



## Effect of kick velocity on the gamma-ray burster distribution

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### ABSTRACT

The effect of kick velocity to newly born pulsars on the distribution of  $\gamma$ -ray bursters is examined in the context of the disk origin model and the halo model of the  $\gamma$ -ray bursters. The conversion formula from a two-dimensional velocity distribution function to a three-dimensional distribution function is derived and it is applied to reproduce the distribution function of the kick velocity of radio pulsars. Monte Carlo simulations of the kicked neutron stars show that the disk neutron star model of the  $\gamma$ -ray bursters still needs unnatural assumptions if the velocity distribution of the  $\gamma$ -ray bursters is same with that of neutron stars; only the neutron stars with very high kick velocity could become the  $\gamma$ -ray bursters and there are silent majorities. On the other hand, the core radius of  $\gamma$ -ray bursters is found not to be extended by the kick velocity if the core-halo structure similar to the Galactic dark matter distribution is assumed on the initial distribution of the halo neutron stars. Thus the kick velocity to the neutron stars do not improve the statistics both of the disk model and the halo model. Two possibilities to save the Galactic models are suggested: (1) The  $\gamma$ -ray bursters are old neutron stars which were accelerated to be faster than  $750 \text{ km s}^{-1}$  by jet propulsion and passed the death line for pulsars due to the spin down caused by the rotational energy loss by the jet ejection. (2) The initial distribution of the neutron stars is fairly uniform and the extent of the halo is large enough for the halo model to be consistent with the observations. It implies that the initial star formation burst of the Galaxy occurred fairly uniformly in the extended halo region.

Key words: Stars: neutron — Galaxy: halo — Galaxies: formation — dark matter — Gamma rays: bursts

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## 1 Introduction

Though many models have been proposed for  $\gamma$ -ray bursters, they are essentially classified into two categories; the cosmological model and the Galactic model. The extreme isotropy of the bursters observed by BATSE seems to be favoring the cosmological model. However, the GINGA observations of the cyclotron absorption lines, which are the unique reliable clue to solve the nature of the  $\gamma$ -ray bursters (Murakami et al. 1988, Fenimore et al. 1988, and Yoshida et al. 1991), are strongly supporting the Galactic model since the same features have been detected in Galactic X-ray pulsars (e.g. Trümper et al. 1978, Makishima & Mihara 1992). The difficulty in the Galactic model is to reproduce the observed isotropy and the non-uniform distribution of the bursters implied by  $V/V_{max}$  simultaneously. Recently it has been reported that radio pulsars are born with very high space velocities (Lyne & Lorimer 1994). If old neutron stars are the source of the  $\gamma$ -ray bursters and neutron stars generally have high kick velocities as implicated by the radio pulsar observation, the Galactic origin model may be saved since the distribution of neutron stars can be extended by the kick velocity. In order to examine the effect of the kick velocity on the distribution of the bursters, we perform Monte Carlo simulations of the distribution of the kicked neutron stars on the basis of the Galactic model of the  $\gamma$ -ray bursters.

First of all, the conversion formula from the observable two-dimensional velocity distribution function in a projected space to the three-dimensional velocity distribution function in a real space is derived and it is applied to the kick velocity distribution of radio pulsars. The orbits of kicked neutron stars in the Galactic potential are traced with a high precision and the various statistical values are evaluated to be compared with the observations.

## 2 Distribution function of kick velocities to neutron stars and neutron star orbits in the Galactic potential

As long as the radio pulsars concerned, only proper motion is observable and hence the transverse velocities of pulsars can be estimated. We need the three-dimensional velocity distribution function of radio pulsars in the real space in order to incorporate the effect of the kick velocity to the neutron stars. First of all, we show how the velocity distribution function in the real space is restored from the observationally available transverse velocity distribution function.

It is straightforward to show that the two-dimensional velocity distribution function  $f_{2D}$  is related to the three-dimensional velocity distribution function  $f_{3D}$  under the assumption of isotropy as;

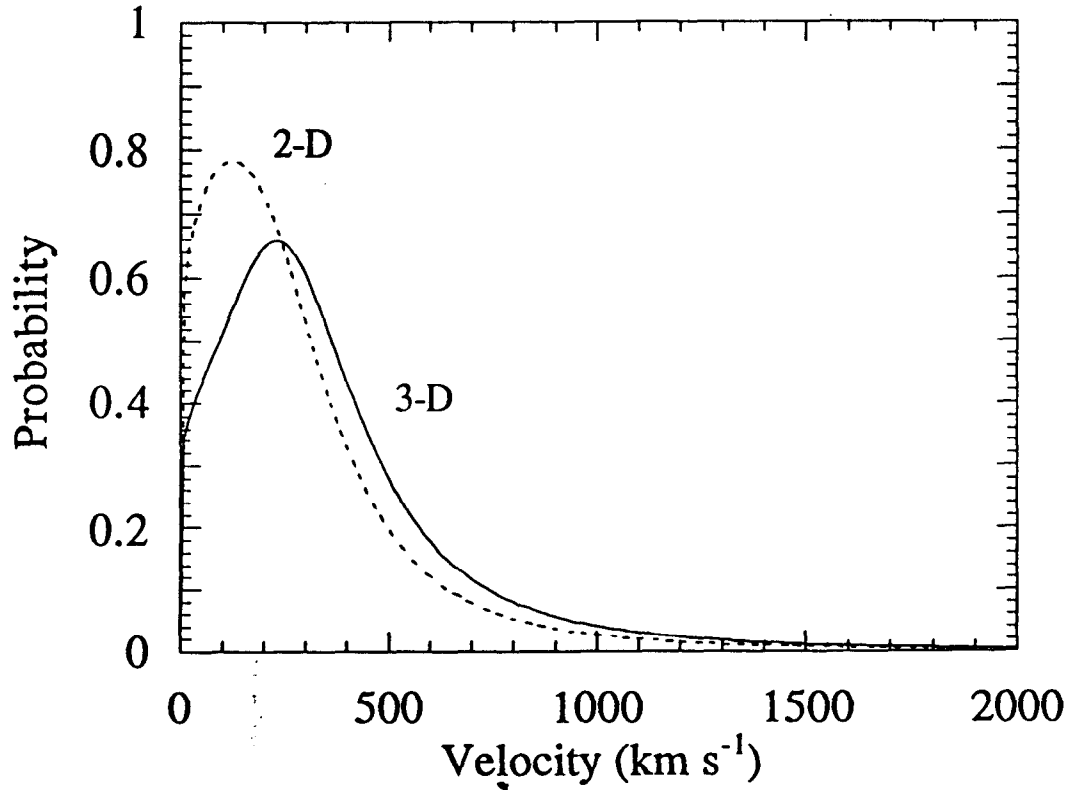


Fig.1. Three-dimensional velocity distribution function reconstructed from the analytical approximation for the two-dimensional velocity distribution of the radio pulsars.

$$f_{2D}(t) = t \int_t^{\infty} \frac{f_{3D}(v)}{v\sqrt{v^2 - t^2}} dv. \quad (1)$$

This Abel integral is inverted to the desired relation of the three-dimensional velocity distribution function to the two-dimensional velocity distribution function by the Laplace transformation,

$$f_{3D}(v) = -\frac{2v^2}{\pi} \int_v^{\infty} \left( \frac{f_{2D}(t)}{t} \right)' \frac{dt}{\sqrt{t^2 - v^2}}, \quad (2)$$

where the prime means the derivative concerned to the transverse velocity  $t$ .

This formula is applied to reproduce the three-dimensional velocity distribution function of the kick velocity of radio pulsars. Lyne and Lorimer (1994) found that the distribution of the observed two-dimensional transverse velocities of radio pulsars is approximated by,

$$f_{2D}(t)dt \propto \frac{t^{0.13}}{1+t^{3.3}}, \quad t = \frac{v}{v_0}, \quad v_0 = 330 \text{ km s}^{-1}. \quad (3)$$

The three-dimensional velocity distribution function given by the transformation of eq. (2) is plotted in Fig. 1. The derived 3-D velocity distribution function has a mean of  $410 \text{ km s}^{-1}$  and r.m.s. value of  $689 \text{ km s}^{-1}$ .

We examine the effect of the kick velocity to newly born neutron stars for several models of gamma-ray bursters: The Galactic disk origin model, the disk origin model with the assumption that only the high velocity neutron stars can be the  $\gamma$ -ray bursters, and the halo origin model.

The neutron stars originated in the Galactic disk are assumed to be born in the stage of the initial star formation burst and/or born with a constant birth rate from the formation of the Galaxy to the present time. The distribution of the disk neutron stars, the Galactic potential, and the calculation procedure of the orbits of the neutron stars are essentially same with those employed by Paczyński (1990). Important difference is that the kick velocities are given with the Monte Carlo method according to the velocity distribution function of the radio pulsars.

The random velocities generated according to the 3-D velocity distribution function are added to the velocities of newly born neutron stars. We assume that the progenitors of neutron stars born in the Galactic disk have a rotational velocity adopted by Paczyński (1990) (Burton & Gordon, 1978, Binney & Tremaine, 1987). On the other hand, the progenitors in the Galactic halo are assumed to be at rest. This assumption is justified since the lifetime of massive stars as the progenitor of neutron stars is much shorter than the dynamical time of the halo.

The probability distribution for the place of the disk neutron stars is assumed to be given by,

$$p_z dz = e^{-z/z_{exp}} \frac{dz}{Z_{exp}}, \quad (4)$$

$$p_R dR = a_R e^{-R/R_{exp}} \frac{R}{R_{exp}^2} dR, \quad (5)$$

$$a_R := [1 - e^{-R_{max}/R_{exp}} (1 + R_{max}/R_{exp})]^{-1} = 1.0683,$$

where  $z$ , and  $R$  stand for the distance from the Galactic plane and the distance from the Galactic center respectively. For the values of  $z_{exp}$ , and  $R_{max}$ , we adopt  $z_{exp} = 75 \text{ pc}$ ,  $R_{max} = 4.5 \text{ kpc}$  (van der Kruit 1987).

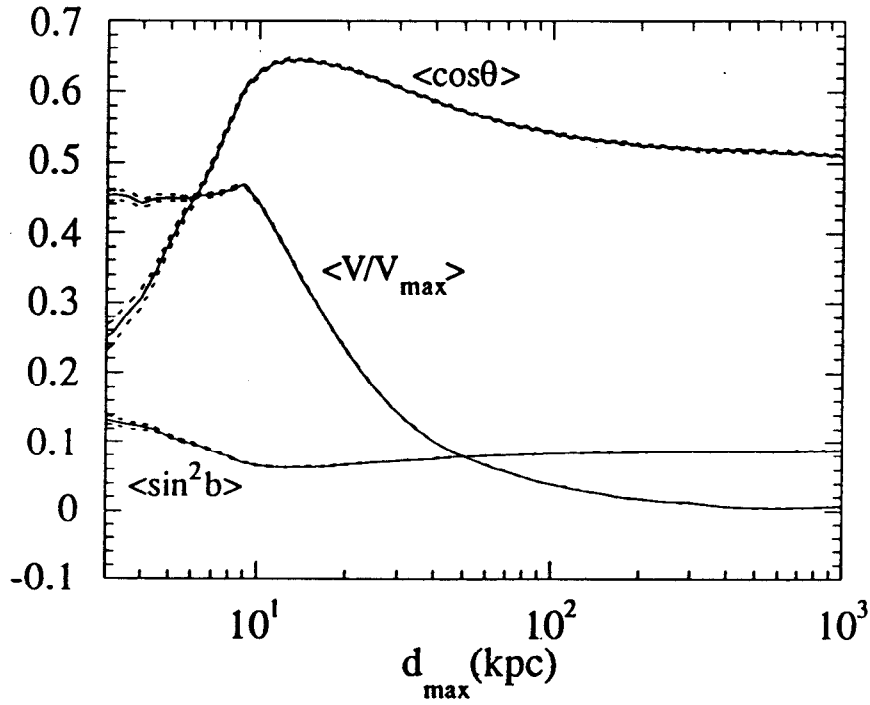


Fig. 2a

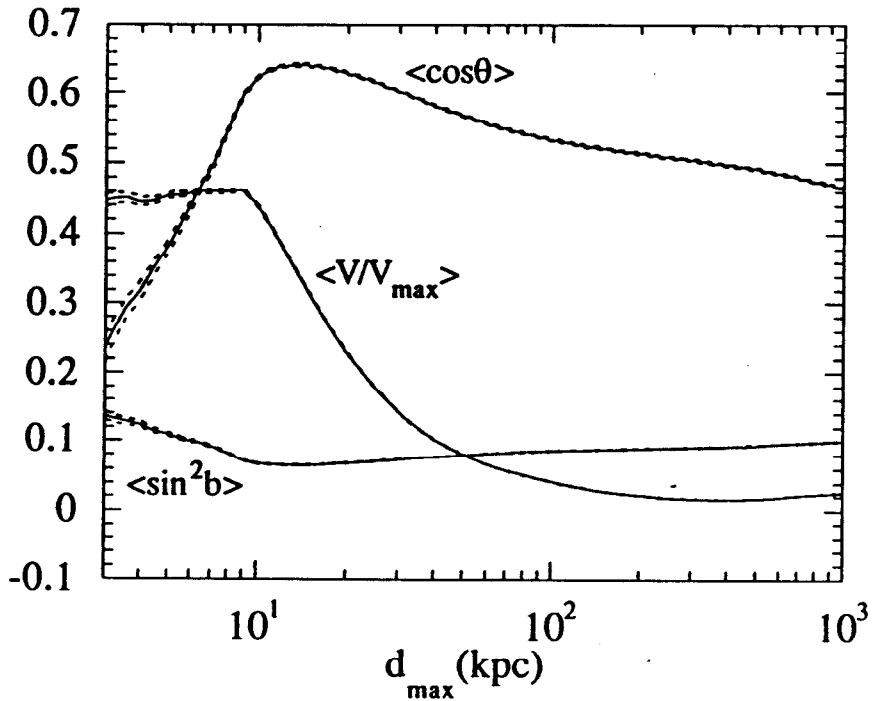


Fig. 2b

Fig. 2a–b. Dipole moment  $\langle \cos\theta \rangle$ , quadrupole moment  $\langle \sin^2 b \rangle$ , and  $\langle V/V_{max} \rangle$  of the  $\gamma$ -ray bursters plotted against the sampling distance  $d_{max}$  in the disk origin model. The bursters are assumed to be born in the initial star formation burst (a) and born with a constant rate from the formation of the Galaxy to the present (b).

The distribution of halo neutron stars is assumed to have a core-halo structure and hence the probability distribution is given as,

$$p_n(r)dr = \begin{cases} \frac{p_0}{1 + (r/r_n)^2} \left(\frac{r}{r_{max}}\right)^2 dr & r \leq r_{max} \\ 0 & r > r_{max} \end{cases} \quad (6)$$

where  $r$  is a distance from the Galactic center. The values of the core radius,  $r_n$ , and the cut-off radius,  $r_{max}$ , of the neutron star halo are left as parameters of our simulations.

The Galactic gravitational potential is assumed to be composed of three components,

$$\Phi_i(R, z) = \frac{GM_i}{\{R^2 + [a_i + (z^2 + b_i^2)^{1/2}]^2\}^{1/2}}, \quad (7)$$

$$\Phi_h = -\frac{GM_c}{r_c} \left[ \frac{1}{2} \ln \left( 1 + \frac{r^2}{r_c^2} \right) + \frac{r_c}{r} \tan^{-1} \left( \frac{r}{r_c} \right) \right], \quad (8)$$

$$M_c := 4\pi\rho_c r_c^3, \quad r^2 = R^2 + z^2,$$

where  $i = 1$  corresponds to the Galactic spheroid,  $i = 2$  corresponds to the Galactic disk (Miyamoto & Nagai 1975), and the halo components corresponds to the density distribution given as,

$$\rho_h = \begin{cases} \frac{\rho_c}{1 + (r/r_c)^2} & r \leq 70 \text{ kpc} \\ 0 & r > 70 \text{ kpc} \end{cases} \quad (9)$$

The parameters in the gravitational potential are chosen to be same with those adopted by Paczyński (1990),

$$a_1 = 0, \quad b_1 = 0.277 \text{ kpc}, \quad M_1 = 1.12 \times 10^{10} M_\odot, \quad (10)$$

$$a_2 = 3.7 \text{ kpc}, \quad b_2 = 0.20 \text{ kpc}, \quad M_2 = 8.07 \times 10^{10} M_\odot, \quad (11)$$

$$r_c = 6.0 \text{ kpc}, \quad M_c = 5.0 \times 10^{10} M_\odot. \quad (12)$$

The orbits of neutron stars are calculated numerically by integrating a set of equations of motion with a fourth-order Runge-Kutta method,

$$\frac{dR}{dt} = v_R, \quad \frac{dv_R}{dt} = \left( \frac{\partial \Phi}{\partial R} \right)_z + \frac{j_z^2}{R^3}, \quad (13)$$

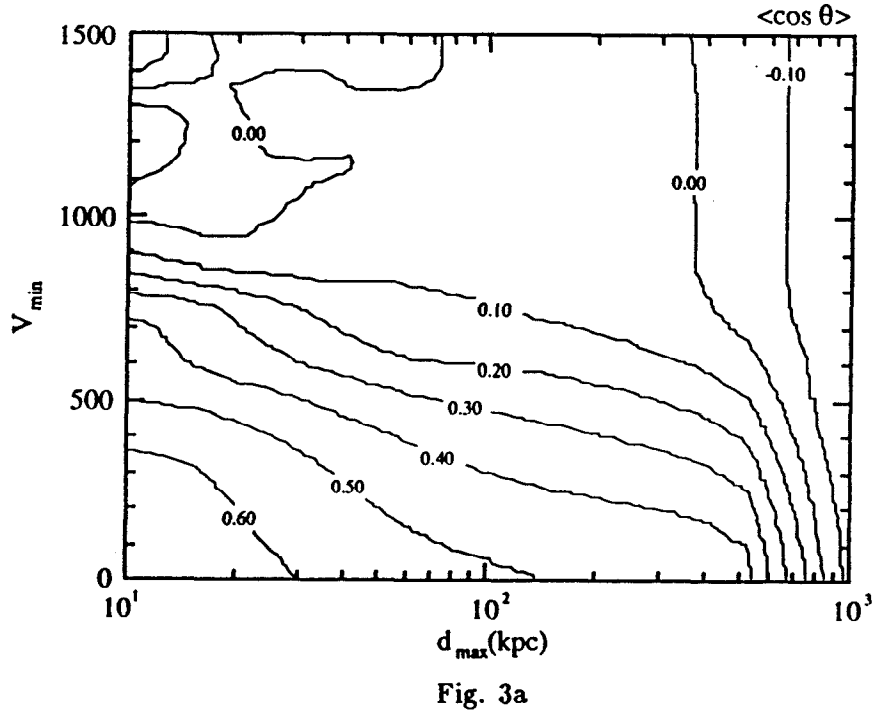


Fig. 3a

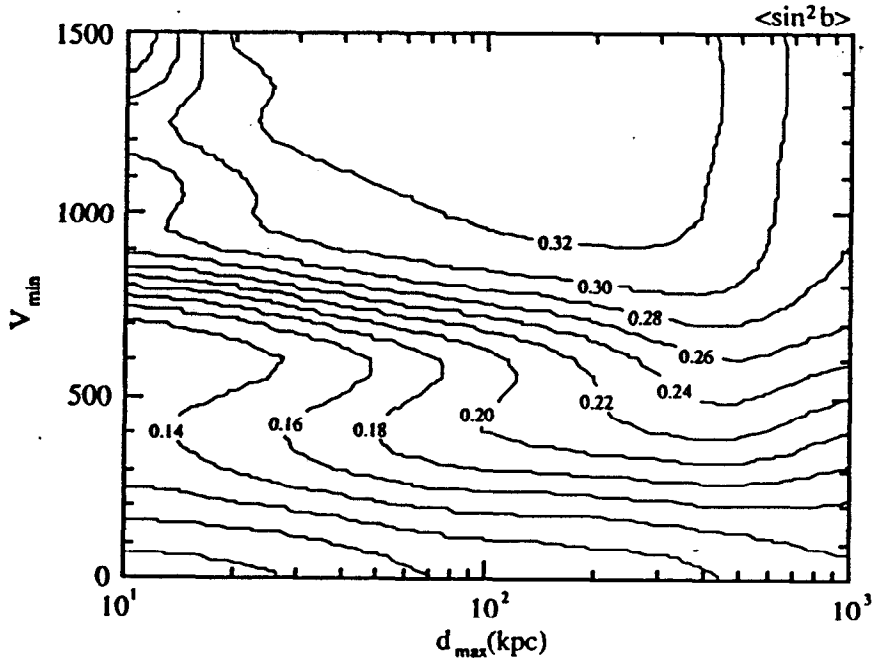


Fig. 3b

Fig. 3a-d Contour maps of the dipole moment (a), the quadrupole moment (b), the  $\langle V/V_{max} \rangle$  (c), and 90% confidence level of the model (d), of the  $\gamma$ -ray bursters on the plane of the minimum velocity cut-off  $v_{min}$  and the sampling distance  $d_{max}$  in the disk origin model. A constant birth rate and the bursting rate function proposed by Li and Dermer (1992) with the time scale of radio pulsars  $\tau = 30$  Myr are assumed. In Fig. 3d, the solid line shows the 90% confidence level from  $\langle \cos \theta \rangle$  and  $\langle \sin^2 b \rangle$ , and the broken line shows the 90% confidence level from  $\langle \cos \theta \rangle$ ,  $\langle \sin^2 b \rangle$ , and  $\langle V/V_{max} \rangle$ .

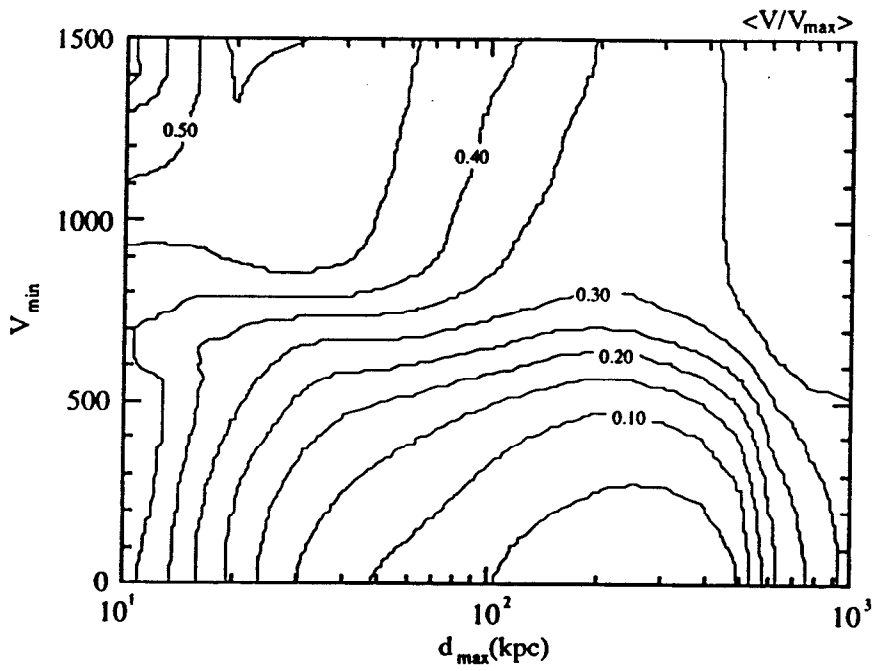


Fig. 3c

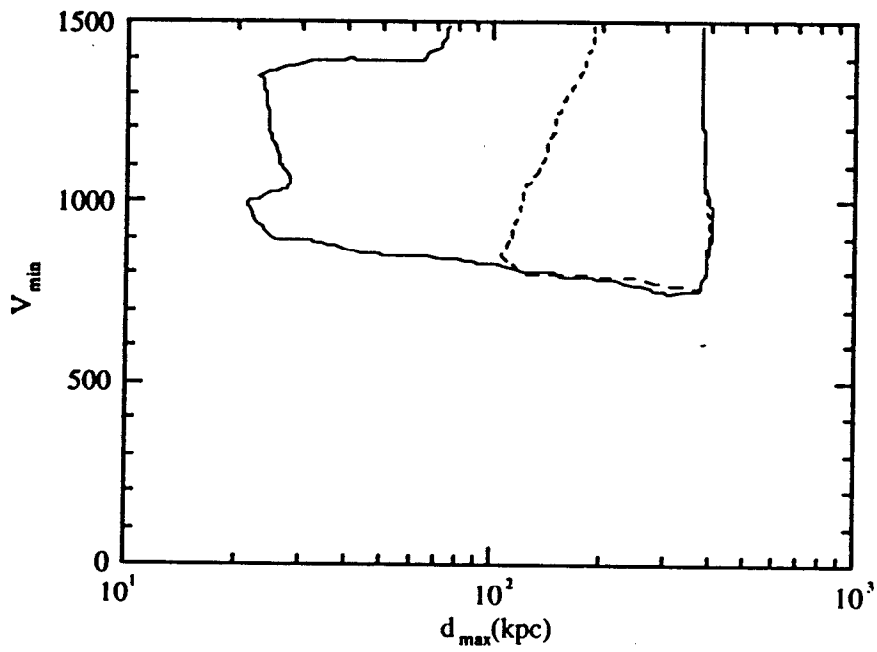


Fig. 3d



$$\begin{aligned}\frac{dz}{dt} &= v_z, & \frac{dv_z}{dt} &= \left(\frac{\partial\Phi}{\partial z}\right)_R, \\ \Phi &= \Phi_1 + \Phi_2 + \Phi_h,\end{aligned}\tag{14}$$

where  $j_z$  is angular momentum. These equations are integrated keeping the accuracy of the energy conservation better than  $10^{-10}$ .

Monte Carlo method is employed to determine the initial position of neutron stars, and the random kick velocity. Total  $10^6$  orbits are traced for each model up to the assumed Galactic age  $10^{10}$ yr. Since the most of high velocity neutron stars escape and few of them are left,  $10^6$  neutron stars are added to the simulation for the samples with  $v_{min} \geq 700$  km s $^{-1}$  and  $v_{min} \geq 1000$  km s $^{-1}$ .

The contributions from the Large Magellanic Cloud (LMC), the Small Magellanic Cloud (SMC), and M31 are incorporated simply by weighting the number of bursters with the luminous mass of them.

### 3 Result of Monte Carlo Simulation

First of all, we show the statistical results for the disk origin models (Fig. 2), which should be compared with the BATSE results for 447 bursts;  $\langle \cos\theta \rangle = 0.034 \pm 0.027$ ,  $\langle \sin^2 b - 1/3 \rangle = -0.017 \pm 0.014$ ,  $\langle V/V_{max} \rangle = 0.324 \pm 0.016$  (Meegan et al. 1992). Both the two cases; 1) neutron stars are born in the initial star formation burst, 2) constant supernova explosions produce neutron stars with a constant rate, clearly contradict with the statistical analysis of the  $\gamma$ -ray bursters observed with BATSE if all the neutron stars can be potentially  $\gamma$ -ray bursters. The dipole moment  $\langle \cos\theta \rangle$ , and the quadrupole moment  $\langle \sin^2 - 1/3 \rangle$  can be consistent with observation only in the very local region around the sun, say,  $r \lesssim 1$  kpc. However, the neutron star distribution is almost uniform and  $\langle V/V_{max} \rangle$  is too large to be consistent with the implication of non-uniform distribution of  $\gamma$ -ray bursters. Hence another component is required for the models in which local neutron stars in the solar neighbor are the source of  $\gamma$ -ray bursters (e.g. Smith & Lamb 1993).

If only the high velocity neutron stars can be  $\gamma$ -ray bursters as in the model proposed by Li and Dermer (1992), the statistics is improved and can be consistent with the BATSE observation (Fig. 3). We find that we still need a minimum velocity cut-off,  $V_{min} \gtrsim 750$  km s $^{-1}$  (Fig. 3d) in spite of the use of the new velocity distribution of the kick-velocity if the velocity distribution of the  $\gamma$ -ray bursters is same with that of radio pulsars. The difficulty of this type of model is clearly that there is no rational physical mechanism to suppress low velocity neutron stars to become  $\gamma$ -ray bursters. Though sharp cut-off in the kick velocity is needed

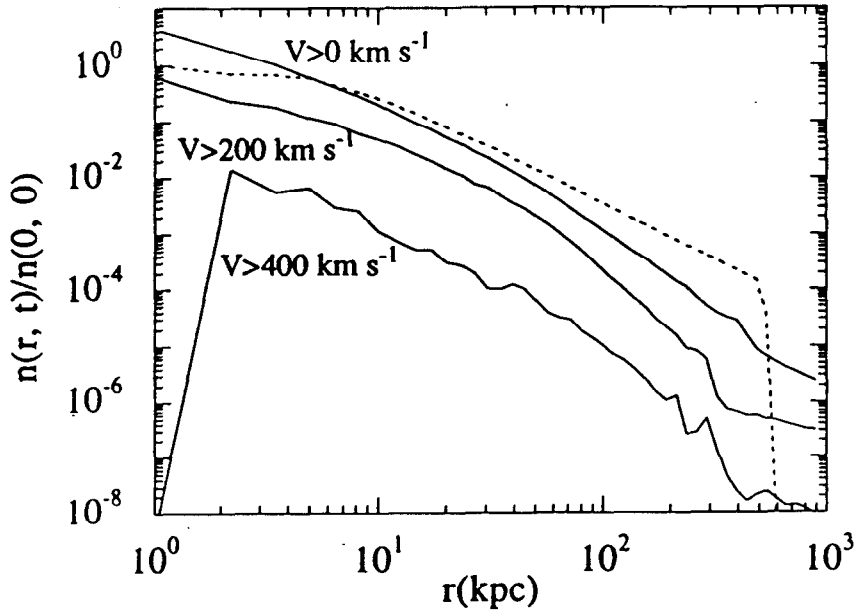


Fig. 4a

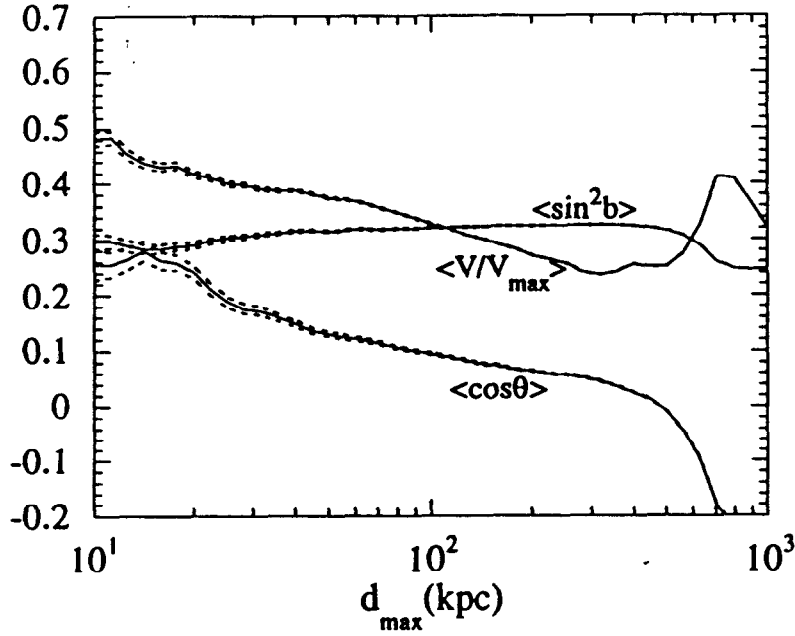


Fig. 4b

**Fig. 4 a–d** Results of the halo model: **a** The initial distribution (dotted line) and the final distribution (solid lines) after  $10^{10}$  yr of the kicked halo neutron stars in the case of the core radius,  $r_n = 6$  kpc, and the initial radius of the neutron star halo,  $r_{max} = 500$  kpc. **b** The dipole moment  $\langle \cos \theta \rangle$ , the quadrupole moment  $\langle \sin^2 b \rangle$ , and  $\langle V/V_{max} \rangle$  plotted against the sampling distance for  $r_n = 100$  kpc and  $r_{max} = 500$  kpc. **c** The coordinate independent dipole moment, Rayleigh-Watson statistic  $W$  (Briggs 1993) for randomly selected 260 orbits in the same case with **b**. **d** The coordinate independent quadrupole moment, Brighton statistic  $B$ . Fishman et al. (1994) found that  $W = 1.1 \pm 0.3$  and  $B = 6.6 \pm 1.2$  for 260 BATSE 1B burst samples.

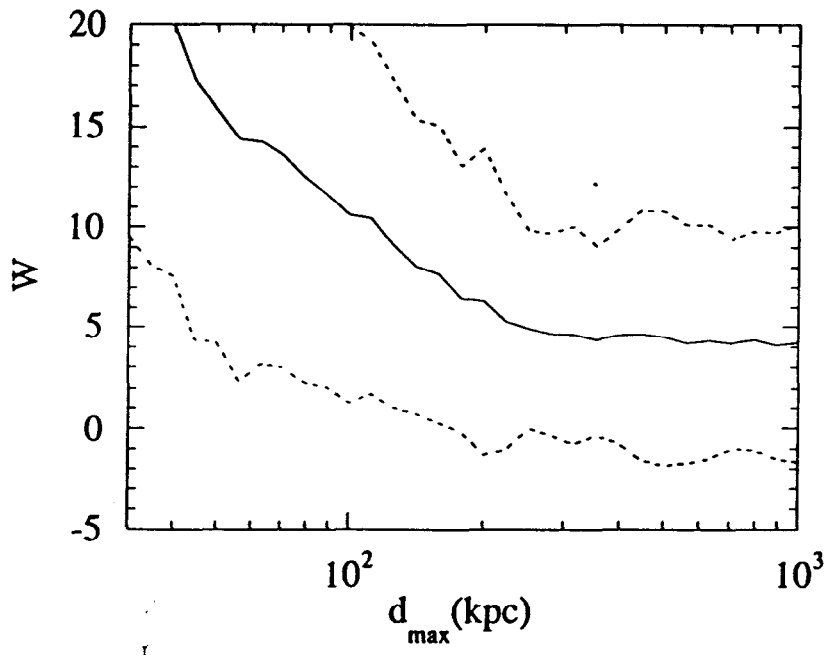


Fig. 4c

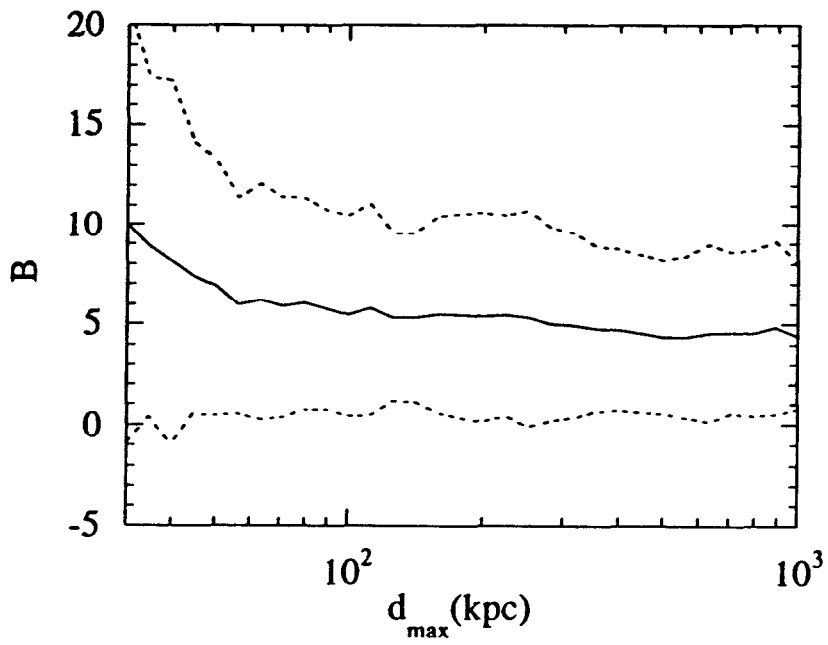


Fig. 4d

for the burster mechanism, it seems difficult to realize such cut-off from, for instance, the observed correlation between the velocity of pulsars and the magnetic field strength (see also the claim that this correlation is merely apparent due to the bias, Itoh & Hiraki 1994). Recently, Markwardt and Ogelman (1995) suggested that the high velocity motion of radio pulsars may be the result of propulsion by the gas jet of pulsars based on their observation. They also suggested that the jet may result in the loss of the rotational energy of pulsars. If neutron stars, which are accelerated further from the pulsar stage and pass the so-called death line for pulsars due to the spin down caused by the rotational energy loss, turn out to be  $\gamma$ -ray bursters, the velocities of the bursters can be much higher than those of the radio pulsars. If the majority of the bursters has the velocity higher than  $750 \text{ km s}^{-1}$ , they can be consistent with the BATSE observations. The fact that older pulsars have lower velocities on average than younger ones, however, seems to contradict with their suggestion though this tendency may be due primarily to a selection effect (Lyne & Lorimer 1994).

The halo neutron star model of  $\gamma$ -ray burster is attractive since it is consistent with the observation of cyclotron absorption lines and it has various theoretical and observational support from the view points of the galaxy formation and the chemical evolution (e.g. Hattori & Terasawa 1993). However the distribution of neutron stars should be fairly uniform in order to be consistent with the observed isotropy of the bursts. If we assume a core-halo structure for the burster distribution, the core radius ( $r_c$ ) should be large enough compared with the distance of the sun from the Galactic center as,  $r_c \gg 8.5 \text{ kpc}$ , and the sampling distance of the bursts must be further larger to be consistent with the non-uniformity of the bursters implied by the value of  $\langle V/V_{max} \rangle$ . We may be able to expect that the kick to neutron stars can extend the core radius and the distribution of the bursters to be consistent with the observations even if we assume the initial distribution of the neutron stars has the same structure and the similar core radius with the standard dark matter halo of the Galaxy. In order to examine the effect of the kick velocity to the halo neutron stars on the distribution of them, we traced the orbits of the halo neutron stars kicked according to the kick velocity distribution of radio pulsars. Contrary to the expectation, the core radius of the neutron star distribution is not extended by the kick velocity (Fig. 4a) when we assume the core-halo structure similar to the Galactic dark matter for the initial neutron star distribution. This result indicates that the initial core radius of the neutron star halo must be large enough compared with the distance between the Sun and the Galactic center and the sampling distance must be still larger than the core radius if we assume a core-halo structure for the initial neutron star distribution. The halo neutron star model of the  $\gamma$ -ray bursters is, however, still consistent with observations when we take the initial distribution of halo neutron stars appropriately, and we need orders of magnitude more samples to exclude the extended halo

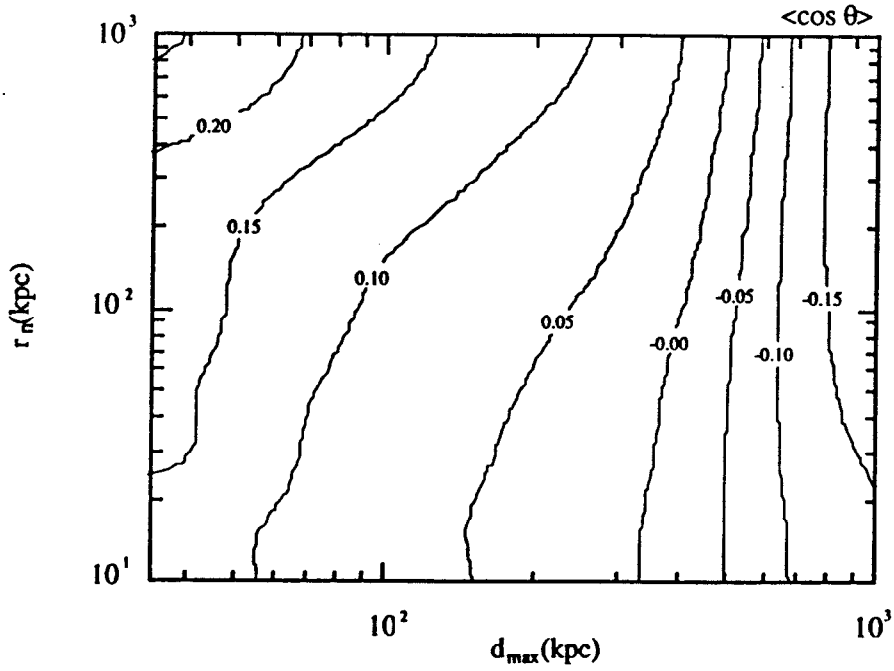


Fig. 5a

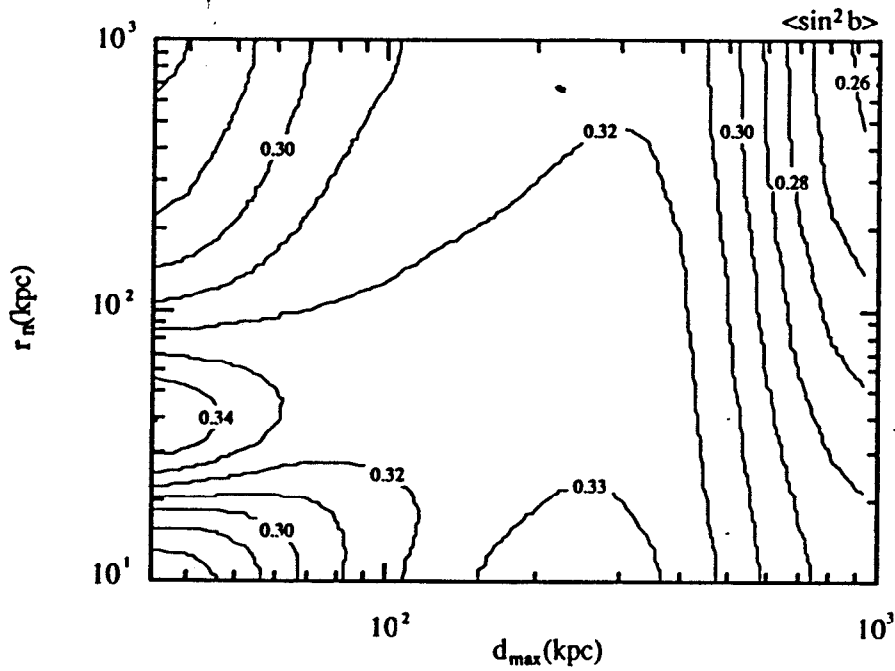


Fig. 5b

**Fig. 5a–e** Constraints on the kicked halo neutron star model defined by core radius ( $r_n$ ) and sampling distance ( $d_{max}$ ). **a**  $\langle \cos \theta \rangle$ . **b**  $\langle \sin^2 b \rangle$ . **c**  $\langle V/V_{max} \rangle$ . **d** 90% confidence levels of the model, the solid line shows the 90% confidence level from  $\langle \cos \theta \rangle$  and  $\langle \sin^2 b \rangle$ , and the broken line shows the 90% confidence level from  $\langle \cos \theta \rangle$ ,  $\langle \sin^2 b \rangle$ , and  $\langle V/V_{max} \rangle$ . **e** The number of bursts which will be need to shrink the parameter space with 90% confidence.

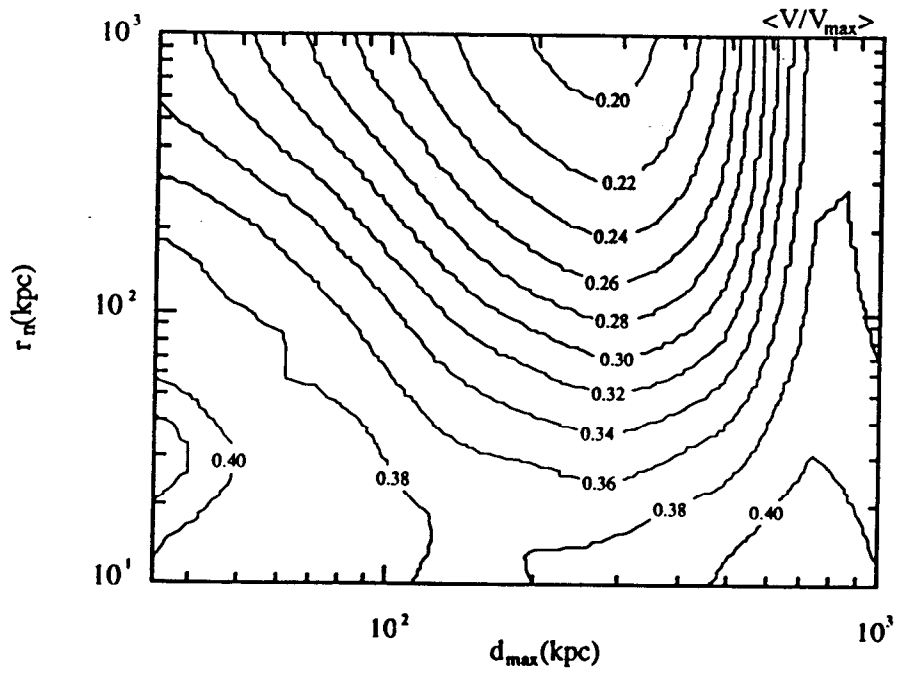


Fig. 5c

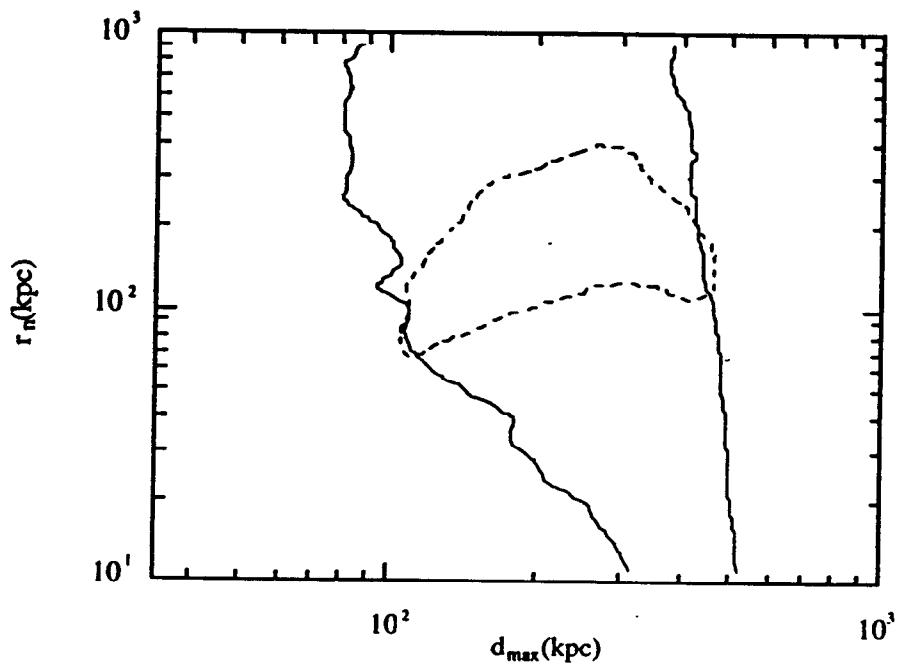


Fig. 5d

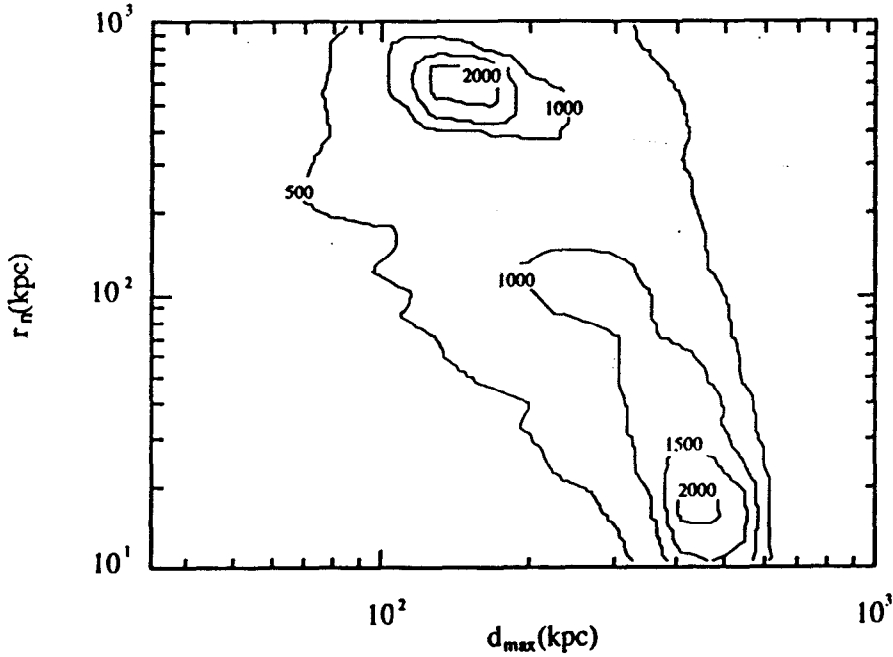


Fig. 5e

model (Fig. 5d, e).

In the halo model, disk origin neutron stars must be contained in the observed burst samples if the natures of the halo origin neutron stars and the disk origin neutron stars are not different as the burster sources. The disk origin bursters make the statistics worse as shown in the above and the halo origin bursts must dominate the number of bursts. If the sampling distances are same for the disk origin bursters and the halo origin neutron stars, the ratio of the number of the halo bursters to that of the disk bursters should be at least 2 orders larger roughly speaking. If the type-II supernova rate is  $0.01\text{yr}^{-1}$ , this means the total number of the halo origin neutron stars is at least around  $10^{10}$  and the total mass of them exceeds 10% of the luminous mass of the Galaxy.

#### 4 Conclusions

In conclusion, the kick velocity to neutron stars do not improve the statistics both of the disk origin model and the halo origin model of the  $\gamma$ -ray bursters. The disk origin model needs unnatural artificial assumptions to be consistent with the observations if the velocity distribution of the  $\gamma$ -ray bursters is same with that of the radio pulsars. If the sampling distance is small enough as  $d_{max} \lesssim 1$  kpc, and local disk neutron stars are the origin of the  $\gamma$ -ray bursters, the dipole moment and the quadrupole moment can be consistent with

observations. Such a model is benefited by the inclusion of the kick velocity since the local structure in the stellar distribution is erased and uniform distribution of the bursters is realized. However the model needs additional component of the bursters in order to be consistent with the observed deviation from uniformity of the burster distribution (e.g. Smith & Lamb 1993) unless slight deviation from uniformity results in the observed small value of  $\langle V/V_{max} \rangle$ .

One possibility to save the Galactic model from the difficulties is that  $\gamma$ -ray bursters are old neutron stars which were accelerated to be faster than  $750 \text{ km s}^{-1}$  by jet propulsion and passed the death line for pulsars due to the spin down caused by the rotational energy loss by the jet ejection. Another possibility is that the initial distribution of the neutron stars is fairly uniform and the extent of the halo is large enough for the halo model to be consistent with the observations. It implies that the initial star formation burst of the Galaxy occurred fairly uniformly in the extended halo region which view is supported by the chemical evolution of clusters of galaxies (Hattori and Terasawa 1993).

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