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Primordial Magnetic Fields generated in the Quark-Hadron Transition

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ABSTRACT

We propose a mechanism by which magnetic fields of magnitude up to 10^8 G are generated at the quark-hadron phase transition. The primordial field is generated by currents formed at the boundaries between the quark and hadron phases, where charge separation occurs. We estimate the magnetic field generated during this phase and discuss its subsequent evolution and possible implications at later times. Today, this seed field would have an amplitude $\lesssim 10^{-10}\mu$G on scales of $\sim 1pc$ in the intergalactic medium, and in the galactic interstellar medium it would correspond to $\lesssim 10^{-5}\mu$G on scales of $\sim 10^3$ AU and $\lesssim 10^{-14}\mu$G on kpc scales. The generated field may act as a seed for galactic and stellar fields.

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I. Introduction

Although magnetic fields permeate most astrophysical systems, from planets and stars to galaxies and clusters, the origin and survival of astrophysical magnetic fields still elude our understanding. It seems clear that in order to ensure the long-term survival of magnetic fields in most astrophysical systems, some form of regeneration (e.g., dynamo action) is necessary [1]. In the absence of a dynamo, magnetic fields present in astrophysical systems today would have dissipated on shorter timescales than the typical lifetimes of these systems. However, dynamos only amplify a previously existent magnetic field; consequently, a *seed* field must be generated by some other physical mechanism. In this paper, we propose a new mechanism for the origin of primordial magnetic fields which may be relevant for seeding present galactic and stellar fields.

Before studying mechanisms for primordial field generation, one should ensure that primordially produced fields can themselves survive to the epoch of structure formation. During the radiation and matter-dominated eras, the universe is a remarkably good conductor, and magnetic fields generated primordially do not decay by the time galaxies are formed [2]. During recombination, the number density of charge carriers decreases dramatically to a residual ionization level, where the number density of free electrons as a function of redshift becomes $n_e(\mathrm{z}) \simeq 3 \times 10^{-10}\mathrm{cm}^{-3}\Omega_0 h(1+\mathrm{z})^3$ [3]. The conductivity, $\sigma_c \simeq n_e e^2/m_e n_\gamma \sigma_T$, is dominated by electrons Thomson scattering off cosmic background photons. An upper limit to the length scale l_{diff} below which diffusion of the magnetic field becomes important is given by $l_{diff} \simeq (\tau_U/4\pi\sigma_c)^{1/2}$, where τ_U is the age of the universe. Using the ionization fraction of the universe today ($n_e(\mathrm{z}=0)$), we get $l_{diff} \sim 3$ AU for the intergalactic medium. As galaxies form, n_e increases significantly, and therefore l_{diff} is even smaller in the interstellar medium. As long as we consider physical scales larger than the diffusion scale, we can ignore the decay of the magnetic field. (Throughout this paper, we set $c = \hbar = 1$ and use λ for comoving scales, physical scales $l = \lambda\, a(t)$, with the scale factor set to one today, $a(t_0) = 1$.)

A number of mechanisms have been proposed for generating a seed magnetic field, B_{seed}, in the early universe, but in order to generate a significant field amplitude with a large coherence length, one is led to invoke contrived inflation models [2,4] or to rely on the existence of topological defects. This is due in part to the difficulty of generating a sufficiently strong magnetic field with coherence length many orders of magnitude larger than the particle horizon at early epochs. For instance, the comoving scale of the density perturbation that gave rise to a galaxy today is about $\lambda_{gal} \sim 1$ Mpc, which is 10^{54} times larger than the Hubble radius at the GUT scale and 10^6 times the Hubble radius at the QCD epoch. Primordial generation of large-coherence length seed fields therefore requires cosmological models that have significant power on super-horizon scales such as inflation or topological defects. Unfortunately, standard inflationary models give rise to only very small magnetic fields [2,4], and one is forced to either break electromagnetic gauge-invariance [2] or change the gravitational couplings [4] to generate a reasonable B_{seed} amplitude.

To date, the focus on generating B_{seed} on large scales has been aimed at seeding the large-scale component of the galactic magnetic field, which would then be amplified by a classical dynamo. However, classical dynamos have recently been found to be less effective in amplifying the large-scale component of the galactic field than previously believed [5], and alternative scenarios for the origin of the coherent component of the galactic field are presently being pursued [6].

On the other hand, seed fields coherent over comparatively smaller scales *can* be generated by sub-horizon scale processes and may be relevant for seeding the small-scale component of the galactic field and stellar fields. In what follows, we describe a novel mechanism for generating B_{seed} during a first-order phase transition in the early universe. In particular, we discuss this scenario in the context of the quark-hadron phase transition, which occurred around 10^{-5} sec after the big bang at a critical temperature $T_c \sim 100$ MeV. The Hubble length at that time was $H_{QCD}^{-1} \sim 4 \times 10^6$ cm, which corresponds to a comoving scale today of $\lambda_{QCD} \simeq 0.6$ pc.

Similar mechanisms to the one described below may also act in transitions that occur earlier than the quark-hadron transition, although the earlier the transition the smaller the typical scale for the field. For instance, the Hubble radius at the electroweak transition corresponds to a comoving scale of about $\lambda_{EW} \sim 10$ AU, comparable to l_{diff}, so dissipation of the generated field plays a role [7]. For transitions at still earlier epochs, only residual effects on super-horizon scales would be of any significance today.

II. The Quark-Hadron Phase Transition

When the universe cooled through a temperature $T \sim 100$ MeV, it underwent a transition from quark-gluon plasma to hadronic matter. Significant effort has been devoted to understanding the detailed dynamics of this transition, but a complete picture is still lacking. In particular, the order of the phase transition remains uncertain, although some recent lattice QCD studies suggest that the transition may be second-order [8]. As there is no firm consensus on the order of the QCD transition, we will simply assume that it is first-order throughout this paper.

In a first-order QCD transition, bubbles of hadronic phase are nucleated as the universe cools below the critical temperature, $T_c \sim 100$ MeV, where the thermodynamic potentials for the quark-gluon and hadron phases are equal. As the bubbles nucleate and grow, shocks form and latent heat is released, reheating the Universe back to T_c. The temperature of the Universe then remains fixed at T_c for the remainder of the transition, during which the two phases coexist while bubbles of hadron gas grow at the expense of the quark phase. The nucleation followed by reheating process is brief ($\sim 0.5\mu s$) compared to the longer

($\sim 30\mu s$) coexistence phase [9]. Toward the end of the coexistence phase, the universe is composed of percolating hadron gas with shrinking bubbles of quark-gluon plasma.

Quashnock, Loeb, and Spergel [10] have studied the possible generation of magnetic fields by the shocks formed during the brief nucleation epoch. They proposed a Biermann battery mechanism that relied on the difference between the positive and negative charge carriers' response to shocks that traverse the quark phase. The field generated by this mechanism reached an amplitude of order 5 Gauss on scales comparable to the mean separation between nucleation sites, r (see below). We focus instead on the dynamics of the coexistence phase and find that much stronger fields can be generated by currents formed on the bubble walls.

The coexistence epoch of the transition can have interesting consequences arising from the difference in baryon number susceptibility of the two phases. As Witten [11] pointed out, baryon number tends to concentrate in the quark phase, giving rise to baryon number inhomogeneities. The baryon number density contrast between the shrinking quark bubbles and the hadron phase, R, can be estimated by approximating the two phases as ideal gases in chemical equilibrium:

$$R = \frac{n_b{}^q}{n_b{}^h} \simeq \frac{N_F}{9}\left[\frac{\pi T_c}{2m}\right]^{3/2} e^{m/T_c} \, , \tag{1}$$

where $n_b{}^q$ and n_b^h are the net baryon number density of the quark and hadron phases, m is the nucleon mass ($m \sim 938\,\mathrm{MeV}$), and N_F is the number of quark flavors. This estimate gives $R = 180$ for $T_c = 100$ MeV and $R = 6$ for $T_c = 200$ MeV.

When the size of a growing hadron bubble becomes larger than the baryon number diffusion length, $r_{diff} \simeq 4.4\mu\mathrm{m}$ (for $T_c = 100$ MeV) [12], diffusion becomes ineffective in maintaining the equilibrium baryon number contrast above. The bubble wall then acts like a snow-plow, piling up a net number of quarks (i.e., an excess of quarks over antiquarks) in the quark phase and leaving a lower baryon number density behind the wall in the hadron phase [13]. Depending on the transparency of the bubble wall to baryon number, this can enhance the baryon density contrast R by a few orders of magnitude, typically $R \sim 10^2 - 10^4$, up to an upper limit of $R \sim 10^6$ [14]. The thickness of such a baryon-excess layer is given by [12]

$$r_d \simeq \frac{r_{diff}^2}{r} \, , \tag{2}$$

where r is the mean separation between nucleation sites.

The mean separation between nucleation sites r was traditionally used as a free parameter (with the obvious constraint $r \lesssim H_{QCD}^{-1}$) in studies of the quark-hadron phase transition and its consequences for nucleosynthesis of the light elements [15]. If the baryon number fluctuations generated at the quark-hadron transition survive to the epoch of nucleosynthesis, we can use the observed light element abundances to constrain r to be below

~ 100 cm [16]. However, subsequent fluid motions may erase the inhomogeneities between the quark-hadron transition and the time of nucleosynthesis [15]. If fluid motions are efficient homogenizers of baryon number inhomogeneities, r may be larger than 100 cm and in principle as large as $H_{QCD}^{-1} \sim 4 \times 10^6$ cm.

Together with baryon number, a net positive charge is also concentrated on the quark side of the boundary between the two phases. The net baryon number density $n_B = (n_u + n_d + n_s)/3$, while the positive charge density $\rho_c^+ = e(2n_u - n_d - n_s)/3$, where n_i is the net number density of the i-quark (by net number density we mean the number density of quarks minus anti-quarks). At the transition temperature, the up and down quark masses can be neglected. If the strange quark mass, m_s, were also negligible, the net charge would be zero, while if $m_s >> T_c$, then $n_s \simeq 0$ and $\rho_c^+ = 0.28en_B$. Reality lies somewhere in between, that is, $m_s \sim T_c$, and thus $\rho_c^+ = en_B\beta$, where $\beta(m_s/T)$ can be found by imposing chemical equilibrium between the up, down, and strange quarks, electrons, and muons. For $m_s/T = 1$, 2, and 3, we have $\beta = 0.002$, 0.075, and 0.15, while $\beta(m_s >> T) = 0.28$.

The positive charge at the bubble wall is compensated by a negative charge carried by electrons and muons. Unlike the baryon number density, however, the lepton number density does not suffer a sharp discontinuity across the bubble wall. If we assume that the baryon number density decays exponentially away from the boundary (as in Ref. 12), we can calculate the distribution of lepton number density using a Debye screening model to relate the electrostatic potential and the charge density at each point. The result is shown in Fig. 1, where we plot the number densities for the positive and negative charge carriers as a function of distance from the boundary wall surface, z, for $R = 100$. In Fig.2, we show the net charge density as a function of z, for different choices of R. This net charge density will form currents when the plasma inside each phase is "stirred", that is, as peculiar motions appear.

To see how peculiar motions arise [17], consider the entropy density during the coexistence epoch to be s_q and s_h for the quark and hadron phases. Since the thermodynamic potentials are fixed during this epoch, the entropy density is constant in time deep within each phase, changing only at the phase boundary. Constant entropy densities together with the continuity equation:

$$\frac{\partial s_i}{\partial t} + \nabla \cdot (s_i \mathbf{v}) = 0 \tag{3}$$

where $i = q, h$ and $\mathbf{v}_i$ is the velocity field in each phase, imply that inside each phase, $\nabla \cdot \mathbf{v}_i = 0$. The velocity field can be decomposed into Hubble flow and a peculiar velocity field: $\mathbf{v}_i = \mathbf{v}^H + \mathbf{v}_i^p$. For the Hubble flow, $\nabla \cdot \mathbf{v}^H = 3\,H_{QCD} \neq 0$; therefore, inside each phase the peculiar velocity field satisfies $\nabla \cdot \mathbf{v}_i^p = -3\,H_{QCD}$. The expansion of the universe mainly takes place at the bubble walls, and peculiar velocities are present throughout each phase. An estimate of the typical peculiar motions then gives $|\mathbf{v}^p| \sim r\, H_{QCD}$.

The exact velocity field structure during the phase transition is quite complex; the requirements of constant entropy away from the wall together with the Hubble expansion in a system of varying sizes and randomly placed bubbles will create peculiar flows throughout the universe. In addition, vorticity will be created by the gravitational attraction between the high-density quark bubbles [17]; shrinking bubbles of quark phase will be moving towards each other, further "stirring" the hadron fluid. The peculiar velocity field is probably best described as a system of eddy currents or even convective flow. If mixing becomes very efficient, the baryon number gradient at the wall eventually decreases and baryon inhomogeneities may be erased by the time of nucleosynthesis [15].

III. The Generated Magnetic Field

We can estimate the induced magnetic field due to the created vorticity by focusing on the flow parallel to the bubble wall, where charge separation is most effective. Assuming that the flow parallel to the bubble wall has velocity $v \sim r\, H_{QCD}$, the charge density on each side of the wall is $\rho_c \sim eR\beta\bar{n}_B$, where $\bar{n}_B$ is the spatial average of the baryon number density, and the thickness of the charged layer is r_d, the induced magnetic field is:

$$B_{QCD} \simeq \frac{8\pi}{3}\rho_c\, r_d\, v = \frac{8\pi\, e\, R\, \beta\, \bar{n}_B\, r_{diff}^2\, H_{QCD}}{3}, \tag{4}$$

where we assumed a spherical geometry for the bubble. Note that B is independent of r, and r only sets a scale for the coherence length of the field.

Substituting $\bar{n}_B = \eta n_\gamma \simeq 10^{-14}\,\mathrm{GeV}^3$, $r_{diff} \sim 4\mu\mathrm{m}$, and choosing a reasonable range for $\beta\, R \sim 0.1 - 10$, we get

$$B_{QCD} \simeq 10^6 - 10^8\,\mathrm{Gauss}. \tag{5}$$

Clearly, B_{QCD} is much stronger than the magnetic field obtained in Ref.[10], but still below the equipartition field, $B_{eq} \simeq 10^{18}$Gauss at 100 MeV. The magnetic Reynolds number is $R_M \lesssim 10^{18}$, and the field will not be readily amplified to the equipartition limit, since $B_{QCD}\sqrt{R_M} < B_{eq}$. If $\beta\, R >> 10$ and the fluid motions reached full turbulence, B_{QCD} could approach B_{eq} on a dynamical timescale [5]. Our mechanism would then be constrained by the requirement that the energy density in the magnetic field be much less than that of the radiation field ($\rho_B = B^2/8\pi << \rho_\gamma$), otherwise primordial nucleosynthesis would overproduce ^{4}He [18].

To see what B_{QCD} corresponds to at later times, we follow the baryon density from the quark-hadron transition to time t, such that

$$B(t > t_{QCD}) = B_{QCD}\Big(\frac{\rho_B(t)}{\rho_B(t_{QCD})}\Big)^{\frac{2}{3}}\ , \tag{6}$$

where $\rho_B(t_{QCD}) \simeq 5 \times 10^6 \Omega_B h^2$ g/cm^3 and $\rho_B(t)$ is the density in baryons as a function of time. Simultaneously, we should rescale the coherence length by $l_{coh}(t) \simeq f\,\lambda_{QCD} a(t)$, where $f = r\,H_{QCD} \sim 10^{-4} - 1$.

For the intergalactic medium today, we take the mean density of baryons to be $\rho_{IGM} \simeq \rho_c \Omega_B$, where $\rho_c \simeq 2 \times 10^{-29} h^2$ g/cm^3 is the critical density, and the Hubble parameter $H_0 = 100h$ km/sec/Mpc. Therefore, we find a present magnetic field in the intergalactic medium of $B_{IGM} \simeq 10^{-10} \mu$G on a lengthscale $\sim f$ pc. This field is well below the intergalactic magnetic field limits [1,21].

For the field within galaxies, we need to take into account the collapse of the protogalactic cloud which simultaneously reduces the coherence length to $l_{coh} \sim 10^{-2} f \lambda_{QCD}$ and increases the baryon density in the disk by a factor of $\sim 10^6$ times the mean. Therefore, $B_{gal} \simeq 10^{-5} \mu$G, smaller than the observed field by 5 orders of magnitude. This may serve as a seed field for amplification in stars since the coherence length is about $10^3 f$ AU.

To find the field strength on scales larger than $f\,\lambda_{QCD}$, we need the spectrum of the generated field on scales larger than the bubble separation. Hogan [19] argued that such processes generate a field with a white noise spectrum: in this case, the rms field on a scale $L > f\lambda_{QCD}$ is given by $B_{rms}(L) \simeq (\lambda_{QCD}/L)^{3/2} B_{rms}(\lambda_{QCD})$. This spectrum gives a field on scales $L \sim f$ kpc in the galaxy of $B_{gal}(kpc) \sim 10^{-14} \mu$G; this is quite small but it may be of interest if subsequent amplification is very effective.

IV. Conclusion

We have shown how a simple process during a first-order phase transition in the early universe can generate magnetic fields which can later seed stellar and small-scale galactic fields. The mechanism is based on charge separation that occurs in the bubble walls and the peculiar velocity flows generated at the quark-hadron transition. We speculate that a similar mechanism might take place at the electroweak transition if it is first-order and if electroweak baryogenesis produces a net charge at the bubble walls.

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Figure Captions

Fig.1 - Baryonic (dotted line) and leptonic (solid line) number densities as a function of the distance from the bubble wall z. The hadronic phase is at $z < 0$ and the quark phase at $z > 0$. We plot them relative to the asymptotic values in the hadron phase.

Fig.2 - Net charge as a function of z for $R = 100$ (solid line) and 10^3 (dotted line).

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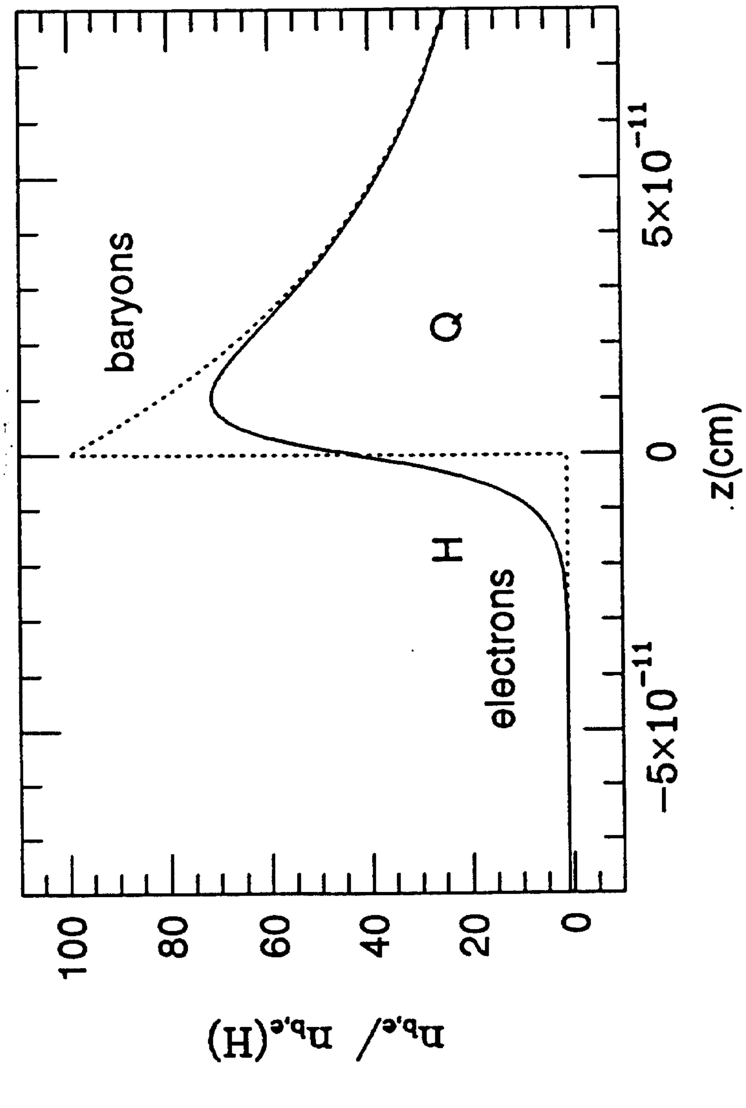
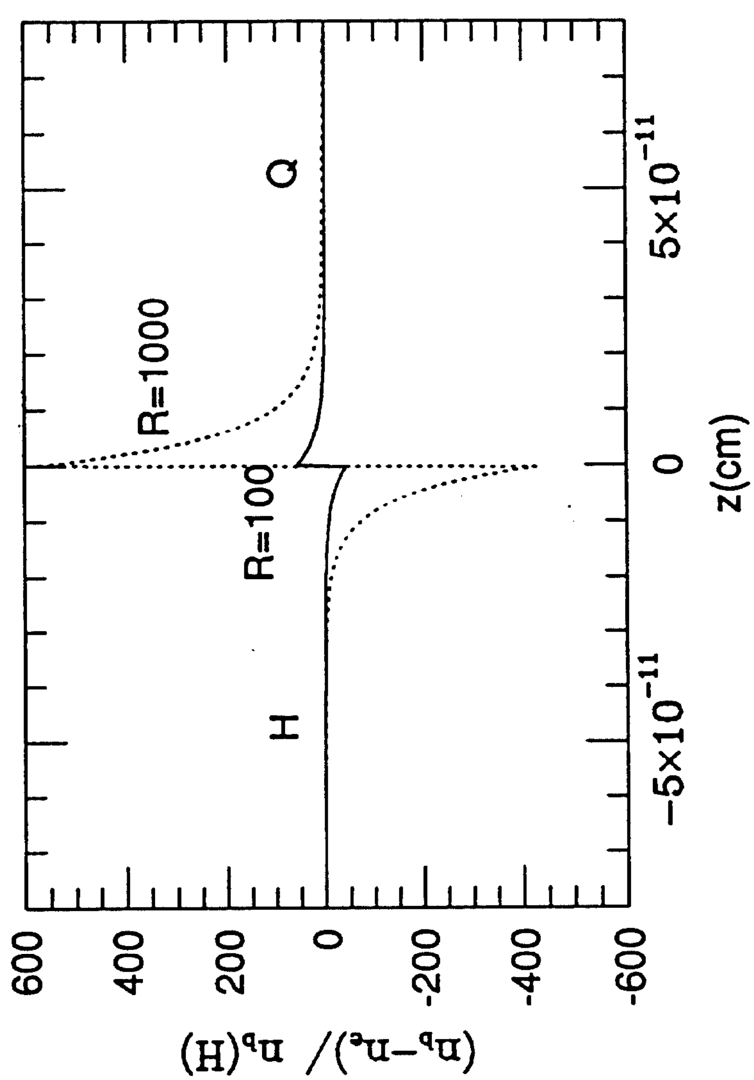


baryons
Q
H
electrons
z(cm)
−5×10^−11
0
5×10^−11
100
80
60
40
20
0
n_{b,e}/ n_{b,e}(H)

FIGURE 1



600
400
200
0
-200
-400
-600
$(n_b - n_e)/\ n_b(H)$
R=1000
R=100
H
Q
-5×10^{-11}
0
5×10^{-11}
z(cm)

FIGURE 2