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On Identifying the Present-day Vacuum Energy with the Potential Driving Inflation

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Abstract

There exists a growing body of observational evidence supporting a non-vanishing cosmological constant at the present epoch. We examine the possibility that such a term may arise directly from the potential energy which drove an inflationary expansion of the very early universe. To avoid arbitrary alterations in the shape of this potential at various epochs it is necessary to introduce a time-dependent viscosity into the system. The evolution of the effective Planck mass in scalar-tensor theories is a natural candidate for such an effect. In these models there are observational constraints arising from anisotropies in the cosmic microwave background, large-scale galactic structure, observations of the primordial Helium abundance and solar system tests of general relativity. Decaying power law and exponential potentials are considered, but for these models it is very difficult to simultaneously satisfy all of the limits. This may have implications for the joint evolution of the gravitational and cosmological constants.

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1 Introduction

The solution to vacuum general relativity with a cosmological constant Λ is de Sitter space and this constant and solution have often been invoked to reconcile theory with observation. Originally Einstein believed the universe to be static and introduced a constant Λ-term into his field equations to cancel the expansionary behaviour found when $\Lambda = 0$.[1] Some decades later the steady state scenario based on de Sitter space was developed because observations of Hubble's constant suggested the Earth was older than the universe itself.[2] More recently the inflationary scenario has been proposed to solve some of the problems of the hot, big bang model.[3] During inflation the potential energy of a quantum scalar field dominates the energy-momentum tensor and behaves as a cosmological constant for a finite time.

Realistic inflationary models predict that the current value of the density parameter, Ω_0, should be very close to unity.[4] There are a number of problems associated with the $(\Lambda = 0, \Omega = 1)$ universe which can be resolved if $\Lambda \neq 0$. Firstly, its age is $t_0 \approx 6.52h^{-1}$ Gyr, where h is the current expansion rate in units of 100 km sec^{-1} Mpc^{-1}. This is very close to the age of the oldest globular clusters in the galaxy, $t_{GC} = (13 - 15) \pm 3$ Gyr, if $h \geq 0.6$ as suggested by some observations.[5] If $\Lambda \neq 0$, however, the expansion rate is increased and t_0 may exceed t_{GC} if $\Omega_{vacuum} \approx 0.8$.[6] Moreover, most dynamical determinations of Ω_0 suggest $\Omega_0 = 0.2 \pm 0.1$ up to scales 30 Mpc and the apparent inconsistency with inflation is again resolved if $\Omega_{vacuum} \approx 0.8$.[7] Finally, the introduction of vacuum energy into the standard cold dark matter model of galaxy formation accounts for the extra large-scale clustering observed in galaxy surveys.[8]

Hence, there are a number of reasons for supposing that a cosmological constant may be influential at the present epoch.[9] This work investigates whether the potential energy that drove the inflationary expansion could be such a term. This has been investigated previously within the context of general relativity by Peebles and Ratra,[10] but their models required the form of the potential to change drastically at various epochs and therefore suffered from an element of ad-hoc fine-tuning. If the potential is relevant at the current epoch, it must either have a minimum at $V \neq 0$ or contain a

non-vanishing decaying tail. Although the first possibility is not ruled out, it requires severe fine–tuning, so we focus on the second. This implies that thermalization of the false vacuum will not proceed via rapid oscillations of the scalar field about some global minimum and the form of the potential must change at various epochs. In general relativity the potential must be sufficiently flat for the strong energy condition to be initially violated, but must then become steep enough for reheating to proceed. But the energy density of the field must redshift at a slower rate than the ordinary matter components at late times if the vacuum energy is to once more dominate the dynamics.[54]

Instead of altering the shape of the potential we extend the gravitational sector of the theory beyond general relativity and investigate whether inflation was driven by potentials which are (a) too steep to lead to inflation in general relativity and (b) do not contain a global minimum. We shall refer to these as *steep* potentials. A number of unified field theories lead to scalar field potentials which exhibit both of these characteristics. The mechanism leading to inflation is very simple. In pure Einstein gravity containing a single, minimally coupled scalar *inflaton* field, σ, the strong energy condition is violated if the condition $\dot{\sigma}^2 < V$ holds, *i.e.* the potential energy dominates over the field's kinetic energy. Clearly this condition must break down at some point as steeper potentials are considered. But a finite interval of inflation is possible with steep potentials if one introduces a viscosity into the inflaton field equation which decays as the universe expands. This viscosity slows the field down and can lead to inflation. As the viscosity becomes weaker, however, the inflaton's kinetic energy increases significantly and a natural exit from inflation proceeds as the expansion becomes subluminal. We identify the dilaton field which arises in scalar-tensor theories as the natural source of this viscosity.

This paper is organised as follows. We survey theories that lead to inflation with steep potentials in section 2. In section 3 we derive expressions for the amplitudes of the primordial fluctuation spectra and discuss the most stringent observational constraints which any viable model of this type must satisfy. These limits arise from the observations of large–scale galactic structure,[11] anisotropies in the cosmic microwave background radiation (CMBR),[12] nucleosynthesis calculations[14,15] and time–delay ex-

periments in the solar system.[16] Numerical results for both decaying power law and exponential potentials are presented in section 4 and we conclude that successful inflation based on this mechanism is unlikely for the examples considered. Some possible implications of this conclusion are discussed in section 5. Unless otherwise stated, units are chosen such that $c = \hbar = 1$, and the present day value of the Planck mass is normalized to $m_{\rm Pl} = 1$.

2 Inflation with steep potentials

2.1 Scalar-Tensor theories

A suitable source of the viscosity is the dilaton field which arises in the Bergmann-Wagoner class of theories.[17] This field plays the role of a time-dependent gravitational constant. The field equations for these scalar-tensor theories are derived by varying the action

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[h(\psi)\mathcal{R} - \frac{1}{2}(\nabla\psi)^2 - U(\psi) - 16\pi\left(\frac{1}{2}(\nabla\sigma)^2 + V(\sigma) + \mathcal{L}_{\rm matter}\right)\right], \tag{2.1}$$

where $g = \det g_{\mu\nu}$, $\mathcal{R}$ is the Ricci scalar, $h(\psi)$ is some arbitrary function of the dilaton ψ, $U(\psi)$ is the dilaton self-interaction and $\mathcal{L}_{\rm matter}$ is the Lagrangian of a perfect baryotropic fluid, which we assume to be relativistic matter with an effective equation of state $p_r = \rho_r/3$. The dilaton and inflaton field equations are coupled and the extra viscosity on σ is due to the dynamical evolution of the effective Planck mass in the theory. The strength of gravity is determined by the magnitude of $h(\psi)$ and a fraction of the inflaton's potential energy is converted into the dilaton's kinetic energy rather than contributing to $\dot{\sigma}^2$.

It is well known that theories of this type appear as the low energy limits to a number of unified field theories and can be expressed in the Jordan-Brans-Dicke (JBD) form with a variable ω-parameter by defining a new scalar field $\Psi \equiv h(\psi)$.[18]

The action (2.1) becomes

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[\Psi\mathcal{R} - \frac{\omega(\Psi)}{\Psi}(\nabla\Psi)^2 - U(\Psi) - 16\pi\left(\frac{1}{2}(\nabla\sigma)^2 + V(\sigma) + \mathcal{L}_{\mathrm{matter}}\right)\right] \tag{2.2}$$

where

$$\omega(\Psi) = \frac{h(\psi)}{2(\mathrm{d}h/\mathrm{d}\psi)^2}. \tag{2.3}$$

The theory of general relativity is recovered whenever a turning point exists in the functional form of $h(\psi)$ (see section 3.3), because $\omega(\Psi)$ tends to infinity and the dilaton's kinetic energy decouples. Although the theories (2.1) and (2.2) are equivalent mathematically, there is a philosophical difference which arises in deciding which function $h(\psi)$ or $\omega(\Psi)$ should be treated as the fundamental quantity.[19] In general, if we consider a simple form for $h(\psi)$ such as a truncated Taylor series, this leads to a very complicated form for $\omega(\Psi)$ and vice-versa.

For example we can expand $h(\psi)$ as some power series

$$h(\psi) = \sum_{i=0}^{\infty}\alpha_i\psi^i \tag{2.4}$$

for arbitrary constant coefficients α_i. To lowest order $h(\psi) \approx \alpha_0$, but this corresponds to a 'constant' gravitational constant and is not interesting. Moreover the linear term may always be eliminated by a simple field redefinition, so the lowest order of interest is the quadratic contribution and this is simply the standard JBD theory with constant $\omega(\Psi)$. In principle, given suitable initial conditions, inflation will then occur as σ slowly rolls down its potential causing ψ to increase. Eventually higher-order ($i > 2$) terms in the expansion (2.4) will become important and for appropriate choices of α_i, such as $\alpha_2 > 0$ and $\alpha_3 < 0$, one can easily arrange for a local maximum in $h(\psi)$ to exist at some value $\psi = \psi_0$. At this point the theory will be identical to general relativity if we normalise $h(\psi_0) = 1$. It is clear that as ψ approaches ψ_0, inflation will end because the gravitational friction weakens and the inflaton speeds

up. This model is a chaotic version of the hyperextended scenario and proceeds via a second-order phase transition.[18]

Another possibility is the induced theory of gravity where $h(\psi)$ is quadratic but the dilaton potential is non-vanishing and contains a global minimum at ψ_0. Inflation driven by σ could occur if ψ is initially displaced from ψ_0, but will clearly end as Einstein gravity is recovered and the dilaton settles into this minimum. After spontaneous compactification to four-dimensions, some Kaluza-Klein theories have this structure, where the dilaton is identified as the logarithm of the radius of the internal space.[20] If monopole and Casimir effects due to non-trivial field configurations are also considered, a classically stable ground state is possible at ψ_0.[21]

Theories (2.1) and (2.2) are conformally equivalent to general relativity with a matter sector containing two interacting scalar fields.[22] By redefining the graviton and dilaton fields as

$$\tilde{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}, \qquad \Omega^2 \equiv 2\kappa^2 h(\psi) \tag{2.5}$$

and

$$\kappa\Phi = \int \mathrm{d}\psi \left(\frac{3(\mathrm{d}h/\mathrm{d}\psi)^2 + h(\psi)}{2h^2(\psi)} \right)^{1/2}, \tag{2.6}$$

where $\kappa^2 = 8\pi m_{\mathrm{Pl}}^{-2}$, the action (2.1) for $U(\psi) = 0$ may be rewritten in the Einstein-Hilbert form

$$S = \int \mathrm{d}^4x\sqrt{-\tilde{g}} \left[\frac{\tilde{R}}{2\kappa^2} - \frac{1}{2}(\tilde{\nabla}\Phi)^2 - \frac{1}{2}A(\Phi)(\tilde{\nabla}\sigma)^2 - C(\Phi)V(\sigma) \right], \tag{2.7}$$

where

$$A^{-1}(\Phi) \equiv \Omega^2 = 2\kappa^2 h(\psi), \qquad C(\Phi) \equiv A^2(\Phi) \tag{2.8}$$

and we assume that $\{h, \mathrm{d}h/\mathrm{d}\psi\} > 0$ for consistency. The viscosity appears through the non-standard coupling, $A(\Phi)$, of the Φ-field to the inflaton's kinetic term. This coupling evolves towards unity as Φ settles into a minimum of the effective potential $C(\Phi)V(\sigma)$, thus causing the accelerated expansion to end. In principle one can

therefore realise this scenario in general relativity if the matter sector of the theory is modified in an appropriate fashion. Some higher–dimensional, higher–order gravity theories exhibit this conformal structure upon compactification of the extra dimensions.[23]

In the following discussions we refer to $g_{\mu\nu}$ as the Jordan-Brans-Dicke (JBD) frame and $\tilde{g}_{\mu\nu}$ as the Einstein-Hilbert (EH) frame. In the former it is the matter contributions which are canonical, whereas gravity is canonical in the latter. The evolution of the Planck mass in the JBD frame is translated into a time-dependence for gauge and Yukawa couplings in the EH frame.[24]

2.2 Field Equations

Extremizing the action (2.2) with respect to arbitrary variations of the metric produces the gravitational field equations

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}\mathcal{R} = -\frac{1}{2}g_{\mu\nu}\frac{U}{\Psi} - \frac{8\pi}{\Psi}T_{\mu\nu} - \frac{\omega(\Psi)}{\Psi^2}\left[\nabla_\mu\Psi\nabla_\nu\Psi - \frac{1}{2}g_{\mu\nu}g^{\rho\lambda}\nabla_\rho\Psi\nabla_\lambda\Psi\right]$$

$$- \frac{1}{\Psi}\left[\nabla_\mu\nabla_\nu\Psi - g_{\mu\nu}\Box\Psi\right], \tag{2.9}$$

where $\Box \equiv g^{\mu\nu}\nabla_\mu\nabla_\nu$ and the energy-momentum tensor $T_{\mu\nu}$ is defined as the functional derivative of the matter lagrangian including the inflaton field.[25] To proceed we assume a space-time with isotropic and homogeneous spatial sections (the Friedmann-Robertson-Walker universes) where the fields are functions of cosmic time only. In this case minimising (2.2) with respect to variations in σ and Ψ leads to the equations

$$\ddot{\sigma} + 3H\dot{\sigma} = -\frac{\mathrm{d}V}{\mathrm{d}\sigma} \tag{2.10}$$

and

$$\mathcal{R} = 2\omega\left(\frac{\ddot{\Psi}}{\Psi} + 3H\frac{\dot{\Psi}}{\Psi}\right) - \omega\left(\frac{\dot{\Psi}}{\Psi}\right)^2 + \frac{\mathrm{d}\omega}{\mathrm{d}\Psi}\frac{\dot{\Psi}^2}{\Psi} + \frac{\mathrm{d}U}{\mathrm{d}\Psi}, \tag{2.11}$$

respectively. Combining Eq. (2.11) with the trace of (2.9) we arrive at the dilaton field equation

$$(2\omega+3)\left(\ddot{\Psi} + 3H\dot{\Psi}\right) = 2U - \Psi\frac{\mathrm{d}U}{\mathrm{d}\Psi} - \frac{\mathrm{d}\omega}{\mathrm{d}\Psi}\dot{\Psi}^2 + 8\pi T, \tag{2.12}$$

where $T = \rho_{total} - 3p_{total}$ is the trace of $T_{\mu\nu}$. Finally, the time-time component of (2.9) gives the Friedmann equation

$$3\left(H^2 + \frac{k}{a^2}\right) = \frac{\omega}{2}\left(\frac{\dot{\Psi}}{\Psi}\right)^2 + \frac{1}{2}\frac{U}{\Psi} - 3H\frac{\dot{\Psi}}{\Psi} + \frac{8\pi}{\Psi}\left(\frac{1}{2}\dot{\sigma}^2 + V(\sigma) + \rho_r\right). \tag{2.13}$$

where $k = -1, 0, +1$ determines the spatial curvature.

It is necessary to reheat the universe shortly after the expansion has become subluminal. The couplings of the inflaton to other relativistic matter fields should then become important because the potential is steep. However, reheating in these theories is difficult to model so we follow Morikawa & Sasaki[26] by introducing a phenomenological dissipation term into the field equations. It was shown that for an exponential inflaton potential and a Yukawa type coupling the inflaton field equation (2.10) becomes[27]

$$\ddot{\sigma} + 3H\dot{\sigma} = -\frac{\mathrm{d}V}{\mathrm{d}\sigma} - C_v\dot{\sigma}, \tag{2.14}$$

where to zeroth order the dissipation

$$C_v \sim fM_\sigma = f\sqrt{\frac{\mathrm{d}^2V}{\mathrm{d}\sigma^2}} \tag{2.15}$$

is proportional to the effective mass of the inflaton and $f = \mathcal{O}(1)$. This form for C_v should be appropriate for any steep potential. The Bianchi identity $\nabla_\mu T^{\mu\nu} \equiv 0$ then implies that

$$\dot{\rho}_r = -4H\rho_r + C_v\dot{\sigma}^2, \tag{2.16}$$

where ρ_r is the energy density of relativistic particles. The decay product must lead to baryogenesis, and we assume the reheat temperature is significantly larger than its rest mass m_X. We may then treat it as a relativistic fluid in local thermodynamic equilibrium. Once the inflaton's effective mass falls below m_X the dissipation becomes negligible.

2.3 Change of Independent Variable

Numerical solutions must be found which trace the evolution of the universe from the Planck time to the current epoch. This corresponds to some 60 orders of magnitude in the independent variable. The codes used to produce these solutions become inefficient when integrating over more than 15 orders of magnitude. Therefore a change of independent variable was used to speed up the process of generating solutions.

The number of e-foldings $N = \ln a$ was used as the new independent variable since this is clearly a monotonically varying function for expanding universes. From the relation $H = \dot{a}/a = \mathrm{d}(\ln a)/\mathrm{d}t$ it can be seen that $\mathrm{d}t = \mathrm{d}N/H$. Hence for a variable $x(t)$ we must make the substitutions: $\dot{x} \rightarrow Hx'$ and $\ddot{x} \rightarrow H^2x'' + HH'x'$ where a prime denotes differentiation with respect to N. We now have the following equations for $k = 0$:

$$\sigma'' + \left(3 + \frac{HH'}{H^2}\right)\sigma' + \frac{1}{H^2}\frac{\mathrm{d}V}{\mathrm{d}\sigma} + \frac{C_v}{H}\sigma' = 0 \tag{2.17}$$

$$\rho_r' + \rho_r - HC_v\sigma'^2 = 0 \tag{2.18}$$

$$(2\omega + 3)\left[\Psi'' + \left(3 + \frac{HH'}{H^2}\right)\Psi'\right] - 2\frac{U}{H^2} + \frac{\Psi}{H^2}\frac{\mathrm{d}U}{\mathrm{d}\Psi} + \frac{\mathrm{d}\omega}{\mathrm{d}\Psi}(\Psi')^2 = 8\pi\left[\frac{4V(\sigma)}{H^2} - \sigma'^2\right] \tag{2.19}$$

$$t = \int \frac{\mathrm{d}N}{H} \tag{2.20}$$

and

$$H^2 = \left[\frac{1}{2}\frac{U}{\Psi} + \frac{8\pi}{\Psi}(V + \rho_r)\right]\left[3 - \frac{\omega}{2}\left(\frac{\Psi'}{\Psi}\right)^2 + 3\frac{\Psi'}{\Psi} + \frac{8\pi}{2}\frac{\sigma'^2}{\Psi}\right]^{-1} \tag{2.21}$$

We now need an extra equation to calculate H'. This is found by taking (2.11) and substituting for $(\ddot{\Psi} + 3H\dot{\Psi})$ from (2.12) leaving

$$6\dot{H} + 12H^2 = \left(1 - \frac{2\omega}{2\omega + 3}\right)\left(\frac{\mathrm{d}\omega}{\mathrm{d}\Psi}\frac{\dot{\Psi}^2}{\Psi} + \frac{\mathrm{d}U}{\mathrm{d}\Psi}\right)$$

$$+ \quad \frac{2\omega}{2\omega+3}\frac{1}{\Psi}\left(2U + 8\pi\left(4V(\sigma) - \dot{\sigma}^2\right)\right) - \omega\left(\frac{\dot{\Psi}}{\Psi}\right)^2, \qquad (2.22)$$

or converting to independent variable N:

$$\begin{aligned} HH' &= \frac{1}{6}\left(1 - \frac{2\omega}{2\omega+3}\right)\left(H^2\frac{\mathrm{d}\omega}{\mathrm{d}\Psi}\frac{(\Psi')^2}{\Psi} + \frac{\mathrm{d}U}{\mathrm{d}\Psi}\right) \\ &+ \frac{1}{6}\frac{2\omega}{2\omega+3}\frac{1}{\Psi}\left(2U + 8\pi\left(4V(\sigma) - H^2\sigma'^2\right)\right) \\ &- H^2\omega\left(\frac{\Psi'}{\Psi}\right)^2 - 2H^2. \end{aligned} \qquad (2.23)$$

Defining $y = \sigma'$ and $z = \Psi'$ we obtain a set of first order, ordinary differential equations which can be integrated given initial conditions on $\{t, \rho_r, \sigma, \sigma', \Psi, \Psi'\}$ at some initial $N = N_i$. When certain slowroll conditions, such as $\ddot{\Psi} \ll H\dot{\Psi}$ and $\dot{\sigma}^2 \ll V$ are valid, this coordinate transformation will simplify any analytical approach considerably.

Before proceeding to solve these equations for specific inflaton potentials, we shall investigate the most important observational constraints that any successful scenario of this type must satisfy. We present a detailed discussion of how such limits arise since they will always be relevant in any future analysis.

3 Constraints from the CMBR, nucleosynthesis and the solar system

3.1 Amplitudes of scalar and tensor perturbations in the conformal frame

The purpose of this section is to derive the general formulae for the amplitude of density fluctuations which arise in models based on theory (2.1) and then discuss the constraints which any scalar–tensor scenario based on a steep potential must satisfy.

We generalize the calculation of Berkin and Maeda[28] by considering an arbitrary functional form for $h(\psi)$ or equivalently for $\omega(\Psi)$. If one wishes to employ Bardeen's[29] analysis for the evolution of super-horizon sized perturbations, it is necessary for consistency to perform the calculation in the EH frame where the Planck mass is truly constant. To proceed analytically, however, it is also necessary to assume certain 'slow-roll' approximations for the two fields and ignore all quadratic first-derivative and linear second-derivative terms in the field equations. The full equations derived from theory (2.7) were presented in Ref. 30 and for slowly rolling fields they reduce to

$$\tilde{H}^2 \approx \frac{\kappa^2}{3} CV \tag{3.1}$$

$$3\tilde{H}\mathrm{d}_\eta\Phi \approx -C_\Phi V \tag{3.2}$$

$$3\tilde{H}\mathrm{d}_\eta\sigma \approx -AV_\sigma, \tag{3.3}$$

where subscripts Φ and σ denote differentiation with respect to Φ and σ respectively, $\mathrm{d}_\eta \equiv \mathrm{d}/\mathrm{d}\eta$, $\eta \equiv \int \mathrm{d}t\Omega(t)$ denotes cosmic time in the conformal picture and tildes refer to quantities defined in the EH frame. In general, the total energy density of the system is defined by

$$\tilde{\rho} \equiv \frac{1}{2}(\mathrm{d}_\eta\Phi)^2 + \frac{1}{2}A(\mathrm{d}_\eta\sigma)^2 + A^2V. \tag{3.4}$$

and the expression for the density spectrum can be found by extending the results of Lyth[31] to the case of two interacting scalar fields. If we denote by η_1 and η_2 the times a perturbation leaves and re-enters the horizon in the EH frame, then

$$\left[(1+\beta)\frac{\delta\tilde{\rho}}{\tilde{\rho}}\right]_{\eta_1} = \left[(1+\beta)\frac{\delta\tilde{\rho}}{\tilde{\rho}}\right]_{\eta_2} \tag{3.5}$$

where $\beta = 2/[3(1+\omega)]$ and $\omega = \tilde{p}/\tilde{\rho}$ is the ratio of the pressure to the energy density at the two epochs. In this discussion, the term 'horizon' refers to the inverse Hubble scale at the times η_i and horizon crossing is defined in terms of comoving wavenumber, $\tilde{k}(\eta)$, by the expression $\tilde{k}(\eta) = \tilde{a}(\eta)\tilde{H}(\eta)$.

During inflation, $\omega_1 \approx -1$ which implies

$$\left[\frac{\delta\tilde{\rho}}{\tilde{\rho}}\right]_{\eta_2} \approx \frac{2}{3}\left(\frac{1}{1+\beta_2}\right)\left[\frac{\delta\tilde{\rho}}{\tilde{p}+\tilde{\rho}}\right]_{\eta_1}. \tag{3.6}$$

It is necessary to derive an expression for $\delta\tilde{\rho}$ at the first horizon crossing. This is achieved by varying (3.4) and ignoring quadratic terms in $\mathrm{d}_\eta\Phi$ and $\mathrm{d}_\eta\sigma$. Dimensional analysis implies $\delta(\mathrm{d}_\eta\Phi) \approx \tilde{H}(\delta\Phi)$ and $\delta(\mathrm{d}_\eta\sigma) \approx \tilde{H}\delta\sigma$, so

$$\delta\tilde{\rho} \approx -2(\tilde{H}\mathrm{d}_\eta\Phi\delta\Phi + A(\Phi)\tilde{H}\mathrm{d}_\eta\sigma\delta\sigma), \tag{3.7}$$

where the field equations (3.1) have been used to simplify the result. The terms $\delta\Phi$ and $\delta\sigma$ are stochastic quantities arising from quantum fluctuations in the fields. In practise, one uses the two-point correlation functions to estimate their magnitudes.[32] However, in this theory the inflaton has a non-standard kinetic term in the action (2.7). Naively, one would expect $|\delta\sigma| \approx \tilde{H}$, but an additional factor is present in this expression which arises when the second quantization is performed. In general, this factor is given by the inverse square root of the function coupled to the kinetic term of the inflaton. Hence

$$|\delta\Phi| \approx \tilde{H}, \qquad |\delta\sigma| \approx A^{-1/2}(\Phi)\tilde{H}. \tag{3.8}$$

By substituting (3.8) into (3.6), we arrive at the final result

$$\tilde{A}_S \equiv \left[\frac{\delta\tilde{\rho}}{\tilde{\rho}}\right]_{\eta_2} \approx \alpha\tilde{H}^2\left[\frac{|\mathrm{d}_\eta\Phi| + \sqrt{A}|\mathrm{d}_\eta\sigma|}{(\mathrm{d}_\eta\Phi)^2 + A(\mathrm{d}_\eta\sigma)^2}\right]_{\eta_1\approx\eta_{60}} \tag{3.9}$$

where α is a numerical constant of order unity and the right-hand side is evaluated at the start of the last 60 e-folds of inflation. This is a very general expression and is valid for arbitrary functions $\{h(\psi), V(\sigma)\}$.

Two regions may be defined in terms of the relative values of $\mathrm{d}_\eta\Phi$ and $\sqrt{A}\mathrm{d}_\eta\sigma$ as

$$\text{Region I} \quad - \quad |d_\eta\Phi| < \sqrt{A}|\mathrm{d}_\eta\sigma| \tag{3.10}$$

$$\text{Region II} \quad - \quad |\mathrm{d}_\eta\Phi| > \sqrt{A}|\mathrm{d}_\eta\sigma|. \tag{3.11}$$

In these two limiting cases, the amplitude of density perturbations becomes

$$\tilde{A}_{S,\mathrm{I}} \approx \frac{\alpha\kappa^3}{\sqrt{3}} A^{3/2} \frac{V^{3/2}}{V_\sigma} \tag{3.12}$$

$$\tilde{A}_{S,\mathrm{II}} \approx \frac{\alpha\kappa^3}{\sqrt{3}} \frac{C^{3/2}}{C_\phi} V^{1/2}, \tag{3.13}$$

where the field equations (3.1) have been used. In region II, the evolution of the dilaton field, as represented by the Φ-field, dominates the inflationary dynamics and one recovers the results for the original hyperextended scenario.[18]

The general formula for the perturbation spectrum in the EH frame can be expressed in terms of quantities in the JBD frame with the use of equations (2.5) and (2.6). The conformal transformation (2.5) implies that the two-point correlation function of the inflaton in the JBD frame is $|\delta\sigma| \approx H$, as expected. It is straightforward to show that the expression for $\tilde{A}_S$ is given by

$$\tilde{A}_S \approx \alpha A^{1/2} H^2 \left[\frac{|\dot{\Phi}| + A^{1/2}|\dot{\sigma}|}{\dot{\Phi}^2 + A\dot{\sigma}^2} \right]. \tag{3.14}$$

The results of McDonald[33] were derived in the JBD frame and they can be compared to those of Ref. 28 for the special case of the JBD theory by using Eq. (2.5). For the pure JBD theory, Eq. (3.14) reduces to

$$\tilde{A}_S \approx \alpha H^2 \left[\frac{(1+6\epsilon)^{1/2}|\dot{\psi}| + |\dot{\sigma}|}{(1+6\epsilon)\dot{\psi}^2 + \dot{\sigma}^2} \right], \tag{3.15}$$

where $\epsilon \equiv 1/4\omega$ and this is identical to Eq. (47) in McDonald's paper with the translation

$$\mathrm{d}/\mathrm{d}t \quad \longrightarrow \quad (1+6\epsilon)^{1/2}\mathrm{d}/\mathrm{d}t \tag{3.16}$$

for the time derivative of the dilaton. Such a change is very close to unity when $\epsilon \ll 1$. Hence the change in Newton's constant is negligible compared to the evolution of the

perturbations, as assumed by McDonald. (When $\epsilon \geq 1/6$, the slowroll conditions used to derive (3.1) break down and the derivation of Eq. (3.9) is inconsistent).

The close agreement between the expressions for the density spectrum in the two frames suggests that the conformal transformation (2.5) used to recast the JBD theory into the Einstein-Hilbert form may be valid at the semi-classical level in these chaotic models. The two results should be identical in the limit as $\epsilon \to 0$. A detailed comparison of the two different methods was made by Guth and Jain[34] within the context of old extended inflation in which the inflaton is fixed. These authors extended a previous analysis by Kolb *et al.*[35] They also conclude that the technique of conformal transformations is valid up to a numerical factor of order unity when $\epsilon \ll 1$.

Finally the expression for the amplitude of gravitational waves (tensor modes) should also be calculated in the EH frame. One can view a graviton as a minimally coupled massless scalar field with two degrees of freedom corresponding to the two polarization states of the wave. Abbott and Wise[36] have derived an expression for the amplitude at horizon crossing for an arbitrary inflationary solution, so their results are also valid for the theories under consideration in this work. Therefore, to a first approximation, the equivalent expression for the tensor modes is

$$\tilde{A}_G = \frac{\kappa}{4\pi^{3/2}}\tilde{H}. \tag{3.17}$$

To summarize, we have derived the expressions for the scalar ($\tilde{A}_S$) and tensor ($\tilde{A}_G$) modes in the EH frame. For comparison with observation, however, we require the equivalent expressions in the JBD frame, since we are interpreting this as the physical frame. In general, the equivalent expressions A_S and A_G are related to their counterparts in the EH frame by an expression involving the conformal transformation (2.5). The relation therefore depends on the form of $h(\psi)$. However, if the condition

$$\frac{\mathrm{d}_\eta \Omega}{\Omega \tilde{H}} \ll 1 \tag{3.18}$$

holds, and if Ω is a function of t only, it is straightforward to show that the scale factors and expansion rates in the two universes are related by

$$a(t) = \Omega^{-1}(t)\tilde{a}(\eta), \qquad H = \Omega \tilde{H}. \tag{3.19}$$

Eq. (3.19) implies that $\tilde{k} = \tilde{a}\tilde{H} \approx aH = k$, so the scale dependences in the two frames are approximately equal. Moreover, the definition of the energy density, $\rho \equiv g^{00}T_{00}$, implies

$$\frac{\delta\tilde{\rho}}{\tilde{\rho}} = \frac{\delta\rho}{\rho} - 4\frac{\delta\Omega}{\Omega} \tag{3.20}$$

and we conclude that amplitudes are also equal when $\delta\Omega/\Omega \ll 1$, as implied by Eq. (3.18) for $\mathrm{d}\eta \approx \tilde{H}^{-1}$.[37] Therefore, it is sufficient to consider results from the conformal (EH) frame directly.

3.2 The CMBR and large-scale structure

If particle physics specified the unique inflaton potential, the amplitude A_S would be known for all scales. Because there exist many possible models, however, the amplitude of fluctuations must be normalized using observations of large-scale structure. The CfA survey suggests that the *rms* fluctuation in the galaxy counts is unity in a sphere of radius $8h^{-1}$Mpc and we normalize the amplitude by specifying the *rms* fluctuation in the mass distribution to be $\sigma_8 = b_8^{-1}$ within this sphere.[53] b_8 is the biasing factor which we assume to be constant over the scales of interest.

We shall show in section (5) that the inflationary solutions approximately 60 e-foldings before reheating can be expressed as a power law $a(t) \propto t^p$ for $p \gg 1$. The slowroll approximations are valid in this region of parameter space. It is well known that this solution leads to a primordial power law spectrum of scalar fluctuations, $P(k) \propto A_S^2(k)k \propto k^n$, where $n = 1 - 2/(p-1)$ is the spectral index. The spectrum of tensor modes has an identical scale-dependence for this solution when $n \leq 1$, *i.e.* $A_G \propto A_S$, and scale-invariant fluctuations correspond to $n = 1$ ($p = \infty$).[38]

Equations relating σ_8 to n have been derived using results from the CMBR and large-scale structure.[12] The range of parameter space (σ_8, n) consistent with these observations can then be determined.[39] For our purposes, these constraints restrict the value of the effective JBD parameter (2.3) during inflation.

At present the observation of multipole anisotropies by the *COBE* DMR experiment provides the strongest constraint from the CMBR. The root of the variance at 10^o is observed to be $\sigma_T(10^o) = (1.1 \pm 0.2) \times 10^{-5}$, where the mean blackbody

temperature $T_0 = 2.736$ K is taken and limits are to 1-sigma.[12] On 10^o scales reionization processes such as Thompson scattering are not important and the dominant contribution to the anisotropy is from the Sachs-Wolfe effect when photons at decoupling are redshifted as they climb out of potential wells.[41] For power law inflation the observed anisotropy is due to both scalar *and* tensor modes.[42] Following an identical procedure to Ref. 40, a numerical fit relating $\sigma_T^2(10^o)$ to n and σ_8 can be found and equated to the observed 10^o variance to yield

$$\sigma_8 = (1.18 \pm 0.2) e^{2.63(1-n)} \sqrt{(3-n)/(15-13n)}. \tag{3.21}$$

The contribution of the tensor modes arises solely in the rooted factor and it becomes negligible as n approaches unity. It has the effect of *decreasing* σ_8 for a given spectral index, thereby increasing the allowed bias.

A second result was presented in Ref. 39 based on the *IRAS/QDOT*[13] and POTENT[43] galaxy surveys. Quoting a value of $b_I = 1.2 \pm 0.3$ at the 2-sigma level for the *IRAS* biasing factor, these authors deduced that

$$\sigma_8 = (0.91 \pm 0.25) \left(\frac{1.9}{2.9 - n} \right)^{2/3}. \tag{3.22}$$

Eqs. (3.21) and (3.22) can be combined to yield a bias-independent lower limit on the spectral index of $n \geq 0.84$ for consistency between *COBE* and *QDOT*. It is this limit which most strongly constrains the JBD parameter during inflation. If Eq. (3.11) is valid in the pure JBD theory one recovers the old extended inflationary solution.[44] In this regime the spectra have the power law form discussed above with a spectral index

$$n = 1 - \frac{8}{2\omega(\Psi) - 1}. \tag{3.23}$$

Consistency with the CMBR and *QDOT* results therefore requires $\omega > 25$ at the start of the last 60 e-foldings before the end of inflation. It is important to note that the constraint due to bubble collisions, $\omega < 18$, does not apply here because the phase transition is second-order.[19]

3.3 Nucleosynthesis and solar system constraints

After the expansion becomes subluminal the most important constraints on the value of ω and the vacuum energy arise at the nucleosynthesis era and the current epoch respectively. The standard model of primordial nucleosynthesis can be used to constrain the vacuum energy density and the JBD parameter at temperatures $\mathcal{O}(1)$ MeV. Time-varying gravitational and cosmological constants modify the expansion rate of the universe and therefore the nuclear reaction rates during this epoch. At temperatures exceeding the 'freeze-out' temperature $T_F \approx 0.8$ MeV the neutrons and protons are held in local thermodynamic equilibrium by weak interactions.[15] In the standard big bang picture the neutron-to-proton ratio at $T \geq T_F$ is determined by the Boltzmann factor $(n/p) = \exp(-Q/T)$, where $Q = 1.293$ MeV is the n-p mass difference. The energy density of the universe is then $\rho = \pi^2 g_{\rm eff} T^4/30$, where $g_{\rm eff}$ is the number of relativistic degrees of freedom. The nucleons fall out of equilibrium when the reaction rate equals the expansion rate of the universe, *i.e.*

$$G_W^2 T_F^5 \approx H \propto \sqrt{G\rho}. \tag{3.24}$$

The temperature drops below T_F and free neutrons undergo β-decay until the photodissociation of deuterium becomes energetically unfavourable at some temperature T_D. At this point synthesis of He^4 proceeds rapidly.

The abundance of He^4 depends crucially on the number of neutrons present at T_D. There will always be two competing effects in any modification to the standard picture. Increasing the freeze-out temperature increases (n/p) and one might expect the He^4 abundance to be correspondingly higher. However, in many cases it then takes longer for the universe to cool to $T = T_D$. More neutrons can undergo β-decay and this effectively reduces the He^4 abundance.

When vacuum energy is introduced the second effect is dominant and the observed He^4 abundance therefore leads to an upper limit on Ω_V.[14] From detailed numerical calculations it was found that this limit is

$$\Omega_V[1\text{MeV}] \leq 0.08 \tag{3.25}$$

for three neutrino species.[14]

There is also a limit on the change in Newton's constant with time. In the standard JBD theory the strength of gravity is higher at earlier times and this increases T_F as indicated by Eq. (3.24). One therefore expects an upper limit on $\dot{G}/G$ to exist or equivalently a lower limit on ω. Casas *et al.*[45] found $\omega > 250$, but by dropping some of the simplifying assumptions used to obtain this bound, Serna *et al.*[46] found the weaker limit of $\omega > 50$. As a first approximation we shall apply this weaker limit to the more general scalar-tensor theories under consideration here, *i.e.*

$$\omega[\Psi, 1\mathrm{MeV}] \geq 50. \tag{3.26}$$

It should be emphasized that this limit is only strictly valid when ω is constant for all time. Although constraints (3.25) and (3.26) may be weaker than those derivable from more exact calculations, they are sufficient to severely limit the scenario discussed here, as is shown by the numerical calculations in section 4.

Finally, constraints on the magnitude and form of the JBD parameter (2.3) at the present epoch can be obtained from solar system experiments by using the post-Newtonian approximation. In this analysis one considers the time independent spherically symmetric metric around a point mass m, expanded as a series in the gravitational potential $U = m/r$, *i.e.*

$$ds^2 = (1 - 2\alpha U + 2\beta U + ...)dt^2 + (1 + 2\gamma U)d\mathbf{x}^2, \tag{3.27}$$

where $d\mathbf{x}^2 = dx_i dx^i$ $(i = 1, 2, 3)$. Current observational limits on the Post-Newtonian parameters are

$$\alpha = 1 \pm 10^{-4}, \qquad (\gamma + 1)/2 = 1 \pm 10^{-3}, \qquad (2 + 2\gamma - \beta)/3 = 1 \pm 10^{-2}, \tag{3.28}$$

whereas general relativity predicts the values $\alpha = \beta = \gamma = 1$. Nordtvedt[16] analyzed the generalized scalar-tensor theory (2.2), finding that

$$\beta = 1 + \frac{\mathrm{d}\omega/\mathrm{d}\Psi}{(4 + 2\omega)(3 + 2\omega)^2}, \tag{3.29}$$

and

$$\gamma = \frac{1 + \omega}{2 + \omega} \qquad \Longrightarrow \qquad \omega[\psi_0, 3\mathrm{K}] > 500. \tag{3.30}$$

It is important to note that $\omega \to +\infty$ is a necessary but not sufficient condition for the recovery of general relativity at the present epoch. We also require from Eq. (3.29) that $(\mathrm{d}\omega/\mathrm{d}\Psi)/\omega^3 \to 0$ as ω diverges. However, this condition is always satisfied when the dilaton is located within the vicinity of a turning point in $h(\psi)$, because

$$\frac{\mathrm{d}\omega/\mathrm{d}\Psi}{\omega^3} = 4\frac{(\mathrm{d}h/\mathrm{d}\psi)^4}{I^3}\left(1 - 2h\frac{\mathrm{d}^2h}{\mathrm{d}\psi^2}\left(\frac{\mathrm{d}h}{\mathrm{d}\psi}\right)^{-2}\right). \tag{3.31}$$

To summarize, the most important limits are $\omega > 25$ during inflation, $\omega > 50$ at nucleosynthesis and $\omega > 500$ at the present epoch.

4 Numerical Results

4.1 Initial Conditions

A number of plausible models were considered by numerically integrating the system of equations (2.17)-(2.21) plus (2.23). The analysis does not depend strongly on the precise form of $h(\psi)$. One only requires that it be at least C^2 continuous, contain a turning point and be normalized to unity at this point. In this sense the scenario we are investigating is rather generic. To ease the calculation we specified

$$\Psi = h(\psi) = \sin^2\beta\psi, \qquad \beta = \text{constant}, \tag{4.1}$$

because this leads to a simple form for $\omega(\Psi)$ given by

$$\omega(\Psi) = \frac{1}{8\beta^2}\frac{1}{1-\Psi^2}. \tag{4.2}$$

Although there are no known particle physics models which lead directly to Eq. (4.1), it has a simple analytical form which can be viewed as an approximation to a more complete theory. Indeed, for small Ψ one can treat Eq. (4.2) as a perturbation to the JBD theory up to terms including $\mathcal{O}(\Psi^2)$. General relativity is recovered as $\Psi \to 1$. Following Linde, the initial conditions were specified by treating the quantum boundary (QB) as the most natural set of initial conditions for chaotic

inflation in scalar-tensor theories.[47] This boundary corresponds to the hypersurface at the Planck density and represents the earliest times at which initial conditions can be placed on classical fields. In theory (2.1) the Planck mass is related to the dilaton by $m_{\rm Pl}^2(\psi) = h(\psi)$ so the QB is reached when $V(\sigma_i) \approx h^2(\psi_i) = m_{\rm Pl}^4$. There is therefore a 1-jet family of initial conditions relating the values of σ and ψ. Indeed, the probability that the universe can be created from nothing via quantum tunneling is

$$p \approx \exp(-3m_{\rm Pl}^4(\psi)/8V(\sigma)), \tag{4.3}$$

which is only high on the QB.[48]

In the numerical calculations we employ this argument as a first approximation. To be more accurate, though, one should also consider the effects of quantum fluctuations in the two fields. Once it has started inflation always occurs at some point in the universe if the change in the fields due to quantum fluctuations with wavelength larger than H^{-1} exceeds the classical change in the time interval H^{-1} due to the equations of motion.[47] An island universe resembling our own (almost) flat Friedmann space-time may only form when such a situation is reversed, and the point of equality is a more accurate estimate of initial conditions.

The evolution of the quantity $\mathrm{d}\ln a/\mathrm{d}\ln t \equiv Ht$ with respect to the number of e-foldings, $N = \ln a$, was investigated because this allows a number of interesting features to be shown diagrammatically. For example, if the scale factor expands as a power law, $a \propto t^p$, $Ht = p =$ constant and the graph is a horizontal line. On the other hand, the exponential (de Sitter) expansion may be written as $N = Ht +$ constant and this is a straight line of gradient $\pi/4$. The critical solution for inflation is the Milne universe, $Ht = 1$, and the strong energy condition is violated above this line. The end of inflation can therefore be defined as the point where the graph cuts the line $Ht = 1$.

4.2 Decaying Power Law Potentials

Witten has shown that potentials of the form $V \propto \sigma^{-\alpha}$, where $\alpha > 0$, can arise when supersymmetry is spontaneously broken.[49] The symmetry breaking occurs when the

potential has non-zero values. Therefore, one would have a natural candidate for the inflaton if σ was the field responsible for breaking supersymmetry. In Einstein gravity the potential is too steep at small σ to lead to inflation, but at large σ an intermediate inflationary solution $a \propto \exp(Ft^q)$ is found, where F and $q < 1$ are positive constants.[50]

The question we address is whether inflation at small σ is possible in the scalar-tensor theory discussed above and the numerical results are shown in Figure (1) for $\alpha = 10$ and $\beta = 0.07$. A power law expansion with $p = 1/(8\beta^2) + 1/2$ arises when $100 < N < 260$. In this region $\omega(\Psi)$ is approximately constant ($\Psi^2 \ll 1$) and the solution corresponds to that found in the JBD theory when the inflaton is held constant. This is the old extended inflationary solution $a \propto t^{\omega+1/2}$.[44] Hence the known results for the form of the perturbation spectra derived when Eq. (3.11) is valid can be employed in this analysis and Eq. (3.13) applies. Consistency with the CMBR and *QDOT* results therefore requires $\beta < 0.07$.

Figures (1a) & (1b)

Unfortunately there is a graceful exit problem in this model. Figures (1a) and (1b) show how the solution temporarily approaches the Milne limit as $\dot{\sigma}^2$ increases and Einstein gravity is recovered at $N \approx 310$. At this point the intermediate solution takes over where $Ht = qN +$ constant. The gradient of this line is consistent with the analytical result $\alpha = 4(q^{-1} - 1)$ derived by Barrow.[50] (This provides a useful check for the numerical calculations.) Because the de Sitter solution is an attractor at infinity in this case, it is not possible to obtain subluminal expansion unless the reheating process is highly efficient. Rather than redshifting to zero, the radiation density grows during the power law phase due to the continual decay of σ. This is a generic feature of these models, but is not sufficient to establish a radiation-dominated phase as required by nucleosynthesis. Naively one may think that $\dot{\sigma}^2$ would increase sufficiently fast as $\Psi \to 1$, but it is too small relative to Ψ during inflation to contribute significantly to the reheating. We considered models up to $\alpha = 100$ and found the same qualitative behaviour.

4.3 Exponential potentials without dissipation

We proceeded to investigate the exponential potential $V(\sigma) = V_0 \exp(-\lambda\kappa\sigma)$ where $\{V_0, \lambda\}$ are independent constants and $\kappa^2 = 8\pi$. In Einstein gravity this potential leads to an attractor power law solution $a \propto t^{2/\lambda^2}$ when $\lambda^2 < 6$ and inflation occurs if $0 < \lambda^2 < 2$. The attractor solution becomes $a \propto t^{1/3}$ for all $\lambda^2 > 6$ and the physical reason for this is as follows. In the spatially flat Friedmann model the $\lambda^2 < 6$ attractor arises because the kinetic and potential energies of σ redshift at the same rate. This is the deep reason why inflation can never end in this model. When $\lambda^2 > 6$, however, the potential is too steep and the field becomes effectively massless as $V \to 0$.[51]

A number of new features arise when the Planck mass is allowed to vary. We find that inflation is possible when $\lambda^2 > 6$ and that the expansion becomes subluminal without any fine-tuning. As an example, we discuss results for $\lambda^2 = 12$ and $\beta^2 = 1/8$. In Figure (2) the dissipation term C_V is removed for all time. The $p = \omega + 1/2$ power law solution is again found, thereby implying that the dilaton dominates the dynamics. The expansion does indeed become subluminal and settles around the value $p = 1/2$. This result is independent of the value of β and is understood by investigating Eq. (2.12). For $U = 0$ and ω approximately constant, this equation can be rewritten in the form

$$\ddot{\Psi} + 3H\dot{\Psi} + \frac{\partial V_{\rm eff}}{\partial \Psi} \approx 0, \tag{4.4}$$

where $V_{\rm eff}$ is an effective interaction potential defined by the integrability condition

$$\frac{\partial V_{\rm eff}}{\partial \Psi} \equiv -\frac{8\pi}{2\omega(\Psi) + 3} T \tag{4.5}$$

and $T \equiv 4V - \dot{\sigma}^2$. Eq. (4.4) resembles the equation of motion for a minimally coupled field and we may view Ψ as evolving along its effective interaction potential. During the inflationary epoch, $V \gg \dot{\sigma}^2$ and $V_{\rm eff}$ has negative gradient. Hence the values of Ψ and $\dot{\sigma}^2$ increase. This means that T, and therefore $\partial V_{\rm eff}/\partial\Psi$, eventually change sign as $4V - \dot{\sigma}^2$ passes through zero, thus causing the dilaton to slow and reverse its motion. Hence, there exists a damped oscillatory behaviour around $4V = \dot{\sigma}^2$ and some constant value of Ψ. The theory soon resembles a rescaled version of general relativity

with a traceless energy-momentum tensor; the solution is therefore equivalent to the radiation-dominated universe $a \propto t^{1/2}$.

Figures (2a) & (2b)

Hence, the effect of a varying Planck mass is to change the attractor for steep exponential potentials from $p = 1/3$ to $p = 1/2$. The above argument should apply to *any* steep potential for which $T < 0$ in Einstein gravity. Including an uncoupled radiation component will not alter the argument since its energy-momentum is identically traceless and does not affect $V_{\rm eff}$.

4.4 Exponential potentials with dissipation

In Figure (3) the dissipation term is included for all time. Figure (3c) shows the radiation density growing during inflation and a sufficient reheating temperature for baryogenesis to proceed is easy to obtain in this model. Moreover, from Figures (3b) and (3d), it is seen that the radiation begins to dominate once Einstein gravity is recovered and the dissipation prevents the quantity $4V - \dot{\sigma}^2$ from passing through zero. However, a new scaling solution is found, where $\rho_\sigma/\rho_{\rm rad} =$ constant, as is shown in the late time behaviour of Figures (3a) and (3d).

Figures (3a) - (3d)

This solution may be derived analytically for the special case where C_V is constant. We consider a model in which a particle species X, with equation of state $p_X = (\gamma - 1)\rho_X$ for some constant γ, is coupled to the inflaton through an interaction $\rho_X \equiv \Gamma_X \dot{\sigma}^2$, where Γ_X is constant. The Bianchi identity and Friedmann equation for this system are

$$(1 + 2\Gamma_X)\ddot{\sigma} + 3(1 + \gamma\Gamma_X)H\dot{\sigma} + V_\sigma = 0 \tag{4.6}$$

$$3H^2 = \kappa^2(\rho_V + \rho_X) = \kappa^2\left[\left(\frac{1}{2} + \Gamma_X\right)\dot{\sigma}^2 + V\right], \tag{4.7}$$

where ρ_V represents the inflaton energy density and we assume Einstein gravity holds. By differentiating Eq. (4.7) with respect to cosmic time and substituting Eq. (4.6)

for $\ddot{\sigma}$ we arrive at the 'momentum' equation

$$\dot{H} = -\frac{\kappa^2}{2}(1+\gamma\Gamma_X)\dot{\sigma}^2. \tag{4.8}$$

This implies that Eq. (4.7) may be rewritten in the Hamilton-Jacobi form by defining a new time coordinate

$$t = -\frac{\kappa^2}{2}(1+\gamma\Gamma_X)\int^{\sigma} \mathrm{d}\,\sigma' \left(\frac{\mathrm{d}H}{\mathrm{d}\sigma'}\right)^{-1}. \tag{4.9}$$

We find

$$3\kappa^2(1+\gamma\Gamma_X)^2H^2 - 2(1+2\Gamma_X)\left(\frac{\mathrm{d}H}{\mathrm{d}\sigma}\right)^2 = \kappa^4(1+\gamma\Gamma_X)^2V, \tag{4.10}$$

which yields the attractor solution

$$H = \sqrt{A}\exp\left(-\lambda\kappa\sigma/2\right), \qquad \lambda\kappa\sigma = 2\ln\left[\frac{\lambda^2\sqrt{A}}{2(1+\gamma\Gamma_X)}t\right], \tag{4.11}$$

where A is a positive constant. Hence the scale factor grows as a power law $a \propto t^p$, where

$$p \equiv \frac{2}{\lambda^2}(1+\gamma\Gamma_X). \tag{4.12}$$

(When $\Gamma_X \to 0$ we recover the standard power law solution with $p = 2/\lambda^2$). The contribution of X to the matter content of this universe may be expressed through the quantity $\Omega_X \equiv \kappa^2\rho_X/3H^2 = \rho_X/(\rho_X+\rho_V)$. For the solution (4.11) we find

$$\Omega_X = \frac{\Gamma_X\lambda^2}{3(1+\gamma\Gamma_X)^2} \tag{4.13}$$

is constant, implying Eq. (4.11) represents a scaling solution and the matter and radiation densities redshift as $\rho_V \propto \rho_X \propto t^{-2} \propto a^{-2/p}$. Moreover, by substituting Eq. (4.11) into Eq. (4.13), the condition $V_0 > 0$ implies this attractor exists only if

$$\Gamma_X > \frac{\Omega_X}{2(1-\Omega_X)}. \tag{4.14}$$

The nucleosynthesis constraint (3.25) then implies $\Gamma_X > 23/4$ which yields the lower limit $\lambda^2 \geq 36$.

In principle this result suggests the nucleosynthesis constraints can be satisfied in this model if λ^2 is sufficiently large. However, if the dissipation is not removed, the scaling solution will survive through to the decoupling era. Numerical results have shown that the lack of observed spectral distortions in the CMBR imply $\Omega_V < 4\times10^{-4}$ if the vacuum decays into low energy photons.[14] For consistency, Eq. (4.14) then implies $\lambda^2 > \mathcal{O}(10^3)$, which is clearly unrealistic. In any case it is physically reasonable to suppose the dissipation becomes negligible once the effective inflaton mass falls below the rest mass of its decay product. Naively one would expect the vacuum energy to rapidly redshift to zero at energies below this mass scale, with the radiation density falling as a^{-4}. In general relativity this would be the case, but the evolution of the Planck mass again significantly alters the arguments, as shown in Figure (4).

Figures (4a) & (4b)

As soon as the dissipation term was removed the vacuum rapidly dominated the universe once more. This feature arises because the peak of reheating occurs while the dynamics is still dominated by the dilaton's motion. The dilaton viscosity still slows the inflaton and thus its energy density does not decay as fast as it would in Einstein gravity. On the other hand, the time–dependence of the Planck mass does not affect the radiation, which still decays as $\rho_r \propto a^{-4}$. This follows because the energy–momentum tensor of the free radiation field is traceless and does not appear in the dilaton field equation (2.12). We suggest that this qualitative behaviour should not depend too strongly on the precise form of the potential unless $V(\sigma)$ is very steep. In this case, though, the vacuum energy would be insignificant at the present epoch. The same qualitative behaviour was observed for values of $\omega = 500$ ($\beta = 0.0158$).

5 Conclusions and Implications

The philosophy behind this work was to identify a decaying cosmological constant at the present epoch *directly* with the vacuum energy which drove the inflationary expansion without altering the form of the potential. This would solve, without severe fine–tuning, the cosmological constant problem and could provide a possible explanation for a number of observational results. The limiting solution for accelerated expansion, $a \propto t$, arises when the vacuum energy density varies as $\rho_V \propto a^{-2}$. Hence, ρ_V must redshift faster than a^{-2} if inflation is to end, but must decay slower than the pressure–free matter component, $\rho_{\rm matter} \propto a^{-3}$, if it is to dominate at late times. This requires viscosity to be present at early times. We derived expressions for the amplitudes of scalar and tensor fluctuations with a general inflaton potential and scalar–tensor theory, and the strongest constraints on any model arise from the CMBR, primordial nucleosynthesis and solar system observations. These constraints are important in any scenario of this type. In general, it is difficult (if not impossible) to satisfy these constraints simultaneously if the viscosity arises due to a time–dependent Planck mass.

We considered two simple forms for a steep potential, $V \propto \sigma^{-\alpha}$ and $V \propto \exp(-\lambda\sigma)$. In the former a graceful exit problem arises for realistic values of α. The latter is more promising and a number of scaling solutions were found both numerically and analytically. In particular, inflation occurs and ends naturally when $\lambda^2 > 2$. This model leads to a power spectrum consistent with observation, but the vacuum does not decay sufficiently fast to satisfy the nucleosynthesis constraint (3.25). It also appears that this constraint cannot be satisfied for very steep potentials, which decay rapidly to zero in Einstein gravity, because the Planck mass is still evolving during reheating. This causes ρ_V to redshift at a much reduced rate relative to the radiation component.

To summarize, the scenario as outlined in section 2 is not viable for the examples considered here and requires more complicated potentials and further extensions. However, with this numerical code, it is possible to develop working hyperextended

chaotic models based on the theories discussed in section 2 if the steep potential has a minimum. One extension to this analysis is to investigate any effects a dilaton self-interaction potential $U(\psi)$ may have. Alternatively one could consider more general couplings of the dilaton and inflaton fields in action (2.1) or alter the form of the dissipation term C_V which models the reheating process.

These results may have implications for the joint evolution of the comological and gravitational constants. A present–day vacuum term may arise if

1. The inflaton settles into a minimum of its potential at $V = 0$ and a second scalar field is located in a non–zero minimum of its own potential.

2. The global minimum of the inflaton potential is located at $V \neq 0$.

3. $V(\sigma)$ decays monotonically for all time.

The third possibility is the most attractive, but based on the above numerical calculations, one may conjecture that the vacuum will generally not decay sufficiently fast relative to the relativistic matter if $m_{\rm Pl}$ is time–dependent during and shortly after inflation. Our analysis therefore favours a reheating process via oscillations of the inflaton about some minimum if the Planck mass varies with time, but it does not rule out other possibilities. This agrees indirectly, and for different reasons, with the conclusions of Ref. 52, in which a detailed quantum mechanical description of the reheating process was given. These authors concluded that particle production is only significant during the oscillating phase of the inflaton. (A direct comparison of conclusions cannot be made, however, since these authors only considered reheating in Einstein gravity, whereas the dilaton is still evolving during reheating in the models discussed here.)

Conversely, if one prefers a continually decaying potential, this suggests a constant Planck mass is required. In any case, it appears that some degree of fine–tuning is necessary if a cosmological constant arising from the inflaton potential is to be non–zero at the present epoch.

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Figure Captions

Figure 1: Numerical solutions for the potential $V(\sigma) = 10^{-10}\sigma^{-10}$, with $\beta = 0.07$ and initial conditions $\dot{\sigma} = \dot{\Psi} = 0$. The dissipation coefficient $f = 1.0$ and the rest mass of the decay product X is $m_X = 10^9$ GeV. Figure (1a) illustrates the evolution of the power index of the solution. Power law and intermediate inflationary expansion is observed, but a graceful exit from the inflationary regime is not found. Figure (1b) illustrates the evolution of the ω-parameter in terms of $\tan^{-1}\omega(\Psi)$. The intermediate solution takes over once ω diverges to infinity (*i.e.* $\tan^{-1}\omega \rightarrow +\pi/2$) and Einstein gravity is recovered.

Figure 2: Numerical solutions for the potential $V \propto \exp(-\lambda\kappa\sigma)$ with $\lambda^2 = 12$, $\beta = 1/8$ and initial conditions $\dot{\sigma} = \dot{\Psi} = 0$. Dissipation effects have been removed and $f = 0$. Figure (2a) shows that inflation is possible and ends naturally as the expansion becomes subluminal. The expansion approaches the attractor $p = 1/2$ as $\omega(\psi)$ settles to the constant value in Figure (2b). A rescaled version of general relativity is recovered but the Planck mass is smaller than the currently observed value.

Figure 3: The same conditions as in Figure (2), but with $\beta = 0.025$ and $f = 1.0$. The vertical dashed line measures where the expansion becomes subluminal. Figure (3c) plots the evolution of the radiation density (ρ_r) and the inflaton energy density (ρ_σ). Figure (3d) plots the ratio of the inflaton energy density to the total energy density. The effects of dissipation are included for all time. A new scaling solution is found once Einstein gravity is recovered, but this violates spectral distortion limits on the CMBR.

Figure 4: As Figure (3), but the dissipation terms are removed at $m_X = 10^{10}$ GeV. In Figure (4a) the inflaton rapidly dominates the expansion after the dissipation is

removed. This follows because the dilaton is evolving during the reheating, as shown in Figure (4b) by the evolution of $\omega(\Psi)$.

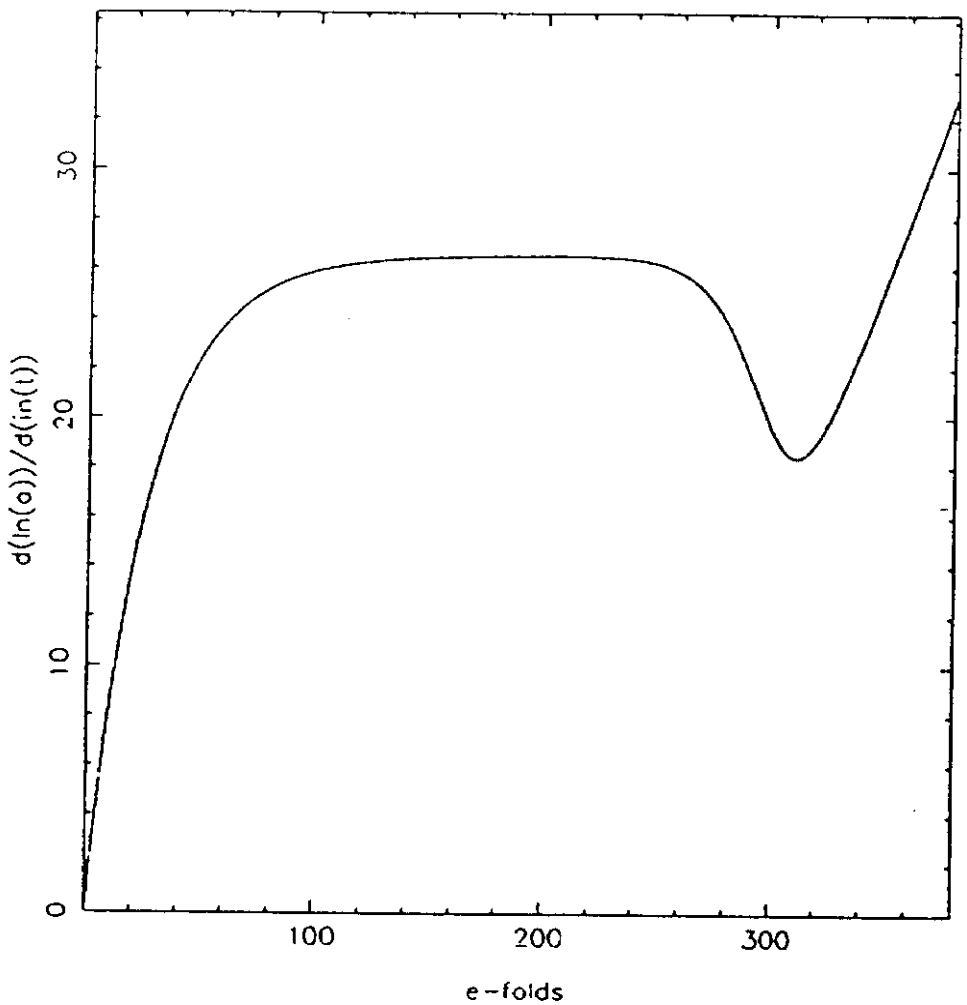

Fig: 19

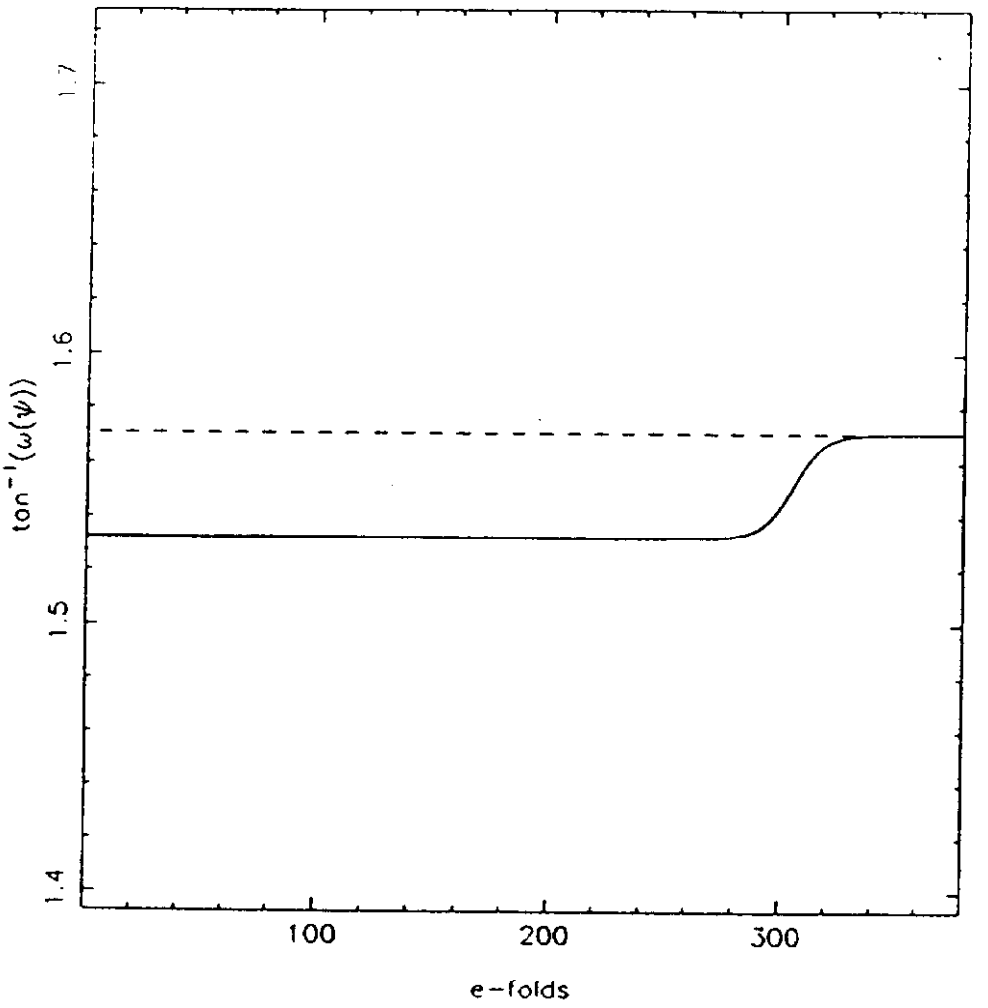

Figure 1b

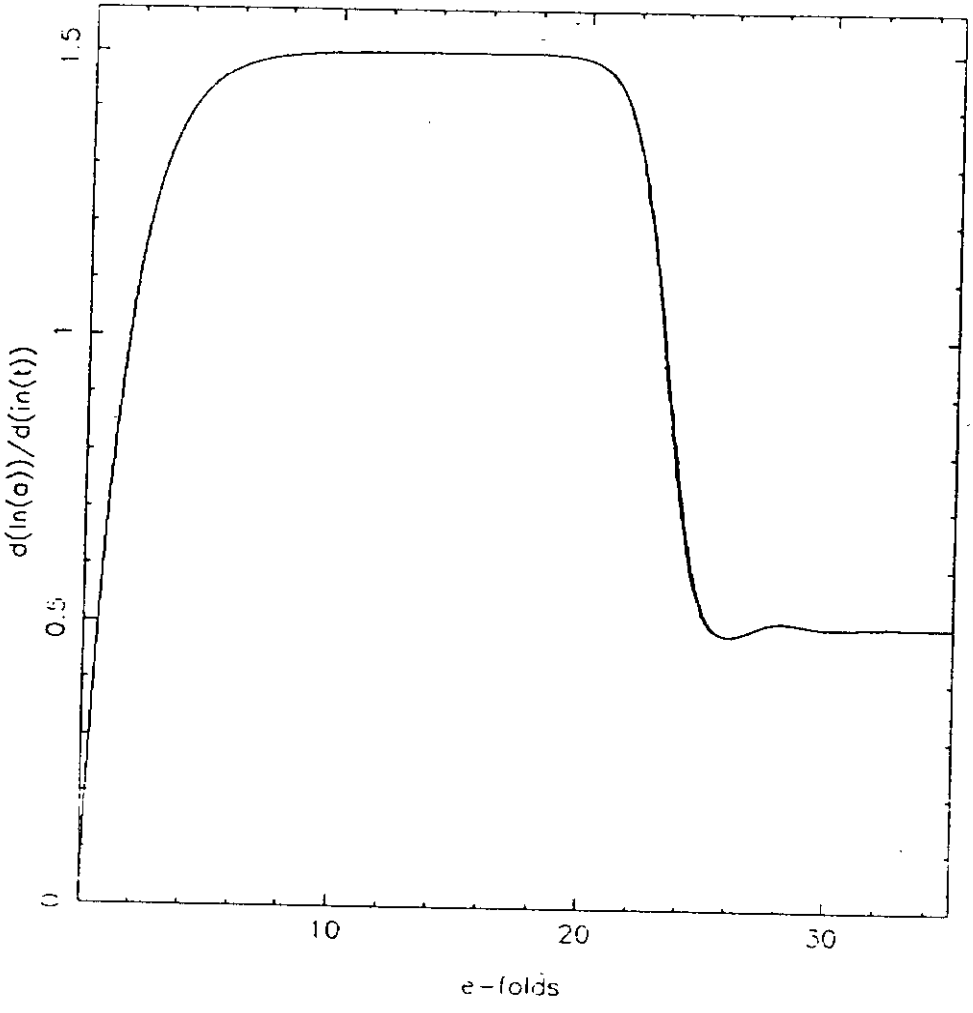

Figure 2a

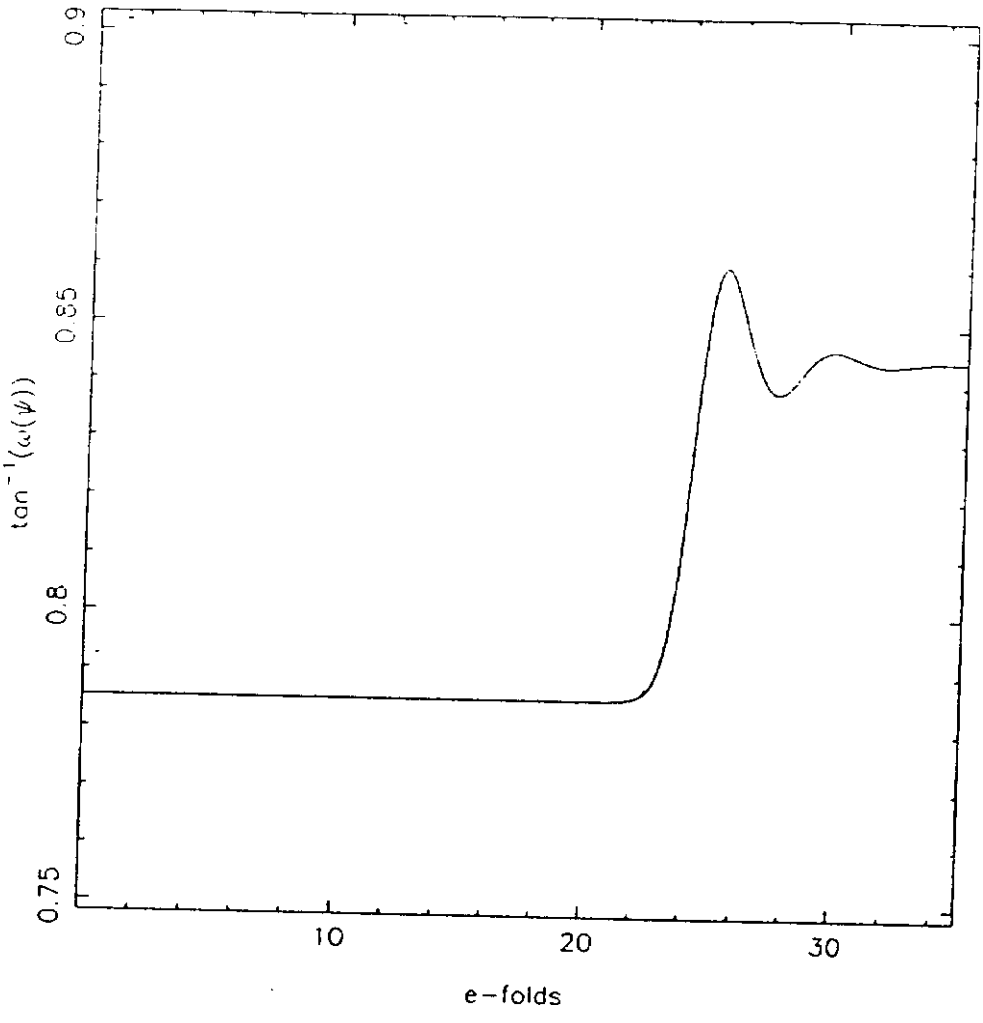

Fig. 2b

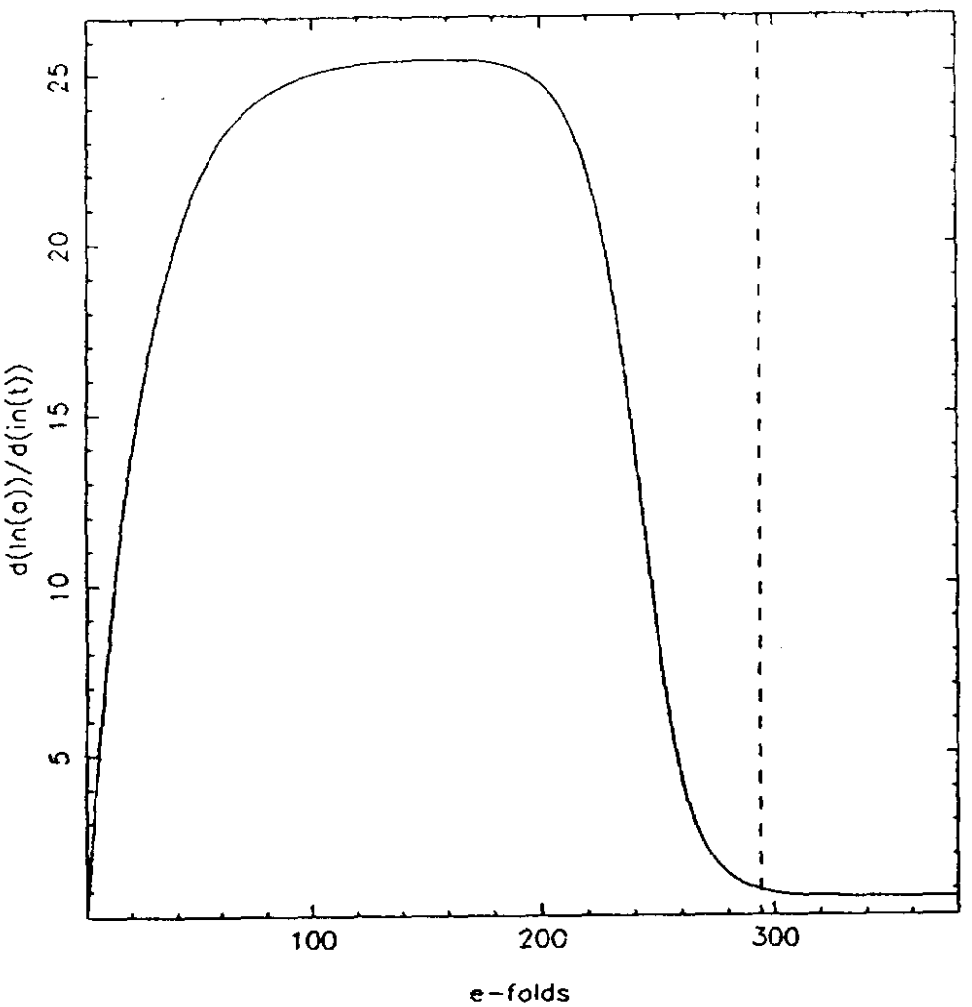

Fig. 3a

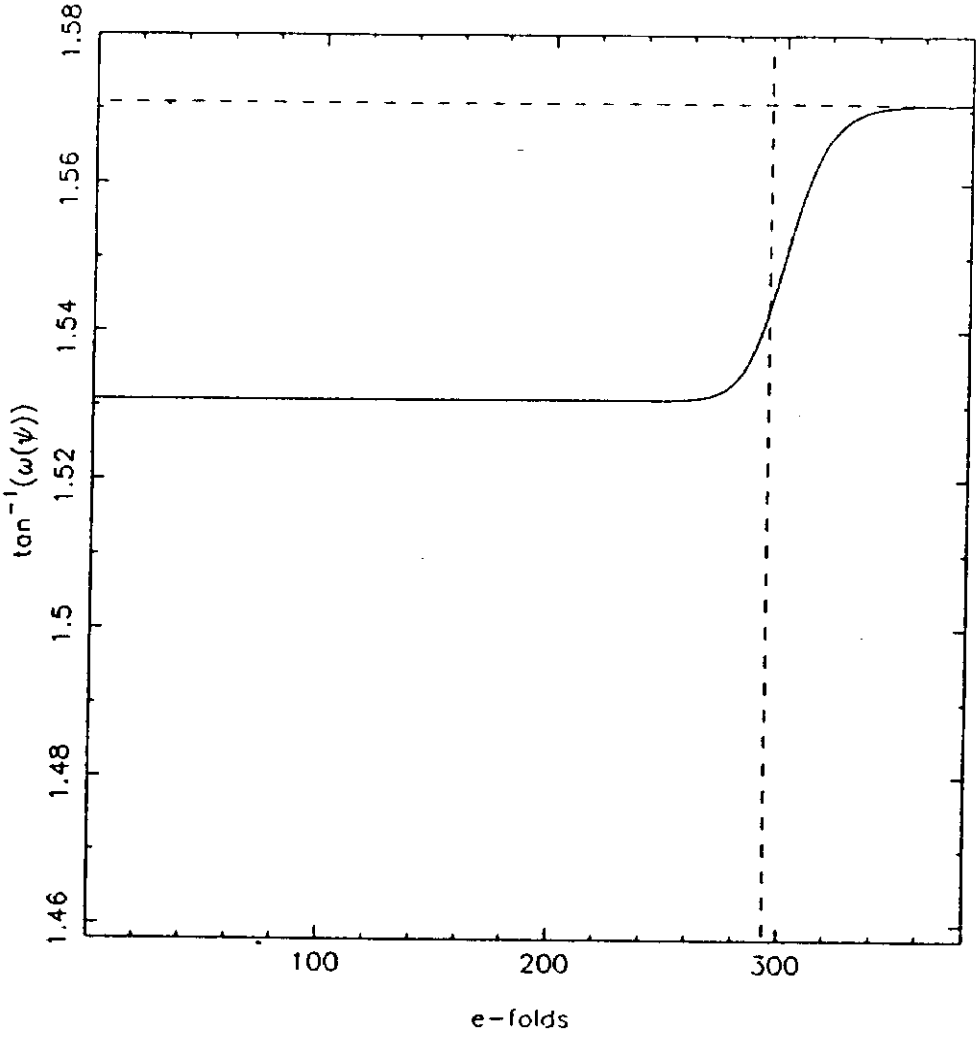

Fig. 3b

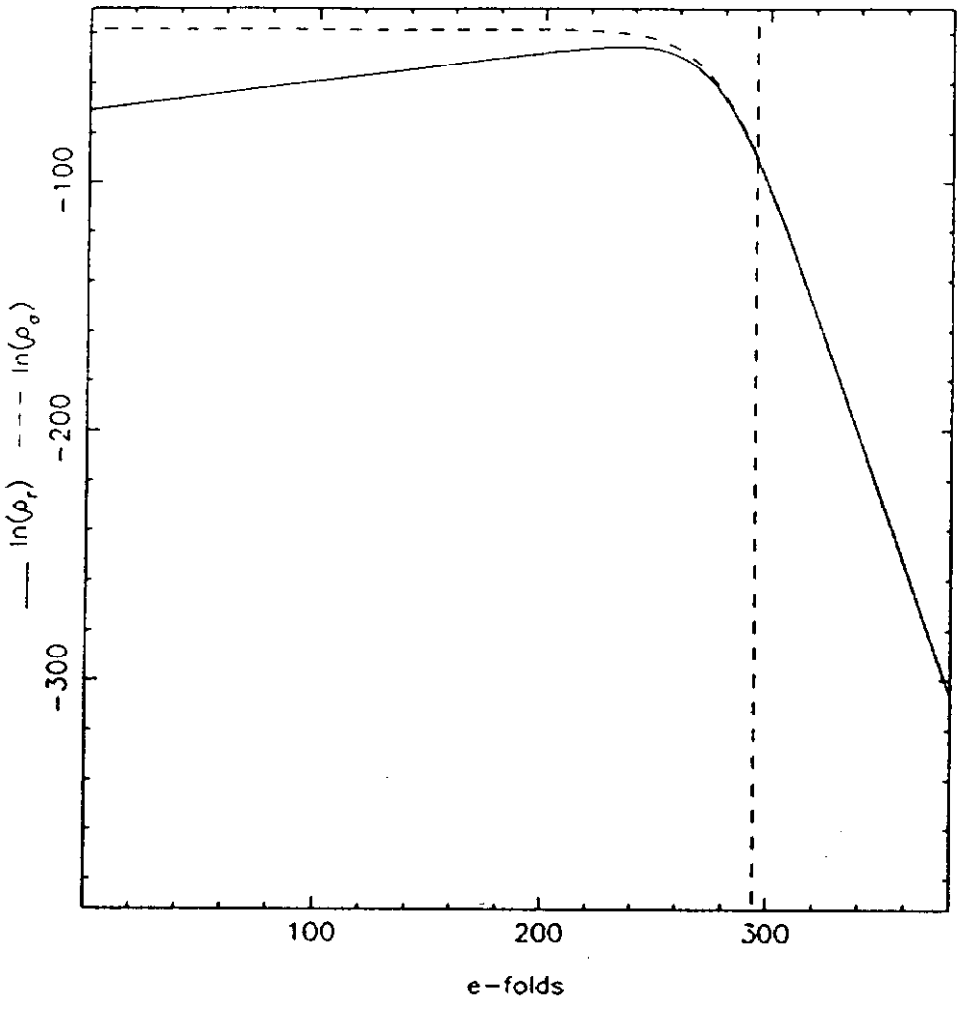

Fig. 3

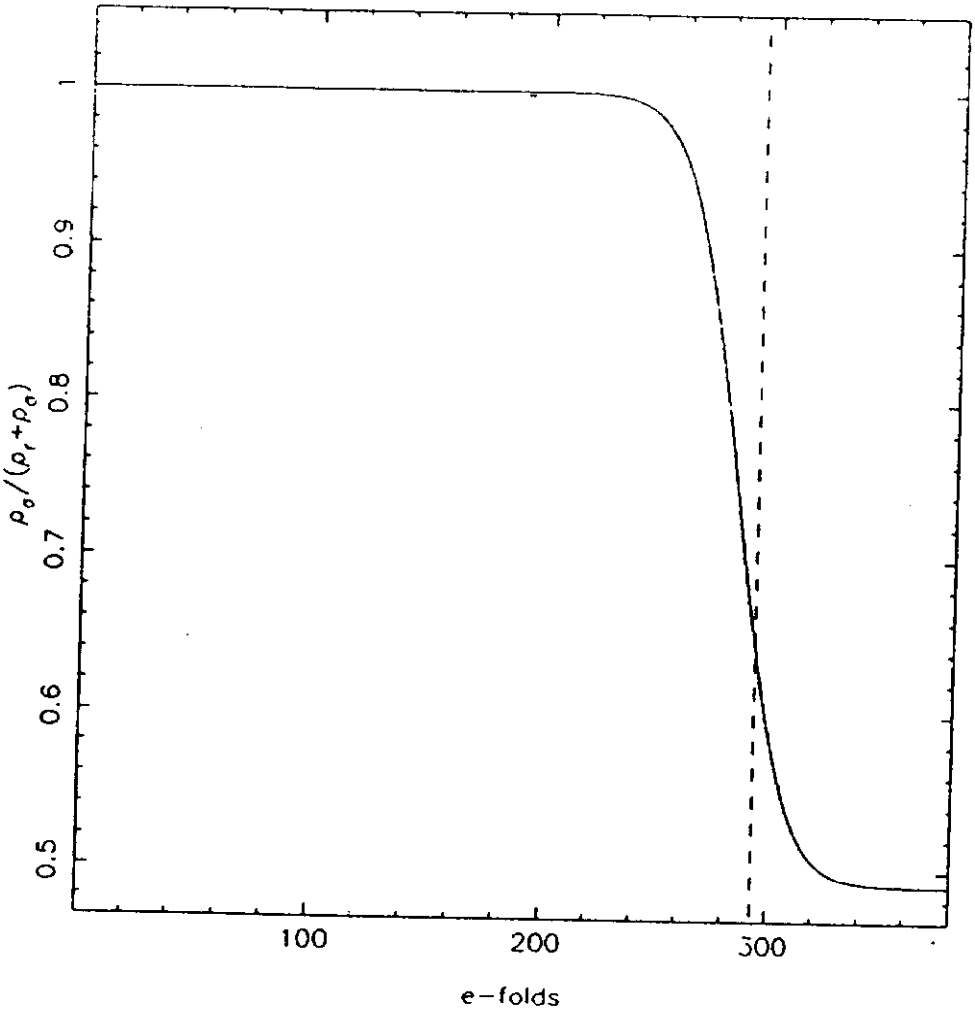

Fig. 3d

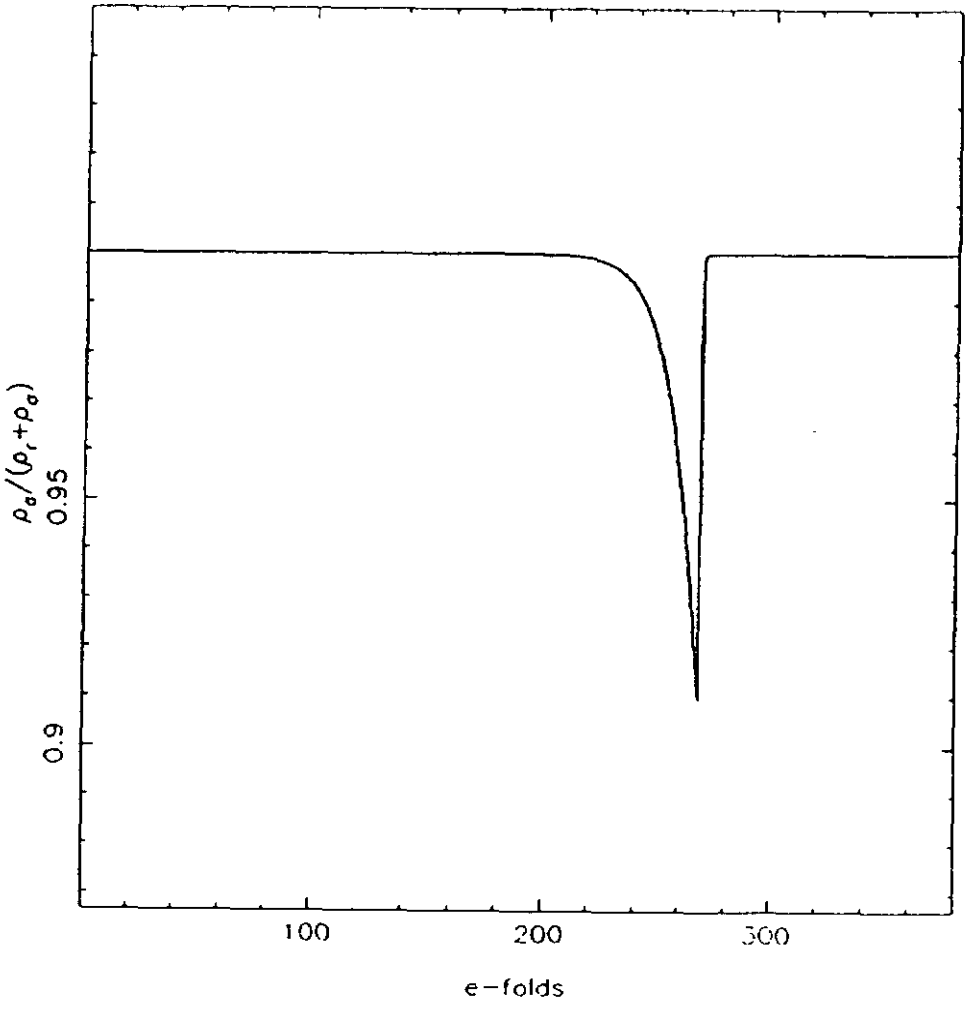

Fig 4a

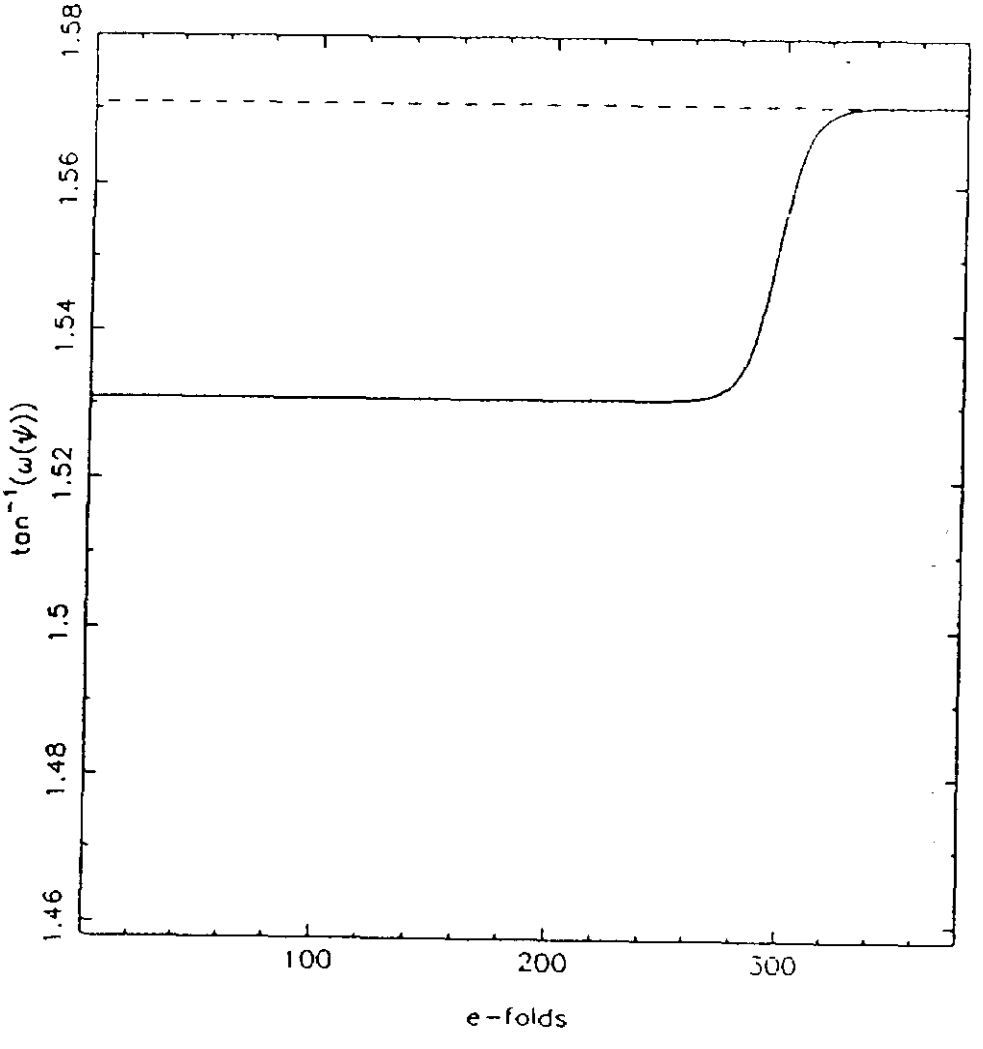

Fig. 4b.