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Abstract

In this paper, a common calorimeter calibration scheme is explored and a hidden bias found. Since this bias mimics a non-linearity in response in the calorimeter, it must be understood and removed from the calibration before true non-linearities are investigated. The effect and its removal are explored and understood through straightforward calculus and algebra.

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Introduction

In the language of a modern high energy physics experiment, a calorimeter is a device that measures energy. Most often, a calorimeter consists of a passive absorber (often a high Z material, used to initiate and propagate showers) and an active material, where at least a portion of the secondary particles' energies are sampled. This energy can be transferred to the medium by a number of techniques (ionization, Cerenkov light, etc.) and the energy is collected and turned into an electrical pulse. Typically, the charge in the pulse is digitized via an ADC and the resultant number is nominally proportional to the energy sampled and therefore the initial particle's energy.

Unfortunately, a realistic calorimeter does not have a delta function response. For many reasons, given a monochrome energy beam input, the signals from a calorimeter invariably follow a distribution, often a gaussian, with a characteristic width. This width complicates calibration, for it is not possible to make a simple one-to-one mapping between signal and energy. People are forced to assign the mean or median signal to correspond to the true energy (see figure (1).) In the event of an asymmetric distribution, the mode might be used. Typically the distribution of signals is basically, if not rigorously, symmetric, so the mean, median, and mode are nearly the same. The process is further complicated by the fact that most real calorimeters are segmented and each segment contains only a portion of the incident energy. For reasons of practicality, the various segments typically do not have perfectly uniform response, so the calibration factor of each segment is unique and must be determined individually.

A common method [1-9] for determining the constants is given below. Assume that there are i segments, each with signal x_1, \dots, x_i and a response function R with j independent calibration constants a_1, \dots, a_j . The true energy is a monochrome E and N measurements are made. Then, approaching the problem in the manner of least squares fitting:

$$\chi^2 = \frac{1}{N} \sum_{k=1}^N (R(x_{1,k}, \dots, x_{i,k}, a_1, \dots, a_j) - E)^2$$

The j derivatives $\frac{\partial \chi^2}{\partial a_1}, \dots, \frac{\partial \chi^2}{\partial a_j}$ are set to zero and the resultant system of equations are solved to determine the a_1, \dots, a_j 's, thus revealing the minimum χ^2 . The intent is to make $\langle R \rangle = E$ and to minimize the width of

the response distribution. It should be noted that χ^2 does not follow the standard statistical definition. It is simply a function to be minimized and it contains some of the structure of the 'standard' χ^2 . When the response is linear, then $i = j$ and the equation becomes

$$\chi^2 = \frac{1}{N} \sum_{k=1}^N (a_1 x_{1,k} + a_2 x_{2,k} + \dots + a_i x_{i,k} - E)^2$$

and the same approach is taken.

For purposes of illustration, the case $i = 1$ is investigated in detail. An explanation of why this is not as restrictive as it seems is given later in this paper. First one notes the relevant equation

$$\chi^2 = \frac{1}{N} \sum_{k=1}^N (ax_k - E)^2 \quad (1)$$

and the following definitions, (\bar{x} is the mean x , and σ is the RMS of the distribution of x)

$$\begin{aligned} \bar{x} &= \frac{1}{N} \sum_{k=1}^N x_k \\ \sigma^2 &= \frac{1}{N} \sum_{k=1}^N (x_k - \bar{x})^2 = \frac{1}{N} \sum_{k=1}^N x_k^2 - (\bar{x})^2 \end{aligned} \quad (2)$$

Finding the a_{\min} that minimizes χ^2

$$\frac{\partial \chi^2}{\partial a} = 0 = 2 \frac{1}{N} \sum_{k=1}^N (ax_k - E)x_k$$

and so

$$a_{\min} = \frac{E\bar{x}}{(\sum_{k=1}^N x_k^2)/N}$$

manipulating equation (2), $(\sum_{k=1}^N x_k^2)/N = \sigma^2 + \bar{x}^2$,

$$a_{\min} = \frac{E\bar{x}}{\sigma^2 + \bar{x}^2} \quad (3)$$

The intended result is $a_{\text{expected}} = E/\bar{x}$, and so the ratio of minimized constant to expected constant is

$$\frac{a_{\min}}{a_{\text{expected}}} = \frac{\bar{x}^2}{\bar{x}^2 + \sigma^2} \leq 1 \quad (4)$$

Implications

Ignoring the effects of equation (4) could have visible consequences, especially at low energies. Recall that the ratio of a 's in equation (4) was expected to be one. Since it is not, the σ dependence (and therefore energy dependence) appears as a non-linearity. For instance, taking a typical hadronic calorimeter resolution $\sigma/E \sim 70\%/\sqrt{E} \rightarrow \sigma(E) \sim 70\%\sqrt{E}$, one finds

$$\frac{a_{\min}(E)}{a_{\text{expected}}(E)} = \frac{E^2}{E^2 + 0.49E} = \frac{E}{E + 0.49} \quad (5)$$

and so

$$E(\text{measured}) = \frac{E(\text{real})}{E(\text{real}) + 0.49} E(\text{real})$$

When the minimization scheme is applied for several energies in order to calibrate a detector, one could be fooled into viewing this behavior as a non-linearity, when in fact it is a bias in the calibration scheme. This effect must be removed before 'true' non-linearities are investigated.

Correcting for this effect is quite easy. Equation (4) is solved for a_{expected} , a_{\min} is returned from the minimization procedure, as is \bar{x} and σ of the minimized signal distribution. Note that scaling \bar{x} and σ by a constant ($(\bar{x}, \sigma) \rightarrow (a\bar{x}, a\sigma)$) does not affect equation (4), so the mean and RMS can be used from the 'expected' distribution, the 'minimized' distribution, or any distribution as long as the dimension of the abscissa is simply scaled linearly.

As a final note, equation (1) appears to be a very special case. In fact, it is more general than it appears. The a factor is a global scale that affects all calibration constants. One could write the general linear case in the following way

$$\chi^2 = \frac{1}{N} \sum_{k=1}^N [a(a_1x_{1,k} + a_2x_{2,k} + \dots + a_ix_{i,k}) - E]^2$$

Since in equation (1) there is no restriction on what the specific x_k 's are, one may define $X_k = a_1x_{1,k} + a_2x_{2,k} + \dots + a_ix_{i,k}$, independent of any details of the individual a_1, \dots, a_i 's. Then the structure of equation (1) is preserved and the results follows. Further, since the various segments enter symmetrically, this effect affects all segments equally.

As a demonstration of this, a simple Monte Carlo was written. A two segment calorimeter with resolution $(\sigma/E)^2 = (70\%/\sqrt{E})^2 + (5\%)^2$ was modelled. For a monochrome energy beam, a signal (x) was randomly selected from a gaussian distribution with this σ and $\bar{x} = E$. The fraction of signal found in segment one (x_1) was chosen from a uniform probability distribution function for each event, the signal in segment two (x_2) was simply $x - x_1$. Response factors (effectively ADC to energy conversion factors) a_1^{real} and a_2^{real} were randomly chosen at the beginning of each run and held constant. The energy signals were converted to ADC signals (e.g. $\xi = x/a^{\text{real}}$) and the following χ^2 function constructed

$$\chi^2 = \frac{1}{N} \sum_{k=1}^N (a_1 \xi_{1,k} + a_2 \xi_{2,k} - E)^2$$

and the minimum a_1 and a_2 were found. The minima were found both by using the CERN minimization package MINUIT [10] and by setting $\frac{\partial \chi^2}{\partial a_1} = \frac{\partial \chi^2}{\partial a_2} = 0$ and solving analytically for a_1^{min} and a_2^{min} . Both methods gave identical results to within the accuracy of the computer. This procedure was repeated 100 times each at particular $(E, a_1^{\text{real}}, a_2^{\text{real}})$'s. The distribution $a_i^{\text{min}}/a_i^{\text{real}}$ were made and the mean and shape of the distribution were identical for both segments. Figure (2) plots $\langle a^{\text{min}}/a^{\text{real}} \rangle$ for a number of energies, along with an overlay of the results of equation (4). The agreement is excellent and it underscores the fact that this effect affects the overall energy scale and not the relative calibration (i.e. $\langle a_1^{\text{min}}/a_2^{\text{min}} \rangle = \langle a_1^{\text{real}}/a_2^{\text{real}} \rangle$.) When the segment sharing was changed such that segment 1 contained a fraction of the energy chosen uniformly between 0.5 and 1.0 (and segment 2 correspondingly between 0.0 and 0.5,) the means of the distributions remained the same, although the RMS of the low fraction segment was wider. This additional width is caused by the fact that the constants associated with small signals are not as well determined and are more likely to be affected by statistical fluctuations.

It should be stressed that the results of equation (4) are quite sensitive to the size of the σ . As a demonstration of this, the reader should remember that for a fractional resolution $\sigma/E = 70\%/\sqrt{E}$, the algorithmic shift was 5% for 10 GeV. Using even a typical electromagnetic resolution ($30\%/\sqrt{E}$), one finds the algorithmic shift at the same energy is much reduced (approximately 1%.)

Further, for higher energies, the fractional resolution is much improved, and the algorithmic effect is therefore less pronounced. In many of the references [1-8] this is the case.

In addition, the reader should be aware that there are other techniques for calibrating that look very similar, but have qualitatively different biases. For instance, reference [11] uses a minimization scheme that, when written in a form similar to equation (1), looks like

$$\chi^2 = \frac{1}{N} \sum_{k=1}^N \frac{(ax_k - E)^2}{ax_k E}$$

When the earlier approach is applied to this χ^2 , one finds that the ratio $a_{\min}/a_{\text{expected}} = \sqrt{\bar{x}} \sqrt{(1/N) \sum_{k=1}^N (1/x)} = \sqrt{\bar{x}(1/\bar{x})}$. This bias yields a_{\min} 's that are larger than a_{expected} , with a magnitude that increases with increasing resolution.

Conclusion

An algorithmic bias has been explored in a common calorimeter calibration scheme. The effect is significant, especially at lower energies for hadronic calorimeters. The cause of the effect has been determined and is shown to be purely mathematical, rather than physics motivated. The bias may be removed from a calibration in a simple way.

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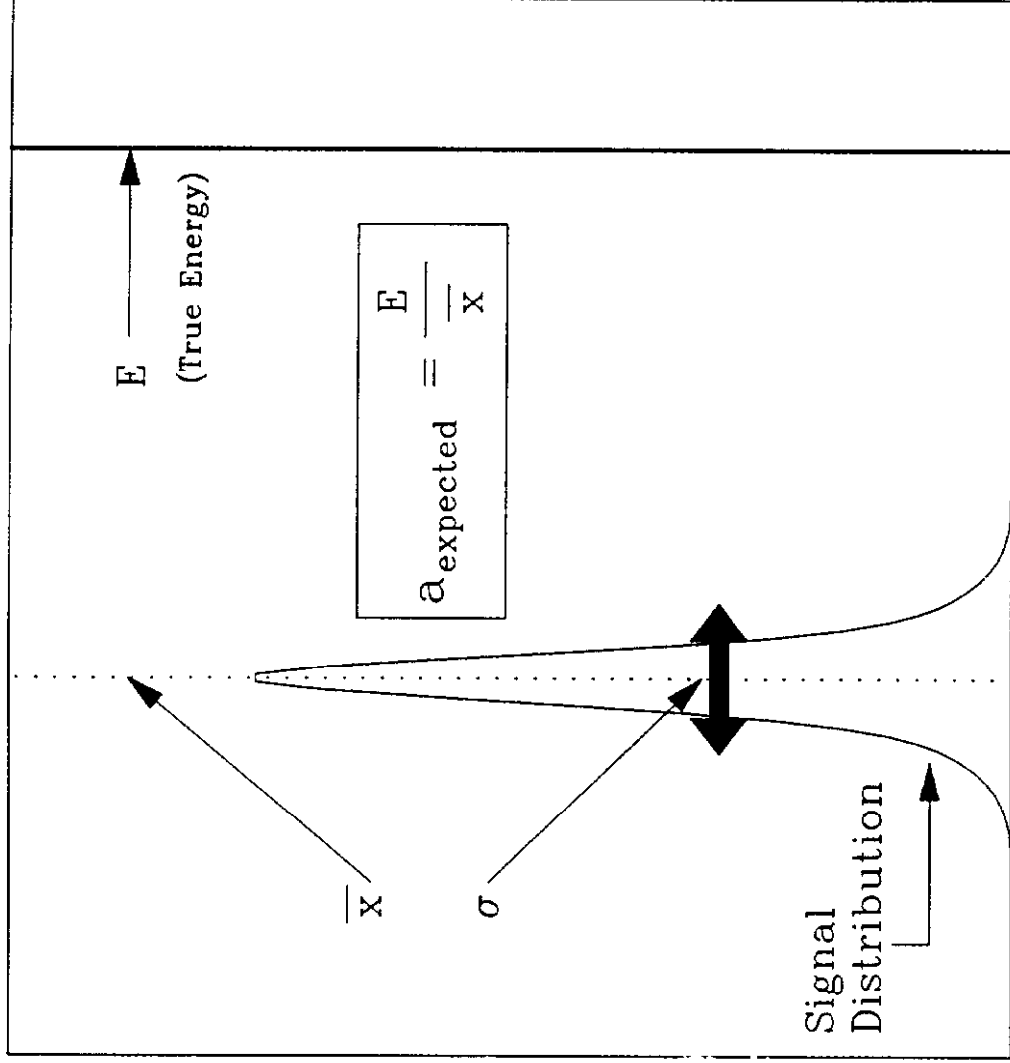
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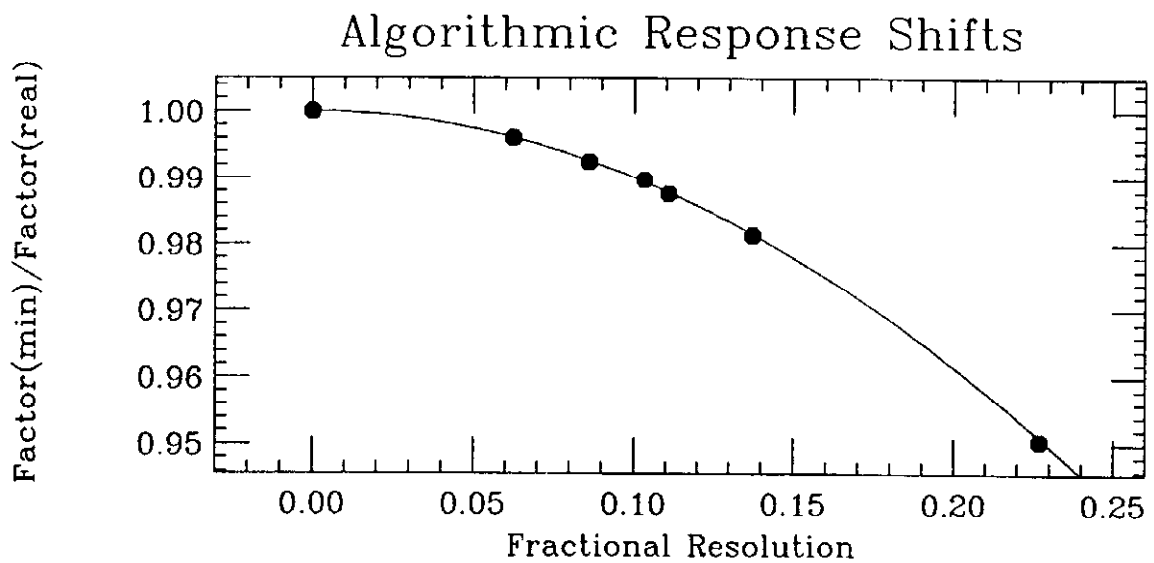
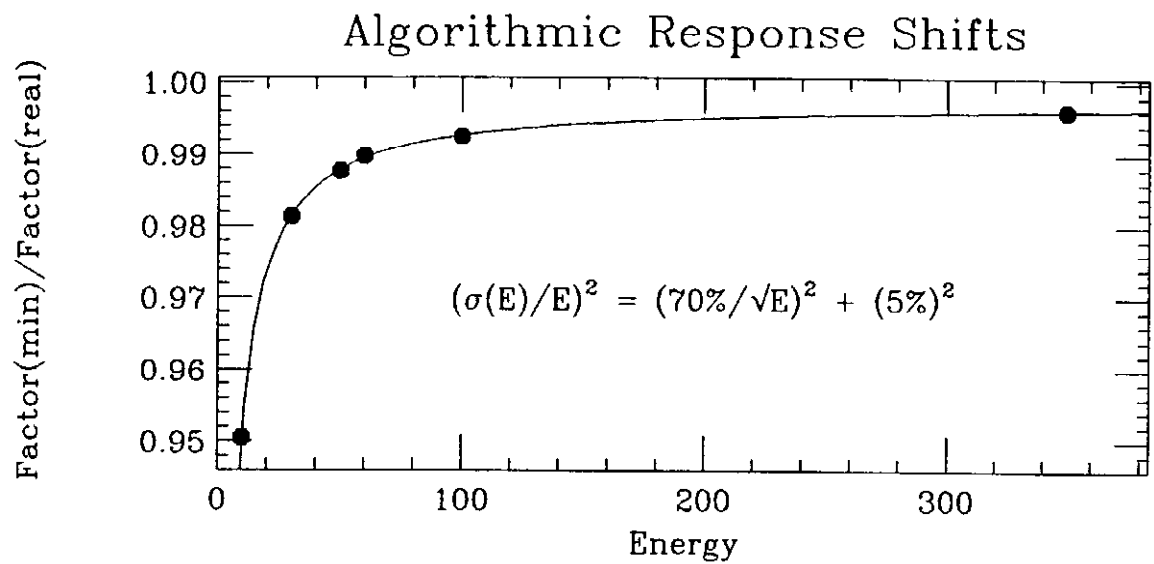
Figure Captions

1. Figure 1. Shown is a distribution of signals that correspond to a monochrome input energy E . In order to get the mean of the distribution \bar{x} equal to the energy, each time the signal is sampled, it should be multiplied by the factor $a_{\text{expected}} = E/\bar{x}$.
2. Figure 2. Shown is the results of a Monte Carlo study of equation (1) with comparison to equation (4). The solid circles show the Monte Carlo data (the errors are substantially smaller than the circles.) The line is the curve given by equation (4.) The σ 's chosen for the Monte Carlo followed the equation: $(\sigma/E)^2 = (70\%/\sqrt{E})^2 + (5\%)^2$. The bottom plot gives results purely in terms of calorimeter resolution, while the top plot shows the size of the effect at various energies for a 'typical' hadronic calorimeter.

Correct Signal to Energy Conversion Factor



(Figure 1)



(Figure 2)