R_{D_s}} = 0.46 \pm 0.04 \,, \tag{20}$$

yields that [35]

$$B(\overline{B} \to D D_s^- X) \approx B(b \to c + D_s^{(*)-}) = (0.7 \pm 0.2) R_{D_s} \,. \tag{21}$$

The remainder of the inclusive D_s yield in B decays $[R_{D_s} - B(\overline{B} \to D D_s^- X) = (0.3 \pm 0.2) R_{D_s}]$ could be a significant fraction of the lower momentum D_s mesons. One sizable source for it could be the $b \to c\bar{c}s$ transition with $\bar{s}s$ fragmentation [22],

$$B(b \to c\bar{c}s + \bar{s}s) \approx 0.01 - 0.03 \,. \tag{22}$$

One generally expects one D_s^- per such a transition, as long as D_s^{**-} and higher D_s^- resonance production in $b \to c\bar{c}s + \bar{s}s$ transitions is negligible. The total D_s^- production in flavor-tagged $\overline{B}$ decays is thus expected to be

$$B(\overline{B} \to D_s^- X) \approx B(b \to c\bar{c}s + \bar{s}s) + B(\overline{B} \to D D_s^- X) \approx 0.1 \,. \tag{23}$$

The D_s^+ yield in flavor-tagged $\overline{B}$ decays is governed by the $b \to c$ transition with $\bar{s}s$ fragmentation, and may be non-negligible

$$B(\overline{B} \to D_s^+ X) = R_{D_s} - B(\overline{B} \to D_s^- X) \sim 10^{-2} \,. \tag{24}$$

For a model of the relative contributions to the D_s^+ yield from $b \to c\bar{u}d, c\ell\nu, c\bar{c}s$ transitions with $\bar{s}s$ fragmentation, we refer the reader to Ref. [22]. The D_s^+ yield in flavor-tagged $\overline{B}$ decays has been traditionally neglected [15,17,9].

In conclusion, by combining reliable theoretical calculations and precise experimental measurements [8], we obtain a more accurate estimate of $B(b \to c\bar{c}s')$ and of n_c than previous investigations [9,17,2,3,5–7]. We predict

$$B(b \to c\bar{c}s') = 0.32 \pm 0.05 \text{ and } n_c = 1.30 \pm 0.05, \qquad (25)$$

which is significantly larger than the low experimental value $n_c|_{exp} = 1.10 \pm 0.06$. We believe that (25) is on firm ground, and expect an increase in the measured n_c in the future. Our prediction can be tested in a variety of ways. First we advocate to measure $B(b \to c\bar{c}s')$ by counting up the number of anticharmed hadrons (the "wrong" charm flavor) per $\overline{B}$-decay. A sizable BR for $\overline{B} \to D\overline{D}\ \overline{K}X$ is our second prediction. It shows up as a large $\ell^-\ D$ and $\ell^+\ \overline{D}$ correlation after removing $B^0 - \overline{B}^0$ mixing effects [28], where the primary lepton comes from one B hadron and the charmed meson from the other B-hadron in the event. It can also be seen by observing the exclusive modes $\overline{B} \to D^{(*)}\overline{D}^{(*)}\overline{K}$, and/or by searching for $D^{(*)}\overline{K}$ (from cs) correlations. There are many additional implications, consequences and tests which we hope to discuss in a forthcoming report [22].

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TABLES

TABLE I. The predicted semileptonic branching ratio, the $B(b \to c\bar{c}s')$ and n_c taken from Bagan et al. [6].

Scheme	$B(\overline{B} \to X_c \ell \nu)$	$B(b \to c\bar{c}s')$	n_c
$\overline{MS}$	0.112 ± 0.017	0.35 ± 0.19	1.35 ± 0.19
Pole mass	0.120 ± 0.014	0.27 ± 0.07	1.27 ± 0.07

TABLE II. The five classes of hadronization of $b \to c\bar{c}s$. $(c\bar{c})$ denotes charmonia not seen in $D\overline{D}X$, and class (e) involves $\bar{s}s$ fragmentation.

Class	Mode	BR	Reference
(a)	$\overline{B} \to D D_s^- X$	$(0.7 \pm 0.2) R_{D_s} \approx 0.08$	
(b)	$\overline{B} \to \Xi_c \overline{\Lambda}_c X$	0.01	[21]
(c)	$\overline{B} \to (c\bar{c}) \overline{K} X$	0.03	[9,22]
(d)	$\overline{B} \to D\overline{D}\,\overline{K} X$	~ 0.2	
(e)	$b \to c\bar{c}s + \bar{s}s$	$\sim few \times 10^{-2}$	
Total:	$b \to c\bar{c}s$	0.31 ± 0.05	

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$$R_{D_s} = (0.12 \pm 0.01)\ \frac{0.035}{B(D_s \to \phi\pi)}\ ,$$

$$R_{\Lambda_c} = (0.041 \pm 0.008)\ \frac{0.044}{B(\Lambda_c \to pK^-\pi^+)}\ .$$

We choose the current central values $B(D_s \to \phi\pi) = 0.035$ and $B(\Lambda_c \to pK^-\pi^+) = 0.044$. We alert the reader that smaller absolute BR's for D_s and Λ_c decays increase the yield of charm per B, and would lessen the discrepancy between experiment and theory regarding n_c. $B(\overline{B} \to \Xi_c \overline{\Lambda}_c X)$ is obtained by combining R_{Λ_c} and the relevant $\ell^\pm \Lambda_c$ measurement [21] where the primary lepton comes from one B and the Λ_c from the other B in the $\Upsilon(4S)$ event. The inclusive BR into $(c\bar{c})$ charmonia is obtained [22] to be 0.026 ± 0.004 by summing over their observed and estimated BR's which is larger than previous estimates [9].

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[35] Although D_s^{**-} resonances are not normally seen as $D_s^- X$, some are broad enough to be produced below the $\overline{D}^{(*)}\overline{K}$ threshold where they decay to $D_s^- X$. That contribution has not been included in the estimate for $\Gamma(b \to c + D_s^{(*)-})$. We approximate $\Gamma(b \to c + D_s^{(*)-}) \approx \Gamma(\overline{B} \to D D_s^- X)$, since $s\overline{s}$ fragmentation at the $b \to c$ vertex is small. Information about this $s\overline{s}$ fragmentation comes from $\overline{B} \to D_s^{(*)+}\overline{K}X\ell\nu$, which has not yet been observed and for which upper limits exist [36]. $B(\overline{B} \to D_s^{(*)+}\overline{K}e\nu)$ is estimated to be $\sim$ few $\times 10^{-3}$ [I. Dunietz, J.L. Goity, and W. Roberts, in preparation].

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FIGURES

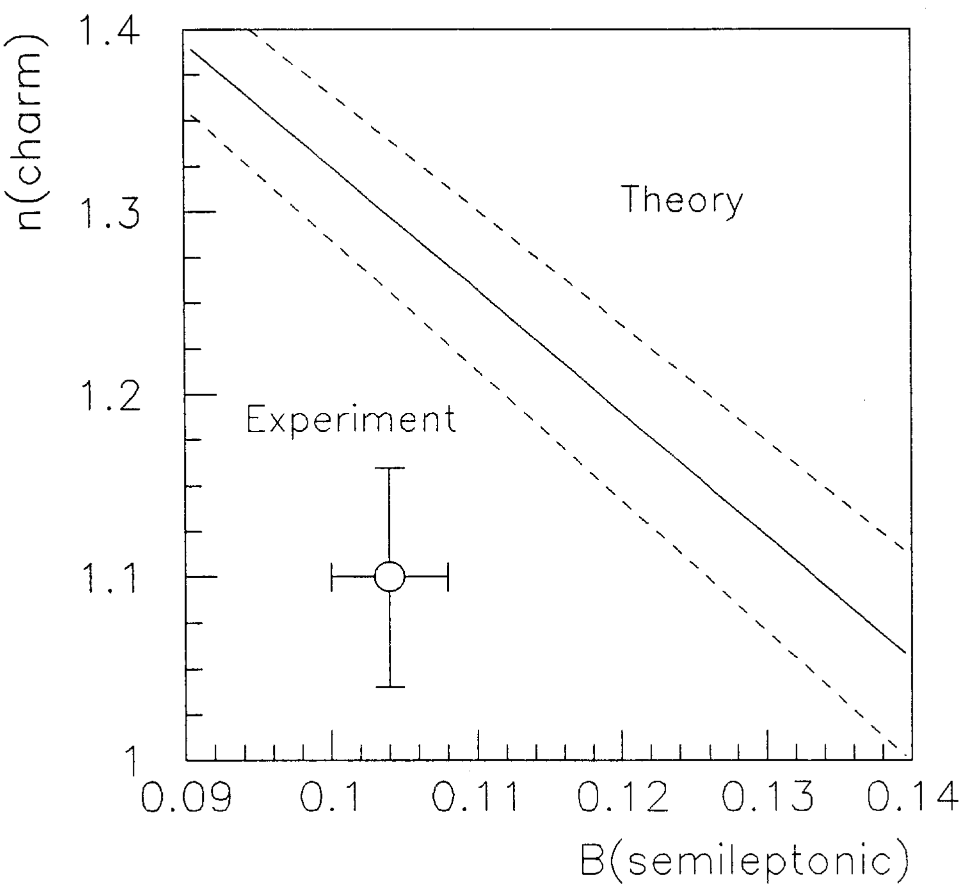

FIG. 1. Number of charm per B decay (n_c) is plotted against the B meson semileptonic branching ratio. The uncertainty in the theoretical prediction is indicated by dashed lines.

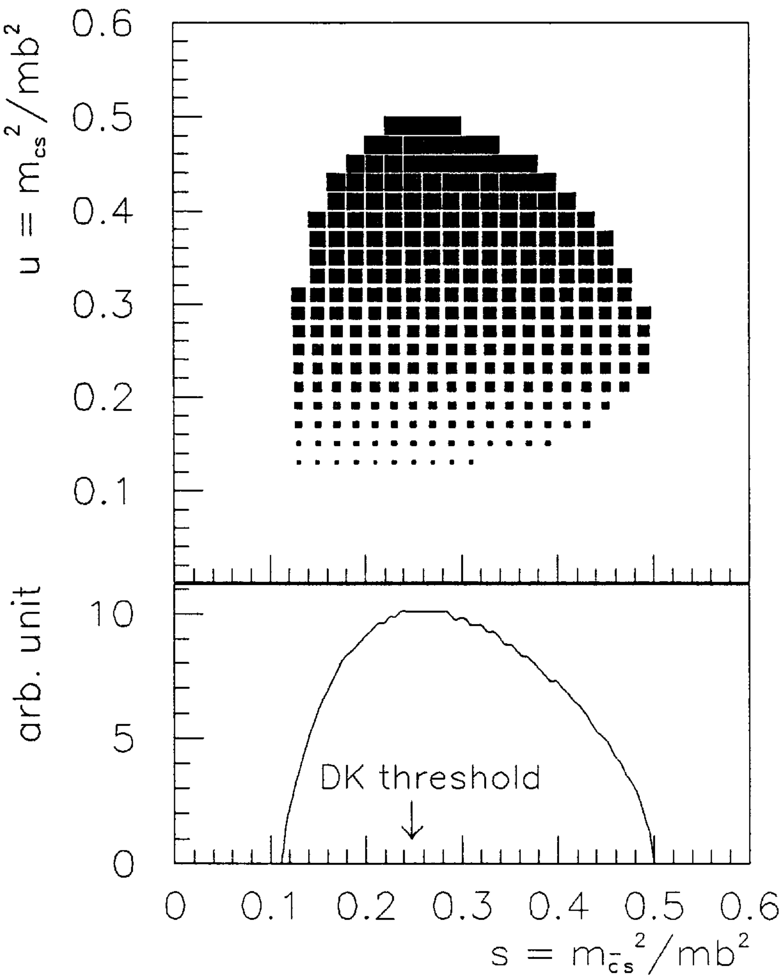

FIG. 2. Dalitz plot of the decay $b \to c\bar{c}s$ as a function of $u(= m_{cs}^2/m_b^2)$ and $s(= m_{\bar{c}s}^2/m_b^2)$. The projection onto the s axis is shown at the bottom where the $\overline{D}\ \overline{K}$ threshold is indicated by an arrow.

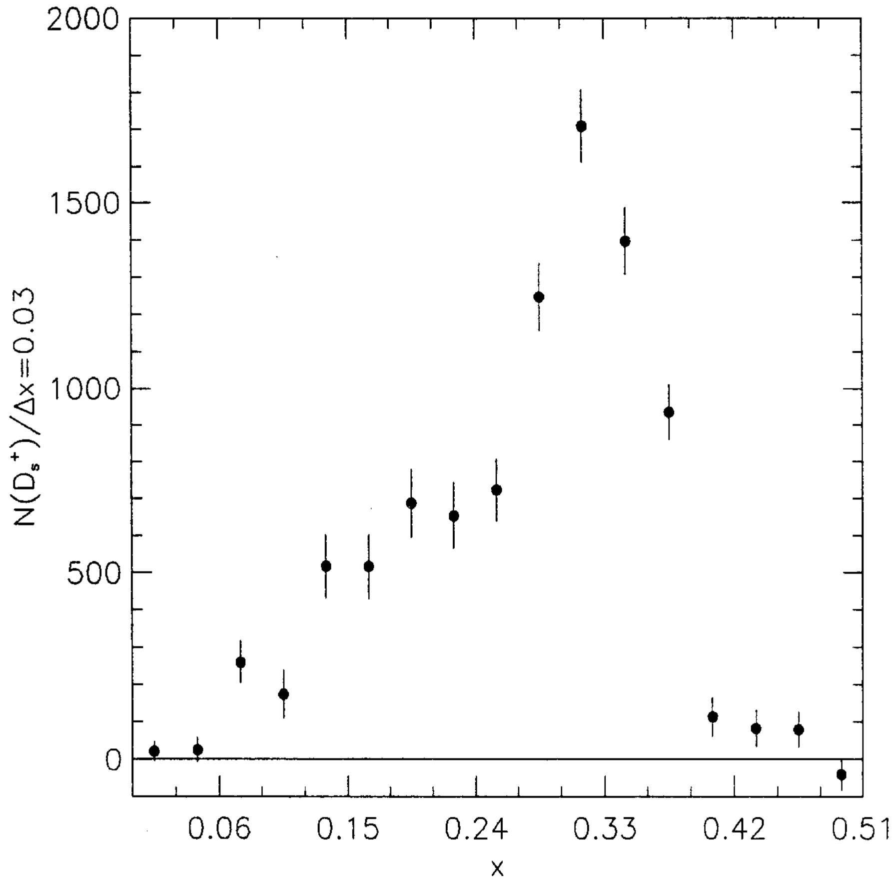

FIG. 3. Momentum spectrum of inclusive D_s mesons produced in untagged B decays at the $\Upsilon(4S)$ as measured by the CLEO collaboration. The parameter x is defined by $x = p_{D_s}/p_{\max}$ where $p^2_{\max} \equiv E^2_{\text{beam}} - M^2_{D_s}$. The continuum background has been subtracted.