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CONSTRAINTS ON NEUTRINO OSCILLATIONS FROM BIG BANG NUCLEOSYNETHESIS

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ABSTRACT

We discuss in detail the effect of neutrino oscillations in Big Bang nucleosynthesis, between active and sterile neutrinos, as well as between active and active neutrinos. We calculate the constraints on mixings between active and sterile neutrinos from the present observation of the primordial helium abundance and discuss the potential implications on various astrophysical and cosmological problems of such oscillations. In particular, we show that large angle sterile neutrino mixing seems to be excluded as a MSW solution to the solar neutrino situation or a solution to the atmospheric neutrino mixing hinted at in some underground experiments. We show how with this constraint, the next generation of solar neutrino experiments should be able to determine the resolution of the solar neutrino problem. It is also shown how sterile neutrinos remain a viable dark matter candidate.

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I. Introduction

Neutrino mixing and oscillations have long been one of the most important and exciting topics in physics since they directly involve physics beyond the standard model [1]. While direct evidence of oscillations has yet to be found, there is some indirect experimental evidence. Perhaps most intriguing are the solar neutrino experiments which observed a solar neutrino flux that is significantly lower than the prediction of the standard solar models [2]. In particular, the result of the Homestake experiment is a factor of 3 and more than 5σ below the predictions of the standard solar model of Bahcall *et al.* [3], and also significantly lower than the standard solar model of Turck-Chièze *et al.* [4]. This observed deficit in fluxes, if real, can be readily explained by neutrino mixing and oscillation [5,6]. Another possible hint of neutrino oscillation (in a different parameter range) comes from the atmospheric neutrino experiments at Kamioka [7] and IMB [8] and recently at Soudan II [9], which observed deficits in the atmospheric ν_μ flux relative to the ν_e flux. Both of these experimental indications are subject to controversy, especially the atmospheric neutrino experiments. However, it is nonetheless useful to explore the effects of neutrino oscillations in various astrophysical and cosmological environments.

There are two basic neutrino oscillation solutions to the solar enutrino problem. One is the vaccum mixing ("just-so mixing"), in which solar neutrinos ν_e simply oscillate into either active (ν_μ and ν_τ) or sterile neutrinos (ν_s) on their way to the earth [10]. It requires a relatively large mixing angle θ ($sin^2 2\theta \gtrsim 0.75$) and a mass square difference between the two species of about 10^{-10} eV^2. The other solution which is much more robust in that it allows a larger set of parameter space, is the MSW matter mixing solution, in which ν_e's go through a resonance inside the Sun and change into other neutrino species (again either active or sterile) [11]. Two possible ranges of mixing parameters exist in this mechanism to simultaneously explain the observed neutrino deficit in each of the operating experiments [12]: the small mixing angle solution with $\delta m^2 = m_2^2 - m_1^2 \sim 10^{-6}$ to 10^{-5} eV^2 and $sin^2 2\theta \sim 5 \times 10^{-3}$, and the large mixing angle solution with $\delta m^2 = m_2^2 - m_1^2 \sim 10^{-6}$ to 10^{-4} eV^2 and $sin^2 2\theta \gtrsim 0.4$. On the other hand, To solve the reported atmospheric ν_μ deficit, a vaccum mixing between ν_μ and either ν_τ or ν_s is needed, with a large mixing $sin^2 2\theta \gtrsim 0.4$ and δm^2 of order 10^{-3} to 10^{-1} eV^2 [13].

Various terrestrial experiments searching for neutrino oscillations have set stringent bounds on the mixing parameters [14]. However, they were unable to explore the mixing parameter space with either small mass square difference (with $\delta m^2 < 10^{-2}$ eV^2 in ν_e-ν_x

mixing, and $\delta m^2 < 10^{-1}$ eV^2 in ν_μ -ν_x mixing), or small mixing angles with $sin^2\theta \lesssim 10^{-2}$ to 10^{-1}, where θ is the mixing angle. Cosmological and astrophysical considerations, however, give more stringent constraints on these parts of the mixing parameter space. One such constraint comes from consideration of Big Bang nucleosynthesis (hereafter BBN).

The effect of neutrino mixing between a sterile neutrino and an active neutrino on BBN has been investigated by various groups. The numerical calculation of Enqvist *et al.* [15] has excluded a large area on the mixing parameter space, including the ν_μ-ν_s mixing solution to the atmospheric neutrino problem and a large portion of the large angle ν_e-ν_s MSW mixing solution to the solar neutrino problem. Analytical calculations exploring the question were done by Barbieri *et al.* [16] and Cline [17]. The possible effect of ν_e-ν_μ(or ν_τ) mixing on BBN has also been estimated by Langacker *et al.*, with a conclusion that the effect might be too small to set constraints [18]. Since the number densities of ν_e and ν_μ (or ν_τ) are nearly equal in the primordial nucleosynthesis epoch, it is anticipated that a mixing between ν_e and ν_μ (or ν_τ) would not change the predictions of standard BBN to a degree that is currently observable.

In this paper we attempt to discuss the neutrino mixing in further detail and calculate the constraints on the mixing parameters using a full numerical BBN study with the latest primordial helium abundance observations. In particular the new ^{4}He abundance determinations have increased with respect to the previously used numbers to an extent that could alter the previous conclusions. We will discuss the implications of the allowed neutrino oscillations on the solar neutrino problem, the atmospheric problem and the dark matter problem of the universe.

II. Neutrino Oscillation Formalism

In the case of a two-family neutrino oscillation between ν_α and ν_β, we assume the mass eigenvalues of the two mass eigenstates ν_1 and ν_2 to be m_1 and m_2, and the mixing angle to be θ. Then the time evolution of a neutrino state is discribed by eq. (1) [19]:

$$i\frac{d}{dt}\begin{pmatrix} C_\alpha(t) \\ C_\beta(t) \end{pmatrix} = M \begin{pmatrix} C_\alpha(t) \\ C_\beta(t) \end{pmatrix}, \tag{1}$$

where in the vacuum mixing,

$$M = \frac{1}{4p}\begin{pmatrix} -\delta m^2 cos2\theta & \delta m^2 sin2\theta \\ \delta m^2 sin2\theta & \delta m^2 cos2\theta \end{pmatrix}. \tag{2}$$

p is the momentum of the neutrino and is assumed to be much larger than m_1 and m_2

(relativistic regime). When there is MSW matter mixing,

$$M = \frac{1}{4p}\begin{pmatrix} -\delta m^2 cos2\theta + 2p(V_\alpha - V_\beta) & \delta m^2 sin2\theta \\ \delta m^2 sin2\theta & \delta m^2 cos2\theta \end{pmatrix}$$
$$= \frac{\delta m_M^2}{4p}\begin{pmatrix} -cos2\theta_M & sin2\theta_M \\ sin2\theta_M & cos2\theta_M \end{pmatrix} \quad (3)$$

where V_α and V_β are the effective potential due to the MSW effect [11,20].

$$\delta m_M^2 = \sqrt{(\delta m^2 sin2\theta)^2 + [\delta m^2 cos2\theta + 2p(V_\alpha - V_\beta)]^2} \quad (4)$$

and θ_M, the matter mixing angle, is given by

$$\theta_M = \frac{1}{2}tan^{-1}\left\{\frac{tan2\theta}{1 - [2p(V_\alpha - V_\beta)/\delta m^2]sec2\theta}\right\}. \quad (5)$$

The effective potentials are obtained by calculating the corrections to the neutrino self-energy from the ambient matter [20]. In the stellar interior (e.g., the sun), the effective potentials can be calculated using zero temperature field theory [20]:

$$V_e = \sqrt{2}G_F(N_e - 0.5N_n), \quad V_\mu = V_\tau = \sqrt{2}G_F(-0.5N_n), \quad (6)$$

where N_e and N_n are the number density of electrons and neutrons. In the early universe after $T < 100$ MeV when μ and τ are absent, and in the case that neutrinos are in kinetic equilibrium but may not be in chemical equilibrium, finite temperature field theory gives [20]

$$V_e = \sqrt{2}G_F N_\gamma[L^{(e)} - 12.61\frac{pT_\gamma}{M_W^2} - 12.61\frac{pT_{\nu_e}^4}{4M_Z^2T_\gamma^3}[n_{\nu_e} + n_{\bar{\nu}_e}])], \quad (7)$$

$$V_\mu = \sqrt{2}G_F N_\gamma[L^{(\mu)} - 12.61\frac{pT_{\nu_\mu}^4}{4M_Z^2T_\gamma^3}(n_{\nu_\mu} + n_{\bar{\nu}_\mu})], \quad (8)$$

where

$$L^{(e)} = (\frac{1}{2} + 2x_W)L_e + (\frac{1}{2} - 2x_W)L_p - \frac{1}{2}L_n + 2L_{\nu_e} + L_{\nu_\mu} + L_{\nu_\tau}, \quad (9)$$

$$L^{(\mu)} = L^{(e)} - L_e - L_{\nu_e} + L_{\nu_\mu}, \quad (10)$$

and

$$L_\alpha = \frac{N_\alpha - N_{\bar{\alpha}}}{N_\gamma}, \quad (11)$$

where $x_W = sin^2\theta_W = 0.23$. N_α is the number density of the species α. $n_\nu = (7N_\nu)/(8N_\gamma)$. A similar expression exists for V_τ if one substitutes μ to τ in eq. (8) and (10). For anti-neutrinos, one simply substitutes $L^{(\alpha)}$ by $-L^{(\alpha)}$ to get their effective potentials. If $L^{(a)}$

is at the level of 10^{-10} as η $(=N_B/N_\gamma = (N_n + N_p)/N_\gamma)$ is, the terms involving $L^{(a)}$ can always be neglected as long as $T_\gamma \sim T_\nu > 0.1$ MeV, which is the range in which we are interested. In all cases, since ν_s is sterile, $V_s \equiv 0$.

When the neutrino ensemble is incoherent, the phase of the neutrino states are not of concern. Therefore an oscillation is characterized by the oscillation length

$$L_{osc} = \frac{4\pi p}{\delta m_M^2} \tag{12}$$

and the mixing angle θ_M (where δm_M^2 and θ_M reduce to δm^2 and θ in vaccum oscillation). If one denotes $P_{\alpha\alpha}$ as the probability of finding a ν_α in a state that is initially ν_α, in vaccum (or a constant medium where V_α and V_β are constant), after averaging over phases,

$$P_{\alpha\alpha} = 1 - \frac{1}{2} sin^2 2\theta (or\ 2\theta_M) \quad \text{for the vaccum (or a constant medium).} \tag{13}$$

In a varying medium where V_α and V_β are also varying, the neutrino encounters a resonance when

$$\delta m_M^2 = 0 \quad \text{or} \quad \delta m^2 = 2p(V_\alpha - V_\beta). \tag{14}$$

Then [19]

$$P_{\alpha\alpha} = \frac{1}{2} + (\frac{1}{2} - P_{jump}) cos2\theta_{Mi} cos2\theta_{Mf} \tag{15}$$

where to linear order

$$P_{jump} = \exp\Big[\frac{-\pi \delta m^2 sin^2 2\theta (V_\alpha - V_\beta)}{4cos2\theta} \Big(\frac{d|p(V_\alpha - V_\beta)|}{dr}\Big)^{-1}\Big|_{res}\Big], \tag{16}$$

where r is the position of the neutrino. We put the neutrino momentum p inside the derivative because in cases such as the expanding universe, p changes with time. When $P_{jump} \ll 1$ ("adiabtic"), because $cos2\theta_{Mi}$ and $cos2\theta_{Mf}$ are of opposite sign on each side of the resonance (at resonance, $\theta_M = 45°$), the neutrinos mostly change their flavor; while if $P_{jump} \sim 1$ ("nonadiabtic"), the neutrinos mostly retain their initial flavor. Therefore P_{jump} serves as a indicator of the significance of the resonance.

III. Effects of Neutrino Oscillation on Primordial Nucleosynthesis

The agreement of the predictions of Big Bang primordial nucleosynthesis with the observed light element abundances has been one of the great successes in Big Bang cosmology [21]. Therefore primordial nucleosynthesis has become an indispensable probe of the early universe, and thereby tests theories of cosmology and particle physics. For example, it sets the best-known limit on the baryon to photon ratio η ($=N_B/N_\gamma$ as defined previously) or equivalently the baryon density, ρ_B [21]:

$$2.8 \times 10^{-10} < \eta < 4 \times 10^{-10} \tag{17}$$

It also rigorously constrains the total number of light neutrino species ($m \lesssim 1$ MeV), N_ν, that is in equilibrium at $T \sim$ MeV [21,22]:

$$N_\nu < 3.6. \tag{18}$$

Eq. (18) has been essentially confirmed by the result of the Z^0 decay experiments at LEP, $N_\nu = 2.99 \pm 0.05$ [23]. The constraints from BBN and that from LEP are complimentary because the latter only limits the active neutrino species and the former limits both active and sterile neutrino species [24].

What constrains the number of light neutrino species is the observed ^{4}He abundance $Y = 0.235 \pm 0.01$ [25]. Assuming only three known neutrino species, standard Big Bang nucleosynthesis yields $0.236 < Y < 0.243$ [21]. When extra neutrino species (therefore extra degrees of freedom) are introduced at the epoch of $T \sim$ MeV in the early universe, the expansion of the universe would be faster due to a higher energy density, which in turn leads to a higher neutron to proton ratio when the ratio freezes out ($T \approx 0.7$ MeV), and thus a higher helium yield [24]. Therefore the constraint of eq. (18) can be obtained by requiring the calculated Y assuming extra species not to exceed the observations. An extra neutrino species, if it exists, must interact weakly enough with the Z^0 to accomodate the LEP result. Furthermore, it has to interact weakly enough so that it will not be counted as one full species at the primordial nucleosynthesis epoch [24]. Therefore, a sterile neutrino might be allowed.

A sterile neutrino, i.e., a gauge group singlet, is the simplest extension of the standard model [1]. It cannot be generated thermally in the epoch of nucleosynthesis due to its sterility. However, if it mixes with active neutrinos, the active-sterile neutrino oscillation would produce sterile neutrinos in the early universe, especially when MSW resonances were present. The intensive production of sterile neutrinos through oscillation or resonance before electron neutrinos get out of chemical equilibrium may bring one more light neutrino species into chemical equilibrium, and thus exceed the bound from the nucleosynthesis. Furthermore, in the case of ν_e-ν_s oscillation, if the mass production of ν_s occurs after ν_e freezes out but before the freeze-out of the neutron to proton ratio, the deficit in ν_e

relative to that counted as a full neutrino species will cause the ratio to freeze out earlier and increase the ^{4}He yield. Therefore the mixing parameters between the sterile and active neutrinos have to be constrained so that such an intensive production of sterile neutrinos would not occur.

In the case of ν_e and ν_μ (or ν_τ) mixing, the neutrino oscillation can only show its signature on primordial nucleosyntheis if there is number density asymmetry around $T \sim$ MeV between ν_e and the species it oscillates into. Within the framework of the standard model, a number density asymmetry between ν_e and other active neutrino species can indeed be generated at the 10^{-2} level by the larger branching ratio of $e^{\pm}$ annihilating into $\nu_e\bar{\nu}_e$ relative to other neutrinos [26]. Then one expects that a resonant oscillation between ν_e and another active species but not between their antiparticles, or vice versa, will interchange their number densities and lead to an asymmetry between ν_e and $\bar{\nu}_e$ [18]. This process doesn't change the total energy density of the universe, so the expansion rate is unchanged. But it changes the rates of weak interactions that interchanges neutrons and protons. More ν_e than $\bar{\nu}_e$ leads to a higher rate for neutrons to be converted into protons, and vice versa. However, as we will show later, in the case of an oscillation between ν_e and another active species, if the lepton number asymmetry is negligible, ν_e and $\bar{\nu}_e$ will encounter resonances simultaneously and the asymmetry between them will not be produced. Furthermore, the asymmetry between the number densities of ν_e and the other two active species may be washed out in the case of large ν_e-active neutrino oscillation.

The formalism of evolving the light neutrino ensemble with neutrino oscillations in the early universe has been carried out by various groups [15-17,27]. We will adopt the formalism of Enqvist *el al.* [15]. At temperatures below 100 MeV and before the primordial nucleosynthesis epoch, the universe was composed of photons, electron-positrons, and neutrinos. (Nucleons are present only at the 10^{-10} level relative to other particle species. Their contribution to the evolution of the neutrino ensemble is negligible.) A natural hierachy in interation rates exist among these particle species. Photons and electron-positrons interact via the electromagnetic interaction, they reach equilibrium much faster than the neutrinos which only interact weakly. Therefore in the temperature region we are concerned with, we can always assume photons and electron-positrons are in chemical equilibrium. The neutrinos, however, get out of chemical equilibrium at temperatures of several MeV (about 3 MeV for ν_e and $\bar{\nu}_e$, 5 MeV for ν_μ and ν_τ, and their antiparticles) [21]. If assuming a two-flavor oscillation between ν_α and ν_β, the density matrix of weak-interacting species can be expressed in the block diagonal form:

$$\rho_W = \rho_\nu \oplus \rho_{\bar{\nu}} \oplus \sum_{\substack{i=e,\nu's \\ i\neq\nu_\alpha,\nu_\beta}} (\oplus n_i \oplus n_{\bar{i}}) \tag{19}$$

where

$$\rho_\nu = \begin{pmatrix} \rho_{\alpha\alpha} & \rho_{\alpha\beta} \\ \rho_{\beta\alpha} & \rho_{\beta\beta} \end{pmatrix} \tag{20}$$

In cases of active-sterile neutrino mixing, evolution of the neutrino ensemble becomes very simple due to the sterility of ν_s. By rewriting

$$\rho_\nu = \frac{1}{2}P_0(I + \mathbf{P}\cdot\sigma), \quad \rho_{\bar{\nu}} = \frac{1}{2}\bar{P}_0(I + \bar{\mathbf{P}}\cdot\sigma), \tag{21}$$

and taking a thermal average by assuming all species are in thermal equilibrium, the evolution equation for ν_e-ν_s oscillation is (analogous expressions arise for the antineutrinos) [15]:

$$\begin{aligned} \frac{d}{dt}\mathbf{P} =& \langle\mathbf{V}\rangle \times \mathbf{P} + (1 - P_z)(\frac{d}{dt}\ln P_0)\hat{\mathbf{z}} \\ &- (D^E + D^I + \frac{d}{dt}\ln P_0)(P_x\hat{\mathbf{x}} + P_y\hat{\mathbf{y}}), \end{aligned} \tag{22}$$

$$\frac{d}{dt}P_0 = \sum_{\alpha=e,\nu_\mu,\nu_\tau} \langle\Gamma(\nu_e\bar{\nu}_e \to \alpha\bar{\alpha})\rangle(\lambda_\alpha n_\alpha n_{\bar{\alpha}} - n_{\nu_e}n_{\bar{\nu}_e}) \tag{23}$$

$$\frac{d}{dt}n_{\nu_\mu} = \sum_{\alpha=e,\nu_e,\nu_\tau} \langle\Gamma(\nu_\mu\bar{\nu}_\mu \to \alpha\bar{\alpha})\rangle(\lambda_\alpha n_\alpha n_{\bar{\alpha}} - n_{\nu_\mu}n_{\bar{\nu}_\mu}) \tag{24}$$

and similarly for n_{ν_τ}.

Here $\lambda_\nu = 1$, $\lambda_e = 1/4$ and the brackets $\langle\ldots\rangle$ indicate an average over the momentum distribution of the ensemble. n_α is the number density relative to that of a fully relativistic fermion with single helicity. Therefore $n_e \doteq 2$ when $T \gtrsim m_e$, and n_ν=1 if they are in chemical equilibrium. The contributions of right-handed neutrinos are neglected since we assume the neutrinos are light and they have only the weak interaction.

$\mathbf{V}$ (and analogously for $\bar{\mathbf{V}}$) of ν_e-ν_s mixing is

$$\mathbf{V} = 2\omega_{es}\hat{\mathbf{x}} + (V_e + \omega_{ee} - \omega_{ss})\hat{\mathbf{y}} \tag{25}$$

in which

$$2\omega_{es} = \frac{\delta m^2}{2p} sin2\theta, \quad \omega_{ee} - \omega_{ss} = -\frac{\delta m^2}{2p} cos2\theta. \tag{26}$$

We assume the sterile neutrino is heavier than the active neutrino if $\delta m^2 > 0$, and vice versa.

D^I and D^E are the dumping coefficients due to inelastic and elastic collisions of the active neutrinos

$$\begin{aligned} D^E = & \frac{1}{2}\Big[\langle\Gamma(\nu_e e^- \to \nu_e e^-)\rangle n_{e^-} + \langle\Gamma(\nu_e e^+ \to \nu_e e^+)\rangle n_{e^+}\Big] \\ & + \frac{1}{2}\sum_{\alpha=e,\mu,\tau}\Big[\langle\Gamma(\nu_e\nu_\alpha \to \nu_e\nu_\alpha)\rangle n_{\nu_\alpha} + \langle\Gamma(\nu_e\bar{\nu}_\alpha \to \nu_e\bar{\nu}_\alpha)\rangle n_{\bar{\nu}_\alpha}\Big] \end{aligned} \tag{27}$$

$$D^I = \frac{1}{2}\Big[\langle\Gamma(\nu_e\bar{\nu}_e \to e^+e^-)\rangle + \sum_{\alpha=\mu,\tau}\langle\Gamma(\nu_e\bar{\nu}_e \to \nu_\alpha\bar{\nu}_\alpha)\rangle\Big] n_{\bar{\nu}_e}. \tag{28}$$

$\bar{D}^I$ and $\bar{D}^E$ for antineutrinos can be deduced by interchanging ν_e and $\bar{\nu}_e$.

The above formalism can be anologously inferred for ν_μ-ν_s or ν_τ-ν_s oscillation by interchanging ν_μ or ν_τ with ν_e.

The thermal averaged reaction rate Γ's has been calculated previously, e.g. in ref. 15 and ref. 26. Table 1 lists their values in units of F_0, where

$$F_0 = \frac{G_F^2\langle p\rangle^2}{6\pi} N_\gamma(T) \tag{29}$$

Since T_γ and T_ν differ by at most 1% in the epoch in which we are interested [26], from now on we shall ignore their difference and denote them by T.

Before going ahead and solving eqs. (22)–(24) numerically, it is helpful to do an analytical estimate of the problem. Because ν_s is only produced through oscillation, its production rate is approximated by

$$\Gamma_{\nu_s}^{prod} \simeq \frac{1}{2} sin^2 2\theta_M \Gamma_\nu \approx 2G_F^2 T^5 sin^2 2\theta_M. \tag{30}$$

At high temperature the oscillation is suppressed by the matter effect which is proportional to T^6. The mixing angle at high temperature is,

$$\begin{aligned} tan2\theta_M \approx -\frac{sin2\theta}{2p(V_\alpha - V_\beta)/\delta m^2} & \sim sin2\theta\Big(\frac{5\times 10^{20}\text{eV}^2\delta m^2}{G_F p^2 T^4}\Big) \quad \text{for } \nu_e{-}\nu_s \text{ mixing,} \\ & \sim sin2\theta\Big(\frac{2\times 10^{21}\text{eV}^2\delta m^2}{G_F p^2 T^4}\Big) \quad \text{for } \nu_{\mu,\tau}{-}\nu_s \text{ mixing.} \end{aligned} \tag{31}$$

If one takes $p \sim 3T$, when

$$T(\mathrm{MeV}) \gtrsim (\frac{\delta m^2}{10^{-8}\mathrm{eV}^2})^{1/6}, \quad \text{for } \nu_e - \nu_s \text{ mixing,}$$
$$(\frac{\delta m^2}{10^{-9}\mathrm{eV}^2})^{1/6}, \quad \text{for } \nu_{\mu,\tau} - \nu_s \text{ mixing,} \tag{32}$$

the mixing is suppressed at the $sin^2 2\theta_M \sim 10^{-4} sin^2 2\theta$ level so that no significant amounts of ν_s can be produced. Only when the temperature of the universe falls significantly below this temperature, does the ν_s production occur noticably. Therefore mixings with $\delta m^2 < 10^{-8}$ to 10^{-9} eV^2 will never produce appreciable amount of ν_s's before the weak freeze out of baryons at ~ 0.7 MeV which affect primordial nucleosynthesis.

The ν_s will be brought into chemical equilibrium if $\Gamma^{prod}_{\nu_s}$ is larger than the Hubble constant before active neutrinos lose thermal contact with the ambient matter, which occurs at $T \sim 1$ MeV. For $\delta m^2 > 0$, $\Gamma^{prod}_{\nu_s}/H$ has a maximum at

$$T_{max}(\mathrm{MeV}) = B_{\nu_\alpha}(\frac{\delta m^2}{1\mathrm{eV}^2})^{1/6}(\frac{p}{T})^{-1/3}, \tag{33}$$

where $B_{\nu_e} \approx 7$ and $B_{\nu_{\mu,\tau}} \approx 9$. The requirement that

$$\frac{\Gamma^{prod}_{\nu_s}}{H}\Big|_{T_{max}} < 1 \quad \text{and} \quad T_{max} \gtrsim 1\mathrm{MeV} \tag{34}$$

gives

$$\delta m^2 sin^4 2\theta \gtrsim 10^{-4}\mathrm{eV}^2 \quad \text{for } \nu_e(\nu_\mu \text{ , or } \nu_\tau) - \nu_s \text{ mixing.} \tag{35}$$

For $\delta m^2 < 0$, a resonance occurs at

$$T_{res}(\mathrm{MeV}) = C_{\nu_\alpha}\Big(\frac{|\delta m^2 cos 2\theta|}{p^2/T^2}\Big)^{1/6}, \tag{36}$$

where $C_{\nu_e} \approx 19$ and $C_{\nu_{\mu,\tau}} \approx 23$. The transition probability P_{jump} has to be sufficiently small so that sizable numbers of ν_s can be converted from active neutrinos. A rough requirement is that $P_{jump} < e^{-1}$. Therefore

$$\Big(\frac{\pi \delta m^2 sin^2 2\theta}{4 cos 2\theta}\Big)\Big|\frac{6p}{T}\frac{dT}{dt}\Big|^{-1}_{T_{res}} > 1 \quad \text{and} \quad T_{max} \gtrsim 1\mathrm{MeV}, \tag{37}$$

from which we get

$$\delta m^2 sin^4 2\theta \gtrsim 10^{-9}\mathrm{eV}^2 \quad \text{and} \quad \delta m^2 \gtrsim 10^{-7}\mathrm{eV}^2 \tag{38}$$

for all three oscillation cases.

When the major production of ν_s through ν_e-ν_s oscillation occurs after the thermal freeze out of ν_e and before the weak freeze out of baryons, additional limits can be obtained. The resultant ν_e deficit relative to a full relativistic species, δn_{ν_e} (defined to be positive if it is a deficit), will increase the neutron to proton ratio, and hence the ^{4}He yield. The constraint that $N_\nu < 3.6$ is replaced approximately by [15]

$$N_\nu + 5\delta n_{\nu_e} < 3.6. \tag{39}$$

In this case, ν_μ and ν_τ are still approximately two full relativistic species. ν_e plus ν_s are roughly counted as one. Therefore $N_\nu \approx 3$. In the case of $\delta m^2 > 0$, The deficit δn_{ν_e} is roughly $0.5 sin^2 2\theta_M$. Hence mixings with

$$\begin{aligned} sin^2 2\theta &\gtrsim 0.25 \quad \text{for} \quad \delta m^2 > 10^{-7}\text{eV}^2 \\ \delta m^2 sin^2 2\theta &\gtrsim 10^{-8} \quad \text{for} \quad \delta m^2 < 10^{-7}\text{eV}^2 \end{aligned} \tag{40}$$

are then excluded by eq. (39). For $\delta m^2 < 0$, the requirement that the resonance occuring between 1 MeV and 0.7 MeV has to be sufficiently nonadiabatic to satisfy eq. (39), sets a stronger limit; δn_{ν_e} is roughly $1 - P_{jump}$. Therefore $\delta n_{\nu_e} \lesssim 0.12$, which translates into

$$\delta m^2 sin^4 2\theta \gtrsim 10^{-10}\text{eV}^2 \quad \text{for} \quad 10^{-8}\text{eV}^2 \lesssim \delta m^2 \lesssim 10^{-7}\text{eV}^2. \tag{41}$$

Our results in eqs (35), (38), (40) and (41) are similar to the result of Enqvist *et al.* [15] and Cline [17]. It should be noted that the Γ_ν in eq. (30) is the total scattering rate of active neutrinos, instead of only the inelastic scattering rate as adopted by Barbieri *et al.* [16]. Elastic scatterings reduce the mixed neutrino states into either pure active neutrino states or pure sterile neutrino states (just as a polarizer reduces a beam of light into either parallel polarized or perpendicularly polarized). The pure active neutrinos will develope sterile components via oscillations. (It is also true that the pure sterile neutrinos will oscillate into mixed states. But if $sin^2 2\theta_M$ is small, the active components in the resultant mixed states are negligible.) The net effect is the production of sterile neutrinos through elastic scatterings of neutrinos.

Figures 1 to 4 show the constant helium yield, Y, contours from numerical integrations of eq. (22) to (24) in different cases. Fig. 1 also shows the ν_e-ν_s MSW mixing solution to the solar neutrino problem at 95% C. L. (using a Monte Carlo calculation of 1000 solar models similar to that in ref. 12, which represents the astrophysical uncertainties in the solar models). The ν_μ-ν_s mixing solution to the atmospheric neutrino problem is shown in

Fig. 3 and 4. Parameter spaces on the upper right of the $Y = 0.245$ contour yield a higher ^{4}He abundance than found by observation, and thus are excluded. *The excluded regions include the large angle ν_e-ν_s MSW solution to the solar neutrino problem and the ν_μ-ν_s mixing solution to the atmospheric neutrino solution.* We express the constraints in terms of the primordial helium yield Y instead of N_ν as in ref. 15 because N_ν gradually changes in the course of the early universe. It is the final helium yield that is to be directly compared with observation. In the integration of eqs. (22)–(24), we assumed particle-antiparticle symmetry, i.e., $L_\alpha \equiv 0$. The justification of such an assumption has been investigated by Enqvist *et al.* [28]. They concluded that as long as there is no resonance, an initial CP asymmetry will be driven toward $L_\alpha = 0$, which is then a fixed point of eqs. (22) to (24). With a large lepton number asymmetry (e.g., larger than 10^{-7}), the first terms in the effective potentials eq. (7) and (8) would dominate. Constraints would then be modefied accordingly. Nonetheless, The large angle mixing ($sin^2 2\theta \sim 1$) between sterile neutrinos and active neutrinos could still be confidently ruled out for $\delta m^2 \gtrsim (\frac{L}{10^{-7}})10^{-6}\mathrm{eV}^2$, where L is the lepton asymmetry.

Another concern is that eqs. (22)–(24) are only accurate as long as the neutrinos maintain a thermal spectrum. Without the oscillations, the neutrino ensemble will have a thermal spectrum through out the history of primordial nucleosynthesis if one neglects the $e^\pm$ heating at the 10^{-2} level. Possible thermal distortion may be generated from those neutrinos with different momentum that begin to oscillate into ν_s at different temperatures with different amplitudes, and the weak interaction cannot fully compensate for the difference. However, we will see that such oscillations are not very sensitive to the neutrino momentum, and as a result, they don't generate significant spectral distortions.

When the oscillation occurs before the freeze out of active neutrinos and $\delta m^2 > 0$, we can see from eq. (33) that the maximal oscillation temperature depends weakly on momentum, $T_{max} \propto p^{-1/3}$. Moreover, Γ_s^{prod} is always less than the weak interaction rates of active neutrinos. Therefore the weak interactions can always keep the active neutrinos in thermal contact. When $\delta m^2 < 0$, It is also shown that P_{jump} is independent of p, while $T_{res} \propto p^{-1/3}$ [15]. Therefore, the possible spectral distortion can only be a higher order effect.

When the large numbers of oscillations between ν_e and ν_s occur after the chemical freeze out of ν_e, a possible distortion occurs due to the dependence of the magnitude of the oscillation $0.5 sin^2 2\theta_M$ on momentum. When $\delta m^2 \gtrsim 10^{-7}\mathrm{eV}^2$, the matter effect is small compared to the vaccum mixing, $sin^2 2\theta_M \approx sin^2 2\theta$, and the thermal distribution assumption is still valid. Below 10^{-7} eV^2, we are only excluding very large angle mixings and the p dependent matter effect is not of major concern.

Overall, for the mixing parameters we probe, a thermal distribution is a reasonable approximation with which to evolve the neutrino ensemble. The effect of a possible non-thermal distribution may be estimated by inserting different p's into V instead of a thermal average. We found that the constraint is not sensitive to the p value we put in, which confirms the validity of a thermal distribution.

In cases of active-active neutrino oscillation, the evolution of the neutrino ensemble is much more complicated to calculate. In the standard Big Bang cosmology, the number densities of ν_μ and ν_τ are identical, and the number density of ν_e is slightly higher (at the 10^{-2} level at MeV temperatures) than the other two species due to the charged current heating of the electron positron annihilations [26]. Therefore the introduction of a ν_μ-ν_τ mixing will have no effect on Big Bang nucleosynthesis. The effect of a ν_e-ν_μ mixing (the same with ν_e-ν_τ oscillation) has been estimated previously [18]. But a careful analysis and calculation has yet to be done. It was expected that a resonance between ν_e and ν_μ, if sufficiently adiabatic and occuring between ~ 0.7 MeV and ~ 3 MeV, will swap the number densities between the ν_e and ν_μ but not $\bar{\nu}_e$ and $\bar{\nu}_\mu$ (or vice versa), and lead to a small deficit in ν_e with respect to $\bar{\nu}_e$, and therefore change the neutron to proton ratio. However, the previous analysis neglected the finite temperature terms in the effective potential eq. (7) and (8), which actually dominate at $T \gtrsim 1$ MeV for a small lepton asymmetry at the 10^{-9} level. Therefore the resonance between ν_e and ν_μ and the resonance between $\bar{\nu}_e$ and $\bar{\nu}_\mu$ occur almost simultaneously with similar strength and no sizable asymmetry between ν_e and $\bar{\nu}_e$ can be produced, even at the 10^{-2} level.

We further found that in the presence of a mixing between ν_e and ν_μ, the existance of such an asymmetry between the number densities of ν_e and ν_μ, is questionable. Using the same formalism as in the eqs. (21), this aymmetry corresponds to a non-zero positive P_z. The rate of increasing P_z, i.e., the rate of generating the asymmetry is roughly $(\Gamma_{e^-e^+\to\nu_e\bar{\nu}_e} - \Gamma_{e^-e^+\to\nu_\mu\bar{\nu}_\mu})\Delta n_e$, where Δn_e is the number density of electrons annihilated with respect to a fully relativistic species; At $T \sim$ MeV, $\Delta n_e \sim 0.1$. The rate of damping P_z by the weak interactions between the flavor eigenstates is roughly $0.5(\Gamma_{\nu_e} + \Gamma_{\nu_\mu}) sin^2 2\theta_M$, where Γ_{ν_e} and Γ_{ν_μ} are the rates of the ν_e and ν_μ's weak interaction. As long as the damping rate is larger than the production rate, P_z will remain 0.

Since $(\Gamma_{\nu_e} + \Gamma_{\nu_\mu}) > 10^2(\Gamma_{e^-e^+\to\nu_e\bar{\nu}_e} - \Gamma_{e^-e^+\to\nu_\mu\bar{\nu}_\mu})\Delta n_e$, to keep P_z at zero requires

$$sin2\theta_M \gtrsim 10^{-1} \quad \text{at } T \sim MeV. \tag{42}$$

Therefore for mixings with

$$\delta m^2 \gtrsim 10^{-7}\text{eV}^2 \quad \text{and} \quad sin2\theta \gtrsim 10^{-1}, \tag{43}$$

the number density asymmetry between ν_e and ν_μ will be damped to a much lower level than 10^{-2}. The "miraculous" cancellation mentioned in ref. 26 would then no longer exist. The net change in ^{4}He yield with respect to standard Big Bang nucleosynthesis considered in ref. 26 would then be $\sim -10^{-4}$ to -10^{-3} instead of $\sim 10^{-5}$. The large angle ν_e-ν_μ MSW mixing solution to the solar neutrino problem clearly belongs to this case, but a change in the helium abudance at the 10^{-3} level can hardly put any constraints on the mixing parameter space of ν_e-ν_μ oscillations.

IV. The Solar Neutrino Implications

From Fig. 1 we can see that the large angle ν_e-ν_s MSW mixing solution to the solar neutrino problem is clearly ruled out. It has been a concern before that the next generation solar neutrino experiments may not unambigously resolve various new neutrino physics solutions from a possible solar model solution to the solar neutrino problems. In particular, the large angle ν_e-ν_s MSW mixing and modefied solar model solutions all fail to exhibit spectral distortions with respect to the β-decay spectra nor do they show enhanced neutral current events. Now that the large angle ν_e-ν_s MSW mixing solution has been excluded by primordial nucleosynthesis, this ambiguity is removed. Two major next generation solar neutrino experiments, the Sudbury Neutrino Observatory (SNO) and Super-Kamiokande (Super-K) [2], both planning to start operation in 1996, will be sensitive to spectral distortions with respect to the β-decay spectra. Furthurmore, SNO will be capable of measuring neutral current events and hence identify ν_e-ν_μ (or ν_τ) oscillations. With the operation of these two experiments, as well as other next generation solar neutrino experiments (Borexino, ICARUS, etc., [2]), one may in principle resolve the different solutions to the solar neutrino problem, in particular, the conflict between new neutrino physics solution and the modefied solar model solution (which is currently still allowed if there is a problem with the chlorine experiment) [6]. In table 2, we list the signatures of six different solutions in next generation experiments. All ν_e-ν_μ (or similarly ν_τ) mixing solutions will exhibit extra neutral current events in SNO, due to the neutral current scattering of $\nu_\mu + \text{d} \rightarrow \nu + \text{p} + \text{n}$. The small angle MSW mixing between ν_e-ν_μ or ν_e-ν_s, will exhibit significant spectral distortion with respect to a β-decay spectrum. Part of the large angle ν_e-ν_μ MSW mixing parameter space also exhibits spectral distortion to some degree, when $sin^2 2\theta \sim 0.5$. But when $sin^2 2\theta$ is close to 1, where most of the large angle mixing solution lies, the spectrum is more or less uniformly suppressed with respect to the normal β spectrum [10]. The vacuum mixing will also distort the neutrino spectrum, due to its strong energy dependence. Furthermore, its strong distance dependence leads to uniquely large seasonal variation in the observed ^{7}Be neutrino flux (besides the usual $1/R^2$ dependences of neutrino flux), which can be detected in the Borexino experiment [10]. The modefied solar model solutions, on the other hand, are immune to all the above anomolies. The resonant spin precession solution (based on neutrino magnetic moments) is not very well understood since the magnetic field in the sun is largely unknown. But, it is not strongly viable due to the bounds on the solar magnetic field and the need for a large neutrino magnetic moment. However, one also expect spectral distortions and 11-year variation since the adiabaticy of the resonance is sensitive to the neutrino energy, and the field configuration [29].

V. Dark Matter Implications

It is interesting to attempt to relate three astrophysical and cosmological problems that involve neutrinos: the solar neutrino problem, the atmospheric neutrino problem, which were discussed above, and the cosmological dark matter problem. Big Bang nucleosynthesis constrains the baryon density to be less than 10% of the critical density. The remaining 90% of the mass must be non-baryonic dark matter [30], assuming a critical density universe. One possible candidate for dark matter is a 30 eV neutrino species (e.g., ν_τ), Hot Dark Matter (HDM) model [31]. However, HDM with adiabatic density fluctuations fails to explain structures on small scales of the universe (~Mpc). Its alternative, the Cold Dark Matter (CDM) with adiabatic density fluctuations lacks power on large scales to simultaneously fit the data from COBE, APM and IRAS well when normalized to the galaxy distributions at 10 Mpc [32]. It was suggested recently that a reasonable fit to the data is a cocktail of a 7 eV neutrino and some cold dark matter [33]. Therefore, to consider a $\sim$ 10 eV neutrino as the dark matter of the universe is still well motivated. Also, pure HDM with topological defects as seeds still seems to work reasonably well [34].

It has been known [35] that with only 3 generation of neutrinos and with the most natural particle models that generate neutrino masses, namely the see-saw mechanism, it is difficult to simultaneously reconcile the three astrophysical and cosmological problems. The solar neutrino deficit observed by the current solar neutrino experiments suggests a mixing between ν_e and another species ν_x with $m^2_{\nu_x} - m^2_{\nu_e}$ of order 10^{-6} to 10^{-4} eV^2 (or 10^{-10} eV^2 in the case of vaccum mixing), While the ν_μ deficit, if real, in the atmospheric neutrino experiments, suggests a large mixing between ν_μ and ν_y with $|m^2_{\nu_\mu} - m^2_{\nu_y}|$ of the order 10^{-3} to 10^{-1} eV^2. If ν_x and ν_y were active neutrinos, i.e., ν_μ or ν_τ, one immediately sees that, unless the three masses are almost degenerate, (which may not be natural theoretically,) no neutrino species can provide a $\sim$10 eV mass to serve as dark matter. The see-saw mechanism predicts a mass hierachy $m_{\nu_e} \ll m_{\nu_\mu} \ll m_{\nu_\tau}$ [36]. Therefore it seems that a sterile neutrino ν_s must be introduced to reconcile the three astrophysical/cosmological problems without conflicting with the constraint from LEP that only three active neutrino species exist [23]. There are three sterile neutrino options (assuming all three problems are real and require neutrino solutions):

1. ν_x is the sterile neutrino ν_s, and ν_y is ν_τ. The matter mixing or the vaccum mixing of ν_e and ν_s explains the solar neutrino deficit. Our calculation above has excluded the large mixing angle MSW solution. The small angle solution or the vaccum solution will then be the solution to the solar neutrino problem, and will show very distinct features in future solar neutrino experiments, as seen from Table 2. The ν_μ-ν_τ mixing with $\delta m^2 \sim 10^{-3}$ to 10^{-1} eV^2 implies $m_\mu \approx m_\tau \sim 10$ eV. Models with this prediction require ν_μ and ν_τ to be Zeldovich-Konopinski-Mahmoud (ZKM) type neutrinos [1].

2. ν_x is ν_μ and ν_y is ν_s. Then in order to solve the atmospheric neutrino problem, a large angle mixing between ν_μ and ν_s is needed, as in Fig. 3 and 4. This clearly conflicts with the nucleosynthesis bound we obtained and is no longer viable.

3. The final option is that ν_x and ν_y are ν_μ and ν_τ respectively, and ν_s provides the dark matter. As seen above, ν_s can be produced by mixing with active neutrinos in the early universe. With a mass of $\sim$ 10 eV or more, the ν_s can serve as the dark matter without conflicting with the nucleosynethesis bound, given appropriate mixing angles. The energy density Ω_{ν_s} of ν_s relative to the critical density, is roughly [37]

$$\Omega_{\nu_s} h^2 \sim \frac{\Gamma^{prod}_{\nu_s}}{H}\Big|_{T_{max}} (\frac{m_{\nu_s}}{30\mathrm{eV}}), \tag{44}$$

where $h = H_0/(100\mathrm{km/sec/Mpc})$ and H_0 is Hubble constant today. To have $\Omega_{\nu_s} \sim 1$ requires

$$m_{\nu_s} sin2\theta \sim 0.3h \text{ eV} \quad \text{and} \quad 10\mathrm{eV} \lesssim m_{\nu_s} \lesssim 1\mathrm{keV} \tag{45}$$

The cap on m_{ν_s} comes from requiring T_{max} in eq. (33) to be less than the mass of muons or the temperature of the quark-hadron phase transition, which is ~100 MeV, so that the total number of degrees of freedom (excluding the produced sterile neutrinos) is roughly a constant during the oscillation, and the previous formalism applies [24]. For a sterile neutrino with a mass larger than 1 keV, large amount of ν_s production through the oscillation occurs before the quark-hadron phase transition. Since the degrees of freedom before the quark-hadron phase transition are $\sim 10^2$ vs. ~ 10 after, the sterile neutrino population would be significantly diluted. Therefore the presence of such a neutrino as a full species before the quark-hadron phase transition will not exceed the nucleosynthesis bound. The mixing parameters that provide $\Omega_{\nu_s} \sim 1$ are not simple to calculate due to the complication of the quark-hadron transition and the presence of muons, pions and quarks.

As we can see, there exists a range of sterile neutrino masses and mixing angles that can provide a condidates for either hot dark matter models or cocktail models (consisting of both hot dark matter matter and cold dark matter). Also, compared with conventional ~ 10 eV active neutrino dark matter, a heavier sterile neutrino is allowed and has a smaller free streaming length λ_{FS}. In other words it won't damp out fluctuations on as small a scale as a 10 eV neutrino would do. Since [30]

$$\lambda_{FS} \sim 20\text{Mpc}(\frac{m_\nu}{30\text{eV}})^{-1}, \tag{46}$$

several hundred eV sterile neutrino dark matter [37] will then have a free streaming length of ~ 1 Mpc, which is the scale of a galaxy, and might allow neutrinos to serve as dark matter even with adiabatic density fluctuations.

VI. Summary

In summary, we examined the effect of neutrino mixings in Big Bang Nucleosynthesis, and calculated the constraints from the primodial ^{4}He abundance on the mixing parameters of sterile-active neutrino mixings. We discussed the implications of these constraints on the solar neutrino problem, the atmospheric neutrino problem, and the cosmological dark matter problem. We conclude that future solar neutrino experiments may unambiguously differentiate solutions to the solar neutrino problem, in particular, the new neutrino physics solutions and solar model solutions. We also conclude that a ν_τ of ~ 10 eV or a sterile neutrino of 10 eV to 1 keV with proper mixing with active neutrinos may provide the cosmological dark matter, depending on whether ν_e-ν_s mixing or ν_e-ν_μ mixing solves the solar neutrino problem.

Acknowledgement

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Figure captions:

Fig. 1. A contour plot of constant primordial helium yield contours for ν_e-ν_s mixing with $\delta m^2 > 0$. The parameter space to the right of the $Y = 0.245$ contour is excluded by the current observations. The area enclosed by dashed lines is the MSW solution to the solar neutrino problem at 95% C. L., with astrophysical uncertainties taken into account.

Fig. 2. The same contour plot as in Fig. 1 for ν_e-ν_s mixing with $\delta m^2 < 0$.

Fig. 3. The same contour plot as Fig. 1 for ν_μ-ν_s mixing with $\delta m^2 < 0$. The area enclosed by the dashed line is the solution to the atmospheric neutrino problem.

Fig. 4. The same contour plot as Fig. 3 for ν_μ-ν_s mixing with $\delta m^2 > 0$.

TABLE 1

Reactions	$\langle\Gamma\rangle/F_0$
$\nu_\alpha\bar{\nu}_\alpha \leftrightarrow e^-e^+$	$8x_W^2 \pm 4x_W + 1$
$\nu_\alpha\bar{\nu}_\alpha \leftrightarrow \nu_\beta\bar{\nu}_\beta$	1
$\nu_\alpha e^- \leftrightarrow \nu_\alpha e^-$	$8x_W^2 \pm 6x_W + \frac{3}{2}$
$\nu_\alpha e^+ \leftrightarrow \nu_\alpha e^+$	$8x_W^2 \pm 2x_W + \frac{1}{2}$
$\nu_\alpha\nu_\alpha \leftrightarrow \nu_\alpha\nu_\alpha$	6
$\nu_\alpha\nu_\beta \leftrightarrow \nu_\alpha\nu_\beta$	3
$\nu_\alpha\bar{\nu}_\alpha \leftrightarrow \nu_\alpha\bar{\nu}_\alpha$	4
$\nu_\alpha\bar{\nu}_\beta \leftrightarrow \nu_\alpha\bar{\nu}_\beta$	1

Table 1. The weak reaction rates of neutrinos averaged over a thermal spectrum. The plus signs correspond to $\alpha = e$, the minus signs correspond to $\alpha = \mu, \tau$; $\alpha \neq \beta$.

TABLE 2

Solutions	Spectral distortion (Super-K, SNO)	Neutral Current Event (SNO)	Seasonal variation[1] (Borexino)
Small angle MSW $\nu_e \leftrightarrow \nu_\mu$	Yes	Yes	No
Large angle MSW $\nu_e \leftrightarrow \nu_\mu$	No	Yes	Possible
Vaccum mixing $\nu_e \leftrightarrow \nu_\mu$	Yes	Yes	Yes
Small angle MSW $\nu_e \leftrightarrow \nu_s$	Yes	No	No
Vaccum mixing $\nu_e \leftrightarrow \nu_s$	Yes	No	Yes
Solar model solution	No	No	No

Table 2. Experimental signatures of different solutions to the solar neutrino problem in Super-Kamiokande and SNO.

[1]Other than the usual $1/R^2$ variation.

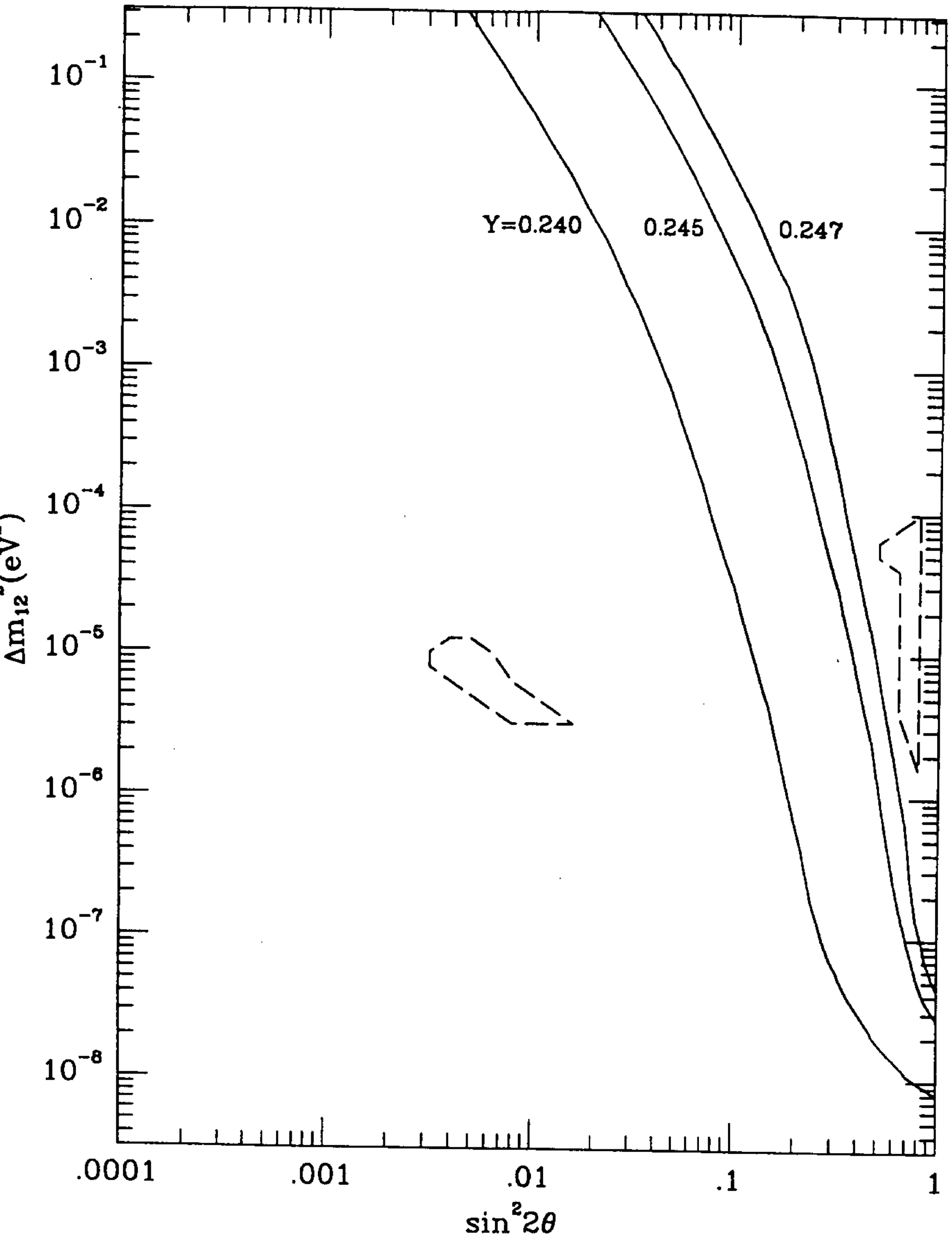
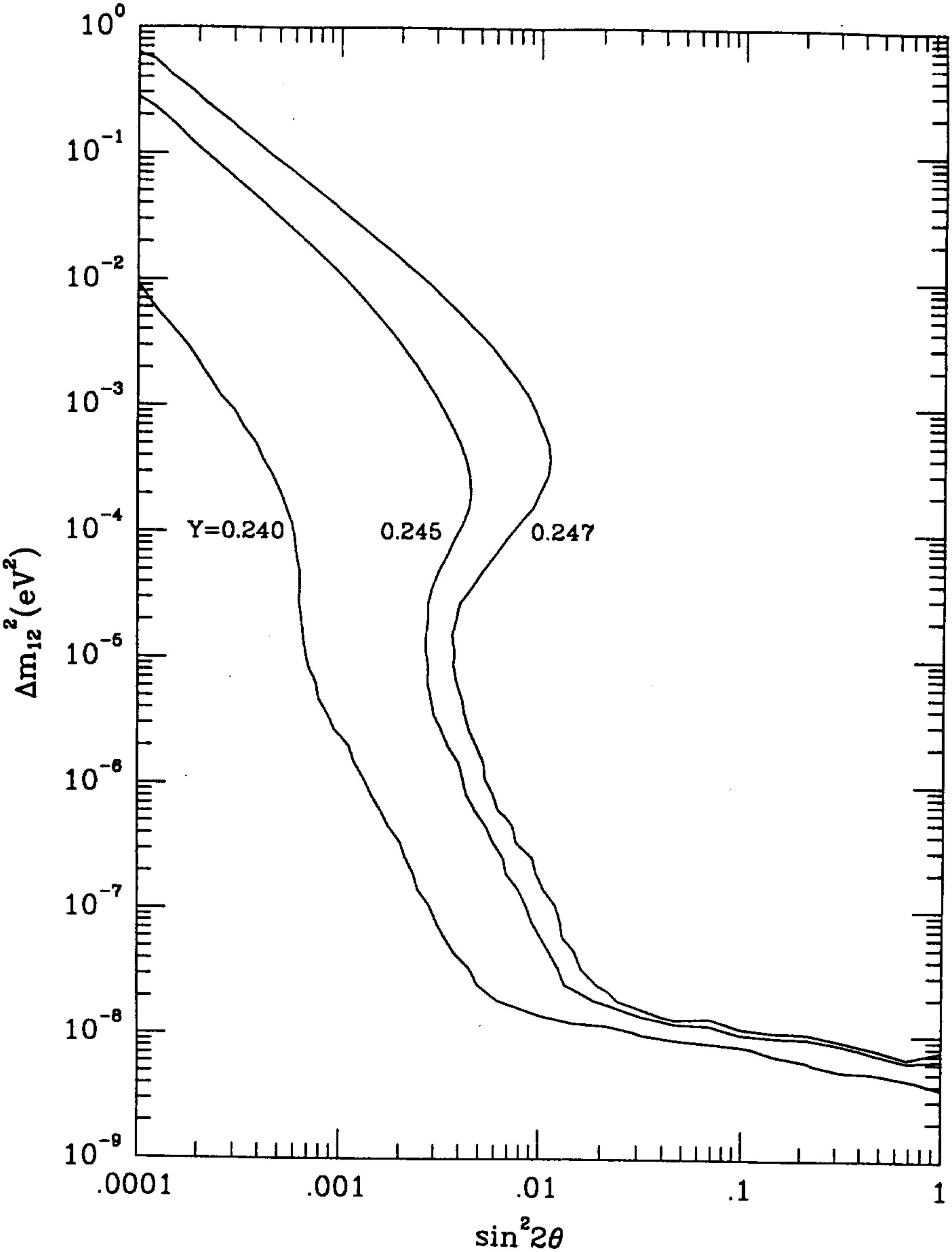
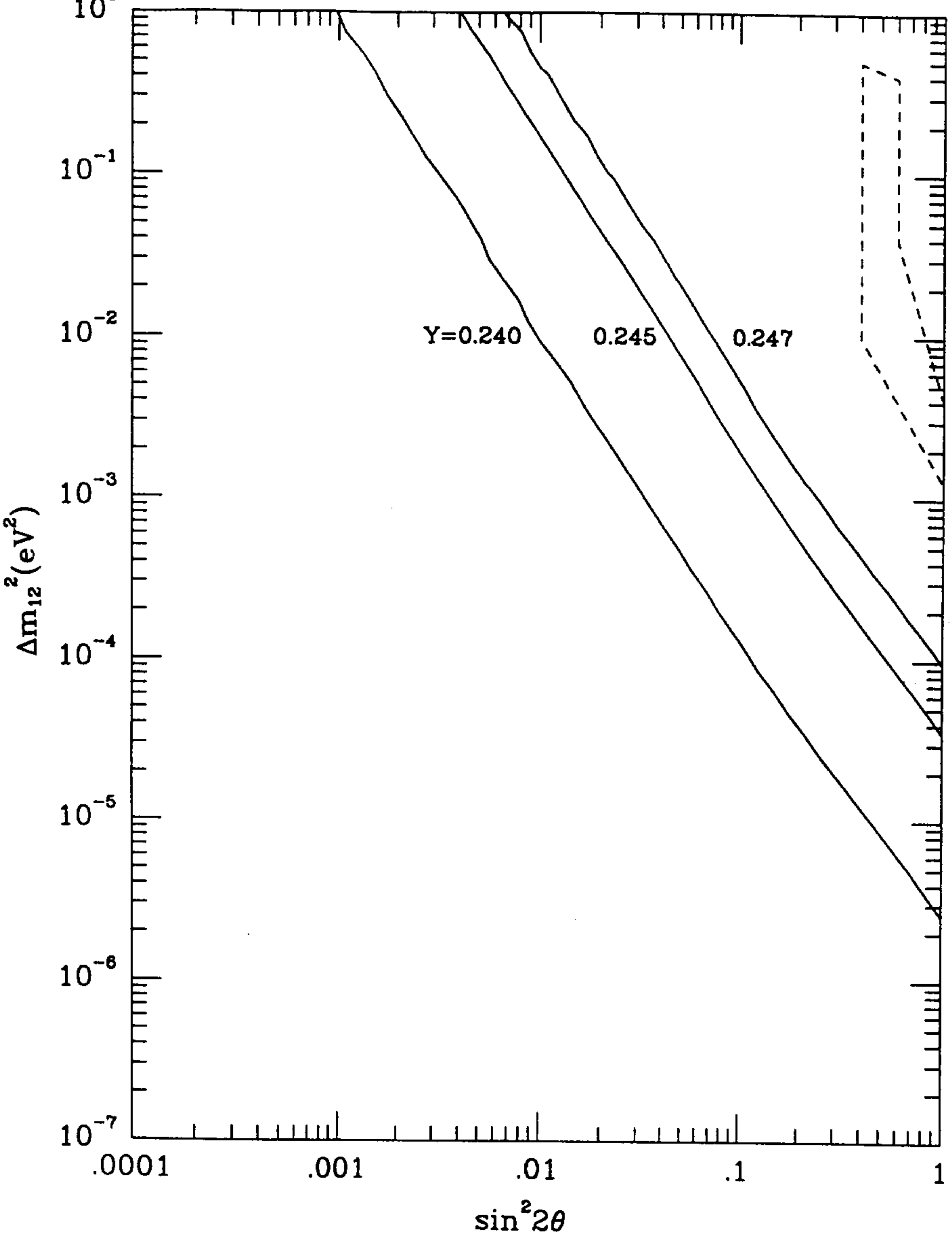

Y=0.240
0.245
0.247
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
$\Delta m_{12}^2 (eV^2)$
.0001
.001
.01
.1
1
$\sin^2 2\theta$

Fig. 1



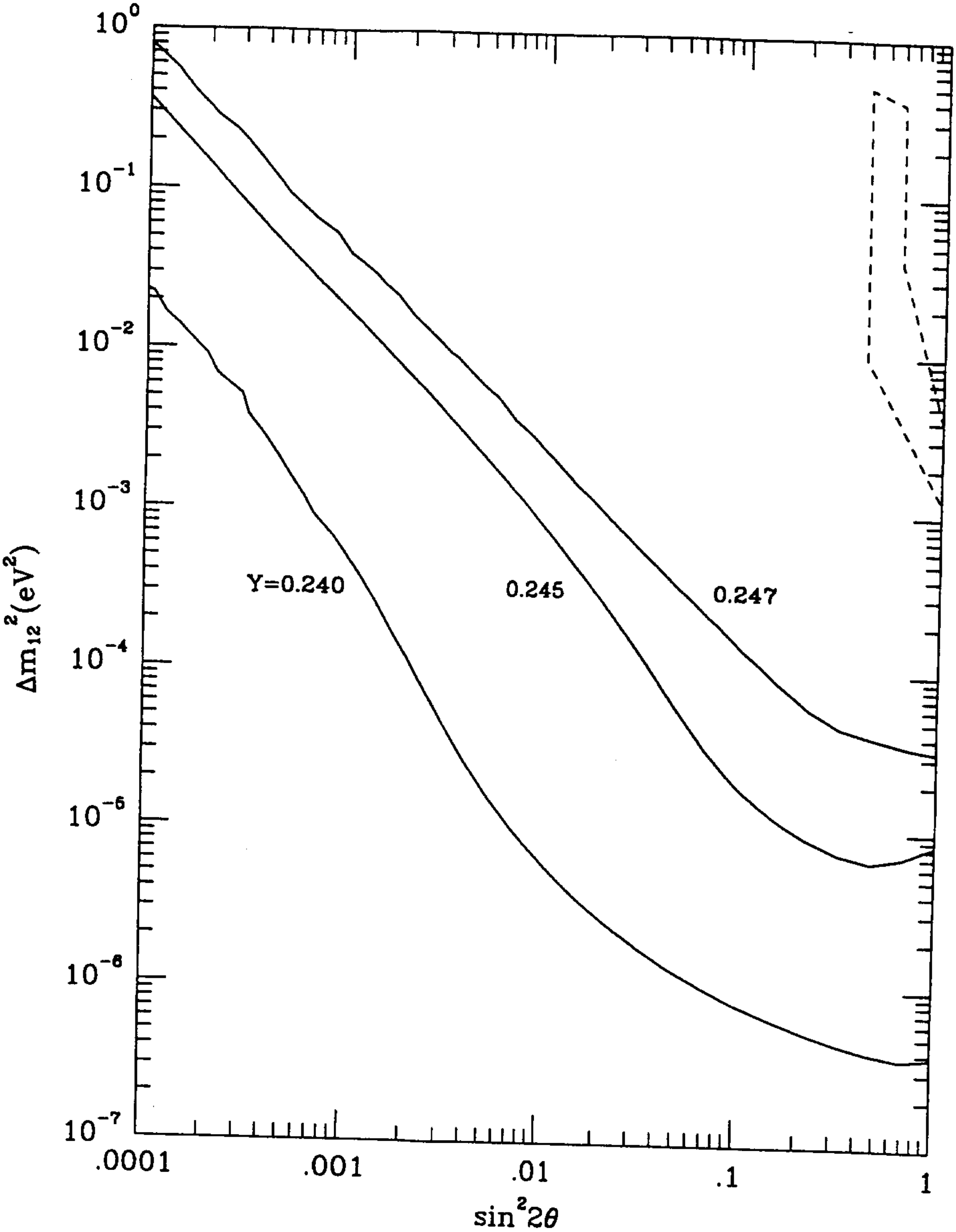
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-9}
$\Delta m_{12}^2 (eV^2)$
Y=0.240
0.245
0.247
.0001
.001
.01
.1
1
$\sin^2 2\theta$

Fig. 2



10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
$\Delta m_{12}^2 (eV^2)$
Y=0.240
0.245
0.247
.0001
.001
.01
.1
1
$\sin^2 2\theta$

Fig. 3



10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
$\Delta m_{12}^2 (eV^2)$
Y=0.240
0.245
0.247
.0001
.001
.01
.1
1
$\sin^2 2\theta$

Fig. 4