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Quarkonium Wave Functions at the Origin

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Abstract

We tabulate values of the radial Schrödinger wave function or its first nonvanishing derivative at zero quark-antiquark separation, for $c\bar{c}$, $c\bar{b}$, and $b\bar{b}$ levels that lie below, or just above, flavor threshold. These quantities are essential inputs for evaluating production cross sections for quarkonium states.

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$$V(r) = -0.6635 \text{ GeV} + (0.733 \text{ GeV}) \log(r \cdot 1 \text{ GeV}) \ , \tag{4}$$

with

$$m_c = 1.5 \text{ GeV}/c^2 \quad m_b = 4.906 \text{ GeV}/c^2 \ ; \tag{5}$$

and a Coulomb-plus-linear potential (the "Cornell potential") [8],

$$V(r) = -\frac{\kappa}{r} + \frac{r}{a^2} \ , \tag{6}$$

with

$$m_c = 1.84 \text{ GeV}/c^2 \quad m_b = 5.18 \text{ GeV}/c^2 \tag{7}$$

$$\kappa = 0.52 \quad a = 2.34 \text{ GeV}^{-1} \ . \tag{8}$$

For quarks bound in a central potential, it is convenient to separate the Schrödinger wave function into radial and angular pieces as $\Psi_{n\ell m}(\vec{r}) = R_{n\ell}(r)Y_{\ell m}(\theta,\phi)$, where n is the principal quantum number, ℓ and m are the orbital angular momentum and its projection, $R_{n\ell}(r)$ is the radial wave function, and $Y_{\ell m}(\theta,\phi)$ is a spherical harmonic [9]. The Schrödinger wave function is normalized, $\int d^3\vec{r}|\Psi_{n\ell m}(\vec{r})|^2 = 1$, so that $\int_0^\infty r^2 dr |R_{n\ell}(r)| = 1$. The value of the radial wave function, or its first nonvanishing derivative at the origin,

$$R_{n\ell}^{(\ell)}(0) \equiv \left. \frac{d^\ell R_{n\ell}(r)}{dr^\ell} \right|_{r=0} \ , \tag{9}$$

is required to evaluate production rates through parton fragmentation. The quantity $|R_{n\ell}^{(\ell)}(0)|^2$ is presented for four potentials in Table I for the narrow charmonium levels and in Table II for the narrow Upsilon levels. For ease of reference, we reproduce in Table III our predictions [3] for the B_c wave functions, with some computational improvements in the entries for the Cornell potential. The strong Coulomb singularity of the Cornell potential is reflected in spatially smaller states.

In view of the efforts [10] to resolve the ψ' anomaly with cascades from above-threshold states, we quote values for the $c\bar{c}$ 3D, 3P, and 3S levels that lie near 3.8, 3.9, and 4.0 GeV/c^2, respectively, and for the $b\bar{b}$ 4F, 4D, 4P, and 4S levels that lie near 10.35, 10.45, 10.52, and

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[9] We adopt the standard normalization, $\int d\Omega\, Y^*_{\ell m}(\theta,\phi) Y_{\ell' m'}(\theta,\phi) = \delta_{\ell\ell'}\delta_{mm'}$. See, for

TABLES

TABLE I. Radial wave functions at the origin and related quantities for $c\bar{c}$ mesons.

Level	$\lvert R_{n\ell}^{(\ell)}(0)\rvert^2$			
	QCD (B–T), Ref. [5]	Power-law, Ref. [6]	Logarithmic, Ref. [7]	Cornell, Ref. [8]
1S	0.810 GeV^3	0.999 GeV^3	0.815 GeV^3	1.454 GeV^3
2P	0.075 GeV^5	0.125 GeV^5	0.078 GeV^5	0.131 GeV^5
2S	0.529 GeV^3	0.559 GeV^3	0.418 GeV^3	0.927 GeV^3
3D	0.015 GeV^7	0.026 GeV^7	0.012 GeV^7	0.031 GeV^7
3P	0.102 GeV^5	0.131 GeV^5	0.076 GeV^5	0.186 GeV^5
3S	0.455 GeV^3	0.410 GeV^3	0.286 GeV^3	0.791 GeV^3

TABLE III. Radial wave functions at the origin and related quantities for $c\bar{b}$ mesons.

Level	$\lvert R_{n\ell}^{(\ell)}(0)\rvert^2$			
	QCD (B–T), Ref. [5]	Power-law, Ref. [6]	Logarithmic, Ref. [7]	Cornell, Ref. [8]
1S	1.642 GeV^3	1.710 GeV^3	1.508 GeV^3	3.184 GeV^3
2P	0.201 GeV^5	0.327 GeV^5	0.239 GeV^5	0.342 GeV^5
2S	0.983 GeV^3	0.950 GeV^3	0.770 GeV^3	1.764 GeV^3
3D	0.055 GeV^7	0.101 GeV^7	0.055 GeV^7	0.102 GeV^7
3P	0.264 GeV^5	0.352 GeV^5	0.239 GeV^5	0.461 GeV^5
3S	0.817 GeV^3	0.680 GeV^3	0.563 GeV^3	1.444 GeV^3