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String Cosmology and Inflation.

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Abstract

Following a suggestion by Gasperini and Veneziano, that String Cosmology can be reconciled with Inflation and, hence, with the Standard Big Bang, we display an analytical solution which possesses four interesting properties: (1) it is non-singular; (2) it distinguishes the dynamics of the external scale factor, $a(t)$, from that of the internal one, $b(t)$; (3) it exhibits a non-monotonic behavior of $a(t)$; and (4) it stabilizes both Newton's constant and $b(t)$ (the latter to a finite, non-vanishing value). The interest of the non-monotonic evolution of $a(t)$ consists in the fact that it contains three phases of accelerated expansion, contraction and expansion before the final decelerated expansion which eventually becomes the Standard Big Bang. The total number of e-folds of the three accelerated eras can be calculated and tuned to fit the requests of observational astronomy.

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I. INTRODUCTION.

String Theory (ST) (see [1] for a review) is at the moment the most attractive candidate for a unified description of the basic constituents in nature and their interactions. Despite tremendous progress in our understanding of fundamental strings in the past decade, we are still very far from a single quantitative prediction to be observed in experiments. The main reason for this unsatisfactory state of affairs is that the natural scale in which string effects become important (Planck scale) is much smaller than the scale we can probe in high energy scattering experiments. However, as observational cosmology provides a test of fundamental physics, since the scales of particle physics become relevant as the universe grows to its present size, the most likely arena for a confrontation between ST and experiments lies in the cosmological inferences which may be measurable today. To begin with, in general relativity, singularities in curved space-times are often unavoidable. In fact, the well-known singularity theorems [2] prove the existence of singularities under very general physical properties of the matter energy momentum tensor. For example, the Standard Big Bang (SBB) scenario (see [3]) exhibits an initial singularity at $t = 0$. In ST, there are several reasons to believe that singularities in target space do not occur. Heuristically, this belief is based on the fact that the ST possesses a "minimal length scale" set by the extension of the string itself. One of the keys in understanding the meaning of singularities and the minimal length scale in ST may be given by the so-called duality symmetry [4,5]. Duality symmetry is the most important string symmetry from many points of view. Let us suppose we have a string propagating in a target space $\mathbf{R}^{d-1} \times S^1$ where we have set the radius of the compactified dimension equal to R. It is a well known fact that every correlation function $A(1,\ldots,N)$ can be written as a topological expansion in the string coupling constant

$$A(1,\ldots,N) = \sum_{g=0}^{\infty} g_{st}^{2(g-1)} A_g(1,\ldots,N)\,, \tag{1.1}$$

where A_g is the correlator at fixed genus. Duality symmetry means that $A(1,\ldots,N)$ as a function of R and g_{st} is invariant under the replacement

$$R \to \frac{\alpha'}{R} \qquad g_{st} \to \frac{\sqrt{\alpha'}}{R} g_{st}\,, \tag{1.2}$$

together with an interchange between the momentum and the winding modes of the external states. In other words, we are unable to distinguish between small and large R provided we change the string coupling properly. Since no string scattering experiment is able to tell us whether we are living in a universe with size R and string coupling constant g_{st} or in a universe with the dual values, this defines in fact a minimal measurable length at the self-dual distance $\sqrt{\alpha'}$.

The duality symmetry is not limited to flat backgrounds; its existence was shown [6] for curved, time-dependent backgrounds, which is of particular interest in the context of cosmological singularities [7,8]. In addition to solve the initial singularity problem, a Theory of Everything must also be able to explain the low energy universe. In particular in describing the history of the whole space-time it must be able to make contact with the SBB in the attempt to describe properly the recent evolution of the 4-dimensional manifold. The main difficulty we have to deal with is that ST is defined on a D-dimensional manifold (with $D = 26$ or 10 for the bosonic and supersymmetric version respectively) while we have experience of only three spatial plus one temporal dimensions. Then the theory must be able to describe the decoupling of the external and internal manifold. In particular we must require $\mathcal{M}^D = \mathcal{M}^4 \times \mathcal{K}^{D-4}$ where $\mathcal{M}^4$ is the external 4-dimensional space-time and $\mathcal{K}^{D-4}$ is a $(D-4)$-dimensional internal compact manifold with typical physical dimension of the order of Planck scale.

Early attempts to find cosmologically interesting string scenarios able to eliminate the singularity in the early history of the universe dates back to the pioneering work of Alvarez, Leblanc, Brandenberger and Vafa, Alvarez and Osorio [7], and were based on the hypothesis of target space duality from thermodynamical considerations.

More recently Gasperini and Veneziano [8] and Antoniadis, Rizos and Tamvakis [9] proposed a different dynamical approach to the solution of the singularity problem, based on Scale Factor Duality (SFD) and the solution of the string effective equations.

Both thermodynamical and dynamical approaches proposed a suitable scenario in which the evolution of the scale factor is monotonic as time runs from $-\infty$ to $+\infty$, while the Hubble parameter is positive and *bell shaped* as a function of time.

In this paper, starting from dynamical considerations, we show a different, richer, non-singular scenario of string cosmology able to make contact with the SBB for the late evolution of the universe. In particular, after introducing external and internal scale factors, $a(t)$ and $b(t)$, we find that the evolution of $a(t)$ is *not* monotonic; a contraction phase is present in the early history of the universe, while the effective 4-dimensional gravitational coupling naturally converges to a constant value corresponding to the present value of Newton's constant. As during the evolution of the universe there are different phases of accelerated dynamics, the scenario presented in this paper, in addition to solving the problems of the singularity and of the constancy of the fundamental constants, offers a natural framework in which to accomodate inflation (which is the solution of the other well known problems of the SBB, i.e. horizon, flatness and structure formation) (see, however, Ref. [10]).

This paper is organized as follows: in Sect. II we discuss the low energy string effective action and the SFD symmetry. Sect. III is devoted to a general and qualitative description of solutions of the string field equations. In Sect. IV we present the general solution to the string cosmology equations which interpolates smoothly the dual evolution of the 'pre-big-bang' and 'post-big-bang' phases, where here by big bang (in low case) we mean the epoch of transition between the two dual phases. In Sect. V we summarize the main conclusions. The Appendixes are devoted to the proof of the non-singular behavior of the solution reported in Sect. IV, and to the representation of the same solutions in the Einstein frame, respectively.

II. LOW ENERGY STRING EFFECTIVE ACTION AND SCALE FACTOR DUALITY.

Let us consider the propagation of a bosonic string in the presence of a background consisting of a D-dimensional metric $g_{\mu\nu}$ ($\mu,\nu = 0,1,\ldots,D-1$) and a dilaton Φ. It is described by the two-dimensional σ-model

$$S_\sigma = \int d^2x\sqrt{h}\left[h^{ab}\partial_a X^\mu \partial_b X^\nu g_{\mu\nu}(X^\rho) + \alpha' R^{(2)}\Phi(X^\rho)\right] , \tag{2.1}$$

where h_{ab} is the world-sheet metric tensor and $R^{(2)}$ is the Ricci scalar constructed with h_{ab}. The requirement of conformal invariance of S_σ (i.e. the vanishing of the β-functions) leads naturally to the determination of the massless modes' dynamics. In particular we have the following tree level effective action for the background fields [11]

$$S_{\text{eff}} = -\frac{1}{2\kappa^2}\int d^Dx\sqrt{-g}e^{-\Phi}\left(R + \partial_\mu\Phi\partial^\mu\Phi + c\right) , \tag{2.2}$$

a multidimensional Brans-Dicke (BD) theory. The cosmological constant represents the central charge deficit of the theory, $c = -2(D_{\text{eff}} - D_{\text{crit}})/3\alpha'$ depending on details of particular ST ($D_{\text{eff}} = D$, $D_{\text{crit}} = 26$ in the bosonic version, $D_{\text{eff}} = \frac{3}{2}D$, $D_{\text{crit}} = 15$ in the supersymmetric version). The effective action (2.2) leads to the following equations of motion

$$0 = (\partial\Phi)^2 - 2\Box\Phi - R - c, \tag{2.3a}$$

$$0 = \left(R_{\mu\nu} + \nabla_\mu\nabla_\nu\Phi\right)e^{-\Phi} \tag{2.3b}$$

(∇_μ is the covariant derivative and $\Box = \nabla_\mu\nabla^\mu$ is the D-dimensional d'Alambertian). Now, it is known [6,12] that if the metric and dilaton fields do not depend on the coordinate x^i, the field equations (2.3) are invariant under the SFD transformation

$$g_{ii} \rightarrow \tilde{g}_{ii} = g_{ii}^{-1} , \tag{2.4a}$$

$$\Phi \rightarrow \tilde{\Phi} = \Phi - \ln|g_{ii}| , \tag{2.4b}$$

The non-trivial duality tranformation behavior of the dilaton field implies that the coordinate-dependent string coupling constant is trasformed like $g_{st}^2(x) = e^\Phi \rightarrow g_{st}^2(x)g_{ii}^{-1}$. This change of the string coupling constant agrees with the transformation of g_{st}^2 in the static case (equation (1.2)) when one considers the genus expansion of the string partition function [5].

This transformation is just a particular case of a more general global $O(d,d)$ covariance of the theory [13–15]: $O(d,d)$ covariance means that, if the theory is independent of d spatial coordinates, the dilaton tranforms as

$$\Phi \rightarrow \Phi - \ln|\det g_{ij}| , \tag{2.5}$$

and the components of the metric and of the antisymmetric[1] tensors mix according to

[1]The third massles mode of the bosonic string.

$$M \to \Omega^{\rm T} M \Omega\,, \tag{2.6}$$

where $\Omega \in O(d,d)$ and

$$M = \begin{pmatrix} G^{-1} & -G^{-1}B \\ BG^{-1} & G - BG^{-1}B \end{pmatrix} \tag{2.7}$$

($G \equiv g_{ij}$ and $B \equiv B_{ij} = -B_{ji}$ are matrix representations of the $d \times d$ spatial part of the metric and antisymmetric tensor, in the basis where the $O(d,d)$ metric is off-diagonal.)

In reference [14] it was shown that $O(d,d)$ covariance holds even if the equations (2.3) are supplemented by a phenomenological source term corresponding to bulk string matter.

The importance of SFD in the context of string cosmology is that, when combined with time reversal (the most obvious symmetry of the theory), it allows us to associate at every phase of 'post-big-bang' evolution (for $t_c < t < +\infty$) a dual phase, called 'pre-big-bang' [8], ($-\infty < t < t_c$) with a completely different dynamics for the fields. In fact SFD is not a simple reparametrization of the fields, nor are its implications trivial. For example, if we start with a scale factor $a(t)$ that is expanding, the dual scenario $a(t) \to a^{-1}(t)$ describes a contracting universe. When combined with time reversal, SFD maps, for example, a background with decreasing curvature to the dual one characterized by a curvature that is increasing.

Finally it is important to stress the necessity of the presence of the dilaton for SFD to be a symmetry of the low energy string effective action. In fact for $\Phi \equiv 0$ the action (2.2) reduces to conventional general relativity, and we are left simply with the symmetry of time reversal. Only the existence of the dilaton field and its non trivial behavior under SFD (equation (2.4)) allows us to realize the transformation $a \to a^{-1}$.

III. GENERAL COSMOLOGICAL SOLUTIONS.

To make contact with observational cosmology, we take the D-dimensional space-time as a direct product of the external pseudo-Riemannian manifold $\mathcal{M}_4$ with an $n(= D-4)$-dimensional compact Riemannian manifold $\mathcal{K}_n$. We take for $\mathcal{M}_4$ a flat Friedman-Robertson-Walker space-time with scale factor $a(t)$, and we assume that the dilaton field Φ and the radius b of the internal space (which we take to be an n-torus) depend only on the temporal coordinate:

$$g_{\mu\nu} = {\rm diag}(1, -a^2(t)\delta_{ij}, -b^2(t)\delta_{ab})\,, \tag{3.1}$$
$$\Phi = \Phi(t) \tag{3.2}$$

($i,j = 1,2,3;\ a,b = 1,\ldots,n = D-4$). With these choices for the fields and adding a source term representing a primordial string gas with energy momentum tensor

$$T_\mu^\nu = {\rm diag}(\rho(t)\,,\ -p(t)\delta_i^j\,,\ -q(t)\delta_a^b)\,, \tag{3.3}$$

the field equations (2.3) (with $c = 0$) which now change in[2]

$$(\partial\Phi)^2 - 2\Box\Phi - R = 0\,, \tag{3.4a}$$

$$\left(R^\nu_\mu + \nabla_\mu\nabla^\nu\Phi\right)e^{-\Phi} = \kappa^2 T^\nu_\mu\,, \tag{3.4b}$$

$$\nabla^\mu T^\nu_\mu = 0\,, \tag{3.4c}$$

can be written as

$$-2\ddot{\bar{\Phi}} + \dot{\bar{\Phi}}^2 + 3H^2 + nF^2 = 0\,, \tag{3.5a}$$

$$\dot{\bar{\Phi}}^2 - 3H^2 - nF^2 = \kappa^2 e^{\bar{\Phi}}\bar{\rho}\,, \tag{3.5b}$$

$$\dot{H} - H\dot{\bar{\Phi}} = \kappa^2 e^{\bar{\Phi}}\bar{p}\,, \tag{3.5c}$$

$$\dot{F} - F\dot{\bar{\Phi}} = \kappa^2 e^{\bar{\Phi}}\bar{q}\,, \tag{3.5d}$$

$$\dot{\bar{\rho}} + 3H\bar{p} + nF\bar{q} = 0\,. \tag{3.5e}$$

Here we have introduced the Hubble parameters $H = \dot{a}/a$ and $F = \dot{b}/b$ for the external and internal space respectively and we have denoted with barred symbols the $O(D-1, D-1)$-invariant expressions for the dilaton and the matter energy density[3]

$$\bar{\Phi} = \Phi - 3\ln a - n\ln b\,, \tag{3.6}$$

$$\bar{\rho} = \rho a^3 b^n \tag{3.7}$$

(we also introduce $\bar{p} = pa^3b^n$ and $\bar{q} = qa^3b^n$.) To solve the system (3.5) we must introduce an equation of state for the source term of the form

$$p = \gamma\rho\,, \qquad q = \lambda\rho\,, \tag{3.8}$$

with, at the moment, γ and λ arbitrary functions of time.

Introducing a coordinate time ξ defined by

$$d\xi = \bar{\rho}\ell dt\,, \tag{3.9}$$

(ℓ is an arbitrary constant of dimension of length) the forementioned equations can be integrated [8,15] to obtain ($2\kappa^2 = 1$)

$$\bar{\Phi} - \Phi_0 = -2\int_0^\xi \frac{\xi - \xi_0}{\Delta(\xi)}\,d\xi\,, \tag{3.10a}$$

$$\bar{\rho} = \tfrac{1}{4}\ell^{-2}\Delta(\xi)e^{\bar{\Phi}}\,, \tag{3.10b}$$

$$H = \tfrac{1}{2}\ell^{-1}(\alpha_H + \Gamma)e^{\bar{\Phi}}\,, \tag{3.10c}$$

$$F = \tfrac{1}{2}\ell^{-1}(\alpha_F + \Lambda)e^{\bar{\Phi}}\,, \tag{3.10d}$$

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[2]These three equations are not independent. The third one, in fact, can be obtained by a combination of the gradient of the first and the second.

[3]The behavior of the latter under SFD consists in [14] $\bar{\rho} \to \bar{\rho}$, while for the pressures $p/\rho \to -p/\rho$ and $q/\rho \to -q/\rho$.

where

$$\Gamma = \int_0^\xi \gamma \, d\xi \,, \qquad \Lambda = \int_0^\xi \lambda \, d\xi \,, \tag{3.11}$$

$$\Delta(\xi) = 4\beta + (\xi + \xi_0)^2 - 6\alpha_H \Gamma - 3\Gamma^2 - 2n\alpha_F \Lambda - n\Lambda^2 \,, \tag{3.12}$$

$\Phi_0, \xi_0, \alpha_H, \alpha_F$ are arbitrary constants and

$$\beta = -\tfrac{1}{4}(3\alpha_H^2 + n\alpha_F^2) \,, \tag{3.13}$$

is negative.

From (3.10b), to have a positive definite energy density we must require $\Delta(\xi) > 0$; also the zeroes of $\Delta(\xi)$ correspond to singularities for the fields.

Let us consider the simplest case $\gamma = \hat{\gamma}$ and $\lambda = \hat{\lambda}$, where $\hat{\gamma}$ and $\hat{\lambda}$ are constants. The general solution of (3.10) [16,17] in a form convenient to our discussion reads

$$a(\xi) = a_0 \, |(\xi - \xi_+)(\xi - \xi_-)|^{\hat{\gamma}/\epsilon} \left| \frac{\xi - \xi_+}{\xi - \xi_-} \right|^{\sigma_H} , \tag{3.14a}$$

$$b(\xi) = b_0 \, |(\xi - \xi_+)(\xi - \xi_-)|^{\hat{\lambda}/\epsilon} \left| \frac{\xi - \xi_+}{\xi - \xi_-} \right|^{\sigma_F} , \tag{3.14b}$$

$$H(\xi) = \tfrac{1}{2} \ell^{-1} e^{\Phi_0} (\alpha_H + \hat{\gamma}\xi) \, |(\xi - \xi_+)(\xi - \xi_-)|^{-1/\epsilon} \left| \frac{\xi - \xi_+}{\xi - \xi_-} \right|^{-\sigma_0} , \tag{3.14c}$$

$$F(\xi) = \tfrac{1}{2} \ell^{-1} e^{\Phi_0} (\alpha_F + \hat{\lambda}\xi) \, |(\xi - \xi_+)(\xi - \xi_-)|^{-1/\epsilon} \left| \frac{\xi - \xi_+}{\xi - \xi_-} \right|^{-\sigma_0} , \tag{3.14d}$$

$$e^{\Phi(\xi)} = a_0^3 b_0^n e^{\Phi_0} \, |(\xi - \xi_+)(\xi - \xi_-)|^{-1(1 - 3\hat{\gamma} - n\hat{\lambda})/\epsilon} \left| \frac{\xi - \xi_+}{\xi - \xi_-} \right|^{-\sigma_0 + 3\sigma_H + n\sigma_F} , \tag{3.14e}$$

$$\rho(\xi) e^{\Phi(\xi)} = \tfrac{1}{2} \epsilon \ell^{-2} e^{2\Phi_0} \mathrm{sign}[\Delta(\xi)] \, |(\xi - \xi_+)(\xi - \xi_-)|^{(\epsilon - 2)/\epsilon} \left| \frac{\xi - \xi_+}{\xi - \xi_-} \right|^{-2\sigma_0} , \tag{3.14f}$$

where

$$\xi_\pm = \frac{1}{\epsilon} \left\{ 3\alpha_H \hat{\gamma} + n\alpha_F \hat{\lambda} - \xi_0 \pm \left[\left(\xi_0 - 3\alpha_H \hat{\gamma} - n\alpha_F \hat{\lambda} \right)^2 - \epsilon \left(\xi_0 - 3\alpha_H^2 - n\alpha_F^2 \right) \right]^{1/2} \right\} , \tag{3.15}$$

are the two *real* zeroes of $\Delta(\xi)$ and

$$\begin{aligned} \epsilon &= 1 - 3\hat{\gamma}^2 - n\hat{\lambda}^2 \,, \\ \sigma_0 &= \frac{\xi_+ + \xi_- - 2\xi_0/\epsilon}{\xi_+ - \xi_-} \,, \\ \sigma_H &= \frac{\xi_+ + \xi_- + 2\alpha_H/\epsilon}{\xi_+ - \xi_-} \,, \\ \sigma_F &= \frac{\xi_+ + \xi_- + 2\alpha_F/\epsilon}{\xi_+ - \xi_-} \,. \end{aligned} \tag{3.16}$$

Other solutions of the system (3.5) are also obtained from (3.14) through SFD.

The internal and external Hubble parameters have two singularities, at $\xi = \xi_+$ and at $\xi = \xi_-$. For $\epsilon > 0$ (necessary condition to have a positive energy density today) the range (ξ_-, ξ_+) is not physical because here ρ becomes negative. The evolution of the scale factors depends strongly on the relative sign of α_H and $\hat{\gamma}$, and of α_F and $\hat{\lambda}$; but in any case singularities are always present in the curvature, contrary to what stressed in [18] in the context of Brans-Dicke theory. In Fig. 1 we report a qualitative representation of a generic scale factor for $\alpha_i/\omega_i > 0$ and $\alpha_i/\omega_i < 0$, where $\{\alpha_i\} = (\alpha_H, \alpha_F)$ and $\{\omega_i\} = (\hat{\gamma}, \hat{\lambda})$.

IV. NON-SINGULAR SOLUTIONS.

For the solutions found in Sect. III the growth of curvature, of the effective coupling $e^{\bar{\Phi}}$ and of the effective energy density $\rho e^{\bar{\Phi}}$, are unbounded, which is inacceptable in the light of the discussion of Sect. I and to phenomenological constraints on the graviton spectrum discussed in [8]. The problem is then to find a smooth transition from the pre-big-bang phase to the post-big-bang one. For this purpose we exploit a suggestion of Gasperini and Veneziano [8].

In the vicinity of the Planck scale the low energy string effective action (2.2) does not apply; we expect some modifications. To preserve the symmetry under SFD we must require that these corrections are themselves invariant. Following [8] we introduce a self-dual dilaton potential $V(\bar{\Phi}) = -V_0 e^{2\bar{\Phi}}$. The new field equations can be still reduced to the form (3.10), but with

$$\beta = \ell^2 V_0 - \tfrac{1}{4}(3\alpha_H^2 + n\alpha_F^2)\,, \tag{4.1}$$

which, unlike (3.13), is no longer necessarily negative. For $\epsilon > 0$ (positivity of the source energy density) it is then possible to choose $V_0 > 0$ and large such that $\Delta(\xi)$ does not have zeroes in the real field (a proof is given in Appendix A). This implies that there are no singularities in the curvature, nor in the effective coupling, nor in the effective energy density.

To solve the system (3.10) we must assign the equation of state of the source term. Being interested in solutions which describe a smooth transition from the pre-big-bang era ($\xi < 0$) to the dual post-big-bang ($\xi > 0$) it is worth looking for self-dual solutions, and in order to obtain them, as ξ goes through zero, the external and internal pressures must change sign, [8]. So $\gamma(\xi)$ and $\lambda(\xi)$ must be odd functions of ξ. From phenomenological considerations we must also impose the constraint of stationariety of the functions γ and λ for large values of $|\xi|$. Then, if we introduce the vector of pressure $\{p_i\} = (p, q)$, we must require

$$\frac{p_i}{\rho} \to -\omega_i\,, \qquad \text{for } \xi < -\bar{\xi}\,,$$
$$\frac{p_i}{\rho} \to +\omega_i\,, \qquad \text{for } \xi > +\bar{\xi}\,,$$

(ω_i and $\bar{\xi}$ constants) with a smooth transition between the two phases. It is evident that these properties are shared by a vast class of functions. For example, any of the choices

$$\frac{f(\xi)}{\sqrt{f^2(\xi)+\hat{\xi}^2}}, \quad \tanh(f(\xi)), \quad \frac{2}{\pi}\arctan(f(\xi)), \quad \ldots \tag{4.2}$$

with $f(\xi)$ arbitrary, smooth, odd and asymptotically monotonic function of ξ, would do. But, since, from a qualitative point of view [19], the result is independent of the form of the functions γ, λ for fixed f, for mathematical simplicity, we choose (as in [8])

$$\gamma(\xi) = \frac{\hat{\gamma}\xi}{\sqrt{\xi^2+\xi_1^2}}, \qquad \lambda(\xi) = \frac{\hat{\lambda}\xi}{\sqrt{\xi^2+\xi_2^2}}, \tag{4.3}$$

with $\hat{\gamma}, \hat{\lambda}, \xi_1, \xi_2$ constants.

Equations (3.10) can be solved analytically if $\xi_0 = 0$ and $\xi_1 = \xi_2$. In place of the singular solutions (3.14) we obtain now [19]

$$H(\xi) = \tfrac{1}{2}\ell^{-1}e^{\Phi_0}\left(\alpha_H + \hat{\gamma}\sqrt{\xi^2+\xi_1^2}\right)(\Delta(\xi))^{-1/\epsilon}\exp\left\{-\frac{2\zeta}{\epsilon\sqrt{\chi}}\arctan\left(\frac{\epsilon\sqrt{\xi^2+\xi_1^2}-\zeta}{\sqrt{\chi}}\right)\right\}, \tag{4.4a}$$

$$F(\xi) = \tfrac{1}{2}\ell^{-1}e^{\Phi_0}\left(\alpha_F + \hat{\lambda}\sqrt{\xi^2+\xi_1^2}\right)(\Delta(\xi))^{-1/\epsilon}\exp\left\{-\frac{2\zeta}{\epsilon\sqrt{\chi}}\arctan\left(\frac{\epsilon\sqrt{\xi^2+\xi_1^2}-\zeta}{\sqrt{\chi}}\right)\right\}, \tag{4.4b}$$

$$e^{\bar{\Phi}(\xi)} = e^{\Phi_0}(\Delta(\xi))^{-1/\epsilon}\exp\left\{-\frac{2\zeta}{\epsilon\sqrt{\chi}}\arctan\left(\frac{\epsilon\sqrt{\xi^2+\xi_1^2}-\zeta}{\sqrt{\chi}}\right)\right\}, \tag{4.4c}$$

$$\bar{\rho}(\xi) = \tfrac{1}{4}\ell^{-2}e^{\Phi_0}(\Delta(\xi))^{(\epsilon-1)/\epsilon}\exp\left\{-\frac{2\zeta}{\epsilon\sqrt{\chi}}\arctan\left(\frac{\epsilon\sqrt{\xi^2+\xi_1^2}-\zeta}{\sqrt{\chi}}\right)\right\}, \tag{4.4d}$$

where

$$\zeta = 3\alpha_H\hat{\gamma} + n\alpha_F\hat{\lambda}, \tag{4.5}$$
$$\chi = (4\beta - \epsilon\xi_1^2)\epsilon - \zeta^2, \tag{4.6}$$

and ϵ has been defined in (3.16). For the scale factors, as we are unable to give analytical expressions, we have proceeded to numerical integrations. In any case a qualitative information can be easily obtained from the relative Hubble parameters. In particular, if we define $\{H_i(\xi)\} = (H(\xi), F(\xi))$, it is easy to see that if $\mathrm{sign}[\alpha_i\omega_i] = +1$ then the Hubble parameter H_i never changes sign, while if $\mathrm{sign}[\alpha_i\omega_i] = -1$ then

$$\mathrm{sign}\left[H_i\left(|\xi| < \sqrt{\left(\frac{\alpha_i}{\omega_i}\right)^2 - \xi_1^2}\right)\right] = -\mathrm{sign}\left[H_i\left(|\xi| > \sqrt{\left(\frac{\alpha_i}{\omega_i}\right)^2 - \xi_1^2}\right)\right]. \tag{4.7}$$

Therefore, if for large $|\xi|$, H_i is positive (corresponding to a background that today is expanding), for ξ near zero we have a transition *expansion-contraction-expansion* as depicted in Fig. 2.

These solutions generalize in a non-trivial way the ones obtained in [8]. The latter, in fact, can be easily recovered if we set $\alpha_H = \alpha_F = 0$, $\hat{\gamma} = -\hat{\lambda} = 1/(3+n)$ and $4\beta = \xi_1^2$.

The presence of a contraction phase, very suggestive in itself, has interesting consequences on the present structure of the observable universe. It was recently stressed in [16] that every kind of accelerated evolution, whether during expansion or contraction, naturally solves the kinematical problems of the SBB. Moreover, when expressed in terms of conformal time, the contraints for successful inflation are the same for expansion and contraction. Then our solution offers a natural scenario capable of solving the flatness, horizon and structure formation problems of SBB, without introducing an *ad hoc* inflaton. As for the number of e-folds of accelerated contraction, we have

$$\mathcal{N} \equiv \int_{t_{\rm in}}^{t_{\rm fin}} H(t)\, dt = \int_{\xi_{\rm in}}^{0} H(\xi) \frac{dt}{d\xi}\, d\xi = \int_{\xi_{\rm in}}^{0} W(\xi)\, d\xi \,, \tag{4.8}$$

with $\xi_{\rm in} = -\sqrt{(\alpha_H/\hat{\gamma})^2 - \xi_1^2}$ the negative zero of $H(\xi)$ and the integrand $W(\xi) \equiv 2(\alpha_H + \Gamma)/\Delta(\xi)$. Although the latter integral cannot be performed analytically, it can be seen that $W(\xi)$ has the same shape of $H(\xi)$ and, for the period of accelerated contraction, a very good fitting is a linear interpolation (we have a correlation coefficient $R = 0.99\ldots$). Then we get for the linear fitting function $\widehat{W}$ in the range $\xi \in [\xi_{\rm in}, 0]$

$$\widehat{W}(\xi) = -W(0)\left(\frac{\xi}{\xi_{\rm in}} - 1\right) , \tag{4.9}$$

and

$$\mathcal{N} = \tfrac{1}{2} W(0) \xi_{\rm in} \,, \tag{4.10}$$

where

$$W(0) \equiv W(\xi = 0) = \frac{\alpha_H + \hat{\gamma}|\xi_1|}{4\ell^2 V_0 - (3\hat{\gamma}^2 + n\hat{\lambda}^2)\xi_1^2 - 2(3\alpha_H\hat{\gamma} + n\alpha_F\hat{\lambda})|\xi_1| - 3\alpha_H^2 - n\alpha_F^2} \,. \tag{4.11}$$

We can choose α_H and α_F to tune $\mathcal{N}$ to any desired value. But the contraction phase is not the only period of accelerated evolution of the external space. For sign$[\alpha_H\hat{\gamma}] = -1$ three different phases of accelerated evolution (see Fig. 3) follow each other during the early history of the universe. Then, in addition to solving naturally the kinematical problems typical of the SBB, from the point of view of structure formation, our scenario gives rise to a sort of multiple inflation (but with a single field) capable of breaking the scale invariance of the fluctuation power spectrum, a possible solution to the problem of large scale power in galaxy distribution [20].

Furthemore, in order to make contact with observational cosmology, in addition to explaining why the universe is flat, the entropy is so high, etc., we must also be able to explain why the fundamental constants are effectively constants. In fact in string cosmology the gravitational coupling is dynamical. When reducing the theory from D to four dimensions, we get that the 4-dimensional "Newton's constant" is proportional to the inverse of the volume of the internal space times the inverse of the coupling between the scalar field and the Ricci scalar ($G_N \sim b^{-n}e^{\Phi}$): in general, the latter expression is hardly constant, while Newton's constant must be "constant" at least from nucleosynthesis onward [21]. Really this is the most difficult problem in multi-dimensional and scalar-tensor theories.

If we look at the asymptotic behavior of solutions (4.4), then we can realize that our scenario has just this additional bonus. It is in fact easy to see that for very large ξ we have

$$a(\xi) \sim \xi^{2\hat{\gamma}/\epsilon}, \tag{4.12a}$$

$$b(\xi) \sim \xi^{2\hat{\lambda}/\epsilon}, \tag{4.12b}$$

$$e^{\Phi(\xi)} \sim \xi^{2(3\hat{\gamma}+n\hat{\lambda}-1)/\epsilon}, \tag{4.12c}$$

and

$$G_N \sim b^{-n}e^{\Phi} \sim \xi^{2(1-3\hat{\gamma})/\epsilon}. \tag{4.13}$$

Then for a universe with radiation in the external space $\hat{\gamma} = \frac{1}{3}$, G_N becomes asymptotically constant (see Fig. 4). What is really surprising is that Newton's constant stabilizes *indipendently* of the dynamics of the internal space, which asymptotically can expand (for $\hat{\lambda} > 0$), contract (for $\hat{\lambda} < 0$) or approach a constant value (for $\hat{\lambda} = 0$; the most attractive possibility). For what concerns the asymptotic dynamics of the external space, the choice $\hat{\gamma} = \frac{1}{3}$ naturally leads to the typical behavior of Radiation Dominated model: $a(t) \sim t^{1/2}$.

The same asymptotic scenario is shared also by the singular solutions (3.14), because the dilaton potential strongly modifies the field's dynamics only around $|\xi| \sim 0$, becoming rapidly uninfluential as ξ grows.

V. CONCLUSIONS.

The combination of Einstein's general theory of relativity and of the Copernican principle naturally leads to the formulation of the SBB. Although in the last decades the SBB has been strongly confirmed by astronomical observations, it still presents some "conceptual" difficulties: the existence of an initial singularity and the well known kinematical problems (horizon, flatness, structure formation). It is a common belief that these problems can be solved when a consistent quantum theory of gravitation will be formulated. Today ST seems the most plausible attempt to quantize gravity, and then it is very tempting to study its implications on the early evolution of the universe. Foremost in supporting the belief that string cosmology can solve the singularity problem, is SFD symmetry, one of the most important symmetries of ST. It means that if $a(t)$ solves the string equations then also $a^{-1}(t)$ is a solution of the same dynamical system, thus introducing a minimal length scale. Furthermore ST modifies the commonly accepted lore: the present decelerated expansion is preceded by a dual phase in which the evolution is accelerated. The smooth passage between the two asymptotic phases may be realized by a period of accelerated contraction. Because accelerated contraction is as efficient as accelerated expansion to solve the kinematical problems of the SBB, the ST scenario presents multiple episodes of inflation. The last difficulty we are able to cope with is to justify the constancy of the 4-dimensional Newton's constant without requiring the introduction of a mass term for the dilaton or the formulation of a least coupling principle [22]. In fact the low energy string effective action is a multi-dimensional scalar tensor theory of gravity, presenting then a dynamical gravitational coupling. Astronomical observations imply however that $\dot{G}_N/G_N \lesssim 10^{-11}\text{sec}^{-1}$ [21]: a realistic model must describe the spontaneous stabilization of the Newton's constant, which we obtain.

Obviously the scenario presented here is basically a toy model, which needs more theoretical support. For example we should justify the form of the dilaton potential and the equation of state for string sources in curved backgrounds. Nevertheless we want to stress that some of the positive and new results presented in this paper (presence of a primordial contraction phase, stabilization of the fundamental constats, convergence towards the SBB) are common both to non-singular and to more conventional singular scenarios, representing then a element in favour of string cosmology.

The development of the subject and the deeper study of the astrophysical implications of an early dynamical gravitational coupling and contraction phase, in addition to the spontaneous compactification of the internal dimensions, may represent a solid benchmark to test the theoretical predictions of ST.

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APPENDIX A: DOES $\Delta(\xi)$ VANISH?

In this Appendix we want to prove that the introduction of the dilaton potential, in Sect. IV, allows us to eliminate the singularities in the fields and in the energy density of the string bulk matter. To show this, it is enough to study the zeroes of $\Delta(\xi)$; we will be able to avoid the singularities if $\Delta(\xi) \neq 0$ for ξ real. We have ($\xi_0 = 0$ as in Sect. IV)

$$\Delta(\xi) = \Omega - 2\zeta\sqrt{\xi^2 + \xi_1^2} + \epsilon\xi^2 , \tag{A1}$$

where we have defined

$$\begin{aligned} \Omega &= 4\beta - (1-\epsilon)\xi_1^2 , \\ \beta &= \ell^2 V_0 - \tfrac{1}{4}\sigma , \\ \sigma &= 3\alpha_H^2 + n\alpha_F^2 \\ \zeta &= 3\alpha_H\hat{\gamma} + n\alpha_F\hat{\lambda} , \\ \epsilon &= 1 - 3\hat{\gamma}^2 - n\hat{\lambda}^2 . \end{aligned}$$

As we have yet stressed in Sect. III, we must require $\epsilon > 0$ to have a positive energy density. It is immediately seen that $\Delta(\xi) = 0$ implies

$$\epsilon^2\xi^4 + 2(\epsilon\Omega - 2\zeta^2)\xi^2 + \Omega^2 - 4\zeta^2\xi_1^2 = 0 . \tag{A2}$$

If we introduce

$$y = \xi^2\,,$$
$$a = \epsilon\Omega - 2\zeta^2\,,$$
$$b = \Omega^2 - 4\zeta^2\xi_1^2\,,$$

equation (A2) reduces to

$$\epsilon^2 y^2 + 2ay + b = 0\,. \tag{A3}$$

Because $y = \xi^2$, $\Delta(\xi)$ does not vanish in the real field if and only if one either of the following conditions is satisfied:

$$a) \quad \delta \equiv a^2 - \epsilon^2 b < 0\,,$$
$$b) \quad -a + \sqrt{\delta} < 0 \qquad \text{if } a > 0,\ \delta > 0\,.$$

For what concerns condition $a)$ we have

$$\delta = (\epsilon\Omega - 2\zeta^2)^2 - \epsilon^2(\Omega^2 - 4\zeta^2\xi_1^2)\,,$$
$$= -4\zeta^2(4\epsilon\ell^2 V_0 - \tau^2)\,,$$

where

$$\tau^2 = \epsilon\sigma + \epsilon(1-\epsilon)\xi_1^2 + \zeta^2 + \epsilon^2\xi_1^2 > 0 \tag{A4}$$

(remember that $0 < \epsilon < 1$). Then $\delta < 0$ implies

$$V_0 > \frac{\tau^2}{4\epsilon\ell^2} > 0\,. \tag{A5}$$

Now with condition $b)$. We must take

$$\delta > 0 \quad \text{and then} \quad V_0 \leq \frac{\tau^2}{4\epsilon\ell^2}\,, \tag{A6}$$

and

$$a > 0\,,$$
$$-a + \sqrt{\delta} < 0\,.$$

Let us start to study the condition $a > 0$. It implies

$$\epsilon\Omega - 2\zeta^2 = \epsilon[4V_0\ell^2 - \sigma - (1-\epsilon)\xi_1^2] - 2\zeta^2 > 0\,, \tag{A7}$$

and then

$$V_0 > \frac{\eta^2}{4\epsilon\ell^2} > 0\,, \tag{A8}$$

where we have defined

$$\eta^2 = \epsilon\sigma + \epsilon(1-\epsilon)\xi_1^2 + 2\zeta^2 = \tau^2 + \zeta^2 - \epsilon^2\xi_1^2 > 0\,. \tag{A9}$$

To make conditions (A6) and (A8) compatible we must require $\eta^2 < \tau^2$, which means

$$\zeta^2 < \epsilon^2 \xi_1^2 . \tag{A10}$$

If inequality (A10) is not verified, we have $\eta^2 > \tau^2$ and then the condition $a > 0$ automatically implies $\delta < 0$, which means that $\Delta(\xi)$ never vanishes in the real field for condition a).

For what concerns the second condition, $\sqrt{\delta} < a$, we have

$$16\epsilon^2 V_0^2 \ell^4 - 8\epsilon(\eta^2 - 2\zeta^2) V_0 \ell^2 - 4\zeta^2\tau^2 + \eta^4 > 0 . \tag{A11}$$

Because

$$V_0 = \frac{\theta_\pm}{4\epsilon\ell^2} , \tag{A12}$$

with $\theta_\pm = \eta^2 - 2\zeta^2 \pm 2|\zeta\epsilon\xi_1|$, we must require

$$4\epsilon V_0 \ell^2 < \theta_- \quad \text{and} \quad 4\epsilon V_0 \ell^2 > \theta_+ . \tag{A13}$$

Summing up, condition b) is equivalent to the following conditions on V_0

$$\eta^2 < 4\epsilon V_0 \ell^2 \leq \tau^2 , \tag{A14a}$$

$$\theta_+ < 4\epsilon V_0 \ell^2 , \tag{A14b}$$

$$4\epsilon V_0 \ell^2 < \theta_- . \tag{A14c}$$

Because $\zeta^2 < \epsilon^2 \xi_1^2$ we have also

$$\eta^2 < \theta_+ < \tau^2 ,$$

$$\theta_- < \eta^2 ,$$

and then, conditions (A14) reduces to

$$0 < \frac{\theta_+}{4\epsilon\ell^2} < V_0 \leq \frac{\tau^2}{4\epsilon\ell^2} . \tag{A15}$$

In any case, the request that $\Delta(\xi)$ never vanishes (conditions a) and b)) automatically implies $V_0 > 0$ (inequalities (A5) and (A15)).

APPENDIX B: REPRESENTATION IN THE EINSTEIN FRAME.

It is well known that scalar-tensor and non-linear gravity theories can be reformulated in a more conventional framework: Einstein gravity plus a minimally coupled scalar field [23]. We have only to perform a Weyl rescaling of the metric tensor, $g_{\mu\nu} \to \Omega^2 g_{\mu\nu}$. For ST we must take [11]

$$\Omega^2 = e^{2\Phi/(D-1)} . \tag{B1}$$

Then the action (2.2) reduces to

$$S \to -\frac{1}{2\kappa^2}\int d^Dx\sqrt{-g}\left\{R - \frac{1}{D-2}(\partial\Phi)^2 + ce^{-\frac{D-1}{D-2}\Phi}\right\} . \tag{B2}$$

Although the BD frame,[4] seems the most natural in ST, it is also interesting to study what happens to solutions (4.4) when we perform the Weyl rescaling (B1)[5].

Recently it was stressed [16,17] that the 'pre-big-bang' era of the BD frame is naturally mapped to an accelerated contraction phase in the Einstein frame. But this is not true in general. The situation is more complex and needs a more accurate analysis. We will deal with non-singular solutions, although for the asymptotic behavior the same results can be applied to solutions (3.14) as well.

Because we have not an analytical expression for the scale factor, we can extract qualitative informations looking at the dynamics of the transformed Hubble parameter[6].

After performing the Weyl transformation (B1) we get (a prime denotes differentiation with respect to ξ)

$$\frac{a'}{a} \to \left(\frac{a'}{a}\right)_E = \frac{e^{\Phi/(n+2)}}{n+2}\left(-\bar{\Phi}' + (n-1)\frac{a'}{a} - n\frac{b'}{b}\right) \tag{B3}$$

$$\frac{b'}{b} \to \left(\frac{b'}{b}\right)_E = \frac{e^{\Phi/(n+2)}}{n+2}\left(-\bar{\Phi}' - 3\frac{a'}{a} + 2\frac{b'}{b}\right) , \tag{B4}$$

and then, substituting the equations (3.10) [19],

$$\left(\frac{a'}{a}\right)_E = \frac{2e^{\Phi/(n+2)}}{(n+2)\Delta(\xi)}\left[(n-1)\alpha_H - n\alpha_F + \xi + ((n-1)\hat{\gamma} - n\hat{\lambda})\sqrt{\xi^2+\xi_1^2}\right] , \tag{B5a}$$

$$\left(\frac{b'}{b}\right)_E = \frac{2e^{\Phi/(n+2)}}{(n+2)\Delta(\xi)}\left[2\alpha_F - 3\alpha_H + \xi + (2\hat{\lambda} - 3\hat{\gamma})\sqrt{\xi^2+\xi_1^2}\right] . \tag{B5b}$$

Let us introduce

$$\{A_i\} = ((n-1)\alpha_H - n\alpha_F\, , \, -3\alpha_H + 2\alpha_F) , \tag{B6}$$

$$\{B_i\} = \left((n-1)\hat{\gamma} - n\hat{\lambda}\, , \, -3\hat{\gamma} + 2\hat{\lambda}\right) . \tag{B7}$$

---

[4]The representation in which the scalar field couples non-minimally to the scalar curvature, action (2.2)

[5]For a review on the debate about the two frames see [24]

[6]We could have had informations about the correct (qualitative and quantitative) dynamics of the scale factors by numerically integrating the equations. But, because there are many free parameters which strongly determine the evolution, it is more interesting to study the shape of the Hubble parameter.

Then the asymptotic behavior of the scale factor depends on the sign of $\xi/|\xi| + B_i$: for $\xi/|\xi| + B_i > 0$ we have expansion, $\xi/|\xi| + B_i < 0$ means contraction, while $\xi/|\xi| + B_i = 0$ correspond to a stabilization of the scale factor. The Hubble parameter changes sign in the interval

$$\frac{-A_i - \sqrt{A_i^2 + (B_i^2 - 1)(A_i^2 - B_i^2\xi_1^2)}}{1 - B_i^2} < \xi < \frac{-A_i + \sqrt{A_i^2 + (B_i^2 - 1)(A_i^2 - B_i^2\xi_1^2)}}{1 - B_i^2}, \quad \text{(B8)}$$

if $A_i^2 + (B_i^2 - 1)(A_i^2 - B_i^2\xi_1^2) > 0$. Then it can happen that we start in the string frame with a scale factor which presents a transition expansion-contraction-expansion, but in the Einstein frame it experiences a monotonic expansion. It is then evident that it is not generally true that the expanding pre-big-bang phase is always converted to a contraction by Weyl rescaling, as stated in [16] . It happens only for homogeneous and isotropic models, and when $B_i - 1 < 0$ with $\omega_i > 0$ (we remember that $\{\omega_i\} = (\hat{\gamma}, \hat{\lambda})$). It is worth to notice that after the Weyl rescaling (B1), we have eliminated the non-minimal coupling of the scalar curvature with the dilaton, but we have still a theory with dynamical gravitational coupling, because of the presence of the volume of the internal space. So, if we want that the two representations be in accordance with observational constraints about the variability of the 4-dimensional Newton's constant, we must impose the two conditions $\hat{\gamma} = \frac{1}{3}$ and $\hat{\lambda} = 0$.

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FIGURE 1.

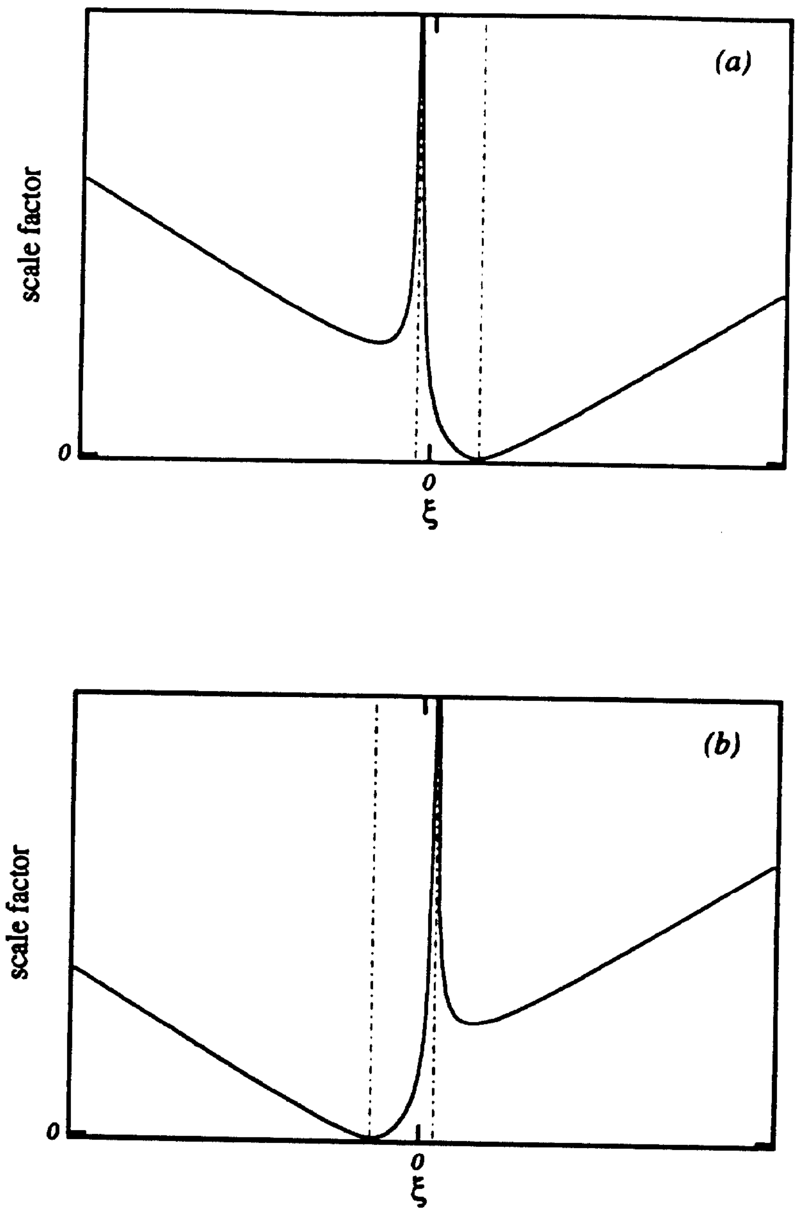

FIGURE 2.

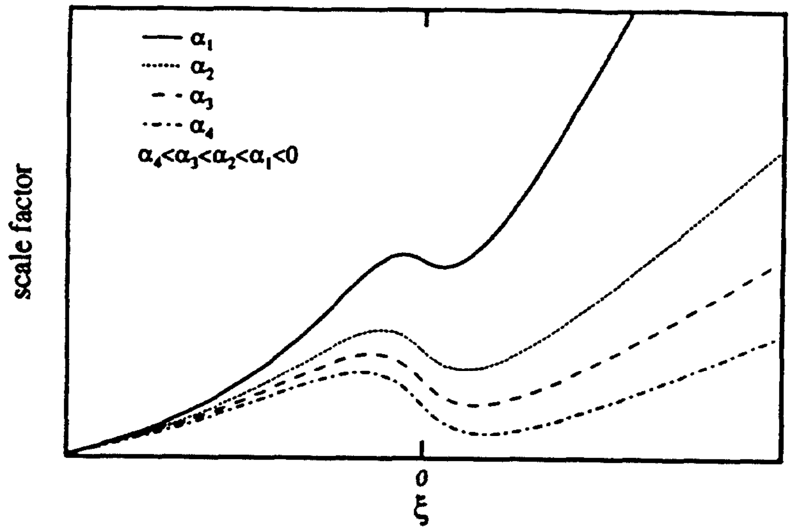

FIGURE 3.

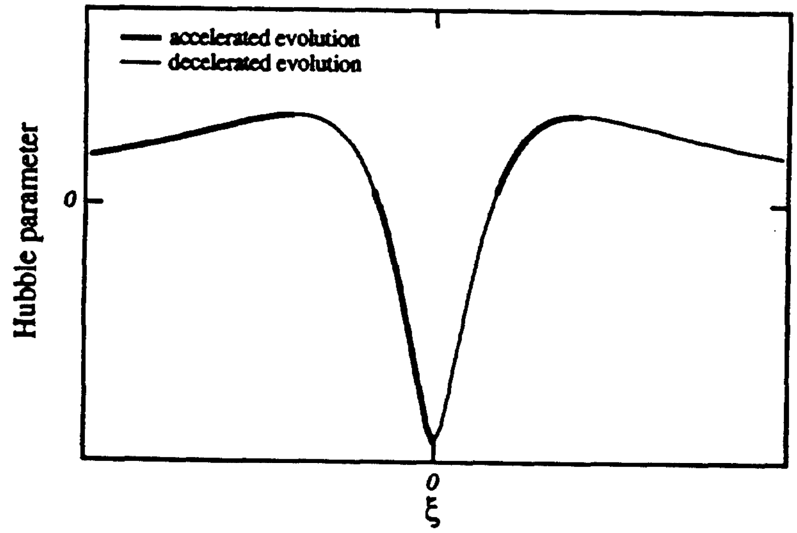

FIGURE 4.