Fermi National Accelerator Laboratory

FERMILAB-PUB-92/191-T

A Program for Analytical Perturbative Calculations in High Energy Physics up to Four-loops for the FORM System

Levan R. Surguladze

Institute of Theoretical Science, University of Oregon
Eugene, OR 97403, USA

and

Fermi National Accelerator Laboratory,
P.O. Box 500, Batavia IL 60510, USA

Abstract

We present a FORM program HEPLoops for analytical computation of arbitrary massless, one-, two- and three-loop Feynman diagrams of the propagator type within dimensional regularization. The program is oriented towards large scale analytical perturbative Quantum Field Theory calculations, within 't Hooft's minimal subtraction prescription, up to four-loop level.

Operated by Universities Research Association Inc. under contract with the United States Department of Energy

PROGRAM SUMMARY

Title of program: **HEPLoops**

Program obtainable from: Institute of Theoretical Science, University of Oregon, Eugene, OR 97403 USA

Computers: IBM-pc, IBM-3090, VAX, Sun and NeXT workstations

Installation: SunOS UNIX at the Institute of Theoretical Science, University of Oregon, Eugene, OR USA.

Programming language used: Algebraic programming system FORM (free for distribution version 1.1) by J.A.M.Vermaseren

Storage required: approx. 400 Kbytes

No. of lines in program: 3380

Keywords:
Perturbative Quantum Field Theory, High energy physics calculations, multiloop Feynman integral, Feynman diagrams, dimensional regularization, minimal subtraction scheme, analytical calculations, algebraic programming systems.

Nature of physical problem:
Multiloop massless Feynman integrals of the propagator type, usually appearing in high energy physics calculations are computed analytically within dimensional regularization.

Method of Solution:
The recursive type algorithm developed by Chetyrkin and Tkachov is used.

Typical running time:
5-10^3 sec. and more in some complicated cases. Running time strongly depends on the topology of the particular Feynman diagram.

1 Introduction

The status of perturbative Quantum Field Theory (QFT) calculations has been intensively discussed during the last 15 years. In particular, significant efforts have been made to develop the methods of perturbation theory to study a wide class of problems including : calculations of renormalization group functions and renormalization constants in various QFT models; calculation of coefficient functions in operator product expansions and calculations of various cross-sections and decay widths (see, e.g., [1]).

After the application of certain theoretical methods , such as the renormalization group, dispersion relations, operator product expansions etc., the problem of evaluation of physical quantities effectively reduced to the problem of evaluation of corresponding Feynman diagrams. In each order of perturbation theory, contributions to the physical observables come from a finite set of Feynman diagrams with the same number of internal momentum integrations (number of loops). Feynman diagrams are usually evaluated within the dimensional regularization method [2] and 't Hooft's minimal subtraction (MS) prescription [3] for ultraviolet renormalization.

Using the recent progress in calculational methods [4], in some cases (in particular, for calculation of renormalization group functions) one can obtain a significant simplification of the problem , by setting to zero dimensional parameters (usually all masses and some of the external momenta). Other physical quantities allow expansion in powers of m^2/Q^2 with massless coefficients and, for high enough energies ($m^2/Q^2 \to 0$, where m is a fermion mass and Q is a transferred momentum), calculation of the first few terms in this expansion gives satisfactory results (e.g., in calculation of total cross-sections and decay widths). Thus we deal with a wide class of problems, which are reduced to the evaluation of massless, multiloop, dimensionally regularized Feynman diagrams of propagator type (with only one external momentum). The analytical recursive type algorithms for one-, two- and three-loop massless propagator type Feynman integrals were developed in [5].

In practice, the calculation of physical quantities within perturbation theory is very cumbersome and tedious already beyond the one- loop level, especially in realistic QFT models, like QCD. However, algorithms [5] allow convenient implementation on the algebraic programming systems like REDUCE [6], SCHOONSCHIP [7], FORM [8] etc., which makes it possible to perform high order perturbative calculations up to the four-loop level.

Several programs were written during the last ten years for analytical calculations of multiloop Feynman diagrams. For better orientation let us briefly review a prehistory of such programs.

The First program for the SCHOONSCHIP system [7] has been worked out in [9]. This program evaluates traces of products of Dirac γ matrices and performs other

simplifications, to reduce the initial diagram to a sum of scalar integrals. Then the program uses a database of precalculated scalar integrals in the form of trivial substitutions. This program is very fast for the very restricted number of problems it was written for. However, a large number of problems are not solvable by this program, since too many scalar integrals have to be precalculated by hand, which is also unreliable. The above program has been used in the first three-loop calculation of QCD β-function [10]. Another program for the REDUCE system, which also uses "database approach" was worked out for some topologies of three-loop integrals [11]. However, such an approach can lead to the wrong results [11]. In any case, more complicated problems cannot be dealt with unless calculation of two- and more loop integrals is made fully automatic.

The first full realisation of algorithms [5] at the two-loop level was done for the REDUCE system [6] in the form of procedures. The program calculates one- and two-loop arbitrary massless propagator type Feynman integrals and allows arbitrary tensor structure in the numerator of integrand. Several two-loop calculations have been done with the help of this program. (e.g., calculation of coefficient functions of gluon and quark condensates in the sum rule method [12]; calculation of two-loop anomalous dimensions of baryonic currents [13] etc.). Later this program was called LOOPS and published in [14]. However, for large scale calculations LOOPS uses comparatively large computer resources.

The implementation of three-loop algorithms for the CDC version of SCHOONSCHIP system [7] first was done in [15]. This program calculates arbitrary massless scalar (with no free Lorenz indices) one-, two- and three-loop Feynman diagrams of the propagator type. The calculation of the following physical quantities has been performed by this program: calculation of total decay width of neutral Higgs boson at the three-loop level [16]; calculation of $O(m^2)$ correction to the correlator of electromagnetic currents at the three-loop level [17]; calculation of four-loop QED β function [18] and most impressive - calculation of $\sigma_{tot}(e^+e^- \to hadrons)$ in QCD at the four-loop level [19].

The next program [20] was written for the IBM and CDC versions of SCHOONSCHIP [7] and fully implements algorithm [5]. This program uses programming experience and some modified parts of [15] and includes a number of original solutions. The structure of the program has been changed according to needs of IBM version of SCHOONSCHIP [7]. As a test, the recalculations of three-loop total decay width of Higgs boson and three-loop $O(m^2)$ contributions to the correlator of electromagnetic currents has been performed by the program [20]. As a result, it was shown in [21, 22] that the results of previous calculations [16] and [17] by the program [15] was incorrect. Later, with help of test runs on the program [20] the source of error was discovered in the program [15] by A.Kataev in collaboration with S.Larin and the author of the present paper. The error was insufficient expansions in Laurent series of one-loop "basic" scalar integrals (for the terminology see [5]). Another independent source of

errors in the program [15] was found by the author of present paper. This source was due to an incorrect logical structure of the program [15] for trace calculations. Fortunately, the program [20] was initially free of these errors. Unfortunately, due to the above mentioned errors, the exciting results of [18] and [19] also turned out to be incorrect. After this the program [20], without changes was published in [23] with the name of MINCER. The recalculations of QED β function and $R(s)$ in electron positron annihilations at the four-loop was done independently with help of program MINCER [23] in [24] and with help of program [15] (the wrong program block was substituted by the corresponding block of MINCER [23]) in [25]. The results of both calculations are in agreement.

During the last decade SCHOONSCHIP was the only system applicable to very large scale calculations (10^5-10^6 terms). However this system is operable on the very restricted class of computers (only IBM mainframe and CDC) [1] and the installation requires some nontrivial experience and knowledge. On the other hand, many different types of fast enough computers, with flexible configurations, are available at present. Thus, it was highly desirable to have a SCHOONSCHIP - type system for various computers, including PC. Recently, the new SCHOONSCHIP -like algebraic programming system FORM has been worked out [8], which is operable on the numerous different types of computers.

In the present paper we will briefly describe the new program HEPLoops for high order analytical perturbative calculations for the FORM system. This program was briefly announced in the review paper [27]. In order to make easier to use this program the reader's acquaintance with the first three sections of [23] (where the algorithm [5] is described) is highly advisable.

2 External Specifications of Program HEPLoops

The program HEPLoops (version 2.0) is intended for analytical computation of dimensionally regularized, massless Feynman diagrams, depending on a single external momentum, with the number of loops $l \leq 3$. The program realizes the algorithms [5] (for the description of algorithms see also [23]) within the algebraic programming system FORM [8]. The program calculates the expansions of Feynman integrals in terms of the parameter of dimensional regularization $\varepsilon = 2 - D/2$ (D is the space time dimension) up to order $O(\varepsilon^{4-l})$, which is suficient for direct evaluation of four-loop integrals.

[1] After the present paper was completed we discovered that the new version of SCHOONSCHIP has been worked out for NeXT workstations by M.Veltman. The new version include number of new tools and much more convenient then previous ones.

2.1 The package.

The program HEPLoops [2] (version 2.0) consists of 10 blocks. (The versions 3 contain also aditional blocks for ultraviolet renormalization procedure and for integration of trivial 4th loop.) The blocks are linked in proper order which should not be changed by the user. However, in large scale calculations it is possible to execute block by block in several runs. The total number of lines in the package is 3380.

The file DECLARE contains all necessary declarations of symbols, functions etc. This file is included in each program block by *include* command. The file FORM.set defines all appropriate system parameters, which usually requires some trivial adjustements for the particular type of computer (see FORM manual [8]). The file INPUT contains the input for initial diagrams. The user will deal mainly with only this file to put the corresponding expressions for the Feynman diagrams. The file INPUT is included in the Block START by the *include* command. Block SCHEME defines a scheme in which the result will be printed. The basic version of program gives the result in the normalizations of momentum integrals corresponding to the $\overline{MS}$-scheme [28]. In this normalization the trivial one-loop scalar Feynman integral looks like:

$$(\mu^2)^\varepsilon \int \frac{d^D P}{(2\pi)^D} \frac{1}{P^2 (P-Q)^2} =$$
$$\frac{1}{(4\pi)^2} \left(\frac{\mu^2}{Q^2}\right)^\varepsilon \left[\frac{1}{\varepsilon} + 2 + 4\varepsilon + (8 - \frac{7}{3}\zeta(3))\varepsilon^2 \right.$$
$$\left. +(16 - \frac{14}{3}\zeta(3) - \frac{13}{4}\zeta(4))\varepsilon^3 + (32 - \frac{28}{3}\zeta(3) - \frac{13}{2}\zeta(4) - \frac{31}{5}\zeta(5))\varepsilon^4 + O(\varepsilon^5)\right] \qquad (1)$$

However it is straightforward to replace the default scheme with any other MS-type scheme. The other program blocks step-by-step implement the algorithm [5]. (More details on the structure of the program blocks will be given in the extended publication.)

[2] For the convenience we have keeped the same names of blocks, some of the variables and terminology from the program MINCER [23]. However, we have changed the well reputed name MINCER to HEPLoops.

2.2 Input Data Formats

One-, two- and three-loop basic diagrams are pictured on fig.1.

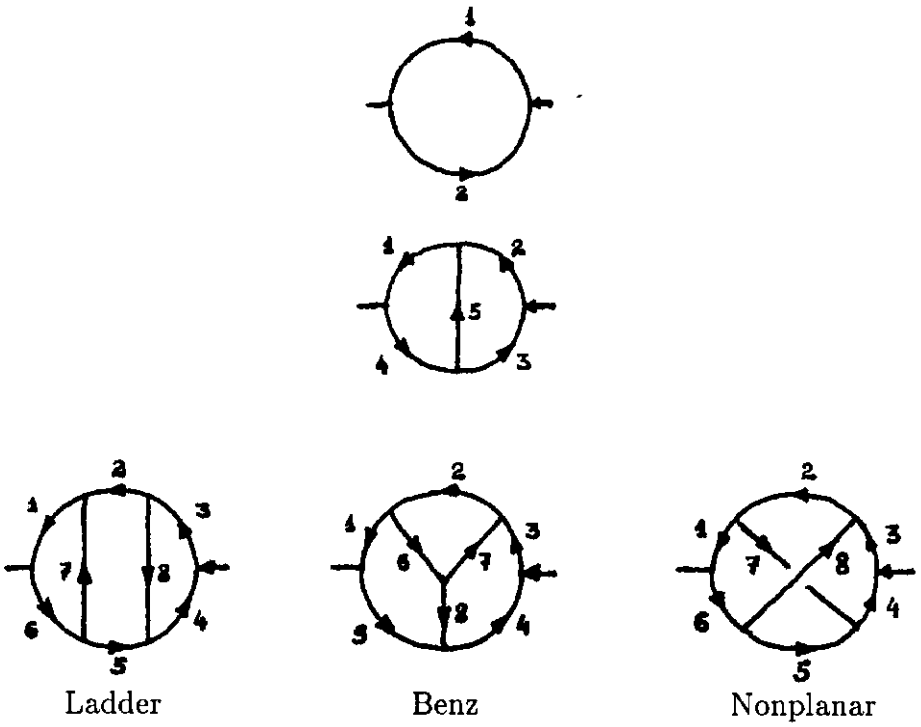

Fig.1 One-, two- and three-loop basic, scalar Feynman diagrams.

Note, that program HEPLoops performs the integrations over the D-dimensional ($D = 4 - 2\varepsilon$) Euclidean space. The directions of the momenta in the program are fixed as pictured in fig.1. It is not difficult to see that arbitrary one-, two- and three-loop diagrams can be turned by a proper choice of the exponents of the propagator powers to one of the types of diagrams pictured in fig.1.

The file INPUT contains five calculable G(lobal) expressions: Ladder; Benz; Nonplanar; Twoloop and Oneloop. R.h.s of each of them should contain an input data (or zero) for the corresponding topology of the diagram and the number of loops (fig.1). Any of the data in the r.h.s. can contain following factors:

- Numerical coefficient;

- Product (including an empty one) of the integer powers of scalar products $P1.P1, P2.P2$ for Oneloop; $P1.P1, ..., P5.P5$ for Twoloop and $P1.P1, ..., P8.P8$ for Ladder, Benz and Nonplanar (the momentum numerations correspond to fig.1);

-Product (including an empty one) of scalar products $P_i.P_j$ and $P_i.QQ$ where $i, j = 1, 2$ for Oneloop; $i, j = 1, ..., 5$ for Twoloop and $i, j = 1, ..., 8$ for all three-loop G-expressions (QQ codes an external momentum for all diagrams);

-Product of Dirac γ-matrices (corresponding to the product of the fermionic propagators and vertices along the fermionic loop), which should be coded within the FORM as follows: $g\_(1,a,P1,m,P2,....) * g\_(2,n,P3,k,P4,...) * ...$ where the first argument indicates the particular fermionic circle (see FORM manual [8]). a, m, n, k,... correspond to γ^a, γ^m, γ^n, γ^k,... and $P1$, $P2$,... correspond to $P1_\alpha\gamma^\alpha$, ... (In the Block START the trace calculations will be performed for $g\_(1,...)$, $g\_(2,...)$,...)

-Products of four-momenta $P_i(\alpha)$ and $QQ(\alpha)$ where $\alpha = m,n,k,l,j,a,r$ and $i = 1,2$ for Oneloop, $i = 1,...,5$ for Twoloop and $i = 1,...,8$ for all three-loop diagrams; (Note, that only two uncontracted Lorenz indices m and n are allowed. In this case the common factor $PRO(m,n)$ should be used at the input)

-Scalar variable EP^A, $A \geq -1$ (EP codes the quantity $\varepsilon = 2 - D/2$);

-Function $PRO(m,n)$, which allows one to calculate the integrals with two uncontracted Lorenz indices m and n in the numerator of integrand; (Thus, if the integrand contains uncontracted Lorenz indices m and n, then the corresponding input should be multiplied by $PRO(m,n)$.)

-Any symbols admitted in FORM which are not used in the program (the list of symbols and functions, which are used in the program can be found in the file DECLARE).

2.3 Output Data Formats

After the execution of program is finished the file Result in the log-file of the block FINAL contains the result of calculation in the form of a polynomial in $1/EP$ (maximum exponent is 3 for three-loop calculations) plus finite part - rational number and Riemann zeta functions $\zeta(3)$, $\zeta(4)$ and $\zeta(5)$ with rational coefficients. The Result may contain an additional symbolic multiplicative factor which was used as a symbolic factor at the input and was transferred to the block FINAL. In the case when the input contains two uncontracted Lorenz indices m and n the result for the diagram should be proportional to the momentum factor: $g^{mn}Q^2A(\varepsilon) - Q^mQ^nB(\varepsilon)$. In this case the output of the program will contain a term which is multiplied by the symbolic factor VA and another term with symbolic factor VB. These terms correspond to the g^{mn} and Q^mQ^n parts of the result. Furthermore, in the working directory the file Result.res will contain the result of calculation, which can be used as an input for another FORM package. Note again, that standard version of the program prints the result in a normalization corresponding to the $\overline{MS}$ scheme [28], without taking into account color group weights, symmetry factors and the factor $(4\pi)^{-2}(\alpha/4\pi)^{(l-1)}(\mu^2/Q^2)^{(l\varepsilon)}$, where $\alpha = g^2/4\pi$ is the coupling, l is the number of loops and μ is 't Hooft's unit of mass.

3 Efficiency of the Program

The test runs of the program HEPLoops and the experience of using it in real calculations show that the program is fast enough to perform any three- and four-loop (with help of infrarred rearrangement procedure [4]) calculations from the class of problems enumerated in the introduction. The program is much faster then the program MINCER [23] and the factor 5-7 was observed in calculations of the three-loop diagrams of the type in fig.2 on the Sun workstation. The program on the IBM-PC/486 (33MHz) is only 3-4 times faster then the MINCER on the IBM compatible 1MFlop ES-1060 mainframe. It seems that this factor will grow together with the growth of the problem complexity. Such an effect was observed during the recalculation of some of the four-loop diagrams contributing to $\sigma_{tot}(e^+e^- \to hadrons)$ [24] by the program HEPLoops.

Recently another program - "The FORM version of MINCER" has been worked out [29]. This program is written in procedure form and calculates three-loop massless propagator type scalar (with no uncontracted Lorenz indices) Feynman diagrams. Some useful optimisations have been done. As was claimed in [29], the program calculates the diagram pictured in fig.2 within 11.5sec. on the NeXT station (25MHz). This is 100(!) times faster then the original MINCER [23]. The essence of such an improvement is that this program extensively uses the database of precalculated three- and two-loop scalar integrals (and other databases). Note, that the program MINCER [23] as well as the HEPLoops evaluate all of the integrals fully automatically. Thus, in this case it is unclear how to compare the efficiency of this programs. On the other hand, as was mentioned in the Introduction, the programs which are based on the "database approach" are very fast only for the problems they are written for and have a restricted area of application (see discussion in [26, 27]). So the exciting factor of "improvement" - 100 seems to be somewhat spurious. (Indeed, we can even increase this factor if we directly use the precalculated value of the diagram in fig.2.)

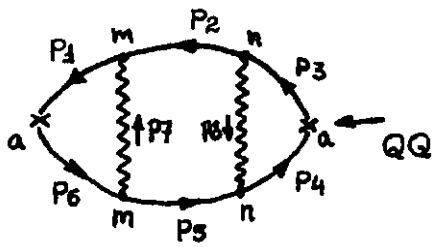

Fig.2 Three-loop diagram contributing to the $\sigma_{tot}(e^+e^- \to hadrons)$. Solid lines correspond to the massless quark propagators, wave lines correspond to the gluon propagators in the Feynman gauge and crosses correspond to the electromagnetic current vertices.

4 Test Run Output

Below we present fragments of a test run output of the program HEPLoops for the diagram pictured on fig.2. The total execution time is 188.2 sec. on the Sun workstation (Compare the input, result and time to the ones from [23] for calculation of same diagram by the program MINCER) Note, that the time shown on the Test Run Output is the time for execution of last block FINAL (not a total time).

```
   FORM by J.Vermaseren. Version 1.1 3-jun-1992

   *                              PROGRAM  HEPLoops
   *                                         by L.R.Surguladze
   * ---------------------------------------- Version 2.0 ----



   *                    --- Block  "START" ---
   #-
   * --------------   INPUT DATA   ------------------

   G Ladder     =    g_(1,P1,m,P2,n,P3,a,P4,n,P5,m,P6,a)
                    /P1.P1/P2.P2/P3.P3/P4.P4/P5.P5/P6.P6/P7.P7/P8.P8;

   G Benz       = 0;
   G Nonplanar  = 0;
   G Twoloop    = 0;
   G Oneloop    = 0;

   * --------------   END of INPUT   -----------------

Time =        0.23 sec    Generated terms =          60
            Ladde         Terms in output =          60
                          Bytes used      =        2282



   *                    --- Block "FINAL" ---
 FINAL loaded

   *                                      --->        HEPLoops
   *                                               --- Version 2.0 ---
   *            ----  R  E  S  U  L  T  ----

Time =        0.07 sec    Generated terms =          44
           Result         Terms in output =           8
                          Bytes used      =         210

   Result =
       + 2583/2 + 207*EP^-1 + 82/3*EP^-2 + 8/3*EP^-3 - 320*dzeta(5) - 144*
         dzeta(4) - 1168/3*dzeta(3) - 96*dzeta(3)*EP^-1;
```

Acknowledgements I would like to express my gratitude to N.G.Deshpande and D.E.Soper for their interest in my work and support.

I am grateful to P.Osland, J.Reid and M.A.Samuel for encouraging interest in the described here program and comments. I am indebted to J.A.M.Vermaseren for discussions during the conference in Dubna (1990) and constructive advice and A.N.Kuznetsov for valuable comments on the problems of programming on the FORM system.

It is pleasure to thank the Directorate of the International Centre of Theoretical Physics (Trieste) for their invitation, where part of this program was written, W.A.Bardeen and P.B.Mackenzie for their invitation and kind hospitality at the Theoretical Physics Department of Fermilab where the final version of this manuscript was completed.

This work was supported by the U.S. Department of Energy under grant No. DE-FG06-85ER-40224.

References

[1] For an introduction to perturbative methods see:
J.C.Collins, Renormalization: An introduction to renormalization group, and the operator product expansion, Cambridge Univ.Press, Cambridge,1984;
T.Muta, Foundations of Quantum Chromodynamics, World Scientific, 1987.

[2] G.'t Hooft and M.Veltman, Nucl.Phys. B44(1972)189.

[3] G.'t Hooft, Nucl.Phys., B61(1973)455.

[4] A.A.Vladimirov, JINR preprint E2-12248 (Dubna,1979); Teor.Mat.Fiz., 36 (1978)271; Teor.Mat.Fiz. 43(1980)210;
O.V.Tarasov and A.A.Vladimirov JINR preprint E2-80-483 (Dubna,1980);
K.G.Chetyrkin and F.V.Tkachov, Phys.Lett., 114B (1982)340.

[5] F.V.Tkacov ,Phys.Lett. 100B (1981)65;
K.G.Chetyrkin and F.V.Tkachov, Nucl.Phys. B192(1981)159;
F.V.Tkachov, Teor.Mat.Fiz. 56(1983)350.

[6] A.C.Hearn, REDUCE, User's Manual, UCP-19, Univ. of Utah (1973).

[7] M.Veltman, SCHOONSCHIP, A CDC 6600 program for symbolic evaluation of algebraic expressions, CERN report, 1967;
H.Strubbe, Comput. Phys.Commun., 8(1974)1.

[8] J.A.M.Vermaseren, FORM, User's Manual, NIKHEP Amsterdam 1989.

[9] O.V.Tarasov, in Proc. Intern. Conf. Systems and Techniques of Analytical computing and their Applications in Theoretical Physics (Dubna 1979) p.150.

[10] O.V.Tarasov, A.A.Vladimirov and A.Yu. Zharkov, Phys.Lett., 93B (1980)429.

[11] J.Reid (private communications).

[12] G.T.Loladze, L.R.Surguladze and F.V.Tkachov, Bull.Acad.Sci. GSSR 116 (1984)509; Phys.Lett. 162B(1985);

[13] A.A.Pivovarov and L.R.Surguladze, Nucl.Phys., B360(1991)97.

[14] L.R.Surguladze and F.V.Tkachov, Comput.Phys.Commun., 55(1989)205.

[15] S.G.Gorishny, S.A.Larin and F.V.Tkachov, INR preprint P-0330 (Moscow 1984).

[16] S.G.Gorishny, A.L.Kataev and S.A.Larin, Yad.Fiz. 40(1984)517.

[17] S.G.Gorishny, A.L.Kataev and S.A.Larin, Nuov.Cim, 92A(1986)119.

[18] S.G.Gorishny, A.L.Kataev and S.A.Larin, Phys.Lett., 194B (1987)429.

[19] S.G.Gorishny, A.L.Kataev and S.A.Larin, Phys.Lett.,212B (1988)238.

[20] L.R.Surguladze, INR preprint P-0643 (Moscow 1989).

[21] L.R.Surguladze, Yad.Fiz., 50 (1989)604 ;
S.G.Gorishny, A.L.Kataev, S.A.Larin and L.R.Surguladze, Mod.Phys.Lett., 5 (1990)2703.

[22] L.R.Surguladze, INR preprint P-0639 (Moscow 1989).

[23] S.G.Gorishny, S.A.Larin, L.R.Surguladze and F.V.Tkachov, Comput. Phys. Commun., 55 (1989)389.

[24] L.R.Surguladze and M.A.Samuel, in Proc. Intern.Conf. "Beyond the Standard Model 2" (Norman, OK, USA 1-3 Nov., 1990) World Scient. 1991, p.206; OSU preprint RN-250, 1990; Phys.Rev.Lett., 66(1991)560.

[25] S.G.Gorishny, A.L.Kataev, S.A.Larin and L.R.Surguladze, in Proc. Intern. Seminar Quarks-90, (Telavi, Rep. of Georgia, May 12-17,1990), World Scient. 1991, p.194; Phys.Lett., B256 (1991)81;
S.G.Gorishny, A.L.Kataev and S.A.Larin, UM preprint UM-TH-91-01; 1991; Phys.Lett., B259 (1991)144.

[26] S.A.Larin, L.R.Surguladze and F.V.Tkachov, in Proc. Intern. Conf. Systems and Techniques of Analytical Computing and their Application in Theoretical Physics, Dubna, June 1990.

[27] L.R.Surguladze and M.A.Samuel in Proc. Intern. Sympos. on Symbolic calculations (ISSAC-91, Bonn, Germany, July 3-7,1991) ACM New York, 1991.

[28] W.A.Bardeen, A.J.Buras, D.Duke, T.Muta, Phys.Rev., D18 (1978)3998.

[29] S.A.Larin, F.V.Tkachov and J.A.M.Vermaseren, NIKHEF preprint NIKHEF-H-91-18, September,1991.