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A MEASUREMENT OF THE QUADRATIC SLOPE PARAMETER IN THE $K_L \to 3\pi^0$ DECAY DALITZ PLOT

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ABSTRACT

We report a value of $-(3.3\pm1.1\pm0.7)\times10^{-3}$ for the quadratic slope parameter h in the Dalitz plot of the $K_L \to 3\pi^0$ decay, where the errors are statistical and systematic, respectively. This result is obtained from a sample of 5.1 million $3\pi^0$ decays from the Fermilab E-731 experiment's ϵ'/ϵ data sample. The validity of the $\Delta I=1/2$ rule in the quadratic term is investigated by comparing our result with a measurement of slope parameters in the charged kaon system. This is the first measurement of the $3\pi^0$ slope parameter, and also the most sensitive measurement to date of any of the quadratic slope parameters for the charged or neutral kaons.

A violation of the elusive $\Delta I=1/2$ rule that applies to strangeness-changing weak hadronic decays has been seen in the linear slope term of the $K \to 3\pi$ decay Dalitz plot, but no violation has so far been reported in the quadratic term[1]. A sensitive measurement of the $3\pi^0$ slope parameter together with the known slope terms of the 3π decays of the charged K probes the $\Delta I=1/2$ rule in the quadratic term. Any information on this topic may allow one to test and better determine the higher order coefficients in the chiral Lagrangian.

The $K_L \to 3\pi^0$ decay is especially suited for the determination of the quadratic term for two reasons. First, since the linear slope term is identically zero, the quadratic amplitude is not contaminated by the square of the linear term. In contrast, the square of the linear term is 3.2 times bigger than the quadratic term for the $K_L \to \pi^+\pi^-\pi^0$ decay[1]. Second, because there are no charged particles in the $K_L \to 3\pi^0$ decay, final state electromagnetic corrections are not required to measure the slope parameter.

An entry in the Dalitz plot for the $K_L \to 3\pi^0$ decay is specified in terms of the distance R from the center, and the angle θ defined by

$$R^2 = \frac{4}{m_{\pi^+}^4}[s_0^2 - \frac{s_1 s_2 + s_1 s_3 + s_2 s_3}{3}], \qquad (1)$$

$$tan(\theta) = \frac{1}{\sqrt{3}}[\frac{s_2 - s_1}{s_3 - s_0}], \qquad (2)$$

$$s_i = (P_K - P_i)^2; \qquad i = 1,2,3, \qquad (3)$$

where P_i is the observed four-vector of the i^{th} π^0, P_K is the four-vector of the kaon, and s_0 is the average of the three s_i's. The choice of m_{π^+} instead of m_{π^0} for normalization in Eqn. (1) makes this definition of R compatible with the Particle Data Group's[2] definition for $K \to 3\pi$ decays with at least one charged pion in the final state. Qualitatively, R is a dimensionless measure of the kinematic asymmetry in a given $K_L \to 3\pi^0$ decay. For example, if all the three π^0's have identical momenta, R is zero, but if one of the π^0's is at rest in the center-of-mass frame, R is maximal. At first glance, the definition of θ in Eqn. (2) appears to be asymmetric in that it distinguishes between the different π^0's. However, the symmetry is manifest when θ is confined between 0^0 and 60^0, as is appropriate for the $K_L \to 3\pi^0$ decay.

Writing $|M_{000}|^2 \sim 1 + hR^2$, where $|M_{000}|^2$ is the density of the $3\pi^0$ decays in the Dalitz plot, the dimensionless quantity h is the slope parameter. The linear term in R vanishes because of the identity of the final state particles. A positive (negative) value for the slope parameter would imply that asymmetric (symmetric) decays are favored.

There appear to be no ab-initio theoretical predictions for the $3\pi^0$ slope parameter in the literature. However, there are two independent global fits to measurements of Dalitz plot parameters for other K decays[3,4] which infer the value of the $3\pi^0$ slope parameter. The first[3] of the two analyses does not allow for the presence of $\Delta I=3/2$ amplitude in the quadratic slope parameters, and infers a value of $-(8.3\pm2.4)\times10^{-3}$ for the $3\pi^0$ slope parameter. However, a more recent analysis of similar nature[4] includes the $\Delta I=3/2$ contribution to the quadratic term, and concludes that the $3\pi^0$ slope parameter should be $-(1.2\pm4.0)\times10^{-3}$, consistent with zero.

The $3\pi^0$ data sample used in this analysis is from the Fermilab E731 ϵ'/ϵ experiment. In E731, two parallel K_L beams entered the decay volume, approximately 124 m downstream of the production target. A regenerator was placed in one of the beams at the start of the decay volume to produce K_S's. The energies and positions of the photons from the π^0 decays were measured with a 1.8m diameter electromagnetic (EM) calorimeter array of 804 lead-glass blocks. The gain of each block was tracked with a flash lamp system. The position resolution of the calorimeter was approximately 3 mm on average, and the energy resolution was $(1+5/\sqrt{E/GeV})\%$ for electrons. Photons, due to greater fluctuation in their conversion depth, had a 2% constant term in the resolution. Eleven planes of vetoes were used to reject photons outside the acceptance of the calorimeter. The $3\pi^0$ trigger was formed by demanding that there be six clusters together with at least 30 GeV energy deposit. A detailed description of the apparatus and data taking can be found elsewhere[5].

During the run, the E731 experiment wrote approximately 5000 nine-track magnetic data tapes. The data sample was split into several subsets to reflect differences in the running conditions. Besides the 2π modes necessary for the ϵ'/ϵ analysis and the $3\pi^0$ sample used here, the summary sample consisted of approximately 120 million Ke3 decays, 3.5 million $\pi^+\pi^-\pi^0$ decays, and 7.5 million calibration electrons. These high statistics samples were used for calibration, aperture determinations, and acceptance studies.

The understanding of the detector acceptance was crucial for the ϵ'/ϵ measurement. Hence, a large sample of $3\pi^0$'s was collected to check the understanding of the detector acceptance for the $2\pi^0$ decays. Also, the detector was simulated in detail, allowing only a small number of adjustable parameters. The simulation was judged by comparing many high statistics histograms for data with those for simulated events.

The reconstruction of a neutral decay used the photon energy and position information from the EM calorimeter. The response of the calorimeter to electrons was calibrated first with electrons from ten calibration samples taken throughout the run, and then with well-selected electrons from the large sample of Ke3 decays. The difference in response of the calorimeter to electrons and photons was understood with standalone EGS simulation, and with the information from the well-measured photons of the $\pi^+\pi^-\pi^0$ sample. The final overall energy scale and resolution adjustments for photons were made using the $K_S\to2\pi^0$ sample from the regenerator beam as follows. The regenerator, with its two interaction lengths of boron carbide followed by a sheet of lead and scintillator at the end, defined the start of the decay region in the K_S beam. Since the reconstructed value of the decay vertex (Z) depends linearly on the measured photon energy, the energy scale adjustment is obtained by matching the sharp edge in the Z distribution for the decay mode $K_S \to 2\pi^0$ at the location of the regenerator against the same edge for the simulated events. By the same procedure, matching the sharpness of the edge gave the final energy resolution adjustment. With this technique, a discrepancy as small as 0.05% in the energy scale clearly stood out. The same was true when a 1.0% fluctuation in the photon energy was introduced on top of the nominal photon resolution of the EM calorimeter. (Upon adding both contributions in quadrature, the 1.0% additional fluctuation represents approximately a 5% fractional change in the photon resolution.)

For the purposes of this analysis, the energy of each of the six photon clusters was required to be above 1.5 GeV, and the K energy to be between 40 and 160 GeV. Also, a subset of the data had a thin lead sheet approximately 14 m downstream of the regenerator. The K decays from this subset downstream of the lead sheet were not used since the gain in statistics was not enough to justify the increase in complexity of the simulation. The fiducial region extended 12 meters further downstream for the remaining subsets. To avoid the two overlapping photons, the cluster shape was required to be consistent with that of a single photon. To reduce the accidental activity, the number of hits in the drift chambers and energy deposited in the photon veto planes were required to be small. The photons were paired into candidate π^0's, and the π^0's were in turn used to reconstruct the K decay vertex (Z). All fifteen pairing possibilities for the $3\pi^0$ decays were considered, and the best pairing was picked based on the χ^2 for the hypothesis that all the π^0's and the K share the same vertex in Z and that the K/π mass ratio for the pairing be the same as the known value of the ratio. A small mispairing background was eliminated by requiring bad χ^2 for the second best pairing. Fig. (1) shows the K mass for the largest data subset.

A total of 5.3 million data events and 6.8 million simulated events passed all the selection criteria. The slope parameter was set to zero during simulation. Fig. (2a) shows the R^2 histograms superposed for the largest data subset and its simulation. Eqns. (1-3) show that the value of the (K/π) mass ratio determines the overall R^2 scale. To prevent the R^2 scale mismatch between the data and the simulation due to small residual non-linearities in the energy, the cluster energies were adjusted on an event-by-event basis to constrain the (K/π) mass ratio. The magnitude of these adjustments was typically 0.2%. After this correction, a small mismatch remained at the falling edge of the R^2 distribution, and was corrected by applying a 0.2% scale factor to R^2. As shown in the figure, the events near the edge were excluded in order to minimize the sensitivity to this scale factor. With this exclusion, the 0.2%

scale factor in R^2 changes the result by 1% of itself, which is negligible.

To evaluate the slope parameter h, the ratio of the data and simulation R^2 histograms in the fiducial region was fitted to a line as shown in Fig. (2b). Since the simulated events were generated with zero h, the slope in the linear fit gives the measurement of h. The χ^2 for the linear fit hypothesis to the ratio of the data and the simulation for the largest subset is 27.7 for 24 degrees of freedom, indicating a good agreement. The value of h averaged over all the subsets is $-(3.3\pm1.1) \times 10^{-3}$. The set to set variation in the value of h is consistent with statistical fluctuations, and the χ^2's for the linear fits are similar.

An important check for this measurement comes from verifying the agreement between data and the simulation for the Dalitz plot angle θ of Eqn. (2). Fig. (3) shows the θ histograms for the largest data subset. The χ^2 for the hypothesis that the ratio of data to simulation for θ is a constant is 53 for 59 degrees of freedom.

The data sample had no backgrounds to speak of, as seen from Fig. (1). Possible sources of systematic error were studied by subdividing the data in several ways. There were no appreciable effects due to the subdivisions based upon the kaon energy, decay vertex, photon cluster energies, pairing χ^2, distance of the clusters from the center of the EM calorimeter, K/π mass ratio, participation of the lead glass blocks near the beam pipes, etc. No appreciable change in the R^2 data versus simulation match was seen upon subdivision according to the Dalitz plot angle θ, and vice versa. The robustness of the result is not surprising given the uniformity of detector acceptance in R^2 - Fig. (2a) shows that the slope due to acceptance in the fiducial region is only 1.5% per R^2.

The largest change in the result arose when the energy resolution for the simulated events was adjusted. Due to effects such as calorimeter radiation damage, different data sets needed slightly different resolution adjustments. As described above, the shape of the regenerator edge in Z provided these small corrections. Amongst all the data sets, the maximum such adjustment was in the form of a 1.1% additional fluctuation of the photon energy. The resultant change of 0.69×10^{-3} in the slope parameter was taken to be the systematic error due to resolution. Since this resolution change is readily apparent in the vertex distribution, this systematic error is conservative. The second source of error was the energy scale, and a systematic error of 0.23×10^{-3} was obtained from a 0.2% energy scale change with analogous reasoning. The third and last systematic error of 0.11×10^{-3} was ascribed to the residual energy nonlinearities in the EM calorimeter. It was obtained by changing energy of each photon by 50 MeV, a change that was approximately five times as large as the uncertainty in the pedestal subtractions. The result was even less sensitive to other energy non-linearities. For example, the photon energies were modified with a power-law function of the energy for the entire calorimeter, and in a separate study for only selected regions of the calorimeter with a little change in the result. The result was also insensitive to several other non-linear distortions of this nature. The total systematic error due to the three sources above added in quadrature is 0.74×10^{-3} and is dominated by the resolution understanding.

The result for the Dalitz plot quadratic slope parameter h for the decay $K_L \rightarrow 3\pi^0$ from the Fermilab E731 experiment is $-(3.3\pm1.1\pm0.7) \times 10^{-3}$, where the errors are statistical and systematic, respectively. There is one work in progress[6] to calculate the $3\pi^0$ slope parameter h in the chiral Lagrangian framework. This group predicts a value of $-(1.2\pm0.4) \times 10^{-2}$ for h, which is almost four times larger than our result. However, given the large uncertainty in their estimate, the disagreement between the two values has only a two standard deviation significance. The source of disagreement is unclear at the present time.

In order to check the validity of the $\Delta I=1/2$ rule in the quadratic term, we combine our result with the linear and quadratic slope parameter values from the most sensitive charged K experiment[7] according to the isospin formalism of reference 5. We use the values from the experiment instead of the Particle Data Group averages since the latter are calculated separately for the K^+ and K^- decays, whereas the experimental values are calculated for the combined data set taking into account the significant correlations that exist. We obtain a value of (31±10)% for the $\Delta I=3/2$ to $\Delta I=1/2$ amplitude ratio ζ_3/ζ_1, where the contribution to the error from the charged K data is a little larger than that from our result. This atypically large value of ζ_3/ζ_1 suggests a violation of the $\Delta I=1/2$ rule in the quadratic term, and may provide a clue to the dynamical origin of the $\Delta I=1/2$ rule. However, this conclusion is subject to the theoretical uncertainties in the Coulomb corrections applied to the charged K data[7]. Further improvement in verifying the status of the $\Delta I=1/2$ rule in the quadratic term will require simultaneous improvements in the slope parameter measurements in both the neutral as well as the charged K sector, and the reduction of systematic uncertainties in the charged K sector.

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FIGURE CAPTIONS

1. The kaon mass calculated from the six photons of the $K_L \to 3\pi^0$ decays for the largest data subset, showing negligible background.

2. The superposition of Dalitz plot radius R^2 distribution for the data and simulated events for the largest subset. The histogram is data and the points are from the simulation. The arrow indicates the end of the fiducial region. Lower figure is the ratio of data and simulation distributions with a linear fit. Since the simulated events were generated with zero slope parameter, the observed slope is the $K_L \to 3\pi^0$ slope parameter.

3. The distribution in the angle θ in the Dalitz plot for the data and simulated events for the largest subset. The histogram is data and the points are from the simulation.

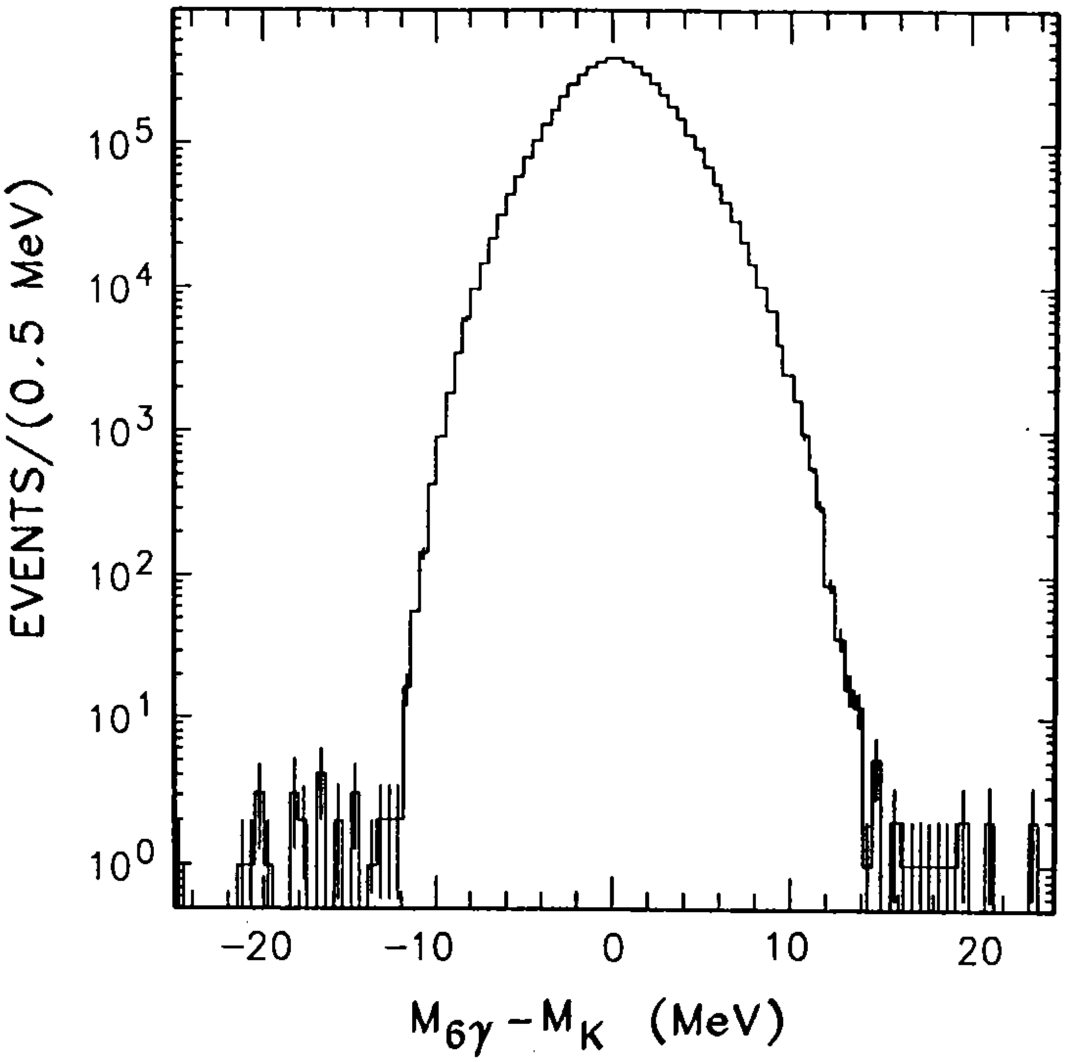
EVENTS/(0.5 MeV)
10^5
10^4
10^3
10^2
10^1
10^0
−20
−10
0
10
20
$M_{6\gamma} - M_K$ (MeV)

Fig. 1

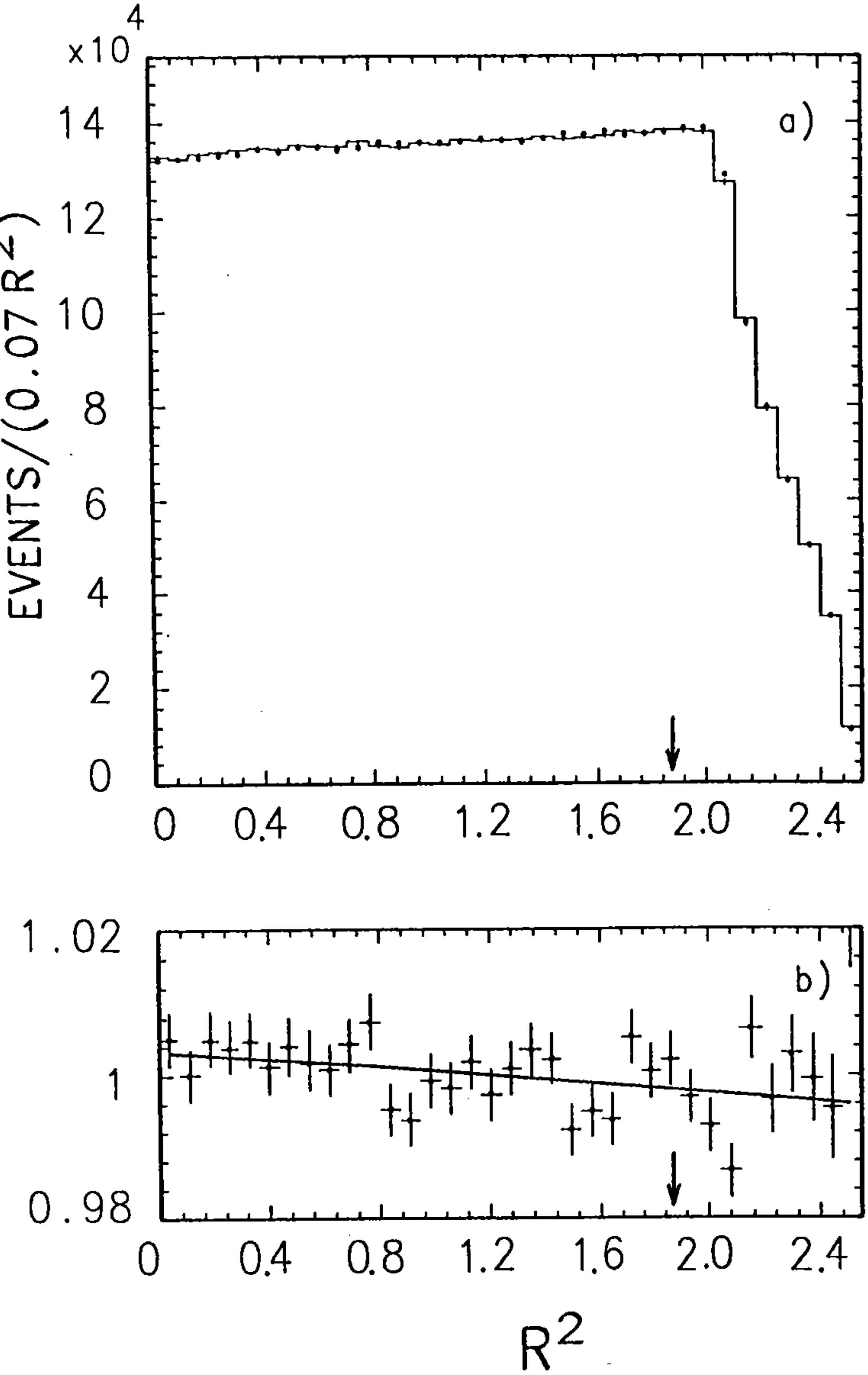
x10^4
14
12
10
8
6
4
2
0
EVENTS/(0.07 R^2)
a)
0
0.4
0.8
1.2
1.6
2.0
2.4
1.02
1
0.98
b)
0
0.4
0.8
1.2
1.6
2.0
2.4
R^2

Fig. 2

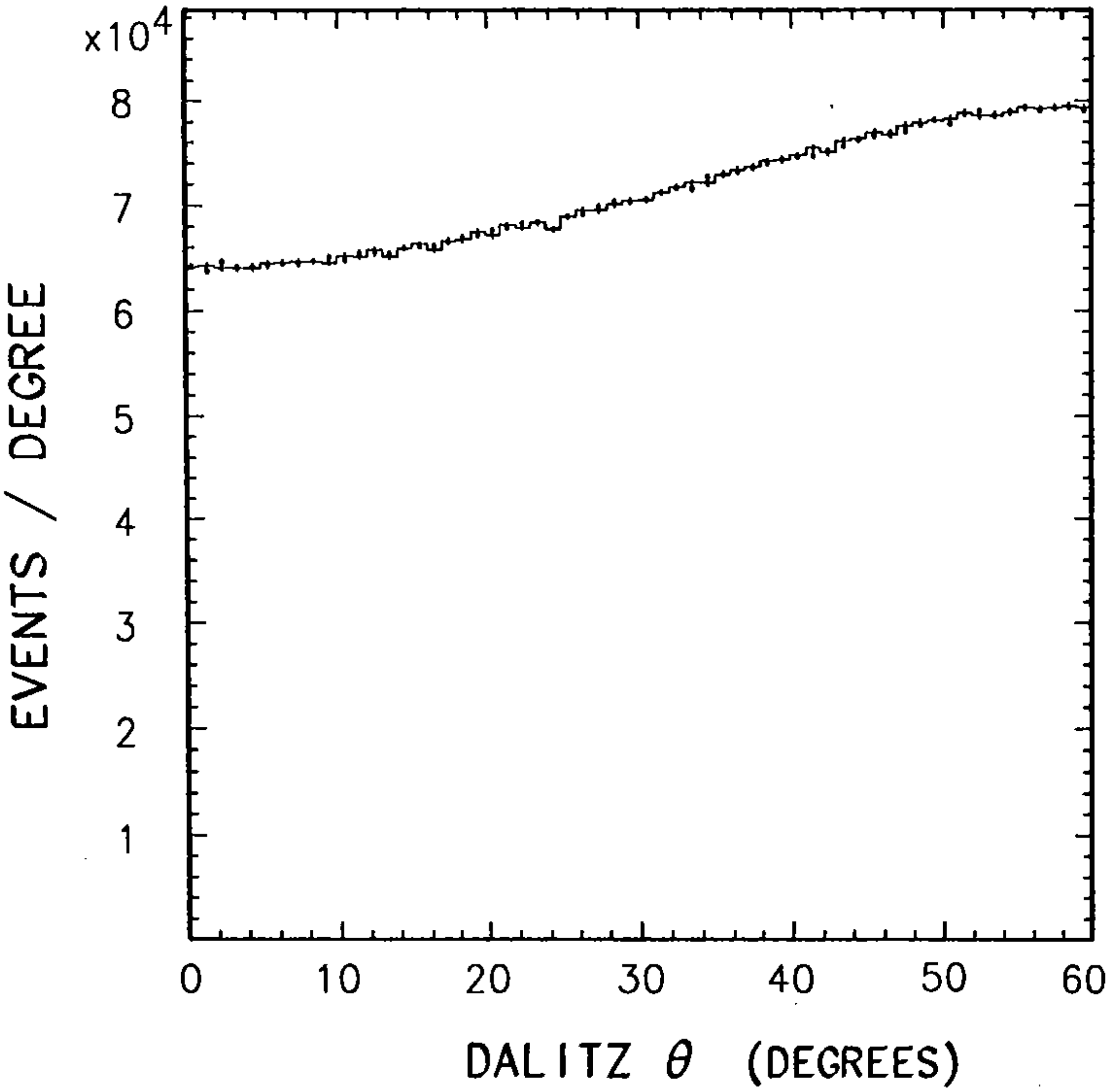
x10^4
EVENTS / DEGREE
8
7
6
5
4
3
2
1
0
10
20
30
40
50
60
DALITZ θ (DEGREES)

Fig. 3