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Spontaneously Broken Technicolor and the Dynamics of Virtual Vector Technimesons

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Abstract

We propose spontaneously breaking technicolor, thus liberating techniquarks and suppressing large resonance contributions to the electroweak S parameter. The dynamics is modeled by a fermion bubble approximation to a single massive technigluon exchange potential. This contains a Nambu–Jona-Lasinio model with additional interactions. "Virtual" vector mesons occur and contribute to S, and their effects are studied. Models of broken technicolor are discussed.

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I. Introduction

Technicolor is the collective name for a class of models attempting to explain electroweak symmetry breaking by condensates of fermion-antifermion pairs, driven by a new strong interaction [1]. All versions of technicolor (TC) imitate, more or less, the known dynamics of QCD chiral symmetry breaking. The technicolor interaction is assumed to be an unbroken, and thus confining, strong interaction amongst techniquarks, which also carry electroweak quantum numbers. The resulting chiral condensate of techniquarks then breaks the electroweak gauge symmetry. The extension of the theory to give the fermions masses [2] is made somewhat awkward by the twin constraints of flavor-changing neutral current processes and the large top quark mass. Implementing these constraints in extended technicolor (ETC) leads to additional model building requirements, such as a walking technicolor coupling [3] and critical or subcritical extended technicolor [4]. The mass of the top quark can then be generated either by a "subcritical amplification" involving tuning of the ETC four-fermion interactions near to, but below, the critical value, or by generating techniquark and top quark condensates together, through ETC effects.

There is a second difficulty which appears to imply that technicolor theories are no longer viable. Techni-resonances such as the techni-ρ and techni-A_1, arising from confinement of techniquarks, produce large contributions to the electroweak radiative S parameter, a measure of isosinglet electroweak symmetry breaking at the loop level [5,6]. In a QCD-like theory these can be treated by resonance saturation of superconvergence relationships, such as the Weinberg sum rules [6]. Recently, precise measurements of electroweak interactions have placed a new constraint on the S parameter, currently at 90% (95%) to < -0.1 (0.0) [7], relative to the minimal Standard Model with the Higgs mass equal to the Z boson mass. In technicolor theories treated as scaled-up QCD-like theories, we have:

$$S \simeq (0.10) N_{TC} N_{TD} + 0.13, \tag{1}$$

where the two indices are the number of technicolors and electroweak technidoublets, respectively [6]. In walking technicolor theories,

$$S \simeq (0.11a - 0.07b)N_{TC}N_{TD} + 0.13, \tag{2}$$

where a, b are constants of order unity [8]. The final contribution in both cases comes from taking the Higgs mass to $\sim$ 1 TeV. By comparison, the perturbative form of S from fermion loops alone [5] is:

$$S = N_{TC}N_{TD}/6\pi \approx (0.05)N_{TC}N_{TD}. \tag{3}$$

If technicolor theories are to be viable, their contribution to S must be minimized somehow. In the present article, we wish to present an alternative realization of technicolor as a spontaneously broken gauge theory and study to what extent this can suppress techni-resonance contributions to the S parameter. In such a theory, the effective coupling constant must be just large enough to drive the formation of electroweak condensates, but the technicolor interaction kept short-range, approximately s-wave and not confining. While scalar bound states such as the Nambu-Goldstone bosons are formed, the unconfined technifermions do not form the offending vector resonances which give large contributions to the coefficients appearing in S. There are, however, "virtual" resonance poles formed at a scale above the breaking scale. The effects of these virtual resonances upon S must be included. We see below, in a large-N analysis, how the intuition of a suppressed S in spontaneously broken technicolor is achieved. While we find that there is suppression, S never falls below the usual result for free fermion loops, within the domain of validity of our analysis.

The proposal of a spontaneously broken technicolor (SBTC) is not really so radical in light of other recent ideas about electroweak symmetry breaking. Another alternative for dynamical electroweak symmetry breaking is to abandon new technifermions and suppose that the top quark itself forms a condensate and acts alone as a techniquark [9]. A preliminary gauge form of this dynamics has been given [10], and the requirement that the electroweak ρ parameter be approximately unity forces some radical fine-tuning: either (a) the scale of new physics Λ is very large, e.g., 10^{15}

GeV and the fine-tuning very severe for the effective quadratic interactions, or (b) supersymmetry must be invoked, or (c) $m_{top} >> 200$ GeV and an *ad hoc* cancellation must be arranged against the large positive top quark contribution to ρ, such as a new heavy Z' or a new broken $SU(2)_V$. These problems disappear if a fourth generation is invoked [11]. In these cases, the number of degrees of freedom contributing to S is minimized. Moreover, in these models we are dealing with a new strong interaction which is itself broken and does not confine, but which is sufficiently strong to be near or beyond criticality.

We cannot solve strong coupling models without resort to some approximation, such as the leading-N fermion bubbles, or ladder approximations. Our view of the dynamics is that a given SBTC gauge theory is described by a gauge group, G_{SBTC}. The theory must be asymptotically free at high energies. If we turn off the breaking mechanism, then it becomes a confining theory with an infrared confinement scale Λ_{SBTC}. However, we imagine that some mechanism intervenes to break $G_{SBTC} \rightarrow G'$ at a scale $\Lambda' \geq \Lambda_{SBTC}$. We assume to start off that G' is a null group (that is, complete breaking; we could generalize to have unbroken subgroups that contain $U(1)$ factors or are infrared free). Thus with G' null, all of the technigluons acquire a mass $M \sim g(M)\Lambda'$, where $g(\mu)$ is the G_{SBTC} coupling and $g(M) \sim \mathcal{O}(1-10)$.

At the scale M we can integrate out the gauge bosons and replace the gauge interactions with four-fermion effective interactions. Thus at and below this scale we have a Nambu–Jona-Lasinio (NJL) model [12]. Our criterion for the symmetry breaking of the theory we take for the sake of simplicity to be the usual fermion bubble, Nambu–Jona-Lasinio result, that a chiral symmetry breaking scale or mass gap m be spontaneously generated for sufficiently strong coupling. We then study the effects of the virtual resonances ρ and A_1 in the model in leading order of $1/N_{TC}$. This may be viewed as a model calculation which seeks to understand directly the virtual resonance contributions to S without use of the Weinberg sum rules. We find that the resonance contribution to S is always positive, and in the limit of tuning a large hierarchy between M and the chiral beaking scale m, we recover the usual free fermion result.

II. A Simple Model of the Dynamics

Consider an SBTC theory, with a single fermionic flavor isodoublet ($N_{TD} = 1$, $N_{TF} = 2$) of the form $\psi = (U, D)$ which also carries an $SU(N_{TC})$ color index in the fundamental representation. We do not presently concern ourselves with anomalies. We assume that the $SU(N_{TC})$ local gauge theory is broken to a global $SU(N_{TC})$ with N_{TC} colors. (Our discussion can be generalized to $N_{TD} > 1$, and other breaking schemes are mentioned in section IV.) We can write the effective current-current form of the fermion interaction Lagrangian due to single gluon exchange of momentum transfer $q^2 << M^2$ as:

$$\mathcal{L}_{\rm int} = -\frac{g^2}{M^2}\bar{\psi}\gamma_\mu\frac{\lambda^A}{2}\psi\bar{\psi}\gamma^\mu\frac{\lambda^A}{2}\psi. \tag{4}$$

where the λ^A are the $SU(N_{TC})$ generators. Upon Fierz rearrangment, keeping only leading terms in $1/N_{TC}$, this interaction takes the form:

$$\begin{aligned}\mathcal{L}_{\rm int} \ = \ & \frac{g^2}{M^2}\Big(\bar{\psi}_L\psi_R\bar{\psi}_R\psi_L + \bar{\psi}_L\tau^a\psi_R\bar{\psi}_R\tau^a\psi_L \\ & -\frac{1}{8}\bar{\psi}\gamma_\mu\tau^a\psi\bar{\psi}\gamma^\mu\tau^a\psi - \frac{1}{8}\bar{\psi}\gamma_\mu\gamma_5\tau^a\psi\bar{\psi}\gamma^\mu\gamma_5\tau^a\psi \\ & -\frac{1}{8}\bar{\psi}\gamma_\mu\psi\bar{\psi}\gamma^\mu\psi - \frac{1}{8}\bar{\psi}\gamma_\mu\gamma_5\psi\bar{\psi}\gamma^\mu\gamma_5\psi\Big)\,. \end{aligned} \tag{5}$$

where $\psi_L = (1-\gamma_5)\psi/2$, $\psi_R = (1+\gamma_5)\psi/2$. Here τ^a are Pauli matrices acting upon the flavor isospin indices. Notice the $SU(2)_L \times SU(2)_R$ invariance of this interaction, where the full chiral group has an $SU(2)$ vector custodial subgroup. The first two terms are Nambu–Jona-Lasinio interactions, and M^2 plays the role of the cutoff. (This can be rigorously checked by comparing arguments of logs and finite corrections in various amplitudes; it is generally found that M^2 can be identified with the NJL cutoff with small residual corrections.) The vector-vector and axial-vector–axial-vector terms generate the resonance contributions to the S parameter.

We begin by demanding that the theory produce a vacuum condensate at low-energy, or equivalently, a dynamically induced effective fermion mass. The most general induced mass term can be taken to be:

$$m\bar{\psi}\psi + \delta m\bar{\psi}\tau^3\psi. \tag{6}$$

The fermion mass is generated self-consistently by its own self-energy in the NJL sector of the theory. This consists of the first two terms of the *rhs* of eq.(5). In the leading–N approximation, only the NJL interactions can contribute to the mass gap. The resulting gap equations for m and δm are most conveniently written in terms of $m_\pm = m \pm \delta m$:

$$m_\pm = \frac{g^2(M^2)N_{TC}}{8M^2\pi^2}\left[m_+(M^2 - m_+^2\ln(M^2/m_+^2+1)) + m_-(M^2 - m_-^2\ln(M^2/m_-^2+1))\right]. \tag{7}$$

We therefore see that $\delta m = 0$, and the symmetric mass m satisfies the NJL gap equation:

$$m = \frac{g^2(M^2)N_{TC}}{4M^2\pi^2} m\left[M^2 - m^2\ln(M^2/m^2+1)\right]. \tag{8}$$

We ignore the trivial solution $m = 0$. Thus, the condition that a chiral condensate form in the theory at scale M is the usual condition in the Nambu–Jona-Lasinio model:

$$g^2(M^2) = 4\pi^2\eta/N_{TC} \geq 4\pi^2/N_{TC}, \tag{9}$$

or $\eta \geq 1$, insofar as the large–N limit is valid. For marginally critical coupling, $\eta \to 1^+$ and $m \to 0$; while for large η, $m \to \infty$. We implicitly assume $m \leq M$ for an effective field theory, so that $\eta \lesssim 3.3$. We also note that the argument of the logarithms $(1 + M^2/m^2)$, can be replaced by M^2/m^2, since upon expandi ng, the corrections are higher order in m^2/M^2, and we have already truncated operator corrections of this order in writing eq.(4). This implicitly requires m to be small compared to M and η not much larger than unity. Note that the result $\delta m = 0$ is an example of the Vafa–Witten theorem [13], that vector symmetries cannot be dynamically broken. In this case, $\delta m = 0$ preserves the vector custodial SU(2).

Let us argue now on general theoretical grounds that there can exist a spontaneously broken, asymptotically free, theory that is unconfining yet sufficiently strong to form a chiral condensate. If we take $g^2(\mu)$ to be the running coupling constant on scales $\mu >> M$,

$$g^2(\mu) = \frac{8\pi^2}{b_0 \ln(\mu/\Lambda_{SBTC})}, \tag{10}$$

then, if we assume $g^2(M)$ is marginally critical, we have the ratio of M to Λ_{SBTC} given by:

$$\frac{M}{\Lambda_{SBTC}} = \exp(2N_{TC}/b_0) \tag{11}$$

where

$$b_0 = \frac{11N_{TC}}{3} - \frac{2N_{TF}}{3}. \tag{12}$$

Therefore, we can, by judicious choice of N_{TC} and N_{TF}, make the ratio M/Λ_{SBTC} arbitrarily large. This includes a "walking theory" in which $b_0 \sim 0$, or $N_{TF} \sim 11N_{TC}/2$. For N_{TC} very large, we then expect the Nambu–Jona-Lasinio approximation to the chiral dynamics to be very good. Of course, this merely demonstrates that such an SBTC theory can exist as a matter of principle, while our specific model results are expected to be less reliable.

Since the SBTC condensate breaks the electroweak gauge symmetry, there are two charged and one neutral Nambu–Goldstone bosons [12,14] which become the longitudinal W and Z bosons respectively. The decay constants of these Goldstone bosons are denoted by $f_W(p^2)$ and $f_Z(p^2)$ respectively, being generally functions of momentum transfer p^2. The decay constants occur in writing the vacuum polarization tensors for two-point functions of electroweak currents. Let us define:

$$\Pi_{\mu\nu}(p^2) = (g_{\mu\nu} - p_\mu p_\nu/p^2)\Pi(p^2). \tag{13}$$

Then we have (in the convention of writing kinetic terms for gauge fields as $(-1/4g^2)F_{\mu\nu}F^{\mu\nu}$, e.g. see ref.[9]):

$$\Pi_\pm(p^2) \;=\; \left(\frac{1}{g_2^2}p^2 - f_W^2\right)$$

$$\Pi_{33}(p^2) = \left(\frac{1}{g_2^2}p^2 - f_Z^2\right)$$
$$\Pi_{3Q}(p^2) = \left(\frac{1}{g_2^2}p^2\right) \qquad (14)$$

where g_2 is the $SU(2)_L$ gauge coupling. These are left-handed current-current two point functions, with $\frac{1}{2}(1-\gamma^5)$ projections. Since custodial $SU(2)$ is unbroken, we have $m_+ = m_-$ and $\Pi_\pm = \Pi_{33}$, whence:

$$f_W^2 = f_Z^2 = \Pi_{33} - \Pi_{3Q} \qquad (15)$$
$$= -\frac{1}{4}[\Pi_{33}^{VV} - \Pi_{33}^{AA}], \qquad (16)$$

where use has been made of $Q = I_{L3} + \frac{Y}{2}$, and VV (AA) are vector (axial vector) two-point functions, and the factor of $1/4$ arises from the $\frac{1}{2}$ in the left-handed projections, $\frac{1}{2}(1-\gamma^5)$.

Let us make some preliminary comments about the physical meaning of the S, T and U parameters. In general, we can expand $f_W^2(p^2)$ and $f_Z^2(p^2)$ in a Taylor series in p^2:

$$f_W^2(p^2) = f_0^2 + \frac{1}{2}\sigma p^2 + \frac{1}{2}\tau f_0^2 + \frac{1}{2}\omega p^2 + \ ... \qquad (17)$$
$$f_Z^2(p^2) = f_0^2 + \frac{1}{2}\sigma p^2 - \frac{1}{2}\tau f_0^2 - \frac{1}{2}\omega p^2 + \ ... \ . \qquad (18)$$

f_0 is just the Higgs vacuum expectation value in the standard model. We use a normalization in which $f_W^2(0) = 1/4\sqrt{2}G_F$, or $f_W(0) \approx 123$ GeV. Furthermore, note that the (isospin-breaking) τ parameter is just a rewriting of the ρ parameter, since $\rho = f_W^2(0)/f_Z^2(0)$. The parameters σ and ω are the isospin-conserving and isospin-breaking measures respectively of physics contributing to the p^2 evolution of the low-energy effective theory. The p^2 expansion about zero is strictly valid only for heavy contributions to the $f_X^2(p^2)$, because singularities would occur for, e.g., massless neutrinos which give $\sim \ln p^2$ terms in σ, τ and ω. However, the physical electroweak observables actually depend on the $f_X(p^2)$, not on their derivatives, and the $f_X(p^2)$ are not singular, only their derivatives are at $p^2 \to 0$.

With the conventional definitions of S, T and U [6]:

$$S = 16\pi \frac{\partial}{\partial p^2} \left[\Pi_{33} - \Pi_{3Q}\right]_{p^2=0} \tag{19}$$

$$T = \frac{4\pi}{\sin^2\theta\cos^2\theta M_Z^2} \left[\Pi_\pm - \Pi_{33}\right]_{p^2=0} \tag{20}$$

$$U = 16\pi \frac{\partial}{\partial p^2} \left[\Pi_\pm - \Pi_{33}\right]_{p^2=0}, \tag{21}$$

we can obtain the following relations to the decay constant parameters introduced above:

$$\sigma = \frac{2S+U}{16\pi}, \qquad \omega = \frac{U}{16\pi}, \qquad \tau = \frac{\sin^2\theta\cos^2\theta M_Z^2 T}{4\pi v_r^2} = \frac{\alpha}{2} T. \tag{22}$$

Note that in the limit of exact custodial $SU(2)$ symmetry, $T = U = 0$, and S is equivalent to σ and parameterizes the p^2 evolution of f^2 in the theory.

Let us now focus upon the quantity:

$$f^2(p^2) = -\frac{1}{4}\left[\Pi_{33}^{VV} - \Pi_{33}^{AA}\right], \tag{23}$$

defined in terms of the vector and axial-vector current correlators. First we compute Π_{33}^{VV}:

$$\begin{aligned}
[\Pi_{33}^{VV}](p^2)_{\mu\nu} &= i\langle 0|\, T\, \bar\psi\gamma_\mu \frac{\tau^3}{2}\psi\; \bar\psi\gamma_\nu \frac{\tau^3}{2}\psi\, |0\rangle \\
&= \frac{N_{TC}}{4\pi^2}\int_0^1 dx \left[x(1-x)(g_{\mu\nu}p^2 - p_\mu p_\nu)\right] \\
&\qquad \times \ln\left\{M^2/(m^2 - x(1-x)p^2)\right\}.
\end{aligned} \tag{24}$$

Note that this expression is transverse, as it should be, owing to the conserved vector current (CVC), $p^\mu \bar{\psi}\gamma_\mu(\tau_3/2)\psi = 0$. Now we compute $\Pi^{\mu\nu}_{AA}$:

$$
\begin{aligned}
[\Pi^{AA}_{33}](p^2)_{\mu\nu} &= i\langle 0|T\ \bar{\psi}\gamma_\mu\gamma_5\frac{\tau^3}{2}\psi\ \bar{\psi}\gamma_\nu\gamma_5\frac{\tau^3}{2}\psi\,|0\rangle \\
&= \frac{N_{TC}}{4\pi^2}\int_0^1 dx\left[x(1-x)(g_{\mu\nu}p^2 - p_\mu p_\nu) - g_{\mu\nu}m^2\right] \\
&\qquad \times \ln\left\{M^2/(m^2 - x(1-x)p^2)\right\}. \qquad (25)
\end{aligned}
$$

This expression is not transverse, owing to the dynamical symmetry breaking ($m \neq 0$). However, if we now sum the leading large–N_{TC} effects of the NJL interactions, and make use of the gap equation, we form the Goldstone pole in the usual way, and then the full amplitude becomes transverse:

$$
\begin{aligned}
[\tilde{\Pi}^{AA}_{33}](p^2)_{\mu\nu} &= \frac{N_{TC}}{4\pi^2}(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2})\int_0^1 dx\left[x(1-x)p^2 - m^2\right] \\
&\qquad \times \ln\left\{M^2/(m^2 - x(1-x)p^2)\right\}. \qquad (26)
\end{aligned}
$$

These expressions contain the usual pure Nambu–Jona-Lasinio result for the decay constant:

$$
\begin{aligned}
f^2(p^2) &= -\frac{1}{4}\left[\Pi^{VV}_{33} - \tilde{\Pi}^{AA}_{33}\right] \\
&= \frac{N_{TC}}{16\pi^2}\int_0^1 dx(m^2)\ln[M^2/(m^2 - x(1-x)p^2)]. \qquad (27)
\end{aligned}
$$

It is instructive to write the expressions for general $m_\pm$ [9]:

$$
\begin{aligned}
f^2_{Z0}(p^2) &= \frac{1}{2}N_{TC}m_+^2(4\pi)^{-2}\int_0^1 dx\ln[M^2/(m_+^2 - x(1-x)p^2)] \\
&\quad + \frac{1}{2}N_{TC}m_-^2(4\pi)^{-2}\int_0^1 dx\ln[M^2/(m_-^2 - x(1-x)p^2)] \\
f^2_{W0}(p^2) &= N_{TC}(4\pi)^{-2}\int_0^1 dx(xm_+^2 + (1-x)m_-^2) \\
&\qquad \times \ln[M^2/(xm_+^2 + (1-x)m_-^2 - x(1-x)p^2)]. \qquad (28)
\end{aligned}
$$

We write f_{W0}^2 and f_{Z0}^2 to denote that these quantities are obtained in the NJL approximation. Expanding in p^2 and extracting the coefficients we find for the S parameter:

$$S_0 \;=\; \frac{N_{TC}}{6\pi}\left[1-\frac{1}{3}\ln\left\{m_+^2/m_-^2\right\}\right], \tag{29}$$

while T is, modulo an overall factor, just the usual Veltman expression for $\delta\rho$:

$$T_0 \;=\; \frac{N_{TC}}{4\pi\sin^2\theta\cos^2\theta M_Z^2}\left[m_+^2+m_-^2-\frac{2m_+^2m_-^2}{(m_+^2-m_-^2)}\ln(m_+^2/m_-^2)\right]. \tag{30}$$

This is the standard result for free fermion loops. Hence, a spontaneously broken technicolor produces in the NJL approximation the usual free fermion loop result for S. We note that for $m_+ = m_-$, $T = U = 0$.

Next, we include the additional vector-vector and axial-vector–axial-vector terms of the full interaction Lagrangian. These are generated when one performs bubble sums of the vector-vector and axial-vector–axial-vector terms. The full bubble sum of the vector-vector interaction yields:

$$[\overline{\Pi}_{33}^{VV}](p^2)_{\mu\nu} \;=\; \frac{N_{TC}}{4\pi^2}(g_{\mu\nu}-\frac{p_\mu p_\nu}{p^2}) \tag{31}$$
$$\times\frac{\int_0^1 dx\, x(1-x)p^2\ln\left\{M^2/(m^2-x(1-x)p^2)\right\}}{\left[1-(GN_{TC}/4\pi^2)\int_0^1 dx\, x(1-x)p^2\ln\left\{M^2/(m^2-x(1-x)p^2)\right\}\right]},$$

where $G = g^2(M^2)/M^2$.

We also sum the full axial-vector amplitudes (note that this is a double summation, of those interactions producing the Goldstone pole together with the axial-vector–axial-vector vertices):

$$[\overline{\Pi}_{33}^{AA}](p^2)_{\mu\nu} \;=\; \frac{N_{TC}}{4\pi^2}(g_{\mu\nu}-\frac{p_\mu p_\nu}{p^2}) \tag{32}$$
$$\times\frac{\int_0^1 dx\,(x(1-x)p^2-m^2)\ln\left\{M^2/(m^2-x(1-x)p^2)\right\}}{\left[1-(GN_{TC}/4\pi^2)\int_0^1 dx\,(x(1-x)p^2-m^2)\ln\left\{M^2/(m^2-x(1-x)p^2)\right\}\right]}.$$

We now observe that the theory generates *virtual* vector meson poles. We refer to these as *virtual* resonances because the poles occur at $p^2 > M^2$, which is beyond the domain of validity of the effective theory. For $p^2 > M^2$, there are only quasi-free technifermions and no bound states. Nonetheless, the effects of these analogue resonances are real on scales $p^2 < M^2$, and they give a nontrivial result for S.

We can consider the ratio of the denominators of eqs. (31) and (32) at $p^2 = 0$ as a definition of the ratio of the off-shell mass-squares of the vector (ρ) and axial-vector (A_1) resonances, $m_\rho^2/m_{A_1}^2$:

$$\frac{m_\rho^2}{m_{A_1}^2} = \frac{M_0^2}{M_0^2 + m^2 \ln(M^2/m^2)} \tag{33}$$

where $M_0^2 = 4\pi^2/GN_{TC}$. We note that:

$$\eta = \frac{N_{TC} G M^2}{4\pi^2} = M^2/M_0^2. \tag{34}$$

Recall that $\eta = 1$ corresponds to the critical coupling and that condensation requires $\eta \geq 1$. The gap equation then states:

$$1 = \eta \left[1 - \frac{m^2}{M^2} \ln(M^2/m^2) \right] \tag{35}$$

so we obtain:

$$\frac{m_\rho^2}{m_{A_1}^2} = \frac{1}{\eta}. \tag{36}$$

Our model does not give the usual Weinberg sum rule result for QCD, that this ratio is one-half, nor should it. We do not have real ρ and A_1 resonances, and we cannot argue that $\Pi_{VV} - \Pi_{AA}$ is saturated by resonances. We should not expect the present model, which captures the features of chiral symmetry breaking dynamics, to be applicable in general to QCD. In particular, the local interaction does not confine. Thus the model cannot be taken as very accurate for the higher (e.g., vector) states of QCD. On the other hand, the four-fermion interaction eqs.(4,5) is *exact* in the broken case, subject only to the low-energy and large-N approximations. So we should expect the results to be reasonable for broken technicolor.

Now we compute S:

$$\begin{aligned} S &= -4\pi \frac{\partial}{\partial p^2}\left[\overline{\Pi}_{VV} - \overline{\Pi}_{AA}\right]_{p^2=0} \\ &= \frac{N_{TC}}{6\pi}\left[1 + (\ln(M^2/m^2) - 1)(1 - \frac{1}{\eta^2})\right] \end{aligned} \tag{37}$$

where we note that the $\ln(M^2/m^2)$ is a function of η by the gap equation (8, 35). This relation is plotted in Figure 1, and we discuss it in the Conclusions. Notice that when we tune the theory near to criticality, $\eta \to 1^+$, and S reduces to the conventional free fermion loop result, $N_{TC}/6\pi$. The model calculation becomes unreliable as the logarithm becomes small, because the mass gap m approaches the cutoff M. Notice that with $m < M$ the parameter S does not drop below the free fermion loop value.

III. Effective Lagrangian Approach

It is instructive to analyze the virtual vector meson effects by way of an effective action approach. We rewrite the four-fermion interaction Lagrangian by introducing auxiliary fields which correspond to Higgs, techni-ρ and techni-A_1 states. These auxiliary fields are non-dynamical at the scale M, having no kinetic terms, but they become dynamical fields from the effect of the fermion loops as we evolve the effective action to scales $\mu < M$. We cannot argue that these are real propagating fields since the energy scale of the poles for these virtual resonance states is as large as the cutoff scale M. But in computing S one is interested in the low-energy effect of the virtual resonance kinetic terms, and one can reliably obtain the correct answer in this approach. Hence, in this section we briefly explain how to obtain S in the effective action method. We proceed in two steps. First, we introduce auxiliary fields to rewrite the Lagrangian. By integrating out the techni-fermion degrees of freedom, we obtain the effective action for the Higgs, techni-ρ, techni-A_1, and $SU(2)_L \times U(1)_Y$ gauge fields W_μ and B_μ. In the second step, we eliminate the techni-ρ and techni-A_1 using the equation of motion. In this way we obtain the low-energy effective Lagrangian for the $SU(2)_L \times U(1)_Y$ gauge fields. The S parameter is easily read off the W_μ^3 - B_μ mixing term in the low-energy effective Lagrangian.

The full Lagrangian of SBTC, after introducing auxiliary fields, is:

$$\begin{aligned}\mathcal{L} &= \bar{\psi}\left(\gamma^\mu D_\mu - \Phi\frac{1+\gamma^5}{2} - \Phi^\dagger\frac{1-\gamma^5}{2}\right)\psi \\ &\quad -\frac{M^2}{2g^2}\,\mathrm{tr}(\Phi\Phi^\dagger) - \frac{M^2}{g^2}\,\mathrm{tr}(V_\mu V^\mu + A_\mu A^\mu), \end{aligned} \tag{38}$$

where:

$$D_\mu = i\partial_\mu - V_\mu - A_\mu\gamma^5 - g_1 B_\mu\frac{1+\gamma_5}{2} - g_2 W_\mu\frac{1-\gamma_5}{2}. \tag{39}$$

For the sake of brevity we write

$$\begin{aligned}\Phi &\equiv \sum_{\alpha=0}^{3}\Phi^\alpha\frac{\tau^\alpha}{2}, \qquad V_\mu \equiv \sum_{\alpha=0}^{3} V_\mu^\alpha\frac{\tau^\alpha}{2}, \qquad A_\mu \equiv \sum_{\alpha=0}^{3} A_\mu^\alpha\frac{\tau^\alpha}{2}, \\ W_\mu &\equiv \sum_{a=1}^{3} W_\mu^a\frac{\tau^a}{2}, \qquad B_\mu \equiv B_\mu\frac{\tau^3}{2}, \end{aligned} \tag{40}$$

where $\tau^0 = \mathbf{1}$. g_1 and g_2 are respectively the $U(1)_Y$ and $SU(2)_L$ coupling constants.

It is convenient to make a shift of variables as follows:

$$\begin{aligned} V_\mu &\longrightarrow V_\mu - \frac{g_1}{2}B_\mu - \frac{g_2}{2}W_\mu \\ A_\mu &\longrightarrow A_\mu - \frac{g_1}{2}B_\mu + \frac{g_2}{2}W_\mu. \end{aligned} \tag{41}$$

After shifting the fields and integrating out the fermion degrees of freedom, the effective action becomes:

$$I = -iN_{TC}\,\mathrm{Tr}\,\ln\left(i\gamma^\mu D_\mu - \Phi\frac{1+\gamma^5}{2} - \Phi^\dagger\frac{1-\gamma^5}{2}\right)$$

$$+\int d^4x\ \left(-\frac{M^2}{2g^2}\,\mathrm{tr}(\Phi\Phi^\dagger) - \frac{M^2}{g^2}\,\mathrm{tr}(V_\mu - \frac{g_1}{2}B_\mu - \frac{g_2}{2}W_\mu)^2 + \mathrm{tr}(A_\mu - \frac{g_1}{2}B_\mu + \frac{g_2}{2}W_\mu)^2\right). \tag{42}$$

We obtain the kinetic term, cubic interaction term, etc., of the effective action by expanding the above expression in terms of the fields around the vacuum. The vacuum expectation value of $\Phi = m$ satisfies the gap equation:

$$4N_{TC}\int \frac{d^4p}{(2\pi)^4}\frac{1}{p^2-m^2} = \frac{M^2}{g^2}. \tag{43}$$

Using the auxiliary field method, the leading term in the $1/N_{TC}$ expansion is obtained by carrying out the fermion one-loop calculation. The kinetic term which is relevant to calculating the S parameter is:

$$\begin{aligned} I_{kin} = & \int \frac{d^4q}{(2\pi)^4} \\ & \Big[\mathrm{tr}(W_\mu(q)W_\nu(-q))(-q^2g^{\mu\nu}+q^\mu q^\nu) \\ & + \mathrm{tr}(B_\mu(q)B_\nu(-q))(-q^2g^{\mu\nu}+q^\mu q^\nu) \\ & + \mathrm{tr}(V_\mu(q)V_\nu(-q))(-q^2g^{\mu\nu}+q^\mu q^\nu)c_1(q^2) \\ & + \mathrm{tr}(A_\mu(q)A_\nu(-q))((-q^2g^{\mu\nu}+q^\mu q^\nu)c_1(q^2)+g^{\mu\nu}c_2(q^2)m^2) \\ & + \frac{M^2}{g^2}(\mathrm{tr}(V_\mu - \frac{g_1}{2}B_\mu - \frac{g_2}{2}W_\mu)^2 + \mathrm{tr}(A_\mu - \frac{g_1}{2}B_\mu + \frac{g_2}{2}W_\mu)^2)\Big]. \end{aligned} \tag{44}$$

Here:

$$\begin{aligned} c_1(q^2) &= \frac{N_{TC}}{(2\pi)^2}\int_0^1 dx\, x(1-x)\ln[M^2/(m^2-x(1-x)q^2)] \\ c_2(q^2) &= \frac{N_{TC}}{(2\pi)^2}\int_0^1 dx\, \ln[M^2/(m^2-x(1-x)q^2)]. \end{aligned} \tag{45}$$

We have used the unitary gauge so that the Nambu–Goldstone bosons are already abosorbed into the axial–vector fields. The kinetic terms for Higgs and charged scalars are omitted because they are irrelevant to the S parameter.

In order to obtain the low–energy effective action, one has to eliminate techni-ρ and techni-A_1 by using the equations of motion. The solutions of the equations of motion for the previous action are,

$$
\begin{aligned}
V_\mu^a &= \left(\frac{-q^2 g_{\mu\nu} + q_\mu q_\nu}{q^2 - m_\rho^2} + g_{\mu\nu}\right)\frac{g_2}{2}W^{a\nu} \\
A_\mu^a &= -\frac{m_\rho^2}{m_{A_1}^2}\left(\frac{-q^2 g_{\mu\nu} + q_\mu q_\nu}{q^2 - m_{A_1}^2} + g_{\mu\nu}\right)\frac{g_2}{2}W^{a\nu},
\end{aligned} \tag{46}
$$

for $a = 1, 2$, and:

$$
\begin{aligned}
V_\mu^3 &= \left(\frac{-q^2 g_{\mu\nu} + q_\mu q_\nu}{q^2 - m_\rho^2} + g_{\mu\nu}\right)\left(\frac{g_1}{2}B^\nu + \frac{g_2}{2}W^{3\nu}\right) \\
A_\mu^3 &= -\frac{m_\rho^2}{m_{A_1}^2}\left(\frac{-q^2 g_{\mu\nu} + q_\mu q_\nu}{q^2 - m_{A_1}^2} + g_{\mu\nu}\right)\left(\frac{g_1}{2}B^\nu - \frac{g_2}{2}W^{3\nu}\right),
\end{aligned} \tag{47}
$$

where:

$$
\begin{aligned}
m_\rho^2 &= \frac{N_{TC}M^2}{c_1 g^2} \\
m_{A_1}^2 &= \frac{c_2 m^2}{c_1} + m_\rho^2.
\end{aligned} \tag{48}
$$

Substituting this expression into eq.(44),

$$
\begin{aligned}
I_{kin} &= \frac{1}{2}\int \frac{d^4 q}{(2\pi)^4} \\
&\Big[W_\mu^a W_\nu^a [(-q^2 g^{\mu\nu} + q^\mu q^\nu) - \left(\frac{g_2^2 c_1}{4}\right)\frac{m_\rho^2}{q^2 - m_\rho^2}(-q^2 g^{\mu\nu} + q^\mu q^\nu) \\
&+ \left(\frac{g_2^2}{4}\right)\frac{m_\rho^2}{m_{A_1}^2}(c_2 m^2 g^{\mu\nu} + c_1\frac{m_\rho^2}{q^2 - m_{A_1}^2}(-q^2 g^{\mu\nu} + q^\mu q^\nu))]
\end{aligned}
$$

$$+B_\mu B_\nu[(-q^2 g^{\mu\nu} + q^\mu q^\nu) - \left(\frac{g_1^2 c_1}{4}\right) \frac{m_\rho^2}{q^2 - m_\rho^2}(-q^2 g^{\mu\nu} + q^\mu q^\nu)$$

$$+ \left(\frac{g_1^2}{4}\right) \frac{m_\rho^2}{m_{A_1}^2}(c_2 m^2 g^{\mu\nu} + c_1 \frac{m_\rho^2}{q^2 - m_{A_1}^2}(-q^2 g^{\mu\nu} + q^\mu q^\nu))]$$

$$-\frac{g_1 g_2}{2} B_\mu W_\nu^3 [\frac{m_\rho^2}{q^2 - m_\rho^2} c_1(-q^2 g^{\mu\nu} + q^\mu q^\nu)$$

$$+\frac{m_\rho^2}{m_{A_1}^2}(c_2 m^2 g^{\mu\nu} - c_1 \frac{m_\rho^2}{q^2 - m_{A_1}^2}(-q^2 g^{\mu\nu} + q^\mu q^\nu))\Big] . \quad (49)$$

Now we can compute the S parameter. From eq.(19) the S parameter can be written as:

$$S = -16\pi \Pi'_{3Y}(0), \quad (50)$$

where Π_{3Y} is just the coefficient of the $g_1 g_2 B_\mu W^{3\mu}$ term in the effective action. We therefore have:

$$\Pi_{3Y} = \frac{1}{4}\left[\frac{m_\rho^2}{q^2 - m_\rho^2} c_1 q^2 - \frac{m_\rho^2}{m_{A_1}^2}(c_2 m^2 + \frac{m_\rho^2}{q^2 - m_{A_1}^2} c_1 q^2)\right] . \quad (51)$$

It can also be shown that the first term is Π_{VV} and the second term is $-\Pi_{AA}$.

Substituting this expression and eq.(45) into eq.(50), we obtain

$$S = \frac{N_{TC}}{6\pi}\left(1 + (\ln \frac{M^2}{m^2} - 1)(1 - \frac{1}{\eta^2})\right) \quad (52)$$

which is as in eq.(37) with $\eta = m_{A_1}^2/m_\rho^2$.

IV. Breaking of Technicolor

How is the technicolor interaction broken? The breaking can either involve additional interactions, a somewhat epicyclic scenario, or it may occur in certain gauge theories automatically. While we have no compelling self-destructing theories in mind, they can certainly exist ("tumbling" [15]). The unitary, orthogonal and exceptional

Lie groups have complex (chiral) representations and thus in principle can break themselves. (Symplectic groups are real.) Unfortunately, there are no chiral gauge theories, at least with minimal fermion content, that break themselves completely or become null in the sense defined earlier [16].

As a minimal toy example, consider a chiral $SU(3)$ model. Chiral asymptotically free, anomaly-free $SU(N)$ theories were classifed by Eichten, Kang and Koh [17]. The $SU(3)$ model requires seven $\bar{\mathbf{3}}$ and one $\mathbf{6}$ representations. The degenerate most attractive channels for condensation are the $\mathbf{3}$ of the $\bar{\mathbf{3}} \otimes \mathbf{6}$ and the $\mathbf{6}$ of the $\mathbf{6} \otimes \mathbf{6}$. There is, after instanton effects are taken into account, a global symmetry of $SU(7) \times U(1)$. The electroweak theory is embedded by assembling four $\bar{\mathbf{3}}$ into two electroweak doublets with hypercharges $Y = \pm 1$, leaving the remaining three $\bar{\mathbf{3}}$ and the $\mathbf{6}$ as electroweak singlets. The hypercharge group can be embedded in an $SU(2)_R$ symmetry. The theory has no electroweak, TC or mixed anomalies. The electroweak corrections to the effective potential of the vacuum favor the $\mathbf{3}$ of the $\bar{\mathbf{3}} \otimes \mathbf{6}$ as the most attractive channel, where the $\bar{\mathbf{3}}$ are the electroweak doublets, thus preserving the $SU(2)$ custodial symmetry. The $SU(3)_{TC}$ is broken to $SU(2)_{TC}$ by this vacuum, and five of the eight technigluons acquire mass. The electroweak $SU(2)_L$ and the $U(1)_Y$ are broken, as neither is a vector symmetry. The $U(1)_Y$ is not vector-like, in spite of the hypercharge assignments, because the electroweak doublets are not vector-like under the $SU(3)_{TC}$. The electromagnetic $U(1)_Q$ subgroup is unbroken, because it *is* vector-like — the remaining $SU(2)_{TC}$ is pseudoreal, and the $\mathbf{2}$ and $\bar{\mathbf{2}}$ representations can be assembled to form Dirac fermions. The Vafa–Witten theorem again applies here, forbidding the dynamical breaking of vector symmetries.

Applying the NJL techniques and results of section II to this model results in a mass gap of $m \simeq 720$ GeV and a technigluon mass of $M \simeq 770$ GeV, with $\eta = 3$. Since the technicondensates break both the technicolor and electroweak groups, the TC coupling is $g(M) = 2 \cdot 770$ GeV/246 GeV $\simeq 6.3$. This small hierarchy is at the edge of the NJL model's reliability (and the argument of the log has been taken to be $1 + M/m$ to allow larger η).

After the electroweak embedding, the global symmetry is broken to $SU(3) \times SU(2)_L \times [U(1)]^3$; after spontaneous symmetry breakdown, a global $SU(3) \times U(1)$ remains and there are three Goldstone bosons eaten by the W and Z, two EW singlet true Goldstones, and eight pseudoGoldstones of mass 150–200 GeV. The Higgses behave, below the cutoff, like a two-doublet Higgs theory [18]. The charged and light neutral Higgses have mass of about 100 GeV, the pseudoscalar "axion" a somewhat lighter mass, and the heavy neutral Higgs a mass of about 900 GeV. There is also a scalar isotriplet. The technifermions acquire dynamical Dirac masses coupling the $\bar{\mathbf{3}}$ electroweak doublets and the $\mathbf{6}$. There are one massless electroweak/$SU(2)_{TC}$ singlet and two electroweak/$SU(2)_{TC}$ doublets of mass $\sqrt{2}(720 \text{ GeV}) = 1020$ GeV. The $SU(2)_{TC}$ confines the latter at 200 GeV. We can estimate the S from fermion loops using the bubble formula eq.(37). For two doublets, $S = (2/3\pi)(0.79) = 0.17$. From $SU(2)_{TC}$ technigluon exchange across the loop, there are perturbative corrections $\sim 30\%$, but impossible to compute at only lowest order [19]. The non-perturbative $SU(2)_{TC}$ corrections can be estimated using the methods of Pagels and Stokar [20] to be negligible. There are scalar (Higgs and pseudoGoldstone) contributions to S which are next order in $1/N_{TC}$, as well as higher order corrections in g_2^2, and we have no systematic way to include all other corrections of this order. The estimate $S \approx 0.17$ is probably too large for current electroweak limits. There are additional condensates. The $SU(2)_{TC}$ is pseudoreal and thus cannot break itself, but the left-over global $SU(3) \times U(1)$ is broken completely, with nine additional electroweak singlet true Goldstones. This $SU(3)$ model, while not realistic, may be taken as a kind of optimal case, inasmuch as the model is self-contained, and the remnant $SU(2)_{TC}$, while not null, is the smallest non-Abelian group. The problem with self-contained models of this kind is clear: they require too many degrees of freedom contributing to S.

Ultimately, it is necessary to maintain the smallest number of additional degrees of freedom possible in order to minimize S. Thus, a fourth generation or a top-condensate scheme may have some advantages. The arguments presented here do not apply directly to a "topcolor" model [10] since there one has a maximally broken custodial $SU(2)$. There we expect the formula for S to resemble eq.(29), in which case negative S can readily occur. However, the T constraint would seem to disfavor such a model since evidently m_{top} cannot be as large as one would expect if the new physics is at $\sim$ 1 TeV. Nevertheless, it is interesting to ask how large the effects of the virtual resonances are on the T parameter, since one might expect a suppression of T for large m_{top} in such a scheme. An investigation of this question is underway.

V. Conclusions

In Figure 1 we plot the result for S as obtained in the fermion bubble approximation to our model of eq.(4). Here we use eq.(37) together with the gap equation constraint, eq.(35), upon the parameter η. The gap equation implies that η can be viewed as a function of the mass ratio M/m, so it is also plotted. The one-doublet free-fermion loop contribution corresponds to $6\pi S/N_{TC} = 1$. We see that the effect of the virtual vector technimesons is to increase S slightly for strong coupling. The approximations of the model are not valid for $\ln(M^2/m^2) \lesssim 2$, for we then become sensitive to higher orders in m^2/M^2. For example, the maximum value of η permitted, such that the gap equation has a nontrivial solution, depends upon the argument of the logarithm. For a logarithm of the form $\ln(M^2/m^2)$ we find $\eta \lesssim 1.6$, while the form $\ln(M^2/m^2+1)$ we find $\eta \lesssim 3.3$. The physical difference between these forms are terms of higher order in m^2/M^2, and we cannot reliably compute their effects. Indeed, in writing eq.(4) as the effective Lagrangian, we have already discarded the higher order terms in an operator product expansion of the effects of single massive technigluon exchange.

Our model approximates all of the dynamics as pure s–wave. Obviously, a confining theory has strong higher partial waves, and we would expect these to contribute to S. The results of Peskin and Takeuchi [6], which reflect the full confinement effects of QCD through the saturation of the $\Pi_{VV} - \Pi_{AA}$ by the real ρ and A_1 resonances, yield $6\pi S/N_{TC} \approx 2$. The pure s–wave result of our analysis of virtual ρ and A_1 resonances is $6\pi S/N_{TC} \approx 1.5$ when η is maximal. Thus our results do capture some of the effects of a full QCD–like theory. The key point of our present analysis is that we may choose $\eta \to 1$ without drastic fine–tuning, and suppress S somewhat toward the free-fermion loop result. For example, choosing $M/m \sim 10$ yields $6\pi S/N_{TC} \sim 1.3$ and $\eta \sim 1.05$. The behavior of the function of eq.(37) also allows a reduction of S for $\eta \gtrsim 1.4$ with $M/m \lesssim 3$, though this is a less reliable limit of the approximation. (Note that in the extreme example of the top condensate scheme where M/m is fine–tuned to $\sim 10^{15}$ we see that there is negligible virtual resonance effects upon S.)

Spontaneously broken technicolor thus offers a mild advantage in reducing the unwanted resonance contributions to S. In estimating S in an SBTC theory one can rely upon the free–fermion loop result of $N_{TC}/6\pi$ if one is willing to tolerate some fine–tuning near to the critical coupling regime. We emphasize thatwe have not given a method for engineering a model with negative S.

There is no good reason, in light of the strong ETC gymnastics that are required to maintain small flavor–changing neutral current processes and a heavy top quark [3, 4], to argue that technicolor need be an unbroken, QCD -like theory with confined techniquarks. A strong, broken gauge theory would be a novelty in nature, but there is no reason to rule out the possibility. For electroweak symmetry breaking it is clearly necessary to maintain the smallest number of degrees of freedom possible in order to minimize S. A fourth generation scheme, such as the model of ref.[11], or a top–condensate scheme [9, 10], thus has clear advantages, and we have briefly discussed others. This is obviously not an exhaustive list, and there may prove to be other advantages to model building in which technicolor, together with its extension, is spontaneously broken.

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Figure Caption

1. The S parameter, as obtained in the fermion bubble approximation (solid line), eq.(37) is plotted against $\ln(M^2/m^2)$. Here the gap equation constraint, eq.(35), is implemented for the parameter η, which is also plotted (dashed line).

References

1. S. Weinberg, *Phys. Rev.* **D19**, 1277 (1979);
   L. Susskind, *Phys. Rev.* **D20**, 2619 (1979).

2. S. Dimopoulos, L. Susskind, *Nucl. Phys.* **B155**, 237 (1979);
   E. Eichten, K. Lane, *Phys. Lett.* **90B**, 125 (1980).

3. B. Holdom, *Phys. Lett.* **B150**, 301 (1985); **B198**, 535 (1987); T. Appelquist, D. Karabli, L. C. R. Wijewardhana, *Phys. Rev. Lett.* **57**, 957 (1986); M. Bando, *et al.* , *Phys. Rev. Lett.* **59** 389 (1987) ; V. Miransky, *Nuovo Cimento* **90A** 1 (1985).

4. T. Appelquist, T. Takeuchi, M. Einhorn, L.C.R. Wijewardhana, *Phys. Lett.* **B220**, 223 (1989); R. Mendel, V. Miransky, *Phys. Lett.* **B286**, 384 (1991).

5. D. C. Kennedy, B. W. Lynn, *Nucl. Phys.* **B322**, 1 (1989);
   D. C. Kennedy, *Phys. Lett.* **B268**, 86 (1991).

6. M. E. Peskin, T. Takeuchi, *Phys. Rev.* **D46**, 381 (1992);
   T. Takeuchi, private communication.

7. P. G. Langacker, U. Pennsylvania preprint UPR-0492T (1992), and private communication.

8. R. Sundrum, S. D. H. Hsu, Berkeley preprint UCB-PTH-91-34 (1991, revised 1992).

9. W. A. Bardeen, C. T. Hill, M. Lindner, *Phys. Rev.* **D41**, 1647 (1990), and references therein.

10. C. T. Hill, *Phys. Lett.* **B266**, 419 (1991);
S. Martin, *Phys. Rev.* **D46**, 2197 (1992) and *Phys. Rev.* **D45**, 4283 (1992);
N. Evans, S. King, D. Ross, "Top Quark Condensation from Broken Family Symmetry," Southampton Univ. preprint, SHEP-91-92-11 (1992).

11. C. T. Hill, M. Luty, E. A. Paschos, *Phys. Rev.* **D43**, 3011 (1991); T. Elliot, S. F. King, *Phys. Lett.* **B283**, 371 (1992);

12. Y. Nambu, G. Jona-Lasinio, *Phys. Rev.* **122**, 345 (1961); **124**, 246 (1961);
B. Rosenstein, B. Warr, S. H. Park, *Phys. Rep.* **205**, 59 (1991);
S. P. Klevansky, *Rev. Mod. Phys.* **64**, 649 (1992).

13. C. Vafa, E. Witten, *Nucl. Phys.* **B234**, 173 (1984).

14. J. Goldstone, *Nuovo Cim.* **19**, 154 (1961).

15. S. Raby, S. Dimopoulos, L. Susskind, *Nucl. Phys.* **B169**, 373 (1980).

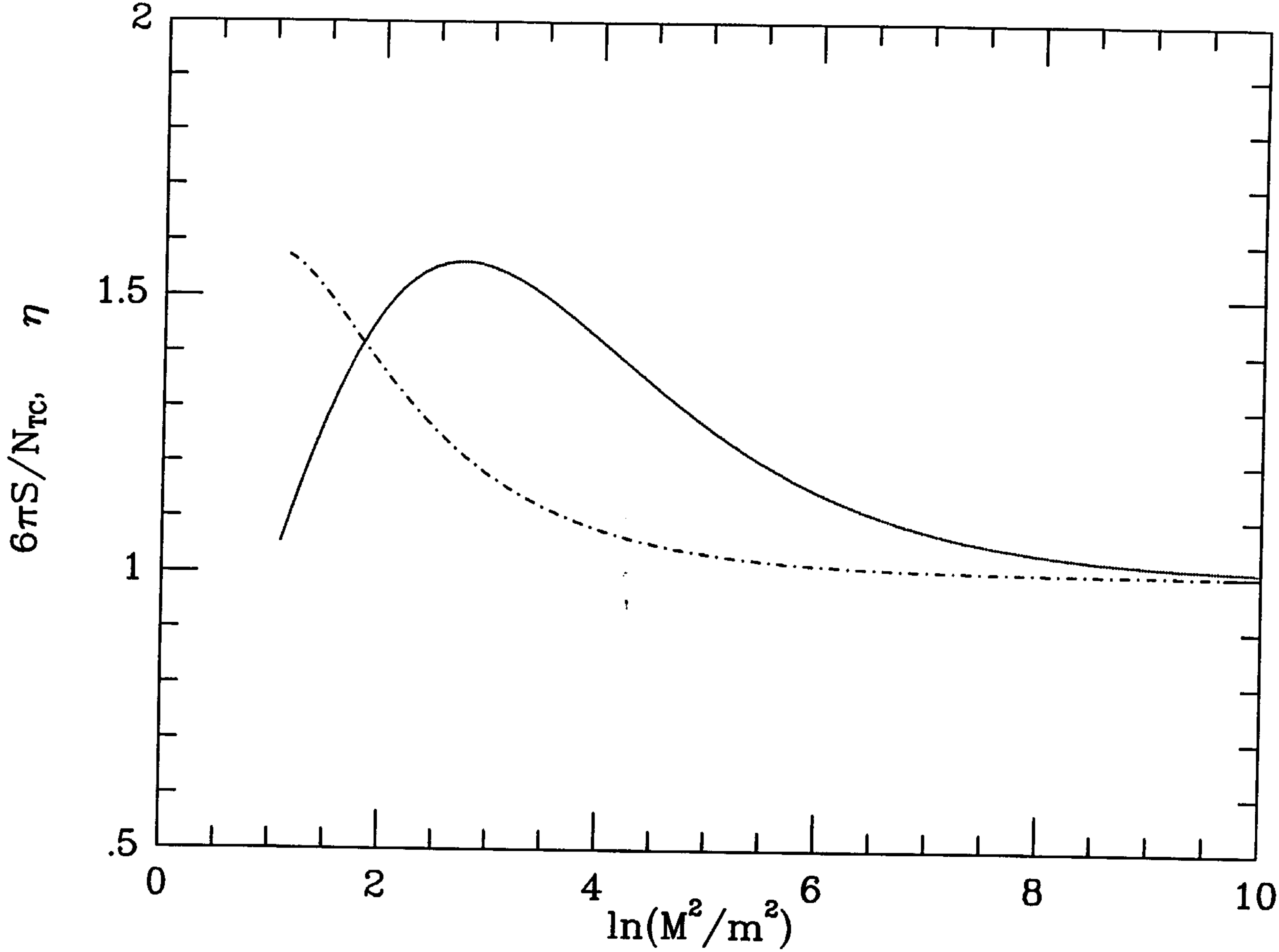
16. R. Slansky, *Phys. Rep.* **79**, 1 (1981).

17. E. Eichten, K. Kang, I.-G. Koh, *J. Math. Phys.* **23**, 2529 (1982);
E. Eichten, R. Peccei, J. Preskill, D. Zeppenfeld, *Nucl. Phys.* **B268**, 161 (1986).

18. J. F. Gunion, H. E. Haber, G. Kane, S. Dawson, *The Higgs Hunter's Guide* (Redwood City, California: Addison-Wesley, 1990) chapter 4;
H. E. Haber, A. Pomarol, UCSC SCIPP preprint 92/29 (1992);
H. E. Haber, UCSC SCIPP preprint 92/31 (1992);
C. T. Hill, C. N. Leung, S. Rao, *Nucl. Phys.* **B262**, 517 (1985).

19. A. Djouadi, C. Verzegnassi, *Phys. Lett.* **B195**, 265 (1987);
A. Djouadi, *Nuovo Cim.* **100A**, 357 (1988);
B. A. Kniehl, *Nucl. Phys.* **B347**, 86 (1990);
F. Halzen, B. A. Kniehl, *Nucl. Phys.* **B353**, 567 (1991).

20. H. Pagels, *Phys. Rev* **D19**, 3080 (1979);
H. Pagels, S. Stokar, *Phys. Rev.* **D20**, 2947 (1979).



2
1.5
1
.5
0
2
4
6
8
10
6πS/N_TC,   η
ln(M²/m²)