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Microwave Anisotropies in the Light of *COBE*

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ABSTRACT

The recent COBE measurement of anisotropies in the cosmic microwave background and the recent South Pole experiment of Gaier *et al.* offer an excellent opportunity to probe cosmological theories. We test a class of theories in which the Universe today is flat and matter dominated, and primordial perturbations are adiabatic parameterized by an index n. In this class of theories the predicted signal in the South Pole experiment depends not only on n, but also on the Hubble constant and the baryon density. For $n = 1$ a large region of this parameter space is ruled out, but there is still a window open which satisfies constraints coming from COBE, measurements of the age of the Universe, the South Pole experiment, and big bang nucleosynthesis. Using the central values of the Hubble constant and baryon density favored by nucleosynthesis and age measurements, we find that, even if the COBE normalization drops by 1σ, $n > 1.2$ is ruled out.

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The recent detection[1] by the *COBE* satellite of anisotropies in the microwave background has important ramifications for the ongoing searches for anisotropies at smaller angular scales. In particular, the *COBE* measurement can be used to normalize the spectrum of primordial perturbations. This normalization, in any given theory, gives an unambiguous prediction for the magnitude of the anisotropy that should be detected in smaller scale experiments. Here we focus on models in which the Universe is flat and matter dominated and perturbations are adiabatic and ask: Does COBE's normalization of these theories imply that a signal should have been seen in the smaller scale Gaier[2] experiment? Although our results have been obtained by assuming cold dark matter (CDM), we expect similar results for hot dark matter or cold + hot dark matter because the Gaier experiment probes scales so large that neutrino free streaming is essentially irrelevant.

The South Pole experiment[2,3,4,5] consists of a beam at fixed zenith angle [$\theta_z = 27.75°$] oscillating back and forth in a given sky patch with period $1/\nu$. Thus the position of the beam is determined by its azimuthal angle: $\phi(t) = \phi_A \sin(2\pi\nu t)$; here $\phi_A \sin\theta_z = 1.5°$. When the beam gets halfway across patch, the "sign" of the signal changes, so that the expected signal is

$$\delta T = 4\nu \int_{-\phi_A}^{\phi_A} \frac{d\phi}{(d\phi/dt)} S(\phi) T(\theta_z, \phi) \tag{1}$$

where S is either plus or minus one depending on the angle and T is the temperature. It is customary to expand the temperature in multipole moments so that $T = T_0(1 + \sum_{l,m} a_{lm} Y_{lm})$, where T_0 is the observed mean temperature of the cosmic microwave background, $2.735°K$, and the a_{lm} are Gaussian random variables. If many measurements are made, the mean value of the a_{lm} should be zero, but with a variance given by $< a^*_{lm} a_{l'm'} >= C_l \delta_{l,l'} \delta_{m,m'}$. After squaring Eq. (1) and inserting these relations, we see that a cosmological theory which predicts a set of C_l's predicts a variance in the Gaier experiment:

$$< \left(\frac{\delta T}{T_0} \right)^2_{th} >= \sum_{l=2}^{\infty} \frac{C_l}{4\pi} (2l+1) W_l. \tag{2}$$

Here the filter function is

$$W_l = \exp \left\{ -(l+.5)^2 \theta_s^2 \right\} \frac{16\pi}{2l+1} \sum_{m=-l}^{l} H_0^2(m\phi_A) Y_{lm}^2(\theta_z, 0) \tag{3}$$

where $\theta_s = 0.425 \times 1.35°$ represents the width of the beam and H_0 is the Struve function of order 0. This filter function peaks at $l \sim 70$ and falls off significantly so that the the contribution from modes greater than $l \sim 250$ is negligible. The Gaier experiment made measurements over nine such patches in each of four frequency channels.

To compare a given cosmological theory with the Gaier experiment, therefore, we must ask it for the C_l's. For the adiabatic, matter dominated models under consideration, generating the C_l's is straightforward[6]: (i) perturb the Einstein and Boltzmann equations about the standard zero order solutions [Robertson-Walker metric; homogeneous and isotropic distributions of photons, neutrinos, ordinary matter, and dark matter][7,8,9]; (ii) Fourier transform these equations after which the perturbations are functions of wavenumber k, time t, and, in the case of photons and neutrinos, the angle between the wavenumber and momentum; (iii) Expand the perturbations to the photons and neutrinos in terms of Legendre polynomials so that the angular dependence, $\Delta(\mu)$, is replaced by the coefficients,

Δ_l; (iv) Evolve these perturbed quantities starting from initial conditions deep in the radiation era: $\delta\rho/\rho(k, t_{\rm init}) \propto k^{n/2}$ where $n = 1$ for the Harrison-Zel'dovich spectrum predicted by inflation; (v) Determine the C_l's today by integrating $C_l \propto \int d^3k |\Delta_l(t_0)|^2$. The proportional signs in the previous two sentences show that these theories do not fix the normalization. That is, there is no prediction for a given C_l; however the ratio C_l/C_2 is unambiguously determined. Therefore, the predicted signal in the Gaier experiment, $< \delta T_{th}^2 >$, depends on only one parameter C_2, or equivalently the quadrupole $Q[= \sqrt{5C_2/4\pi}T_0]$.

Let us take the quadrupole as a free parameter. Then in a given patch we can construct the probability density of a given measurement $[\delta T_{obs} \pm \sigma]$:

$$P(\delta T_{obs}|Q) = \left[2\pi(\sigma^2 + < \delta T_{th}^2(Q) >)\right]^{-1/2} \exp\left\{\frac{-\delta T_{obs}^2}{< \delta T_{th}^2(Q) > + \sigma^2}\right\}. \qquad (4)$$

Naively, if this probability density is significantly lower at a value of Q than it is at its maximum, then we can confidently rule out that particular value of Q. The Gaier experiment has nine patches, so the nine probability densities must be multiplied together to form the *likelihood function*[10]. In fact, things are a little more complicated than this because the nine patches are close to each other [in fact they overlap somewhat], so that the expected signals in the nine patches are correlated. We have included cross-correlations amongst the different patches; this is a straightforward extension of the above[4,11].

Figure 1 shows the likelihood as a function of Q for several different values of the Hubble constant $[H_0 = 100h \text{ km sec}^{-1} \text{ Mpc}^{-1}]$ and baryon density $[\Omega_B$ is the ratio of the baryon density to the critical density]. While $Q = 0$, corresponding to no signal, is the most likely value, clearly values of Q up to about 10μK are allowed. It is also clear that values of Q greater than about 20μK are ruled out. With this range in mind, we note that the COBE-inferred value of Q is[12] $15 \pm 3\mu$K.

Is $Q = 15\mu$K "ruled out" by the Gaier experiment? One way to answer this question[13] is to perform a Bayesian analysis assuming a uniform prior[4]. All this means here is we ask what fraction of the area under the likelihood curve is taken up by $Q > 15$. For $\Omega_B = .05; h = .05$, this fraction is only 4%, so we say that the theory is ruled out at the 96% confidence level. However, this number becomes significantly less impressive as the COBE normalization is lowered. $Q = 12$, which is allowed by COBE at the one sigma level is "ruled out" with only 91% confidence.

Until now we have ignored the dotted line in Fig. 1; the solid lines were drawn using only the highest frequency channel from the Gaier experiment. The lower three channels had larger signals [i.e. larger average values of $|\delta T|$]. The dotted line in Fig. 1 shows what the likelihood function would look like if all four channels were included in the analysis. We see that the most likely value of the quadrupole is about 9μK and no signal, or $Q = 0$, is ruled out on the basis of the four channel data! The difference between analyzing all four channels of data and analyzing only the highest channel is immense: either we say that COBE normalized CDM is on the verge of being ruled out OR there has been a detection at roughly the level expected. The team analyzing the data ran extensive spectral tests and concluded that there is only a 2% probability that the signal in the low channels is cosmic microwave background. [Other contributions, such as Bremsstrahlung and synchrotron radiation, fall off as the frequency increases so the highest channel should be least contaminated by them.] We have run a similar test[14] and also find that the probability that the signal in the four channels is pure cosmic background is very low. So we will follow Gaier *et al.* and consider only the highest channel of data in our analysis.

The three solid lines in Fig. 1 make it clear that we lied when we claimed that the signal expected from CDM depends only on the normalization. Clearly it depends on two other parameters as well, h and Ω_b. Figure 2 shows the allowed region of parameter space for $Q = 15\mu$K, the central value of COBE.

There are two physical effects which lead to the shape of this contour plot. The first effect relates to the imperfect coupling between photons and baryons prior to decoupling. If the coupling were perfect, the intrinsic photon fluctuations would be maximal, and the the anisotropy in the photon temperature would be quite large. Since the coupling is not perfect, photons can diffuse out of perturbations, damping the temperature anisotropy [i.e. the perturbations undergo Silk damping[15]]. Therefore, the weaker the interactions between photons and matter, the smaller is the final photon anisotropy. We know though that the interaction rate increases as the amount of matter increases. Since the matter density scales as $\Omega_b h^2$, we expect the intrinsic anisotropy to *increase*[16] as h increases for fixed Ω_b. This effect shows up at the high h end of Fig. 2, where even relatively low values of Ω_b are ruled out. The second effect depends not on the matter, but rather on the gravitational field through which the photons travel before they reach us. If h is small, the Universe was *not* purely matter dominated since the surface of last scattering. The epoch at which the energy density in matter equals that in radiation comes closer to the epoch of last scattering as h decreases, so that for at least part of the photons' flight to us, the gravitational potential was not constant. This leads to an additional contribution to the anisotropy and hence a larger signal. This explains why for h less than 1/2 or so, even small values of Ω_b are ruled out.

Also shown in Fig. 2 is the allowed region in (h, Ω_b) space from primordial nucleosynthesis considerations[17]. One might combine the allowed BBN regime with the regime favored by measurements of the age of the Universe [e.g. restricting the age to be greater than 10 billion years corresponds to $h < .65$] and direct measurements of h [which all observers would agree is greater than .4]. While the Gaier experiment rules out a large part of the (h, Ω_b) plane, it does *not* rule out this "favored" region of $h \sim .6$ and $\Omega_b \sim .03$.

What happens if the primordial spectrum differs from the Harrison-Zel'dovich spectrum predicted by inflation? Or, perhaps more to the point, What limits do microwave anisotropy experiments place on the spectral index of primordial perturbations? Figure 2 shows the values of the normalization Q and spectral index n allowed by COBE and the South Pole experiment. Large n corresponds to more power on small scales and hence a larger predicted signal on angular scales probed by Gaier, *et al.* Hence, the only way to reconcile the absence of a signal in the Gaier experiment with large n is if the normalization Q is small. For $n > 1.2$ the upper limit on Q is smaller than the region favored by COBE.

To sum up our results: (i) The signal in medium scale anisotropy experiments depends not only on the assumed shape and normalization of the primordial spectrum but also on the Hubble constant and the baryon density; (ii) For CDM-like theories, the Gaier *et al.* experiment, together with the normalization provided by COBE, rules out a large region of the (h, Ω_b) parameter space; (iii) There is still a window open which satisfies constraints coming from COBE, measurements of the age of the Universe, the Gaier experiment, and big bang nucleosynthesis; (iv) COBE and the Gaier experiment rule out values of the primordial spectral index $n > 1.2$.

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REFERENCES

1. G. F. Smoot, *et al.* Astrophys. J. Lett. **396**, L1 (1992).
2. T. Gaier, *et al.* Astrophys. J. Lett. **398**, L1 (1992).
3. P. Meinhold and P. Lubin, Astrophys. J. Lett. **370**, 11 (1991); P. Lubin, P. Meinhold, and A. Chingcuanco in *The Cosmic Microwave Background 25 Years Later*, edited by N. Mandolesi and N. Vittorio (Kluwer,Dodrecht, 1990).
4. J. R. Bond, G. Efstathiou, P. M. Lubin, and P. R. Meinhold, Phys. Rev. Lett. **66**, 2179 (1991).
5. For another analysis of the South Pole data see K. M. Gorski, R. Stompor, and R. Juszkiewicz, YITP preprint 92-36 (1992).
6. S. Dodelson and J. M. Jubas (in preparation). Many of the methods we have used are taken from J. R. Bond and G. Efstathiou, Mon. Not. Roy. Astron. Soc. **226** 655 (1987). For a clear review, see G. Efstathiou in

Physics of the Early Universe, edited by J. A. Peacock, A. F. Heavens, and A. T. Davies (Edinburgh University Press, Edinburgh, 1990).

7. P. J. E. Peebles and J. T. Yu, Astrophys. J. **162**, 815 (1970).
8. M. L. Wilson and J. Silk, Astrophys. J. **243**, 14 (1981).
9. J. R. Bond and A. S. Szalay, Astrophys. J. **274**, 443 (1983).
10. For a clear discussion of likelihoods, see A. C. S. Readhead, *et al.*, Astrophys. J. **346**, 566 (1989), section VIII.
11. N. Vittorio, P. R. Meinhold, P. F. Muciaccia, P. M. Lubin, and J. Silk, Astrophys. J. Lett. **372**, 1 (1991).
12. The error here includes the effects of cosmic variance. For a clear discussion of how to normalize using COBE data, see G. Efstathiou, J. R. Bond, and S. D. M. White, Oxford preprint (1992); F. C. Adams, J. R. Bond, K. Freese, J. A. Frieman, and A. V. Olinto, Phys. Rev. D (in press), section IV.
13. Since χ^2 for this data set is not small [5 for 6 degrees of freedom], we expect most other statistical tests to give similar answers. See Ref. 10.
14. Specifically we found that the χ^2 for the best fit with 27 degrees of freedom is 46. Gaier, *et al.* assumed a Gaussian correlation function while we used the full correlation function induced by CDM.
15. J. Silk, Astrophys. J. **151**, 459 (1968).
16. At any given time, the Silk damping scale is of order $(n_e\sigma_T H)^{-1/2}$. At decoupling then, when the free electron density $n_e \propto n_b^{1/2}$, the Silk damping scale [and hence the magnitude of this effect] remains constant if $\Omega_b h^4$ remains constant.

17. S. Yang, M. S. Turner, G. Steigman, D. N. Schramm, and K. A. Olive, Astrophys. J. **281**, 493 (1984); T. P. Walker, G. Steigman, D. N. Schramm, K. A. Olive, and H. Kang, Astrophys. J. **376**, 51 (1991).

FIGURE CAPTIONS

1) The likelihood function for the South Pole experiment using all four channels (dotted line) and only one channel (solid lines). The likelihood function has been normalized so that it is equal to 1 at its peak.

2) Constraints from the South Pole experiment on h, Ω_B assuming $n = 1$. The region above the dashed line is ruled out at the 95% confidence level if COBE normalization is used ($Q = 15\mu$K). The region allowed by Big Bang Nucleosynthesis is bounded by solid lines.

3) Combined constraints on spectral index n and quadrupole Q from COBE and Gaier *et al.* COBE allows the region between the solid lines [from a combination of their sky noise at 10° and the full correlation function]. The Gaier experiment rules out the region above the dashed [short-dashed] line at the 95(68)% confidence level. Here we have set $h = .5$ and $\Omega_b = .05$.

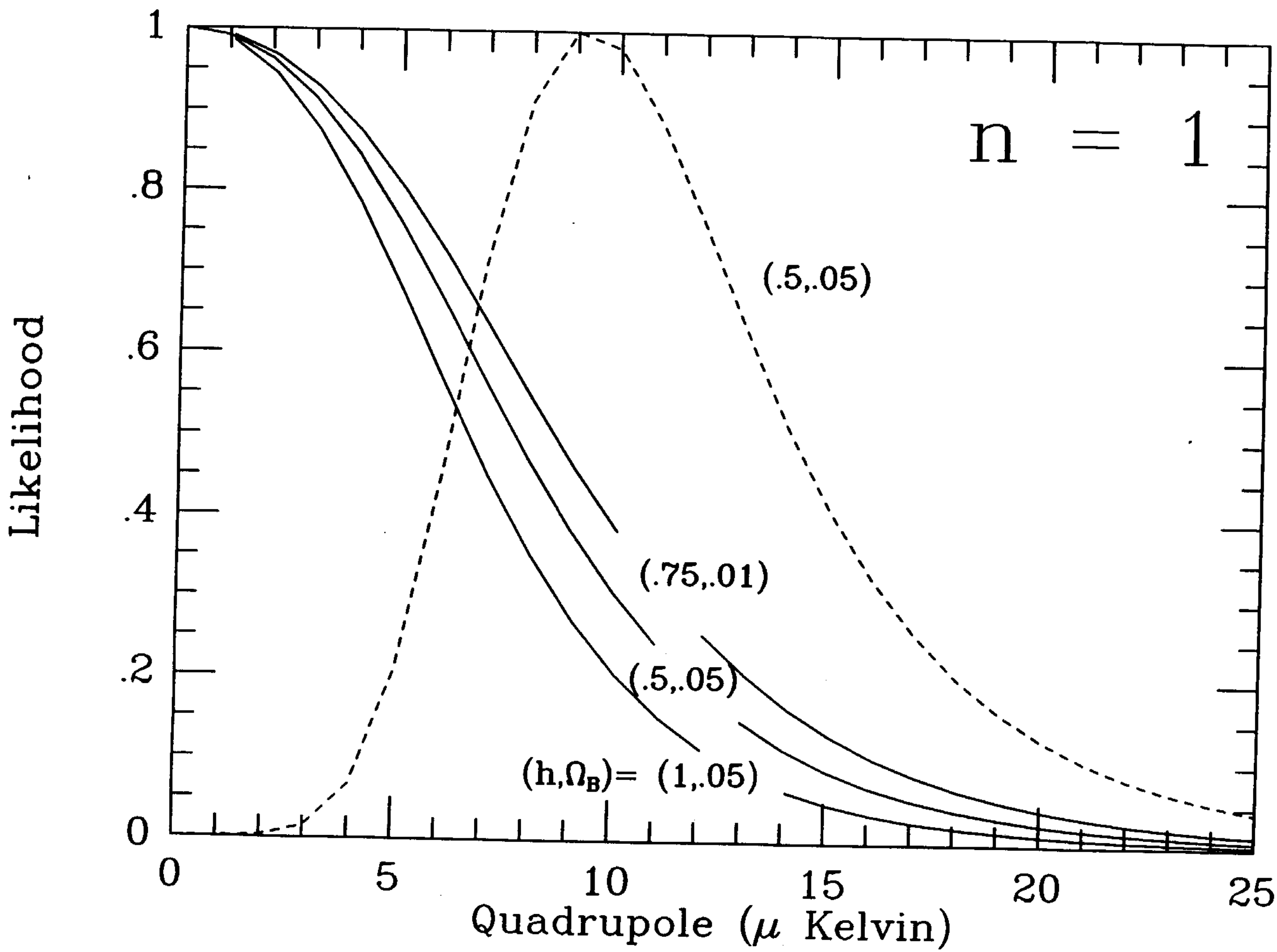
Likelihood
1
.8
.6
.4
.2
0
n = 1
(.5,.05)
(.75,.01)
(.5,.05)
(h,Ω_B)= (1,.05)
0
5
10
15
20
25
Quadrupole (μ Kelvin)

Fig 1

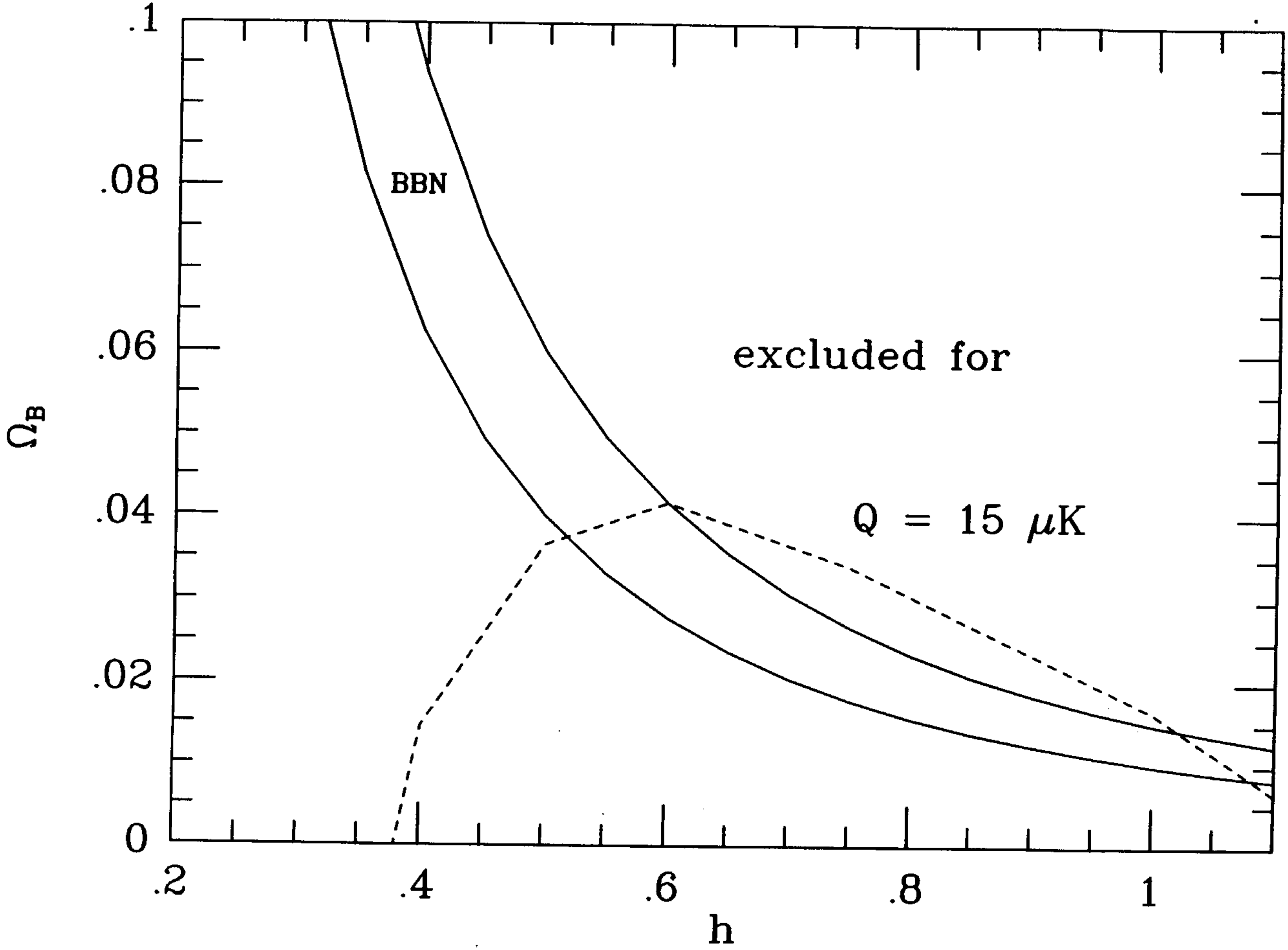

Fig 2

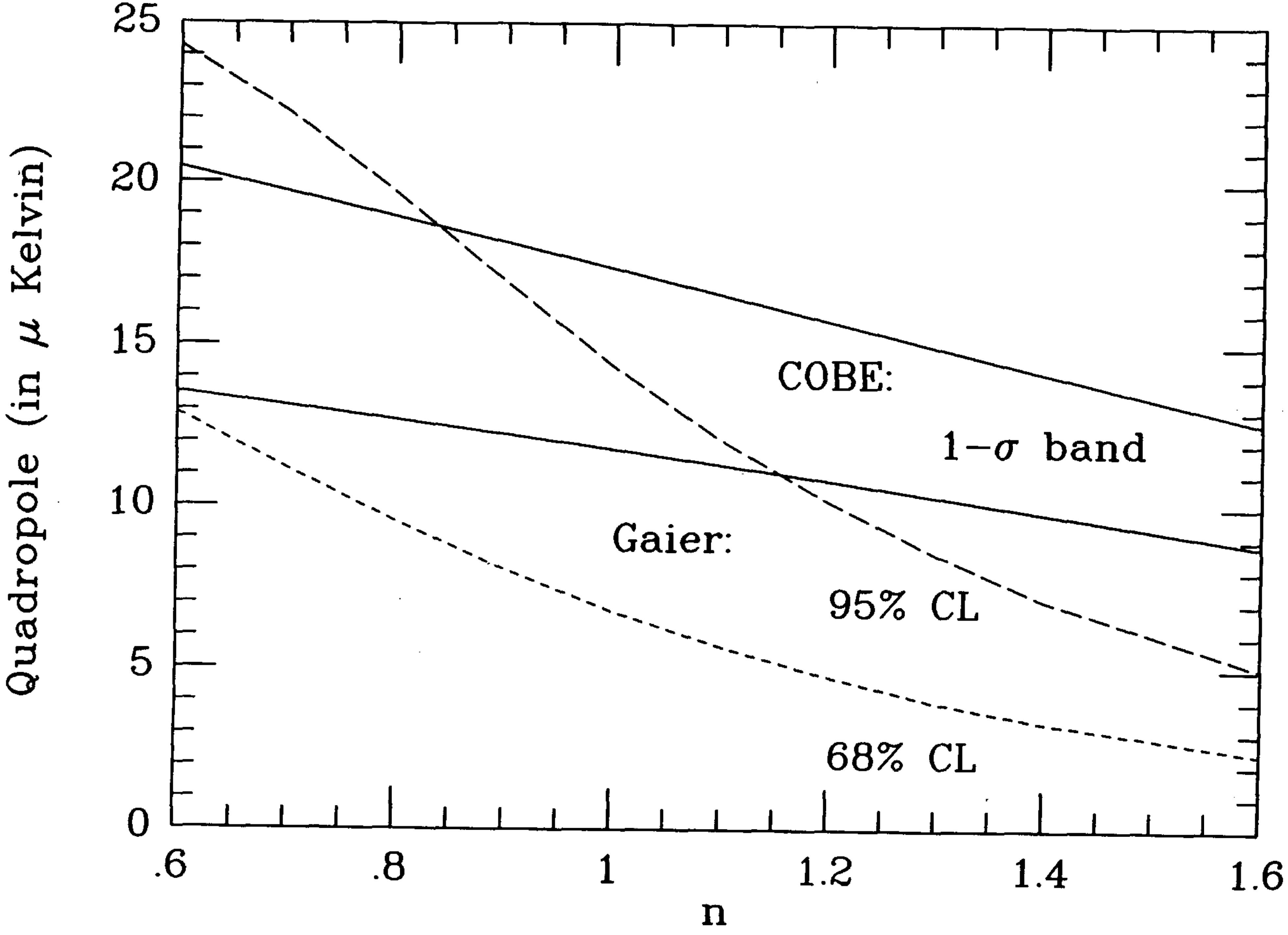
Quadropole (in μ Kelvin)
25
20
15
10
5
0
.6
.8
1
1.2
1.4
1.6
n
COBE:
1−σ band
Gaier:
95% CL
68% CL

Fig 3