



Constraints on cosmic strings due to black holes formed from collapsed cosmic string loops

R. R. Caldwell[†] and Evalyn Gates^{†‡}

†NASA/Fermilab Astrophysics Center
Fermi National Accelerator Laboratory
P.O. Box 500
Batavia, Illinois 60510-0500

‡University of Chicago
5640 S. Ellis Avenue
Chicago, Illinois 60637

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ABSTRACT

The cosmological features of primordial black holes formed from collapsed cosmic string loops are studied. Observational restrictions on a population of primordial black holes are used to restrict f , the fraction of cosmic string loops which collapse to form black holes, and μ , the cosmic string mass-per-unit-length. Using a realistic model of cosmic strings, we find the strongest restriction on the parameters f and μ is due to the energy density in 100MeV photons radiated by the black holes. We also find that inert black hole remnants cannot serve as the dark matter. If earlier, crude estimates of f are reliable, our results severely restrict μ , and therefore limit the viability of the cosmic string large-scale structure scenario.



I. Introduction

The cosmic string scenario for the formation of large scale structure has many observable features. Primarily, cosmic strings may serve to produce perturbations to the cosmological fluid of the necessary magnitude and distribution to seed the formation of galaxies and clusters, as observed today. Cosmic strings leave an observational signature through these perturbations, as well as through the emission of gravitational radiation. Broadly, then, there are two areas of cosmic string research. These are studies of the large-scale structure produced by cosmic strings, and tests of the compatibility of cosmic strings with cosmological observations. Such tests focus, for example, on the anisotropies produced by cosmic strings in the microwave background and the noise in pulsar timing due to the cosmic string stochastic gravitational wave background. Ultimately, the test of compatibility results in a restriction on μ , the mass-per-unit-length and sole free parameter in the cosmic string model. In this report, we will examine the restrictions on black holes formed from collapsed cosmic string loops.

It is well known that a sufficiently smooth, circular cosmic string loop may collapse to form a black hole [1,2,3,4,5]. During the evolution of a network of cosmic strings, some cosmic string loops may collapse to form black holes. In this case, the observational restrictions on a population of primordial black holes may be used to restrict such a cosmic string scenario.

The study of primordial black holes has been vigorously carried out in, for example, [6,7,8,9,10,11,12]. We will take advantage of this work in applying constraints to a population of black holes formed from collapsed cosmic string loops. In turn, we will place restrictions on the cosmic string network. *In this paper, we will find observational restrictions on the cosmic string scenario from cosmic string loops which collapse to form black holes.*

The organization of this paper is as follows. In section II we will summarize previous efforts to estimate the fraction f of cosmic string loops which collapse to form black holes. In section III we will present the models of cosmic strings and black hole evaporation used to calculate the energy density in black holes and black hole radiation. In section IV we will present the observational constraints on a population of black holes formed from collapsed cosmic string loops. We will conclude in section V with a restriction on the parameters f and μ .

II. Collapse of Cosmic String Loops to form Black Holes

A cosmic string which contracts under its own tension to a size smaller than its Schwarzschild radius will form a black hole. In this section, we will conduct a brief review of the analysis of this phenomena. We will present a naive estimate of the probability that a realistic cosmic string loop will collapse to form a black hole. While no conclusive work has been carried out to determine this fraction, our naive estimate will serve as a rough guide for the cosmological analysis in the succeeding sections.

The phenomena by which a cosmic string loop collapses to form a black hole may be best understood by examining a simple case. We will consider a perfectly circular, planar cosmic string loop of mass m . The string equations of motion dictate that such a loop will expand and contract under its own tension, with a maximum radius $R_{max} = m/2\pi\mu$. When it contracts under its own tension to within its Schwarzschild radius $R_S = 2Gm = 4\pi G\mu R$, it will form a black hole. (We use units $\hbar = c = 1$ and $G = m_{\text{planck}}^{-2}$.) A loop may never contract to within its Schwarzschild radius, however, if it is sufficiently non-circular. As well, a loop with a Schwarzschild radius comparable to its thickness may dissipate, by radiating the quanta trapped in the string, before a black hole may form. Thus, not all cosmic string loops collapse to form black holes; in fact we expect only a small fraction to do so.

We are interested in determining which loops formed by a realistic cosmic string network collapse to form black holes. Simplifying this problem, we ask what fraction f of cosmic string loops collapse to form black holes. We will ultimately find that observational restrictions on black holes formed from collapsed cosmic string loops will depend linearly on this fraction f . In this study we will not be able to conclusively determine f . After reviewing past work on the properties and behavior of realistic cosmic string loops, however, we will be able to determine the relevant properties that effect this fraction. In addition, previous attempts to determine this fraction f , along with reasonable assumptions about the loop population, will be shown to indicate a rough value for f . If these estimates are reasonable, we may be able to place severe restrictions on cosmic string scenarios.

The initial investigation of black holes formed from cosmic string loops was carried out by Hawking [1]. Considering a scenario where the loop would have a similar probability of collapsing to form a black hole in each oscillation period, he proposed that the fraction f is a function of the mass per unit length of the loop and the number of kinks (n) on a string loop. His expression for f , however, depends exponentially on n . Numerical simulations of cosmic string evolution suggest that a reasonable range for n is given by $n \sim 2 - 5$ [13], which corresponds to, for $\mu = 10^{-6}$, $f \sim 1 - 10^{-36}$. Thus, unless a more accurate determination of n is achieved, this approach does not provide a conclusive estimate for f that may be useful for constraining cosmic string scenarios.

A numerical analysis of several families of parameterized loop configurations was carried out by Polnarev and Zembowicz [4]. They examined the fraction of parameter space for which Burden and Kibble-Turok families of loops collapse to form black holes. These families model loops which contain cusps, and may not necessarily be representative of realistic loops. Depending on the measure assigned to the configuration parameter space, they found the fraction may lie in the range $f \sim 10^{-9} - 10^{-15}$. For loops with few kinks, this range of values seems to be roughly in accord with Hawking's estimates.

There are two major features of a cosmic string loop which may determine whether the loop will collapse to form a black hole. These are, roughly, the large

(underlying loop configuration or shape) and small (kinks and cusps) scale features of the loop. One may attempt to determine the fraction of loops collapsing to black holes by asking the following questions: (i) what fraction of realistic loops possess an underlying configuration which would lead to the formation of a black hole, and (ii) what fraction of these loops possess kinks and cusps (fluctuations) small enough that a black hole may still form. Previous numerical simulations of cosmic string evolution [13,14,15,16] provide some insight into the relevant issues.

The study of the effects of the gravitational back-reaction on the evolution of cosmic string loops indicates that the gravitational back-reaction will set the minimum scale of structures on long strings. These structures are the predecessors of parent loops which are chopped off the long strings. The parent loops then undergo fragmentation and rapidly evolve towards simple, non-intersecting configurations, containing on the order of 2 – 5 kinks. Quashnock and Spergel [17] found that the kinks and small scale structure on string loops rapidly decay, and the loops then oscillate in a self-similar manner. Cusps, however, are not suppressed by the gravitational back-reaction and persist throughout the evolution of the loop. Thus, except for configurations that contain cusps, the underlying shape of a realistic loop is dominated by low mode or long wavelength oscillations.

The work of Garriga and Vilenkin [18] considered nucleated cosmic string loops, which may collapse to form black holes. The behavior of classical fluctuations on the loops were examined, and it was shown that while transverse perturbations maintain constant amplitude as the cosmic-string loop contracts, radial perturbations shrink by a factor of the perturbation mode number. Since the loops are dominated by low mode oscillations, the maximum allowed amplitude of a perturbation that is consistent with collapse to a black hole is of order R_g . That is, the maximum tolerable fluctuation which will not prevent the formation of a black hole contains only $\sim G\mu$ of the total loop energy. However, while we expect most of the daughter (non-self-intersecting) loops to be relatively simple, we currently have no information regarding the frequency or distribution of fluctuations and loop configurations.

We may nevertheless attempt to estimate the fraction f using Hawking's approach, incorporating the work of [13] on the properties of realistic cosmic string loops. We consider the properties of stable, non-self-intersecting loops, and assume that the number of kinks on a loop is the dominant factor in determining whether such a loop will collapse to form a black hole. Note that since these loops oscillate in a self-similar manner, we are concerned only with whether a given configuration will immediately collapse to form a black hole. We do not integrate this probability over the number of oscillations in the black hole lifetime, as Hawking did in his original calculation. Thus, we then integrate Hawking's expression for the number of loops which collapse *immediately* to form black holes over the distribution of the number of kinks on stable daughter loops. This integral is dominated by the contribution from loops with two kinks. We find

$$f \sim 10^{-12}$$

This estimate is roughly in the same range as that given in the work by Zembowicz and Polnarev. We must stress that although this is just a rough estimate, we may use this fraction as a guide for our study of the observational constraints in the following section.

A careful determination of f will be necessary to conclusively evaluate the observational constraints on cosmic strings. Such a study will be the focus of a future work [19]. In the meantime, we will adopt the working hypothesis that *a fraction f of realistic loops are smooth enough at the time they are chopped off the cosmic string network that they may immediately collapse to form black holes.* We may now proceed to evaluate the observational constraints on black holes formed from collapsed cosmic string loops.

III. Production and Evolution of Black Holes from the Collapse of Cosmic String Loops

The properties of a population of black holes formed from collapsed cosmic string loops are well specified by the properties of the cosmic string network and by the properties of quantum mechanical evaporation by a black hole. In this section we will first present the model of cosmic strings used in this study, focusing on those aspects relevant to the population of black holes produced from collapsed cosmic string loops. Second, we will describe how the quantum mechanical decay of black holes is incorporated into the cosmic string scenario. Third, we will outline the calculation of the physical properties of the population of black holes necessary to make contact with cosmological observations, and subsequently restrict the cosmic string model.

We use the “one-scale” model of kinky cosmic strings. The properties of this model have been well described by [20]. We will repeat the necessary elements of this model.

- i. The background cosmology is a spatially flat FRW spacetime with scale factor $a(t) \propto t^{1/2}$ in the radiation dominated era, and $a(t) \propto t^{2/3}$ in the matter dominated era.
- ii. For simplicity, we will calculate physical quantities in a fiducial, physical volume $V(t) = a^3(t)r^3$ where r is an arbitrary coordinate length.
- iii. We define loops to be closed cosmic strings formed, with an initial size $L(t) = \alpha l(t)$, through the intercommutation of long strings, where $l(t)$ is the horizon radius. All other cosmic string is contained in long strings.
- iv. Loops are considered to be non-self-intersecting. One may argue that a newly formed loop may self-intersect and fragment at a rate proportional to the loop oscillation frequency, producing smaller loops. This rate at which a loop self-intersects, however, is much faster than both the rate at which a loop radiates gravitational waves and the expansion rate. Thus, we are justified in assuming that a loop fragments rapidly; we may consider that $L(t)$ represents the size of the final, non-self-intersecting loops.

v. The rate of loop formation is [21]

$$\frac{dN_{loop}}{dt} = 4 \frac{A}{\alpha} t^{-4} V(t) \quad (III.1)$$

where $A \approx 10$ gives the number of long, horizon-length cosmic strings present in a horizon volume, as determined by numerical simulations [14,15]. The value $\alpha \approx 10^{-4}$ is given by the observational bounds on cosmic string gravitational radiation [21].

We may now obtain the rate of black hole formation from collapsed cosmic string loops. Applying our hypothesis regarding the formation of black holes from cosmic string loops to equation III.1, we find

$$\frac{dN_{bh}}{dt} = f \frac{dN_{loop}}{dt}. \quad (III.2)$$

This equation states that N_{bh} black holes of mass $m(t) = \alpha \mu l(t)$ were formed during the time interval t to $t + dt$. This equation is valid for times $t \gg t_i$, where $t_i \sim \alpha^{-1} \mu^{-3/2} t_{planck}$ gives the time at which the Schwarzschild radius of a newly formed loop is comparable to the thickness of the cosmic string. Thus, the properties of the cosmic string network determine the initial properties of the population of black holes.

The cosmological evolution of a black hole is dominated by quantum mechanical emission of a spectrum of particles [2]. This radiation, which has been well investigated, is the most important cosmological aspect of a black hole. Thus, in order to follow the evolution of the black holes formed from collapsed cosmic string loops we need the black hole decay rate. For a black hole of mass m , the decay into massless particles is given by [22]

$$\frac{d^2 m}{d\omega dt} = \sum_i \Gamma(m, \omega, s) \frac{\omega}{e^{8\pi m \omega} \pm 1} \quad (III.3)$$

where the sum is over all particle species i . The \pm refers to boson or fermion statistical weights, and $\Gamma(m, \omega, s)$ is a dimensionless function of the black hole mass, the radiated particle spin s , and frequency ω . We will be interested primarily in the emission of photons, for which $\Gamma(m, \omega, s) = 64m^4\omega^4/9$. Examining III.3, we see that a black hole emits a burst of thermal radiation, characterized by the black hole temperature, which is inversely proportional to the black hole mass. This emission will continue until the black hole has completely evaporated away, or, as has been suggested [11,23,24], an inert Planck-mass object remains. In such a case, the black hole evaporation rate will truncate when $m \sim m_{planck}$. Adding the black hole decay rate into our model, therefore, we have completely specified the cosmological evolution of a population of black holes.

We may now use the expressions for the rate of black hole formation and the rate of black hole evaporation to calculate the energy density produced in black holes and black hole radiation. The fraction of critical energy density in black holes is

$$\Omega_{bh}(t) = \frac{1}{\rho_{crit}(t)} \frac{1}{V(t)} \int_{t_i}^t dt' \frac{dN_{bh}}{dt'} m(t', t). \quad (III.4)$$

The limits of integration are from the time t_i when the cosmic string loops may first collapse to form black holes, to the present time t . Here, the function $m(t', t)$ gives the mass of a black hole formed at time t' at a later time t . This function may be obtained by integrating the black hole evaporation rate III.3 over frequency and time, applying suitable boundary conditions. Examining III.4, the energy density in black holes at time t is dominated by those surviving black holes which formed earliest, as the rate of black hole production is a rapidly decreasing function of time. The fraction of critical energy density in black hole radiation, in a logarithmic frequency interval, is

$$\frac{d\Omega_{bhrad}(t)}{d \ln \omega} = \frac{1}{\rho_{crit}(t)} \frac{1}{V(t)} \int_{t_i}^t dt' \frac{dN_{bh}}{dt'} \int_{t'}^{\tau(t')+t'} dt'' \frac{\omega(t'') d^2 m}{d\omega(t'') dt''}. \quad (III.5)$$

Here, $\tau(t')$ is the lifetime of a black hole, and $\omega(t'') = \omega a(t)/a(t'')$ gives the relationship between the frequency as emitted at time t'' , $\omega(t'')$, and the frequency observed at time t . This power spectrum is dominated by the contribution from black holes evaporating at the present time. These expressions, III.4 and III.5 have been integrated numerically; the results will be presented in section IV.

We will be interested, as well, in constructing the function $\beta(m)$ in order to evaluate the observational constraints on this population of black holes formed from collapsed cosmic string loops. This function represents the fraction of critical energy density in black holes formed during the time interval t to $t + dt$.

$$\begin{aligned} \beta(m) &= \frac{1}{\rho_{crit}(t)} \frac{m(t)}{V(t)} dN_{bh} \\ &= \frac{256\pi}{3} A \mu f \end{aligned} \quad (III.6)$$

(Here, we have used $\rho_{crit}(t) = 3t^{-2}/32\pi$.) It is not surprising that this function is really a constant. The gross features of the cosmic string network scale with the horizon radius: the gross features of the population of black holes formed from collapsed cosmic string loops scale with the horizon radius. It is important to note that equation III.6 describes a population of black holes different from the black holes described by the function $\beta(m)_{horizon}$ found in the primordial black hole literature (for example, see [10]). There, $\beta(m)_{horizon}$ represents the fraction of critical energy density in black holes which enter the horizon in the time interval t to $t + dt$. Such a black hole will be much smaller than the horizon radius, as

are the black holes formed from collapsed cosmic string loops, at the later time $(2\alpha\mu)^{-1}t$. Therefore, to relate equation III.6 to the function $\beta(m)_{\text{horizon}}$ found in the literature, we write

$$\begin{aligned}\beta(m) &= \beta(m)_{\text{horizon}} \frac{a(t/2\alpha\mu)}{a(t)} \\ &\approx (2\alpha\mu)^{-1/2} \beta(m)_{\text{horizon}}.\end{aligned}\tag{III.7}$$

Here, we have simply accounted for the growth in the black hole energy density over the background radiation energy density from the time the black hole enters the horizon to the time that a black hole of the same mass would be formed from a collapsed cosmic string loop. The function β will be used in section IV, as has been used in [10,12], to evaluate the restrictions on black holes.

We have apparently neglected to consider the loops formed along with the long strings at the time of the cosmological phase transition. These loops, as has been recently shown in [18], may be smoothed by the friction with the cosmological fluid [25]. The loops most smoothed by the friction, however, have Schwarzschild radii smaller than the string thickness; these loops will not collapse to form black holes. The remaining, unsmoothed loops, which are larger than the horizon radius at time t_i , behave simply as long strings. Thus, we argue that we have considered all loops which may collapse to form black holes, and may now proceed to evaluate the observational constraints on black holes.

IV. Observational Constraints on Black Holes from Collapsed Cosmic String Loops

The numerous observational restrictions on a population of primordial black holes are a direct consequence of the richness of the physics of black hole evaporation. Through quantum mechanical decay, a black hole will radiate all particle species. Primordial black holes may be observed then through the emitted particle spectra. Consequently, the observation of spectra produced by such black holes may serve to indicate exotic events which may have taken place in the early universe. Figuratively, the cosmic string energy invested in black holes in the early universe provides a return with observational consequences today. In the following paragraphs we will present the observational constraints on black holes formed by collapsed cosmic string loops. We will begin by evaluating equations III.4-5 for constraints on the energy density in black hole photon radiation and remnants. Next, we will use equations III.6-7 to evaluate more observational constraints. These constraints will be expressed in terms of a restriction on f , using the preferred value $\mu = 10^{-6}$. We will conclude this section with an interpretation of the observational constraints on the cosmic string parameters.

The strongest constraint on this population of black holes formed from collapsed cosmic string loops is due to the γ ray flux observed at 100 MeV [26,10,27]. We require that the fraction of critical energy density in photons emitted by

black holes, with energy in a logarithmic interval at 100MeV , be less than $\Omega_\gamma = 10^{-8}h^{-2}$. Integrating equation III.5, we find

$$\begin{aligned} \frac{d\Omega_{bh\gamma}(t_0)}{d\ln\omega} \Big|_{\omega=100\text{MeV}} &= 10^9 h^{-2} f \leq 10^{-8} h^{-2} \\ &\rightarrow f \leq 10^{-17}. \end{aligned} \quad (\text{IV.1})$$

Black holes of mass $m \sim 10^{16}g$ (with a lifetime $\sim 10^{17}s$) which evaporate today serve as the dominant source of photons at this energy.

In the work of both Hawking [1] and Polnarev and Zembowicz [4] the cosmological constraints on cosmic strings due to the γ rays emitted by black holes formed from collapsed cosmic string loops were evaluated. These calculations used a very rough cosmic string model. Our work improves upon their results by implementing a realistic cosmic string model, as we take advantage of results from numerical simulations to determine the average number of long strings in a horizon volume and the size of newly formed loops. The improved limits are due to our improved model.

Further observational constraints on primordial black holes, which are typically stated as a restriction on the energy density in black holes in a particular mass range, may be easily evaluated analytically using β . We have taken advantage of the literature [10,12], in which the restrictions on black holes are stated in terms of $\beta(m)_{\text{horizon}}$, which we may simply convert into β according to equation III.7. In the following table, then, we list the observational constraint, the restriction on β , and the resultant limit on f , the fraction of collapsing cosmic string loops.

Observational Restrictions on f		
diffuse γ ray background	$\beta(m_{15}) \leq 10^{-21}$	$f \leq 10^{-17}$
interstellar e^+ background	$\beta(m_{15}) \leq 10^{-21}$	$f \leq 10^{-17}$
interstellar \bar{p} background	$\beta(m_{15}) \leq 10^{-21}$	$f \leq 10^{-17}$
interstellar e^- background	$\beta(m_{15}) \leq 10^{-20}$	$f \leq 10^{-16}$
photodissociation of d by photons	$\beta(m_{10}) \leq 10^{-16}$	$f \leq 10^{-14}$
distortion of CMBR	$\beta(m_{13}) \leq 10^{-15}$	$f \leq 10^{-13}$
photon-to-baryon ratio	$\beta(m_{13}) \leq 10^{-14}$	$f \leq 10^{-12}$
n/p by nucleons	$\beta(m_{10}) \leq 10^{-11}$	$f \leq 10^{-9}$
entropy production	$\beta(m_{11}) \leq 10^{-3}$	$f \leq 10^{-1}$
remnants overclose universe	$\beta(m_{-2}) \leq 10^{-18}$	$f \leq 10^{-15}$

In this table, m_X indicates black holes formed with mass $10^X g$. The first four constraints are taken from [10]. The limit due to the observed diffuse γ ray background at $100 MeV$ is the strongest constraint. Uncertainties in the clustering of black holes and the diffusion of charged particles within the galaxy may weaken the constraints on the interstellar e^\pm and \bar{p} backgrounds. The next five constraints are taken from [12]. These constraints focus primarily on the nucleosynthesis restrictions on black hole radiation. None of these nucleosynthesis limits are very strong.

The constraint on black hole remnants requires that the remnants not over-close the universe. However, it has also been suggested [11,24] that inert, Planck mass black hole remnants may provide a substantial fraction of the dark matter. Integrating equation III.4, we find

$$\begin{aligned}\Omega_{bhr}(t_0) &= 10^{15} f \leq 1 \\ &\rightarrow f \leq 10^{-15}.\end{aligned}\tag{IV.2}$$

Black holes of mass $\sim 10^{-2} g$, the first black holes formed from collapsed cosmic string loops at the time t_i , serve as the dominant source of remnants. This limit is a conservative upper bound on f , as the energy density in remnants depends critically on this initial time t_i . Although we have stated earlier that black holes may form only at times $t \gg t_i$, when the cosmic string loop Schwarzschild radius is much greater than the string thickness, we have actually included black holes formed starting at time t_i . Therefore, this limit on f may weaken somewhat depending on the detailed behavior of the collapse of a thick cosmic string to form a black hole. Then, closure density in remnants requires $f \approx 10^{-15}$, which is in disagreement with the γ ray background limit, $f \leq 10^{-17}$. Therefore, the restrictions on a population of black holes formed from collapsed cosmic string loops indicate that these black hole remnants cannot serve as the dark matter.

We may now interpret the restrictions on the population of black holes formed from collapsed cosmic string loops in terms of restrictions on cosmic strings. The γ ray background limit requires

$$f \leq 10^{-17} \quad \text{for} \quad \mu = 10^{-6}.\tag{IV.3}$$

This equation gives the strongest restriction on cosmic strings due to black holes formed from collapsed cosmic string loops. If the rough estimates of the magnitude of $f \sim 10^{-12}$ are reliable, then our results would indicate that $\mu \leq 10^{-11}$ is necessary for compatibility with the γ ray background. In this case, we could rule out the cosmic string scenario of large scale structure formation, which demands $\mu \sim 10^{-6}$. We are not confident in these crude estimates of f , however, as we have indicated in section II. Clearly, a more detailed investigation is necessary to definitively determine f [19].

V. Conclusion

In this work we have analyzed the restrictions on black holes formed from collapsed cosmic string loops. We have found that the requirement that the photon flux due to evaporating black holes does not exceed the observed γ ray background flux serves as the strongest restriction on such black holes formed from cosmic string loops. Using a realistic model of cosmic strings, we find that this observation requires $f \leq 10^{-17}$ for $\mu = 10^{-6}$. Thus, a fraction of no more than 10^{-17} of newly formed cosmic string loops may collapse to form black holes in order that $\mu = 10^{-6}$ remains compatible with observation. This restriction also precludes black hole remnants from serving as the dark matter. We plan to study in greater detail the fraction f of loops which collapse to form black holes [19]. If a lower bound $f \geq 10^{-16}$ is found, the γ ray background limit on evaporating black holes would serve as the strongest observational bound on cosmic strings, and would rule out the cosmic string scenario of large-scale structure formation.

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