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INTERACTION RATES AT HIGH MAGNETIC FIELD STRENGTHS AND HIGH DEGENERACY *

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Abstract

In this paper, we have derived the effects of strong magnetic fields $\vec{B}$ on nucleon and particle reaction rates of astrophysical significance. We have explored the sensitivity to the presence of artbitrary degeneracy and polarization. The possible astrophysical applications of our results are discussed.

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1. Introduction

The effects of intense magnetic fields on various phenomena, in both laboratory and astrophysical dynamical systems, have been investigated by numerous authors[1,2]. Specially, a number of the principal features of high-energy, high-field electromagnetic conversion processes have been well studied, such as magnetic bremsstrahlung (synchrotron radiation), magnetic pair production, direct and indirect trident cascades, photon splitting, and magnetic Cerenkov radiation. However, the effects of an intense magnetic field in astrophysical investigations have often been ignored. In particular, the effects on the the rates of processes which are fundamental to studies of many astrophysical objects (e.g., neutron stars, supernova, and etc.), and even Big Bang Nucleosynthesis, are usually neglected. For example, in neutron stars, where the magnetic field may be as large as $10^{12} \sim 10^{14}$ gauss[3,4], the effects on reaction rates which are relevant to the cooling rates of neutron star can be substantial. A joint work of Ruderman, Cannuto, Lodenquai, and Tsuruta on the general effects of strong magnetic fields and superfluidity on neutron star cooling, has been presented in Ref.[5,6], where they have specially studied the effects of magnetic fields on photon opacities. They found that the major effect of strong magnetic fields is to drastically reduce photon opacities in certain spacial conditions and greatly accelerate the cooling rates. However, the influences on the cooling rates directly from the altered URCA rates by strong magnetic fields still remain. Moreover, in the early universe, the effects on the weak-interaction rates which determine the rate of production of helium and other light elements can also be significant, depending on the possible existence of magnetic fields at that time. In this paper, we examine the effects of a magnetic field on reaction rates (including both weak and strong interactions) in the presence of variable degrees of degeneracy. Some potential astrophysical implications of our results will also be discussed.

2. Relativistic Motion of a Charged Particle in a Magnetic Field

The Dirac equation for the motion of a charged particle in an arbitrary magnetic field is, in conventional notation,

$$i\hbar\frac{\partial\psi}{\partial t} = [i\hbar\vec{\alpha}\cdot(c\vec{\mathbf{p}} + e\vec{\mathbf{A}}) + \gamma_4 mc^2 + e\hbar c\gamma_4\vec{\sigma}\cdot\vec{\mathbf{B}}]\psi, \tag{2.1}$$

where ψ is the wave function of the charged particle, e and m are the electric charge and mass, $\vec{\mathbf{A}}$ is the electromagnetic vector potential, $\vec{\mathbf{B}}$ is the strength of the magnetic fields, $\hbar$ denotes the Planck constant, and c represents the speed of light. The Dirac operators $\vec{\alpha}$

and γ_4 are chosen to be of the following forms

$$\vec{\alpha} = \begin{pmatrix} 0 & \vec{\sigma} \\ \vec{\sigma} & 0 \end{pmatrix}, \quad \gamma_4 = \begin{pmatrix} I & 0 \\ 0 & I \end{pmatrix}, \quad I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \tag{2.2}$$

where $\vec{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ is the usual Pauli spin matrix, with components

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{2.3}$$

For a constant magnetic field oriented along the z axis ($\vec{\mathbf{A}} = 0$, $\vec{\mathbf{B}} = B_z = B =$ const.), the energy eigenvalues E for the charged particle are given by

$$E^2 = c^2p^2 + m^2c^4 + e\hbar cB\,(2n + s + 1), \tag{2.4}$$

where $n = 0, 1, 2, \cdots$ is the principal quantum number, s ($= \pm 1$ refers to spin up and spin down) is the spin variable, and p is the momentum of the particle. The third term in the expression reflects the contribution to the energy of the particle resulting from the interaction of the particle's magnetic moment with the magnetic field. This expression is very useful in determining the influence of the magnetic field on the interaction rates of many processes.

3. Interaction Rates in the Presence of Magnetic Fields and Arbitrary Degeneracy

3.1 Weak reaction rates:

We here consider three fundamental weak interactions which act to determine the critical n/p ratio (see Schramm and Wagoner 1977)[7]in many astrophysical processes, including e.g., big bang nucleosynthesis and neutron star cooling:

$$n + e^+ \rightleftharpoons p + \overline{\nu}, \tag{a}$$

$$n + \nu \rightleftharpoons p + e^-, \tag{b}$$

$$n \rightleftharpoons p + e^- + \overline{\nu}. \tag{c}$$

The cross-sections for these reactions can be computed using standard charged current β - decay theory, with the well-known V-A interaction Hamiltonian [8]

$$H = \frac{g_V}{\sqrt{2}}\left[\overline{u}_p\gamma_\mu(1 - \alpha\gamma_5)u_n\right]\left[\overline{u}_e\gamma^\mu(1 - \gamma_5)\overline{v}_\nu\right] + h.\ c., \tag{3.1}$$

where $g_V = 1.4146 \times 10^{-49}$erg cm^3, and $\alpha = \frac{g_A}{g_V} \simeq -1.262$.[9] Here $\overline{u}_p$, u_n, $\overline{u}_e$, and v_ν stand for the proton, neutron, electron, and neutrino operators, with $\overline{\psi} = \psi^\dagger \gamma_4$.

For the case of a constant B-field[10], we use the exact relativistic wave function in a constant uniform magnetic field for an electron and a free-particle relativistic spinor wave function for an antineutrino. As to the neutron and proton, since the effects of a magnetic field on them are small compared with their rest mass energy difference, they can be treated nonrelativistically. To a good approximation, we can therefore use the free-particle nonrelativistic spinor wave functions for them.

The energy eigenvalues for an electron are obtained from Eq.(2.4)

$$E^2 = c^2p^2 + m_e^2c^4 + e\hbar cB(2n + s + 1), \tag{3.2}$$

where m_e is the rest mass of electron. In the nonrelativistic limit, this becomes

$$E^2 = c^2p^2 + m_e^2c^4 + 2e\hbar cBn. \tag{3.3}$$

If the electron spin is not measured, the matrix element for each of the reactions $((a) \rightarrow (c))$ will be the same, i.e.,

$$\mid H_{fi}(P,p,n) \mid^2 = \frac{g_V^2 m_e^2 c^4 \gamma}{2\pi\hbar^2}(1+3\alpha^2)\{1 - \frac{1}{2}\delta_{n,0}(1-x) + P\Lambda[x + \frac{1}{2}\delta_{n,0}(1-x)]\}, \tag{3.4}$$

where

$$x \equiv cp/E, \quad \gamma \equiv \frac{1}{2}B/B_c, \quad \Lambda \equiv 2\alpha(1-\alpha)/(1+3\alpha^2) \simeq -0.99, \tag{3.5}$$

and the subscripts f and i denote, respectively, the final and initial states. P is the polarization of the neutron source($P = 0$: unpolarized; $P = 1$: completely polarized; $0 < P < 1$: partially polarized). $B_c = \frac{m_e^2c^3}{e\hbar} = 4.4 \times 10^{13}$gauss is the field strength at which magnetic quantum levels, "cyclotron lines," begin to occur[11].

The reaction rate is obtained from Fermi's *golden rule* (cf 12)

$$\lambda_{i\rightarrow f} = \frac{2\pi}{\hbar}\sum_i n_i \int_f dE_f \frac{dn_f}{dE_f}\delta(E_f - E_i) \mid H_{fi}(P,p,n) \mid^2, \tag{3.6}$$

where n_i is the distribution of initial states and $\frac{dn_f}{dE_f}$ is the density of final states.

For convenience, we introduce the following parameters:

$$\epsilon \equiv \frac{E}{m_ec^2}, \quad q \equiv \frac{m_n - m_p}{m_e}, \quad Z_e \equiv \frac{m_ec^2}{kT_e},$$

$$Z_\nu \equiv \frac{m_ec^2}{kT_\nu}, \quad \phi_e \equiv \frac{\mu_e}{kT_e}, \quad \phi_\nu \equiv \frac{\mu_\nu}{kT_\nu}, \tag{3.7}$$

where m_n and m_p are the rest masses of the neutron and proton, respectively, $T_{e\,\mathrm{or}\,\nu}$ represents the temperature of the electron or neutrino, $\mu_{e\,\mathrm{or}\,\nu}$ is the chemical potential of the electron or neutrino, and $\phi_i (i = e,\ \nu)$ is the degeneracy parameter.

Thus, the rate for reaction $n + e^+ \to p + \overline{\nu}_e$, is given by

$$\lambda_a = \frac{g_V^2(1+3\alpha^2)m_e^5c^4\gamma}{2\pi^3\hbar^7}\sum_{n=0}^{\infty}[2-\delta_{n0}(1-P\Lambda)]\int_{\sqrt{1+4\gamma n}}^{\infty}\frac{\epsilon d\epsilon}{\sqrt{\epsilon^2-(1+4\gamma n)}} \times \frac{1}{(1+e^{\epsilon Z_e+\phi_e})}\frac{(\epsilon+q)^2e^{(\epsilon+q)Z_\nu+\phi_\nu}}{(1+e^{(\epsilon+q)Z_\nu+\phi_\nu})}. \tag{3.8}$$

Similarly, for reaction (b), $n + \nu \to p + e^-$, the rate is

$$\begin{aligned}\lambda_b =& \frac{g_V^2(1+3\alpha^2)m_e^5c^4\gamma}{2\pi^3\hbar^7}\sum_{n=0}^{\infty}[2-\delta_{n0}(1-P\Lambda)]\int_{\sqrt{1+4\gamma n}}^{\infty}\frac{\epsilon d\epsilon}{\sqrt{\epsilon^2-(1+4\gamma n)}} \\ &\times \frac{1}{(1+e^{\epsilon Z_e+\phi_e})}\frac{(\epsilon-q)^2e^{\epsilon Z_e+\phi_e}}{(1+e^{(\epsilon-q)Z_\nu-\phi_\nu})} \\ &- \frac{g_V^2(1+3\alpha^2)m_e^5c^4\gamma}{2\pi^3\hbar^7}\sum_{n=0}^{n_{\max}}[2-\delta_{n0}(1-P\Lambda)]\int_{\sqrt{1+4\gamma n}}^{q}\frac{\epsilon d\epsilon}{\sqrt{\epsilon^2-(1+4\gamma n)}} \\ &\times \frac{1}{(1+e^{\epsilon Z_e+\phi_e})}\frac{(\epsilon-q)^2e^{\epsilon Z_e+\phi_e}}{(1+e^{(\epsilon-q)Z_\nu-\phi_\nu})}.\end{aligned} \tag{3.9}$$

where $n_{\max}$ is the largest integer in $(q^2-1)/4\gamma$ and $q = 2.53$. Finally, for reaction (c), $n \to p + e^- + \overline{\nu}_e$, we have

$$\lambda_c = \frac{g_V^2(1+3\alpha^2)m_e^5c^4\gamma}{2\pi^3\hbar^7}\sum_{n=0}^{n_{\max}}[2-\delta_{n0}(1-P\Lambda)]\int_{\sqrt{1+4\gamma n}}^{q}\frac{\epsilon d\epsilon}{\sqrt{\epsilon^2-(1+4\gamma n)}} \times \frac{1}{(1+e^{\epsilon Z_e+\phi_e})}\frac{(q-\epsilon)^2e^{\epsilon Z_e+\phi_e}e^{(q-\epsilon)Z_\nu+\phi_\nu}}{(1+e^{(q-\epsilon)Z_\nu+\phi_\nu})}. \tag{3.10}$$

From these expressions, we can clearly see that, if $\gamma \geq (q^2-1)/4 \simeq 1.35$, (or, equivalently, $B > 10^{14}$gauss), then $n_{\max} = [\frac{q^2-1}{4\gamma}] = 0$, and the rate of β-decay would behave linearly with B. In addition, the rate λ_c (Eq.(3.10)) is exactly cancelled by the second term of λ_b (Eq.(3.9)) if ϕ_ν is negligible.

Finally, we are interested in the total reaction rates for converting neutrons into protons (and vice-versa), which can be obtained by summing the above three rates

$$\begin{aligned}\lambda_{n\to p} =& \lambda_a \quad + \quad \lambda_b \quad + \quad \lambda_c \\ =& \frac{g_V^2(1+3\alpha^2)m_e^5c^4\gamma}{2\pi^3\hbar^7}\sum_{n=0}^{\infty}[2-\delta_{n0}(1-P\Lambda)] \\ &\times \int_{\sqrt{1+4\gamma n}}^{\infty}\epsilon d\epsilon\frac{[\epsilon^2-(1+4\gamma n)]^{-\frac{1}{2}}}{(1+e^{\epsilon Z_e+\phi_e})}\left[\frac{(\epsilon+q)^2e^{(\epsilon+q)Z_\nu+\phi_\nu}}{1+e^{(q+\epsilon)Z_\nu+\phi_\nu}}+\frac{(\epsilon-q)^2e^{\epsilon Z_e+\phi_e}}{1+e^{(\epsilon-q)Z_\nu-\phi_\nu}}\right].\end{aligned} \tag{3.11}$$

For illustrative purpose, we will briefly examine several limiting cases.

1. No magnetic fields:

In the absence of a magnetic field, $B = 0$, which means $\gamma = 0$ and $P = 0$. Thus, the summation over the principal quantum number n needs to be replaced by an integration. We define a variable

$$\theta \equiv 1 + 4\gamma n, \tag{3.12}$$

such that

$$\sum_n^\infty \to \int_0^\infty dn = \lim_{\gamma\to 0} \frac{1}{4\gamma}\int_1^\infty d\theta. \tag{3.13}$$

Substituting Eq.(3.13) into Eqs.(3.8), (3.9), (3.10), and (3.11), and integrating over the variable θ, we finally obtain

$$\lambda_a(B=0) = \frac{1}{\tau}\int_1^\infty \frac{\epsilon d\epsilon\sqrt{\epsilon^2-1}}{(1+e^{\epsilon Z_e+\phi_e})}\frac{(\epsilon+q)^2 e^{(\epsilon+q)Z_\nu+\phi_\nu}}{(1+e^{(q+\epsilon)Z_\nu+\phi_\nu})}, \tag{3.14}$$

$$\begin{aligned}\lambda_b(B=0) = &\frac{1}{\tau}\int_1^\infty \frac{\epsilon d\epsilon\sqrt{\epsilon^2-1}}{(1+e^{\epsilon Z_e+\phi_e})}\frac{(\epsilon-q)^2 e^{\epsilon Z_e+\phi_e}}{(1+e^{(\epsilon-q)Z_\nu-\phi_\nu})}\\ &-\frac{1}{\tau}\int_1^q \frac{\epsilon d\epsilon\sqrt{\epsilon^2-1}}{(1+e^{\epsilon Z_e+\phi_e})}\frac{(\epsilon-q)^2 e^{\epsilon Z_e+\phi_e}}{(1+e^{(\epsilon-q)Z_\nu-\phi_\nu})}.\end{aligned} \tag{3.15}$$

$$\lambda_c(B=0) = \frac{1}{\tau}\int_1^q \frac{\epsilon d\epsilon\sqrt{\epsilon^2-1}}{(1+e^{\epsilon Z_e+\phi_e})}\frac{(q-\epsilon)^2 e^{\epsilon Z_e+\phi_e} e^{(q-\epsilon)Z_\nu+\phi_\nu}}{(1+e^{(q-\epsilon)Z_\nu+\phi_\nu})}, \tag{3.16}$$

and

$$\begin{aligned}\lambda_{n\to p}(B=0) = &\frac{1}{\tau}\int_1^\infty \frac{\epsilon d\epsilon\sqrt{\epsilon^2-1}}{(1+e^{\epsilon Z_e+\phi_e})}\Big[\frac{(q+\epsilon)^2 e^{(\epsilon+q)Z_\nu+\phi_\nu}}{(1+e^{(q+\epsilon)Z_\nu+\phi_\nu})}\\ &+\frac{(\epsilon-q)^2 e^{\epsilon Z_e+\phi_e}}{(1+e^{(\epsilon-q)Z_\nu-\phi_\nu})}\Big],\end{aligned} \tag{3.17}$$

where $\frac{1}{\tau} \equiv \frac{g_V^2(1+3\alpha^2)m_e^5c^4}{2\pi^3\hbar^7} \simeq 6.515\times 10^{-4}\text{sec}^{-1}$. We express our results relative to the measured life-time τ_n of the neutron ($\tau_n \sim 889.6 \pm 2.9$ sec., $\tau_{1/2} \sim 10.277 \pm 0.046$ min.)[13],

$$\tau = I\,\tau_n,$$

where I is the value of the integral.

These formulae are merely the generalized form of the rates derived by Peebles and Wagoner et al[9] and used in standard Big Bang Nucleosynthesis (BBN) calculations.[9] Thus our formalism reduces to the correct limit as $B \to 0$.

2. Weak magnetic fields ($B \ll B_c$, that is $\gamma \ll 1$) :

In this case, we can approximate

$$(\epsilon^2 - 1 - 4\gamma n)^{-1/2} \simeq (\epsilon^2 - 1)^{-1/2} + 2\gamma n(\epsilon^2 - 1)^{-3/2} + 0(\gamma^2). \tag{3.18}$$

Incorporating this, together with equation (3.13), into equations (3.8), (3.9), (3.10), and (3.11), and replacing the summation over n by an integral, we obtain the result

$$\lambda_i(B \ll B_c) \simeq \lambda_i(B = 0)\left[1 + 0(\gamma^2)\right]. \tag{3.19}$$

where $i = a, b, c$, of $n \to p$. Thus, it can be seen that a weak magnetic field ($B \ll B_c$) will have a negligible effect on the rates of the weak reactions.

3. Strong magnetic fields ($B \gg B_c,\ \gamma \gg 1$) :

If the magnetic field is strong, $\gamma \gg 1$, then $n_{\max} \to 0$, which implies that there is only one term ($n = 0$) left in the summation over n. In order to see the effect of the magnetic field, we rewrite the first function in the integrals as

$$(\epsilon^2 - 1 - 4\gamma n)^{-1/2} \simeq \frac{(\epsilon^2 - 1)^{1/2}}{\epsilon^2 - 1}. \tag{3.20}$$

Since ϵ goes from 1 to ∞, in a mathematical sense, it is always true that

$$\int_1^\infty d\epsilon\, \frac{(\epsilon^2 - 1)^{1/2}}{\epsilon^2 - 1}\, g(\epsilon)\bigg|_{B\neq 0} \leq 2\int_1^\infty d\epsilon\,(\epsilon^2 - 1)^{1/2}\, g(\epsilon)\bigg|_{B=0}. \tag{3.21}$$

where $g(\epsilon)\,(\geq 0,$ for any $\epsilon)$ is an arbitrary positive exponential decay function. Thus, by comparing with the rates for zero magnetic field, we have

$$\lambda_i(B \gg B_c) \leq 2\gamma\, \lambda_i(B = 0)\ (> \lambda_i(B = 0)). \tag{3.22}$$

The dependence of the ratio of the total neutron-depletion rate to the free-field rate on the magnetic field parameter and the temperature, in the case of nondegeneracy and nonpolarization, is plotted in Figure 1. From this, we see that the effects of a strong magnetic field are negligible until the temperature drops to the point at which neutron β-decay begins to dominate and large deviations can occur. We therefore conclude that a strong magnetic field can, in some domains, have a significant effect on the rates of the weak reactions.

Here, it is worthwhile to point out that, if the magnetic field is not uniform, but rather is characterized by a distribution function with regions(or magnetic bubbles) ($\vec{\mathbf{A}} \neq 0$ in Eq.(2.1)), the nuclear reaction rates will become inhomogeneous; i.e., the field will vary with spacial variables, and the reaction rates will be different, region by region. Thus a fluctuation in the reaction rate would occur (similar to the introduction of the density fluctuations associated with a first order QCD phase transition[14]).

3.2 Strong reaction rates

Let us now discuss the effects of magnetic fields on strong interactions. In this situation, the nucleons(p and n) and pions(π^+, π^0, π^-) will interact both with themselves, and with the magnetic fields, through their charges and magnetic moments. Among these interactions, the relative interaction strength between themselves ($\alpha_s \sim 1$) is about two orders of magnitude larger than that of the electromagnetic interaction ($\alpha_e \sim 10^{-2}$) and five orders of magnitudes larger than that of the weak interaction($\alpha_w \sim 10^{-5}$). The entirety of the phenomena caused by the strong interaction, for example, the processes below

$$p + p \rightarrow D + \pi^+, \tag{3.23a}$$

$$n + p \rightarrow D + \gamma, \tag{3.23b}$$

$$n + \pi^+ \rightarrow p, \tag{3.23c}$$

$$n + p \rightarrow D + \pi^0, \tag{3.23d}$$

$$3\alpha \rightarrow {}^{12}C, \tag{3.23e}$$

is described by the quantum chromodynamic(QCD) gauge theory. For these systems in the presence of a magnetic field, the Hamiltonian of the particles is composed of two parts:

$$H = H_0 + H_I$$

where

$$H_0 = cp + mc^2 \tag{3.24}$$

and

$$H_I \sim \frac{1}{mc} e\vec{\mathbf{A}} \cdot \vec{\mathbf{p}} + \frac{e\hbar}{2} \vec{\mu} \cdot \vec{\mathbf{B}}, \tag{3.25}$$

respectively, represent the energy of the particle in the field-free case and the coupling energy of the field to the magnetic moment $\vec{\mu}$ of the particle. If the magnetic field is uniform, $\vec{\mathbf{A}} = 0$, the interaction Hamiltonian will become

$$H_I \sim \frac{e\hbar}{2} \vec{\mu} \cdot \vec{\mathbf{B}}. \tag{3.26}$$

Since all baryons and pions involving the strong interactions have mass $m \geq 10\, m_e$ and magnetic moment $|\, \vec{\mu} \,| \leq |\, \vec{\mu}_p \,| \sim \frac{1}{660} \,|\, \vec{\mu}_e \,|$, where $|\, \vec{\mu}_p \,|$ and $|\, \vec{\mu}_e \,|$ stand for the magnetic moment of proton and electron respectively, the interaction energy between the magnetic field and the magnetic moment of the baryons and pions will be

$$H_I \leq \vec{\mu}_p \cdot \vec{\mathrm{B}} \sim 10^{-3} \vec{\mu}_e \cdot \vec{\mathrm{B}} \sim 10^{-6} \frac{B}{B_c} m_p c^2, \tag{3.27}$$

where, again $B_c = 4.4 \times 10^{13}$gauss. It is apparent that this interaction energy is much much smaller than the rest mass energy difference of the associated reactions, if the magnetic field is weaker than 10^{18} gauss.

We thus conclude that, for all magnetic fields of interest ($B \leq 10^{18}$ gauss), the contribution of the interaction energy between nucleons and magnetic fields to the total energy of the system is too small to be significant, and therefore that the effects of magnetic fields ($B \leq 10^{18}$G) on the strong interaction rates are negligible.

3.3 URCA and modified URCA rates

At very high temperatures ($T \geq 10^9$K), e.g., in the core of some massive stars, the so-called URCA rates[15]

$$n \rightarrow p + e^- + \overline{\nu}_e, \tag{3.28a}$$

$$e^- + p \rightarrow n + \nu_e, \tag{3.28b}$$

and modified URCA rates[13]

$$n + n \rightarrow n + p + e^- + \overline{\nu}_e, \tag{3.29a}$$

$$n + p + e^- \rightarrow n + n + \nu_e, \tag{3.29b}$$

$$n + \pi^- \rightarrow n + e^- + \overline{\nu}_e, \tag{3.29c}$$

$$n + n \rightarrow n + p + \mu^- + \overline{\nu}_\mu, \tag{3.29d}$$

$$n + + p + \mu^- \rightarrow n + n + \nu_\mu, \tag{3.29e}$$

(as well as the inverse processes) provide a dominant mode of energy loss via neutrino emission. A typical example of this will be the essential cooling processes involved in the interiors of neutron stars.

Ruderman et al[5,6] have explored the effects of the strong magnetic field on the photon opacities, and in turn, on the cooling rates. Using our calculations in Section 3.1 of the weak interaction rates in the presence of the magnetic field, we are now in a position to examine the rate of the URCA reactions and the modified URCA reactions in the presence of a intense magnetic field. This should lead to important consequences to the cooling of neutron stars since it has been inferred that a strong magnetic field ($\geq 10^{13}$gauss) may exist in the interior of the neutron stars. A detailed calculation will be presented in a coming paper.

4. Astrophysical Applications

In section 3, we have derived the interaction rates as a function of magnetic field B in the presence of variable degrees of degeneracy and polarization. Our calculations have shown that the effects of the magnetic fields on weak reaction rates are significant, if the strength of the magnetic field is comparable with the critical B field $B_c = 4.4 \times 10^{13}$gauss where quantized cyclotron states begin to exist. These results can lead to important astrophysical applications.

(a) Big Bang Nucleosynthesis

If an intense primordial magnetic field existed in the early universe, particularly at or just before the epoch of primordial nucleosynthesis($\sim$ 1min.), then the direct influence on the nuclear reaction rates can be consequential and, in turn, the abundances of light elements produced in BBN can be significantly affected. This phenomenon will enable us to constrain the strength of the primordial magnetic field more accurately by using big bang nucleosynthesis. Detailed discussion and numerical calculations relevant to this issue will be presented in a separate paper[16].

(b) Physics of astrophysical compact objects

If compact objects (e.g., neutron stars, pulsars, and white dwarfs) indeed have intense magnetic fields, as has been implied by a number of authors and observed for a number of objects, then the effects of the fields on the fundamental physical processes involved in these objects, which have often been neglected in earlier studies (e.g., of the (URCA) cooling processes of the neutron stars), should be fully taken into account. The results presented in this paper reveal that, if the magnetic fields in neutron stars are strong enough (as large as $10^{12} - 10^{14}$gauss), the effects of the fields on URCA rates may be substantial and must surely be considered in calculations of the cooling problem of neutron stars. A more detailed examination of this question will appear as a subsequent paper.

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REFERENCES

[1] J. Erber, Rev. Mod. Phys., 38 (1966) 626.

[2] J. J. Eklein, Rev. Mod. Phys., 40 (1968) 523.

[3] C. Woltjer, Astrophys. J., 140 (1964) 1309; J. Trumper, et al, Astrophys. J. Lett., 219 (1978) L105.

[4] W. A. Wheaton et al, Nature, 282 (1979) 240.

[5] S. Tsuruta, V. Cannuto, J. Lodenquai, and Ruderman, Astrophys. J., 176 (1972) 739.

[6] S. Tsuruta, in Physics of Dense Matter, Edited by C. J. Hansen (D. Reidel Publishing Company, Boston, U.S.A., 1974), p209.

[7] D. N. Schramm and R. V. Wagoner, Ann. Rev. Nucl. & Part. Sci., 1977.

[8] C. S. Wu and S. A. Moszkowski, Beta Decay, (Wiley–Inter–Science, Inc., New York, 1966); G. Källén, Elementary Particle Physics (Reading, Mass.: Addison - Wesley Publishing Ca. 1964); J. D. Bjorken and S. D. Drell, Relavistic Quantum Mechanics (New York: McGraw-Hill Book Co., 1964).

[9] P. J. E. Peebles, Astrophys. J., 146 (1966) 542; R. V. Wagoner, W. A. Fowler, and F. Hoyle, Astrophys. J., 148 (1966) 3; R. A. Alpher, J. W. Follin, and R. C. Herman, Phys. Rev., 92 (1953) 1347.

[10] M. H. Johnson and B. A. Lippmann, Phys. Rev. 76 (1949) 828; 77 (1950) 702; R. F. O'Connell, Phys. Rev., 176 (1968) 1433.

[11] L. D. Landau and E. M. Lifshitz, The Classical Theory of Fields (Addison - Wesley Publishing Company, Inc., Reading Mass., 1965), revised 2nd ed., Chap. 9.

[12] G. Baym, Lectures on Quantum Mechanics (Benjamin/Cummings Publishing Company, Inc., Menlo Park, Ca. 1973).

[13] T. P. Walker, G. Steigman, D. N. Schramm, K. A. Olive, and H-S. Kang, Astrophys J., 373 51 (1991).

[14] H. Hurki-Suonio, R. A. Matzner, K. A. Olive, and D. N. Schramm, Astrophys. J., 353 (1990) 406; J. H. Applegate, C. J. Hogan, and R. J. Scherrer, Phys. Rev. D, 35 (1987) 1151.

[15] S. L. Shapiro and S. A. Teukolsky, Black Holes, White Dwarfs, and Neutron Stars, (A Wiley - Interscience Publication, John Wiley & Sons, Inc., New York, 1983) p. .

[16] B. Cheng, D. N. Schramm, and J. W. Truran, in press

Figure Captions

Fig. 1 The ratio of the neutron depletion rate with $B \neq 0$ to $B = 0$.

Fig.1 The ratio of the neutron depletion rate with B≠0 to B=0.