Possibilities of Discovering a Heavy Top Quark in the Lepton-Multijet Channel

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Abstract

As the Fermilab experiments steadily increase the top quark mass limit, one has to study the top quark discovery in an scenario where its mass is larger than 150 GeV. We will show in this paper that one can apply simple and efficient cuts in the single lepton plus inclusive four jets channel which will reduce the QCD background by a factor of at least 5, while the top quark signal efficiency is more than 70%. With these cuts a discovery limit of 180 GeV can be set for an integrated luminosity of 100 pb$^{-1}$. This limit is much higher than previously quoted in the literature and does not use the method of heavy quark tagging. The analysis includes energy smearing of the measured jets to simulate the most important detector effects.
1 Introduction

The top quark has eluded discovery at the $p\bar{p}$ colliders for over ten years. The current top quark mass limit is set at the Fermilab Tevatron collider well over 100 GeV by both the CDF [1] and D0 [2] collaborations. The electro-weak precision measurements at LEP combined with the standard model gives a prediction for the top quark mass of $160 \pm 30$ GeV [3]. This leads to two possible scenarios at the Fermilab collider.

The first scenario is a top quark with a mass around the expected standard model value. The discovery requires a combination of methods, the di-lepton channel [4], heavy quark jet tagging [5], kinematical cuts in the single lepton channel [5, 6, 7, 8] and possibly a combination of kinematical cuts and heavy quark tagging in the all-jet channel [9]. A careful analysis of the upcoming 100 pb$^{-1}$ run should guarantee a top quark discovery within the standard model expectation of $100 \pm 30$ GeV. After the discovery, the mass of the top quark has to be determined as accurately as possible. This, when combined with an accurate measurement of the $W$ vector boson mass will further constrain the standard model. Also it has to be verified that the top quark has standard model decay modes.

The second and more interesting scenario is that the top quark will not be found at the Tevatron collider after the upcoming 100 pb$^{-1}$ run with a mass limit set well beyond the standard model expectations. In this scenario the minimal standard model is failing to explain the physics in the energy scale of around 200 GeV. With this conclusion other non standard model physics is likely to show up at the Tevatron collider and a high luminosity run would be crucial to understand the absence of the
top quark within the standard model expectation by looking for other clues.

In this paper we discuss the single lepton plus multijet decay channel of the top quark pair. We follow closely the analysis of reference [9] where we developed techniques for determining the top quark mass using the all-jet decay mode of the top quark pair. It partly overlaps with a recent publication [7] which applies some of the techniques of ref. [9].

The first part of the paper discusses the kinematical cuts one can apply in the single lepton plus multijet channel. The background rejection using these cuts is not as good as the method of heavy quark tagging which reduces the background by a factor of 20 [5]. However the displaced vertex heavy quark tagging method suffers from a low tagging efficiency (approximately 20%) for the top quark signal. This gives the kinematical cuts method certain advantages over the tagging method [10]. The experimental advantage is that by applying the kinematical cuts we lose less than 30% of the Top Quark signal opposed to the loss of 80% of the signal in the heavy quark tagging method. This means that by using the kinematical cut method at least a factor three more top quark events are expected. This is reflected in the fact that while the heavy quark tagging method gives a much better improvement in the signal over background ratio than the kinematical cuts method, the opposite is true in the improvement of the significance of the signal. From the theoretical side it is clear that by applying the kinematical cuts, which select central events with a high momentum transfer, the reliability of the perturbative expansion is greatly enhanced. Already without these additional kinematical cuts there is a good agreement between leading order QCD predictions for vector boson plus multijet production [11]. Applying the
kinematical cuts will further enhance the reliability of the background estimates as given by the leading order VECBOS monte carlo program [5]. On the other hand the theoretical predictions for heavy quark tagged vector boson plus multijet events are far more uncertain, especially once hard jets are required. Firstly the transverse momentum distribution of heavy quark tagged jets in the vector boson plus multijet system has to be understood (e.g. $W$ boson plus one or two jets and $Z$ boson plus 3 or more jets). Indeed, already the bottom quark inclusive transverse momentum distribution is not understood [12], using even next-to-leading order QCD [13].

While the first part of the paper deals with background reduction, the second part involves the mass reconstruction of the top quark once it is discovered. We neglect the effects of initial state radiation which can lead to an additional jet. This might be important because in approximately 50% of the time one of the top quarks decay jets is not observed due to either merging with another jet or having a low transverse energy. An initial state jet will in this case be misidentified as a decay jet resulting in a wrong mass reconstruction. To resolve this issue one needs a more involved theoretical calculation, which is beyond the scope of the paper. Here we will address only the leading order QCD prediction.

With the method described in this paper it is possible at the end of 1994 with an integrated luminosity of $100 \text{ pb}^{-1}$ to either discover the top quark within the standard model expectations or set a mass limit well beyond the standard model expectations. The main injector upgrade, delivering an integrated luminosity in excess of $1000 \text{ pb}^{-1}$, will then be able to either study the top quark and give the necessary verification of a standard model top quark or explore the new physics at the energy scale beyond
200 GeV.

2 The selection and kinematical cuts

For the top quark signal we use the leading order calculation of ref. [14] with the renormalization/factorization scale chosen to be equal to half the top mass. This value was chosen so that the normalization is close to the next-to-leading order inclusive top quark cross section of ref. [13]. Note that recent calculations of the resummed cross section [15] shows that this is a conservative lower limit of the top quark cross section.

The background was estimated using the leading order VECBOS monte carlo program[5] where the phase space integration was performed using important sampling in the transverse energy distribution of the jets in order to obtain a faster reduction in the monte carlo error. This method is analogous to the method described in ref. [9]. The chosen renormalization/factorization scale is the average transverse momentum of the four jets. As was shown by the CDF collaboration [11], using this scale choice in the used next-to-leading order expression for the strong coupling constant gives a good agreement between the VECBOS monte carlo program and the data for $W$ boson production in association with up to four jets inclusive cross section.

Both the signal and background monte carlo samples were used with the most recent sets of structure functions: CTEQ1M [16] and MRSD0 [17]. There is a small uncertainty of the order of 1% due to the structure function choice. This uncertainty
can be neglected with respect to the larger systematic uncertainty in the theory related to the choice of the renormalization/factorization scale. In the remainder of the paper we will use the MRSD0 parametrization of the parton density function.

It is important to note that all results quoted are for one lepton species (e.g. electron and positron), including a second lepton species (e.g. muon and antimuon) would increase the event rates by at most a factor of two.

In order to study the effects of the hadronic calorimeter we included a gaussian response function:

\[
\frac{\Delta E}{E} = \frac{S}{\sqrt{E}},
\]

where the smearing \( S \) is typically between 0.75 and 1.0 depending on the detector. This smearing will be important for the mass determination method used in the next section. However for the kinematical cuts the smearing does not affect the results. When one applies the above smearing to the true jet energies to get the detected jets in a fixed order calculation one has to remain consistent within the perturbative expansion. The smearing can only take place within the leading order inclusive four jet cross section. Therefore true jets before transverse energy smearing have looser cuts than the smeared jets as observed by the detector. This is because with gaussian smearing the measured transverse energy of the jet can be higher than the true energy. However there is no significant dependence on the looser cut. The actual jet cuts are applied after the smearing.

In the subsequent top quark analysis we will apply the following event selection cuts:
Table 1: Cross Section (pb) for different top quark mass values and cuts on the jet-jet separation $\Delta R_{jj}$, the lepton-jet separation $\Delta R_{ij}$ and minimum transverse momentum $E_{T}^{jets}$ of the jets. A jet smearing of 100% is applied.

- A cut on the transverse momentum of charged lepton of $P_{T}^{lepton} > 20$ GeV.
- A cut on the missing transverse energy of the event $E_{T}^{missing} > 20$ GeV.
- A lepton pseudorapidity cut $|\eta^{lepton}| < 1.5$, with $\eta = \log(-\tan(\theta/2))$ and $\theta$ being the polar angle of the jet axis.

These cuts select central $W$ bosons. Next we apply the following jet cuts:

- At least four jets in the final state.
- A cut on the jet transverse momentum of $E_{T}^{jet} > 15$ GeV.
- A jet pseudorapidity cut of $|\eta^{jets}| < 2.5$.
- A jet separation cut of $\Delta R_{jet-jet} = \sqrt{\Delta \phi^2 + \Delta \eta^2} > 0.7$, where $\Delta \phi$ is the azimuthal angle between the jets and $\Delta \eta$ the pseudorapidity difference of the jets.
A lepton isolation cut of $\Delta R_{\text{jet-lepton}} > 0.7$. This cut mimics the isolation cut of the lepton and reduces the top quark detection efficiency by approximately 25% as is shown in Table 1.

Apart from the last two cuts, which mimic a realistic jet clustering algorithm, the applied cuts are very efficient because the top quark decay produces high momentum jets in the central region.

The choice of the jet defining cuts $E_T^{\text{jet}} > 15$ GeV is such that it has the highest possible efficiency for the top quark signal while keeping the experimental uncertainties acceptable. In table 1 one sees that for the top quark masses above 150 GeV there is at most a 12% loss in efficiency for the signal when we raise the minimal transverse energy cut on the jets from 10 GeV to 15 GeV. Therefore, by choosing the 15 GeV cut we reduce the experimental uncertainties while maintaining a good efficiency for the signal. However when we increase the jet separation cut between the jets from 0.7 to 1.0 we loose around 50% of the signal events. The $\Delta R$ separation cut cannot be chosen smaller than 0.7, as going below this value would induce large energy losses in the reconstruction of the jet transverse energy and therefore introduce large systematical uncertainties in the analysis.

After the event selection cuts described above we apply two simple kinematical cuts to reduce the QCD background with respect to the top quark signal. These two cuts are based on the observation that for the top quark production the jets and lepton are produced through a cascade decay of the top quark pair. This produces hard, well centered jets in contrast to the soft, collinear jets resulting from the QCD bremsstrahlung. To exploit these differences we apply the following cuts to the signal:

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(a) We apply a cut on the summed transverse energy of the charged lepton and the hardest four jets in the event. Note that the transverse momentum sum, $\sum E_t$, does not include possible additional jets because these are most likely generated by bremsstrahlung from the initial state partons and are therefore not associated with the top quark decay products. We could alternatively have chosen not to include the lepton in the sum, this would give similar results. The $\sum E_t$ distribution is shown in figure 1 for both the background and various values of the top quark mass. Because of the top quark cascade decay the majority of events will have a value of the summed transverse energy larger than the top quark mass plus the average transverse energy of the lepton (which is around 40 GeV). The distribution would peak around twice the top quark mass if we would have included the missing transverse energy, however now it peaks around 40 GeV below this value (the average missing transverse energy).

To chose the optimum $\sum E_t$ cut we use fig. 2. Fig. 2a gives the relative improvement in the ratio $S/B$, while in fig. 2b shows the relative improvement in the significance $S/\sqrt{S+B}$ (where $S$ and $B$ are the number of signal and background events respectively) as a function of the $\sum E_T$ cut. As can be seen the optimum cut, at the maximum of the significance curve, is approximately at the top quark mass plus 40 GeV. Note that the cut works better for higher top quark masses. This is because we can remove more background events while losing only a few percent of the top quark events. By applying the $\sum E_t$ cut we select harder events for the background which will lead in general to a smaller value of $\alpha_S$. This is reflected in the average $E_T^{\mu\mu}$ scale choice, which will
increase significantly. The reliability of the leading order prediction is therefore enhanced.

(b) We can in addition exploit the fact that the top quark decay jets are well separated compared to the background bremsstrahlung jets. To quantify the difference we can use one of two possible cuts. The first possibility is a cut on the rapidity of the two highest transverse momentum jets. This cut has the advantage of not using the missing energy measurement in the event. An alternative method with a better background rejection is a cut on the aplanarity $A$ of the jets in the event. The major disadvantage here is that we need to reconstruct the aplanarity in the center-of-mass frame of the collision implying a full reconstruction of the neutrino momentum and the use of the lepton momentum. To get the value of the longitudinal component of the neutrino we constrain the four-momentum of the lepton and neutrino to the $W$ vector boson mass and pick the solution which gives the lowest value for the longitudinal momentum of the $W$ boson. This method gives the right value in about 80% of the cases. The applied aplanarity cut of $A > 0.05$ reduces the signal by no more than 25% while the background is reduced by an additional factor two. This can be seen in fig. 3 which shows the aplanarity distribution for both the background and signal. The aplanarity cut is independent of the top quark mass. Note that the application of this particular cut overlaps with the recent publication of ref. [7].

To conclude this section we quantify our findings in Table 2 which gives the cross sections before and after the kinematic cuts have been applied for several top quark
Table 2: Effect of kinematical cuts on signal and background (signal/background) for several top quark masses. A jet smearing of 100% is applied.

<table>
<thead>
<tr>
<th>$\sigma(m_{top}) / \sigma$(background)</th>
<th>120 GeV</th>
<th>140 GeV</th>
<th>160 GeV</th>
<th>180 GeV</th>
<th>200 GeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma$ (pb) (before kinematic cuts)</td>
<td>1.93/3.46</td>
<td>1.16/3.46</td>
<td>0.65/3.46</td>
<td>0.36/3.46</td>
<td>0.21/3.46</td>
</tr>
<tr>
<td>$\sigma$ (pb) (after $\sum E_t$ cut)</td>
<td>1.68/2.07</td>
<td>1.04/1.55</td>
<td>0.60/1.15</td>
<td>0.33/0.83</td>
<td>0.20/0.60</td>
</tr>
<tr>
<td>$\sigma$ (pb) (after $\sum E_t$ and A cuts)</td>
<td>1.26/1.12</td>
<td>0.79/0.85</td>
<td>0.47/0.63</td>
<td>0.27/0.46</td>
<td>0.15/0.33</td>
</tr>
</tbody>
</table>

masses. Note that for a top quark mass of 160 GeV we obtain an improvement in the signal over background ratio of 3.9, while the significance improvement is 1.6. This has to be compared to the heavy quark jet tagging method which has a tagging efficiency for a top quark event of 20% and a background reduction of a factor of 20. This leads to a signal over background ratio improvement of a factor of 19. However due to the low tagging efficiency for the signal there is little improvement in the significance.

3 Global Constrained Fit

After applying the above cuts we reconstruct the top quark mass from the remaining events by a constrained fit. In the lepton plus four jets channel we assume that the missing energy is due to a neutrino. We therefore want to test the kinematical hypothesis that we have observed $t\bar{t}$ production where each top quark decays through the intermediate vector boson $W$ and a bottom quark (b-quark) and that one $W$ boson decays hadronically to 2 light quarks and the other $W$ boson through the
lepton-neutrino mode. There are 12 combinations of the 4 jets to make up the final state. Each of the 4 jets could be assigned as the b-quark associated with the leptonic $W$ boson decay and in each of these cases one of the remaining three jets can be assigned to be the b-quark associated with the hadronically decaying $W$ boson, the remaining 2 jets forming a $W$ boson mass. Heavy quark jet tagging would reduce the number of combinations to be fitted.

We have adopted the method of kinematical fitting based on lagrange multipliers. In our case the constraints applied are two $W$ boson masses for the lepton-neutrino and the pair of light quarks and a third constraint that the two top quarks have the same mass. We define a $\chi^2$ to be minimized as

$$\chi^2 = \sum_{ij} (y_i - y_i^m) G_{ij} (y_j - y_j^m) + \sum_{\lambda} a_\lambda f_\lambda (y_j^m)$$

where $y_i^m$ and $y_i$ are the measured and fitted kinematic variables respectively, $G_{ij}$ is the covariance matrix and $a_\lambda$ are the lagrangian multipliers. The $f_\lambda (y_j^m)$ are the constraint equations and after fitting $f_\lambda (y_j) = 0$.

For each jet and lepton we choose kinematical variables energy $E$, polar angle $\theta$ and azimuthal angle $\phi$ and assume that these are uncorrelated in the measurement. For the neutrino we choose $E_x$, $E_y$ and $E_z$ variables where $E_x$ and $E_y$ are transverse momenta components and $E_z$ the longitudinal component. A measurement of $E_x$ and $E_y$ is made through a summation over the lepton, jets and underlying event in the detector whereas $E_z$ is unmeasured. In this case it is necessary to take into account the correlation between $E_x$ and $E_y$ and the other kinematic variables by explicit calculation of the off diagonal terms of the covariance matrix.
Differentiating the $\chi^2$ with respect to the $y_i$ we obtain

$$\frac{\partial \chi^2}{\partial y_i} = \sum_j G_{ij} (y_j - y_j^m) + \sum_\lambda \alpha_\lambda \frac{\partial f_\lambda(y_j)}{\partial y_i} \bigg|_{y_j = y_j^m} = 0$$

(2)

Expanding the constraint equations in a Taylor series we get

$$f(y_j) = f_\lambda(y_j^m) + \frac{\partial f_\lambda(y_j)}{\partial y_j} \bigg|_{y_j = y_j^m} (y_j - y_j^m) = 0$$

(3)

The above equations, (2) and (3), may be written in a matrix form $A_{ij} Y_j = B_i$ where $i = 1, m$ and $j = 1, n$ and solved for $Y_j$. Here $n$ is the number of kinematical variables and $m$ is equal to $n +$ the number of constraint equations.

Measurement errors are assigned according to the following

$$\Delta E = S \times \sqrt{E}$$

(4)

$$\Delta \theta = S \times 0.05 \text{ radians}$$

(5)

$$\Delta \phi = S \times 0.05 \text{ radians}$$

(6)

where the smearing $S = 1.0$ for jets and $0.2 \times S$ for the leptons. For the above case of six particles in the final state the number of degrees of freedom of the fit is given by 17 independent measurements plus 3 constraints minus 18 independent variables yielding 2 degrees of freedom. For each combination fitted we calculate a fit probability according to $e^{-\frac{\chi^2}{2}}$. Performing kinematic fits on monte carlo $t\bar{t}$ events where the correct combination is chosen gives gaussian mass distribution centered at the correct value and a flat probability distribution between 0 and 1. Kinematic fits performed with the wrong combinations generally give lower probabilities than the correct combination.
Figure 4 shows the mass distributions for a top quark mass of 160 GeV obtained after kinematic fitting of all 12 combinations and plotting the mass from the combination giving the highest probability and if that probability is greater than 10%. Two curves are given for both the signal $t\bar{t}$ events and the background corresponding to different jet energy resolutions with $S = 1.0$ and 0.75 respectively. The improved resolution gives a bigger mass peak and a slightly reduced background. For the case $S = 1.0$ (0.75), the kinematic fit, on the basis of higher probability selects the correct combination 47% (54%) of the time and still gives the correct mass on average. Part of the reason for this is that in 67% (70%) of the time the correct jet is assigned to be the $b$-quark jet associated with the lepton neutrino. The mass reconstruction for the top quark mass is dominated by the measurement of the other 3 jets in the event as the neutrino is badly measured relative to the jets. The other wrong combinations selected give a much broader mass distribution under the peak. This is also the case for the background. By counting events fitted with a probability greater than 10% in a broad mass bin about the peak (140 to 180 GeV) we observe that for the resolution case $S = 1.0$ (0.75) that 69% (80%) of the signal survives but that only 26% of the background is fitted. In this mass bin the signal to background ratio is now 2 (2.4) to 1 which corresponds to an improvement in the signal to background of a factor 2.6 (3.0) after fitting for the 2 resolution cases respectively.
4 Conclusions

We showed that by applying two simple kinematical cuts, which exploit the decay properties of a heavy top quark, in the single lepton plus four or more jets channel the QCD background is considerably reduced while maintaining a good efficiency for the top quark signal. With these cuts we obtain a signal over background ratio of order 1 for even very heavy top quark masses. Because of the large efficiency for the signal, especially for large top quark masses, we obtain a large improvement in the significance of the signal over background, which is a factor of 2.5 for a top quark mass of 200 GeV. This in contrast to the method of heavy quark jet tagging which, because of the small tagging efficiency for the signal, has little improvement in the significance. The hard kinematical cuts method has also the advantage of being better known from the theoretical point of view.

The luminosity required for a top quark discovery, with a signal to background significance value of 3, in the lepton plus four jets channel with kinematical cuts is shown in figure 5 as a function of the top quark mass.

In addition to the kinematical cuts the constrained fit method allows identification of the decay products of each top quark and a determination of its mass. The hard kinematical cuts will also reduce the influence of the initial state radiation for the mass determination. The signal over background ratio is increased by more than a factor of 2 after the constrained fit.
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[15] "Top quark Production Cross Section",


Figure Captions

Fig. 1 The differential $\sum E_T$ distribution for the QCD background and various values of the top quark mass.

Fig. 2 Signal to background improvement and signal to background significance improvement as a function of the $\sum E_T$ cut for top quark masses of 120 GeV, 160 GeV and 200 GeV.

Fig. 3 The differential aplanarity distribution for the QCD background and the signal for a top quark mass of 160 GeV after the $\sum E_T$ cut.

Fig. 4 The constrained three jet invariant mass distribution for a top quark mass of 160 GeV (solid line). The dashed line is the QCD background. The kinematical cuts are applied.

Fig. 5 Luminosity required for getting a three sigma signal to background ratio as a function of the top quark mass.