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Update on the lattice calculation of $B \to K^*\gamma$

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We present updated results on the calculation of the matrix elements for $B \to K^*\gamma$ in the quenched approximation on a $24^3 \times 48$ lattice at β=6.2, using an $O(a)$-improved fermion action. The scaling behaviours of the form factors $T_1(q^2=0)$ and $T_2(q^2_{max})$ for the decay are examined and pole model ansatzes tested.

1. Introduction

Theoretical interest in the rare decay $B \to K^*\gamma$ as a test of the Standard Model has been renewed by the experimental results of the CLEO collaboration [1]. The viability of calculating the relevant hadronic matrix elements on the lattice was first demonstrated by Bernard, Hsieh and Soni [2] in 1991.

The computational details and results of this work have been described in references [3] and [4].

2. Form Factor Definitions

The hadronic matrix elements can be parametrised by three form factors,

$$\langle K^*(k,\epsilon)|\bar{s}\sigma_{\mu\nu}q^\nu b_R|B(p)\rangle = \sum_{i=1}^{3} C^i_\mu T_i(q^2), \qquad (1)$$

where,

$$C^1_\mu = 2\varepsilon_{\mu\nu\lambda\rho}\epsilon^\nu p^\lambda k^\rho, \qquad (2)$$

$$C^2_\mu = \epsilon_\mu(m_B^2 - m_{K^*}^2) - \epsilon\cdot q(p+k)_\mu, \qquad (3)$$

$$C^3_\mu = \epsilon\cdot q\left(q_\mu - \frac{q^2}{m_B^2 - m_{K^*}^2}(p+k)_\mu\right), \qquad (4)$$

and q is the momentum of the emitted photon.

As the photon emitted is on-shell, the form factors need to be evaluated at q^2=0. In this limit,

$$T_2(q^2{=}0) = -iT_1(q^2{=}0), \qquad (5)$$

and the coefficient of $T_3(q^2{=}0)$ is zero in the on-shell matrix element. Hence, the branching ratio can be expressed in terms of a single form factor, for example $T_1(q^2{=}0)$.

3. Heavy Quark Scaling

We calculate with a selection of quark masses near the charm mass and extrapolate to the b-quark scale. In the heavy quark limit, heavy quark symmetry [5] tells us that,

$$\begin{aligned} T_1(q^2_{max}) &\sim m_P^{1/2} \\ T_2(q^2_{max}) &\sim m_P^{-1/2}, \end{aligned} \qquad (6)$$

where m_P is the pseudoscalar mass. Combining this with the relation $T_2(q^2{=}0) = -iT_1(q^2{=}0)$ constrains the q^2 dependence of the form factors. However, it does not provide a scaling law for $T_1(q^2{=}0)$ without further assumptions about the actual q^2 behaviour of the form factors.

Pole dominance ideas suggest that,

$$T_i(q^2) = \frac{T_i(0)}{(1-q^2/m_i^2)^{n_i}}, \qquad (7)$$

for $i = 1, 2$, where m_i is a mass that is equal to m_P plus $1/m_P$ corrections and n_i is a power. Since $1 - q^2_{max}/m_i^2 \sim 1/m_P$ for large m_P, the combination of heavy quark symmetry and the form factor relation at q^2=0 implies that $n_1 = n_2 + 1$. For example, $T_2(q^2)$ could be a constant and $T_1(q^2)$ a single pole, or $T_2(q^2)$ could be a single pole and $T_1(q^2)$ a double pole. These two cases correspond to,

$$T_1(0) \sim \begin{cases} m_P^{-1/2} & \text{single pole} \\ m_P^{-3/2} & \text{double pole} \end{cases}. \qquad (8)$$

The data appear visually to favour $T_2(q^2)$ constant in q^2 when m_P is around the charm scale. However, we will consider both constant and single pole behaviours for $T_2(q^2)$ below.

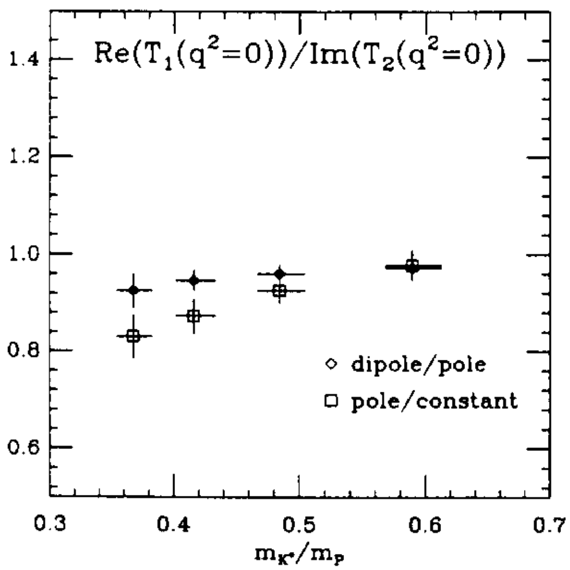

Figure 1. The ratio T_1/T_2 at $q^2{=}0$ for dipole/pole and pole/constant fits.

4. Results

As demonstrated in a previous paper [3], the evaluation of $T_1(q^2;m_P;m_{K^*})$ is relatively straightforward, and T_2 can be determined in a similar way. We fit $T_1(q^2)$ to a pole or dipole model in order to obtain the on-shell form factor $T_1(q^2{=}0)$,

$$T_1(q^2) = \frac{T_1(q^2{=}0)}{1-q^2/m^2}, \quad \frac{T_1(q^2{=}0)}{(1-q^2/m^2)^2}, \tag{9}$$

The difference between the two models was found to be negligible. The form factor T_2 was fitted to a pole model or constant

The ratio T_1/T_2 at $q^2{=}0$ for dipole/pole and pole/constant fits is shown in Fig.(1). The magnitude is found to be consistent with 1 at low masses, in accordance with the identity $T_1(0) = iT_2(0)$, Eq.(5). At higher masses, the dipole/pole fits for T_1/T_2 deviate less than the pole/constant fits.

5. Extrapolation of $T_2(q^2_{max})$ to m_B

In order to test heavy quark scaling, we also extracted the form factor T_2 at maximum recoil, where $q^2 = q^2_{max} = (m_P - m_V)^2$, in the same way as Bernard *et al.* [6]. In the heavy quark limit, $T_2(q^2_{max})$ is expected to scale as $m_P^{-1/2}$, analogous to the scaling of f_B. Higher order $1/m_P$ and radiative corrections will also be present. For convenience, we remove the leading scaling behaviour by forming the quantity,

$$\hat{T}_2 = T_2(q^2_{max})\sqrt{\frac{m_P}{m_B}}\left(\frac{\alpha_s(m_P)}{\alpha_s(m_B)}\right)^{2/\beta_0}. \tag{10}$$

The normalisation ensures that $\hat{T}_2 = T_2(q^2_{max})$ at the physical mass m_B. Linear and quadratic correlated fits for $\hat{T}_2$ were carried out with the functions,

$$\hat{T}_2(m_P) = A\left(1+\frac{B}{m_P}\right), \tag{11}$$

$$\hat{T}_2(m_P) = A\left(1+\frac{B}{m_P}+\frac{C}{m_P^2}\right), \tag{12}$$

and are shown in Fig.(2). Taking the quadratic fit of T_2 at $m_P = m_B$ as the best estimate, and the difference between the central values of the linear and quadratic fits as an estimate of the sytematic error, T_2 was found to be,

$$T_2(q^2_{max};m_B;m_{K^*}) = 0.269^{+17}_{-9} \pm 0.011. \tag{13}$$

If the q^2 dependence of T_2 at m_B were known, this result could be related to $T_1(q^2{=}0)$ via the identity $T_1(0) = iT_2(0)$.

6. Extrapolation of $T_1(q^2{=}0)$ to m_B

For $T_1(q^2{=}0)$ we test the two possible scaling laws in the same way as for T_2, by forming the quantity,

$$\hat{T}_1 = T_1(q^2{=}0)\left(\frac{m_P}{m_B}\right)^n\left(\frac{\alpha_s(m_P)}{\alpha_s(m_B)}\right)^{2/\beta_0}, \tag{14}$$

where $n = 1/2, 3/2$. For $n = 3/2$, a similar scaling relationship has been found using light-cone sum rules by Ali, Braun and Simma [7]. The $n = 1/2$ case has been suggested by other sum rules calculations [8].

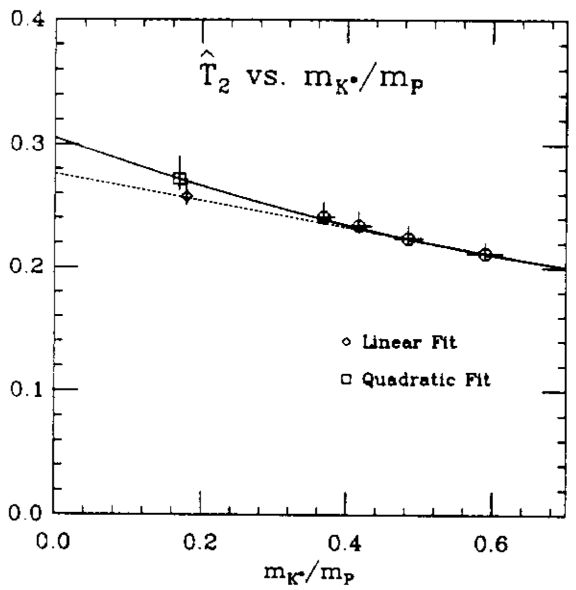

Figure 2. $\hat{T}_2$ extrapolation, with linear and quadratic fits.

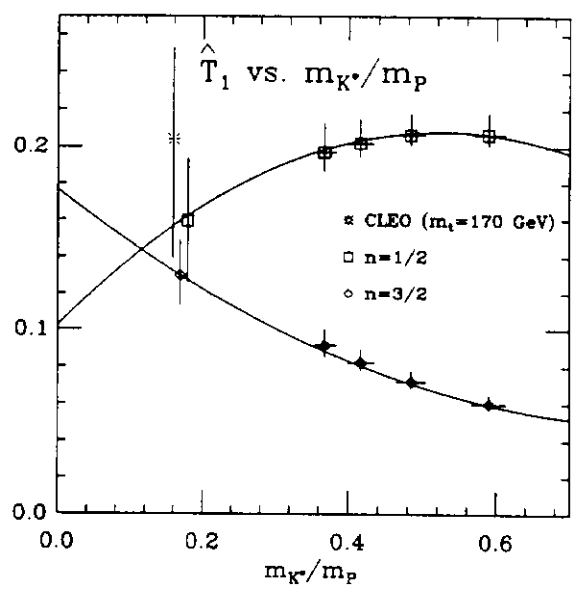

Figure 3. $\hat{T}_1$ extrapolation, for $n = 1/2, 3/2$ (Points displaced slightly for clarity).

Linear and quadratic fits were carried out with the same functions as for T_2. The two cases $n = 1/2, 3/2$ are shown in Fig.(3). The χ^2/d.o.f. are approximately 1 for the scaling laws, indicating that the models are statistically valid in the available mass range.

The final results for $T_1(q^2{=}0; m_B; m_{K^*})$ are taken from the quadratic fit for T_1, with the systematic error estimated as for T_2,

$$T_1(q^2{=}0) = \begin{cases} 0.159^{+34}_{-33} \pm 0.067 & n = 1/2 \\ 0.124^{+20}_{-18} \pm 0.022. & n = 3/2 \end{cases} . \quad (15)$$

7. Conclusions

Further information on the q^2 dependence of T_1 and T_2 is required to remove the uncertainty in obtaining the form factors at the physical point $q^2{=}0$, $m_P{=}m_B$.

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REFERENCES

1. CLEO Collaboration, R. Ammar *et al.*, Phys. Rev. Lett. **71**, 674 (1993).
2. C. Bernard, P. Hsieh, and A. Soni, Nucl. Phys. (Proc Suppl.) **B26**, 347 (1992), note that there is a factor of 2 missing in eq. (4) of this paper.
3. UKQCD Collaboration, K. Bowler *et al.*, Phys. Rev. Lett. **72**, 1398 (1994), hep-lat 9311004.
4. UKQCD Collaboration, K. Bowler *et al.*, hep-lat 9407014 (1994).
5. N. Isgur and M. Wise, Phys. Rev. D **42**, 2388 (1990).
6. C. Bernard, P. Hsieh, and A. Soni, Phys. Rev. Lett. **72**, 1402 (1994), hep-lat 9311011.
7. A. Ali, V. Braun, and H. Simma., hep-ph 9401277 (1993).
8. P. Ball, hep-ph 9308244 (1993). P. Colangelo, C. Dominguez, G. Nardulli, and N. Paver, Phys. Lett. B **317**, 183 (1993), hep-ph 9308264.