Fermi National Accelerator Laboratory

FERMILAB-Pub-92/173-A & UTAPHY-HEP-2 (revised)
July 20, 1992

IMPLICATIONS OF NEW GALLEX RESULTS FOR THE MSW SOLUTION OF THE SOLAR NEUTRINO PROBLEM

James M. Gelb
NASA/Fermilab Astrophysics Center,
Fermi National Accelerator Laboratory, Batavia, Illinois 60510

and

Waikwok Kwong and S. P. Rosen
Department of Physics, The University of Texas at Arlington,
Arlington, Texas 76019

ABSTRACT

We compare the implications for ^{7}Be and *pp* neutrinos of the two MSW fits to the new GALLEX solar neutrino measurements. Small mixing angle solutions tend to suppress the former as electron-neutrinos, but not the latter, and large angle solutions tend to reduce both by about a factor of 2. The consequences for BOREXINO and similar solar neutrino–electron scattering experiments are discussed.

Published in Physical Review Letters
(1992) v.69, p. 1864

The impressive results from the GALLEX solar neutrino experiment[1] clearly imply that neutrinos from the *pp* reaction in the sun have been detected, but the magnitude of the observed signal, 83 ± 21 SNU, leaves unresolved the issue of solar physics versus neutrino physics as the cause of the solar neutrino problem.[2] As pointed out by the collaboration,[3] it is possible to stretch solar models to be in conformance with the GALLEX, Davis,[4] and Kamiokande II[5] experiments, and it is also possible to fit all of the data with the MSW mechanism.[6] Here we wish to explore the implications of the MSW fits for *pp*, ^{7}Be, and ^{8}B neutrinos in order to guide future experiments in the resolution of the basic issue.

In the context of the MSW mechanism, the GALLEX data pick out two small and distinctive regions of parameter space: one with a small mixing angle ($\sin^2 2\theta = 7 \times 10^{-3}$) and $\Delta m^2 =$ few $\times 10^{-6}$, and the other with a large mixing angle ($\sin^2 2\theta = 0.6$) and somewhat larger Δm^2. The small mixing angle solution lies on the so-called 'nonadiabatic' line[7] which is characterised by a falling electron-neutrino survival probability as neutrino energy falls. To accommodate the relatively large GALLEX signal, this fall cannot continue indefinitely into the low energy regime, but must begin to turn upwards at some energy in the neighborhood of the ^{7}Be line at 0.86 MeV or between the ^{7}Be and *pp* neutrinos. Such behaviour occurs in those regions of parameter space in which the adiabatic[8] and nonadiabatic solutions join onto one another;[9] examples of it can be found in Figures (6e and f) of the original paper of Rosen and Gelb.[10] The essential point is that while the Davis and Kamiokande II experiments are sensitive to the high energy end of the solar neutrino spectrum, the GALLEX experiment probes the low energy end which might behave in an entirely different way.

A similar difference between high and low energy neutrinos shows up in the large angle solution.[11] High energy neutrinos are in the asymptotic regime of the adiabatic line in which the electron-neutrino survival probability is given by[8] $\sin^2\theta$, which is roughly 0.2 in the present case, while low energy neutrinos fall on the other end of the adiabatic probability curve which gives a larger value of[8] $(1 - 0.5\sin^2 2\theta)$, about 0.7 in the present case.

Let us first concentrate on low energy neutrinos and the adiabatic MSW survival probability[8] which holds for both the small and large angle GALLEX solutions.[3] With Δm^2 in the range of 10^{-6}, the MSW resonance enhancement occurs deep inside the sun relative to the in vacuo oscillation length of a few $\times 10^6$ meters for these neutrinos; and so we can write the electron-neutrino survival probability at Earth, including oscillations between Sun and Earth, as:

$$P(\nu_e \to \nu_e; \mathrm{Earth}) = \frac{1}{2}\left(1 - \frac{p(r)\cos 2\theta}{\sqrt{p^2(r) + \sin^2 2\theta}}\right) \tag{1}$$

$$p(r) = 1.52 \times 10^{-7}\,\frac{E\,\rho(r)}{\Delta m^2} - \cos 2\theta \tag{2}$$

where E is the neutrino energy in MeV and Δm^2 is the squared mass difference in eV^2. The neutrino is produced at radius r and $\rho(r)$ is the electron density in mol/cc, properly adjusted for Helium and heavy element abundance. Since the low energy neutrinos are produced over a region of roughly 15–20% of the solar radius around the core,[2] we must integrate the survival probability over this region to form:

$$\langle P(\nu_e \to \nu_e)\rangle = \int f(r) P(\nu_e \to \nu_e; \mathrm{Earth})\,dr \tag{3}$$

where f is the fraction of neutrinos produced at radius r.

The solar density decreases fairly slowly over the production regions for the ^{7}Be and pp neutrinos[2] and we can infer the qualitative behaviour of the integrated probability $\langle P(\nu_e \to \nu_e)\rangle$ from the unintegrated expression in Eq. (1). For the ^{7}Be neutrinos, we fix the energy at 0.86 MeV and use the tables of solar electron densities and production fractions as given by Bahcall and Ulrich[2] to study $P(\nu_e \to \nu_e; \mathrm{Earth})$ as a function of Δm^2. In the small angle case, the survival probability behaves almost like a step function around the enhancement point $\Delta m^2 = 10^{-5}$ at which $p(r)$ vanishes, changing rapidly from almost zero below it to close to unity above it. In the large angle case, the variation is more gentle: it passes from a value of $0.5(1-\cos 2\theta) = 0.18$ below the enhancement point, now near $\Delta m^2 = 2 \times 10^{-5}$, to $0.5(1+\cos^2 2\theta) = 0.70$ above it.

The qualitative behaviour of pp neutrinos is exactly the same, except that, because of the lower energies and a production region that extends further out into somewhat lower density zones of the sun,[2] the enhancement points shift to lower values of Δm^2. Taking a typical energy of 0.3 MeV, we find that enhancement occurs at 3×10^{-6} in the small angle case and 6×10^{-6} in the large angle one. In Figures 1 and 2 we show these features in actual calculations of the integrated probabilities $\langle P(\nu_e \to \nu_e)\rangle$ for specific mixing angles. The behaviour of the small angle curves are not very sensitive to the precise value of the mixing angle, while the large angle GALLEX solution[3] clusters closely around the value used in Figure 2.

From the curves in Figure 1, we can read off the survival probabilities for ^{7}Be neutrinos in the small angle solution for the Δm^2 range of (3 to 10) $\times\ 10^{-6}$ eV^2. Between 3 and 6×10^{-6}, $\langle P \rangle$ remains very small, being less than 0.05 at 6.3×10^{-6}; it then climbs rapidly to about 0.15 at 8×10^{-6} and 0.4 at 10×10^{-6}. In the same interval the survival probability for *pp* neutrinos is significantly larger, climbing rapidly from 0.5 to 1.

Curves for the large angle solution, with $\sin^2 2\theta = 0.6$, are shown in Figure 2. The range for Δm^2 in this case is 4×10^{-6} to 3×10^{-5} and the survival probability for ^{7}Be neutrinos gradually varies from 0.2 to 0.6. The survival probability for *pp* neutrinos is somewhat larger over most of the range, increasing from 0.47 to 0.70.

Suppose that we apply these results to experiments which plan to observe ^{7}Be neutrinos via neutrino-electron scattering, for example BOREXINO.[12] For recoil electrons in the kinetic energy range of 250 to 663 keV, the lower end of which excludes *pp* neutrino scattering, the ratio of the rate with ^{7}Be electron-neutrino survival probability $\langle P \rangle$ at Earth to the rate in the standard solar model (SSM) is:

$$R(\langle P \rangle) = 0.787\langle P \rangle + 0.213 \tag{4}$$

The expected SSM signal in BOREXINO is 47 events per day,[12] and so the small angle solution, with $\langle P \rangle$ varying between 0 and 0.4, will yield between 10 and 25 events per day. The large angle solution yields $\langle P \rangle$ in the range of 0.2 to 0.6 and a BOREXINO signal of 17 to 32 events per day. Thus we can conclude that should the BOREXINO signal be below 17 events per day, then the small angle solution will be the correct one. Moreover, the fraction of ^{7}Be neutrinos arriving at Earth as electron-neutrinos will be much smaller than the average fraction for ^{8}B neutrinos, which stands at 40% on the basis of the Kamiokande II experiment, and this would rule out a small change in solar temperature[13] as the cause of the solar neutrino problem.

Should the BOREXINO signal fall in the neighborhood of 17 to 25 events per day, then the situation will be ambiguous with respect to the two solutions, and we will have either to measure the *pp* neutrinos directly or to measure the spectral shape of the high energy neutrinos in order to choose between them. In the small angle solution, *pp* neutrinos remain almost entirely as electron-neutrinos, while in the large angle case the electron-neutrino survival probability is at most 70%. The difference between the two cases should show up in low temperature experiments designed to detect *pp* neutrinos via neutrino–electron scattering.[14]

In the case of high energy neutrinos, those with energy greater than 5 MeV, the small angle and large angle solutions lead to different electron-neutrino survival probabilities. For small angles, the nonadiabatic approximation yields a probability of[9,15] $\exp(-C/E)$ where the constant C is approximately 10 MeV;[10,16] whereas for large angles, the adiabatic approximation (see Eq. 1) gives a constant value at Earth of $\sin^2\theta$. The difference in these probabilities will be reflected in the electron recoil spectra in solar neutrino–electron scattering experiments such as Kamiokande II,[5] SNO[17] and Superkamiokande,[18] especially in the neighborhood of 5 MeV.[19] In general, measured spectral shapes which differ from the SSM predicted shape will also rule out solar physics as the cause of the solar neutrino problem.

A BOREXINO signal of about 30 events per day will point to the large angle solution as being correct. This conclusion would imply that ^{8}B neutrinos in the energy range from 1 to 5 MeV would be roughly 40% electron-neutrinos. It would be interesting to observe them directly.

Besides total rate, BOREXINO also has the recoil electron spectrum as a diagnostic tool: should it observe a shape significantly different from that predicted by SSM, then the solution to the solar neutrino problem must lie in neutrino physics rather than solar physics. The same holds true if a day-night effect,[20] which favors the large angle solution, were to be observed. The SSM spectrum and representative cases of the spectra for the MSW mechanism are shown in Figure 3.

In conclusion, we see that experiments designed to detect ^{7}Be neutrinos, such as BOREXINO, have a good chance of resolving the basic question of the solar neutrino problem and of distinguishing between MSW solutions. As more data accumulates, the GALLEX errors[1] should decrease and thus help to remove those areas of parameter space[3] leading to ambiguous predictions for ^{7}Be neutrinos.

The authors wish to thank W. Hampel, T. Kirsten, J. Bahcall, and J. Weneser for useful conversations. They are grateful to J. Weneser for pointing out an error in the original version of the paper and to S. Parke, S. Petcov, and L. Wolfenstein for discussion of the the adiabatic approximation.

The research of James. M. Gelb is supported in part by the U. S. Department of Energy and by NASA grant NAGW-2381; the research of S. P. Rosen is supported in part by U. S. Department of Energy grant DE-FG05-92ER40691.

REFERENCES

[1] GALLEX Collaboration preprint GX 1-1992, submitted to *Phys. Lett. B*, June 1, 1992.

[2] J. N. Bahcall and R. Ulrich, *Rev. Mod. Phys.* 60, 297 (1988).

[3] GALLEX Collaboration preprint GX 2-1992, submitted to *Phys. Lett. B*, June 1, 1992.

[4] R. Davis *et al.*, in *Proc. 21st ICRC Adelaide*, edited by R. J. Protheroe, Volume 12, 143 (1990).

[5] K. Hirata *et al.*, *Phys. Rev. Lett.* 65, 1297 and 1301 (1990); and *Phys. Rev. D* 44, 2241 (1991).

[6] S. P. Mikheyev and A. Y. Smirnov, *Nuovo Cim.* 9C, 17 (1986); L. Wolfenstein, *Phys. Rev. D* 17, 2369 (1978). A review with references to the original literature is given by T. K. Kuo and J. Pantaleone, *Rev. Mod. Phys.* 61, 937 (1989).

[7] S. P. Rosen and J. M. Gelb, *Phys. Rev. D* 34, 969 (1986); E. W. Kolb, M. S. Turner and T. P. Walker, *Phys. Lett.* B175, 478 (1986).

[8] H. A. Bethe, *Phys. Rev. Lett.* 56, 1305 (1986); V. Barger, R. J. N. Phillips and K. Whisnant, *Phys. Rev. D* 34, 980 (1986); and A. Messiah in *'86 Massive Neutrinos in Physics and Astrophysics*, edited by O. Fackler and J. Tran Than Van (Editions Frontieres, Paris 1986) p.373.

[9] W. C. Haxton, *Phys. Rev. Lett.* 57, 1271 (1986).

[10] see Rosen and Gelb, Reference 7.

[11] S. J. Parke and T. P. Walker, *Phys. Rev. Lett.* 57, 2322 (1986).

[12] R. S. Raghavan *et al.*, *Phys. Rev. D* 44, 3786 (1991).

[13] J. N. Bahcall, *Neutrino Astrophysics* (Cambridge University Press, 1989); S. A. Bludman, D. C. Kennedy, and P. G. Langacker, *Nucl. Phys.* B374, 373 (1992).

[14] B. Cabrera, L. M. Krauss, and F. Wilczek, *Phys. Rev. Lett.* 55, 25 (1985); L. M. Krauss and F. Wilczek, *ibid* 55, 122 (1985); R. E. Lanou, H. J. Maris, and G. M. Seidel, *ibid* 58, 2498 (1987); S. R. Bandler *et al.*, *ibid* 68, 2429 (1992). These papers also discuss the possibility of observing ^{7}Be neutrinos simultaneously with the *pp* ones.

[15] S. J. Parke, *Phys. Rev. Lett.* 57, 1275 (1986).

[16] J. N. Bahcall and H. A. Bethe, *Phys. Rev. Lett.* 65, 2233 (1990); H. A. Bethe and J. N. Bahcall, *Phys. Rev. D* 44, 2962 (1991).

[17] A. B. McDonald in *Franklin Symposium in Celebration of the Discovery of the Neutrino*, Philadelphia, April 30 1992 (to be published).

[18] Y. Totsuka in *Franklin Symposium in Celebration of the Discovery of the Neutrino*, Philadelphia, April 30 1992 (to be published).

[19] W. Kwong and S. P. Rosen, *Phys. Rev. Lett.* 68, 748 (1992).

[20] A. J. Baltz and J. Weneser, *Phys. Rev. D* **35**, 528 (1987) and D**37**, 3364 (1988); M. Cribier *et al.*, *Phys. Lett.* **B182**, 89 (1986); E. D. Carlson, *Phys. Rev. D* **34**, 1454 (1986); A. Dar *et al.*, *ibid* **35**, 3607 (1987).

FIGURE CAPTIONS

Figure 1. Electron-neutrino survival probability as a function of Δm^2 in the small angle solution. The solid curve, marked $E_\nu = 0.862$ MeV, is for the mono-energetic ^{7}Be branch of the solar neutrino spectrum, and the dashed curve is for a typical *pp* neutrino with energy $= 0.300$ MeV. The mixing angle is taken to be $\sin^2 2\theta = 7\times10^{-3}$, but the curves are not sensitive to small changes in the range allowed by Reference 3.

Figure 2. Electron-neutrino survival probability as a function of Δm^2 for the large angle solution $\sin^2 2\theta = 0.6$. The solid curve with energy $E_\nu = 0.862$ MeV is for ^{7}Be neutrinos and the dashed curve, with $E_\nu = 0.300$ MeV is for a typical *pp* neutrino.

Figure 3. Recoil electron spectra for the scattering of ^{7}Be neutrinos by electrons. The solid curve is for the Standard solar model, the dashed one is for a typical large angle solution, and the dash-dotted curve is for a typical small angle solution. The actual shapes are not very different from one another, but the overall normalisations are significantly different.

FIGURE 1

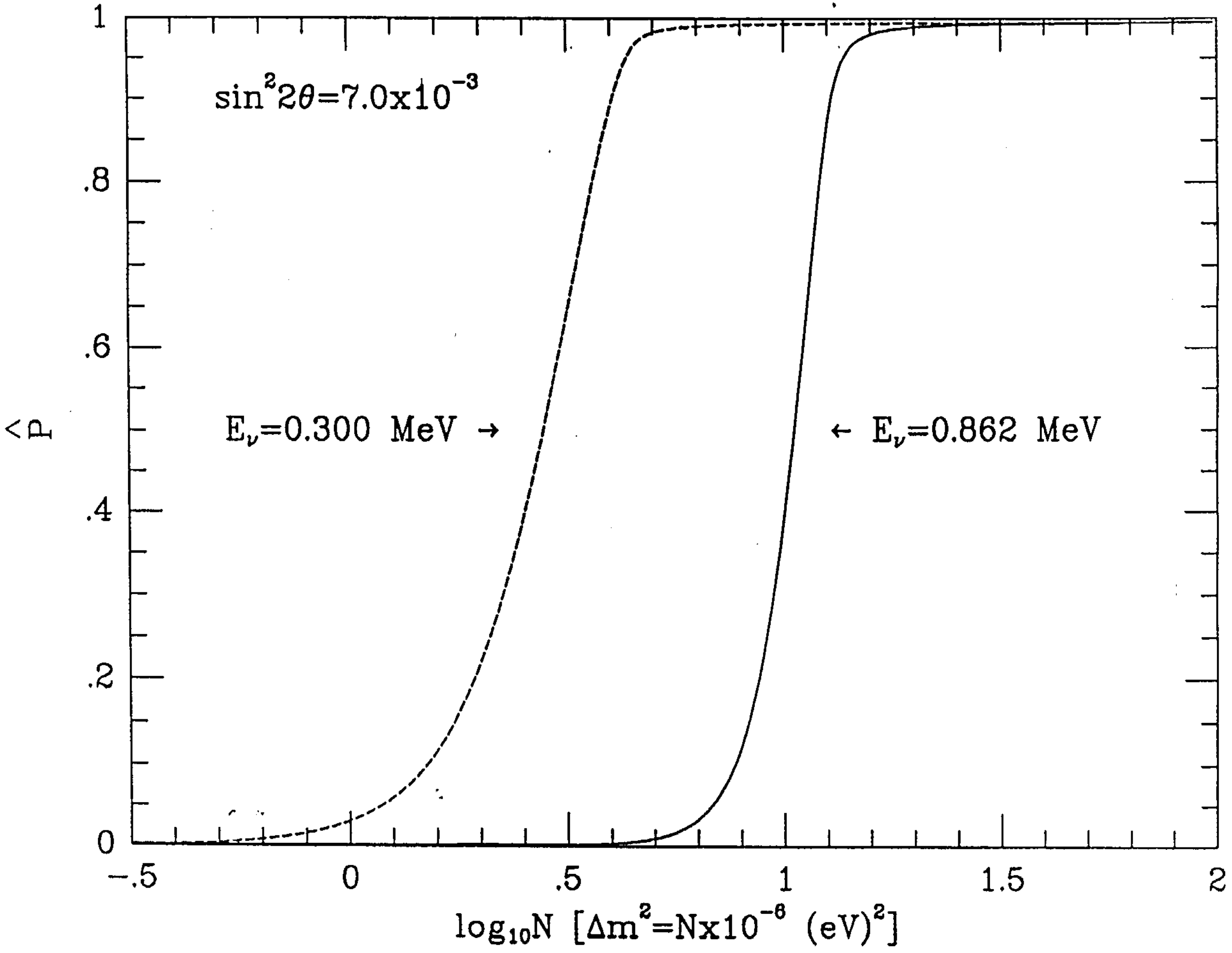

$\sin^2 2\theta = 7.0 \times 10^{-3}$
$E_\nu = 0.300$ MeV →
← $E_\nu = 0.862$ MeV
$\langle P$
1
.8
.6
.4
.2
0
−.5
0
.5
1
1.5
2
$\log_{10} N$ [$\Delta m^2 = N \times 10^{-6}$ (eV)^2]

FIGURE 2

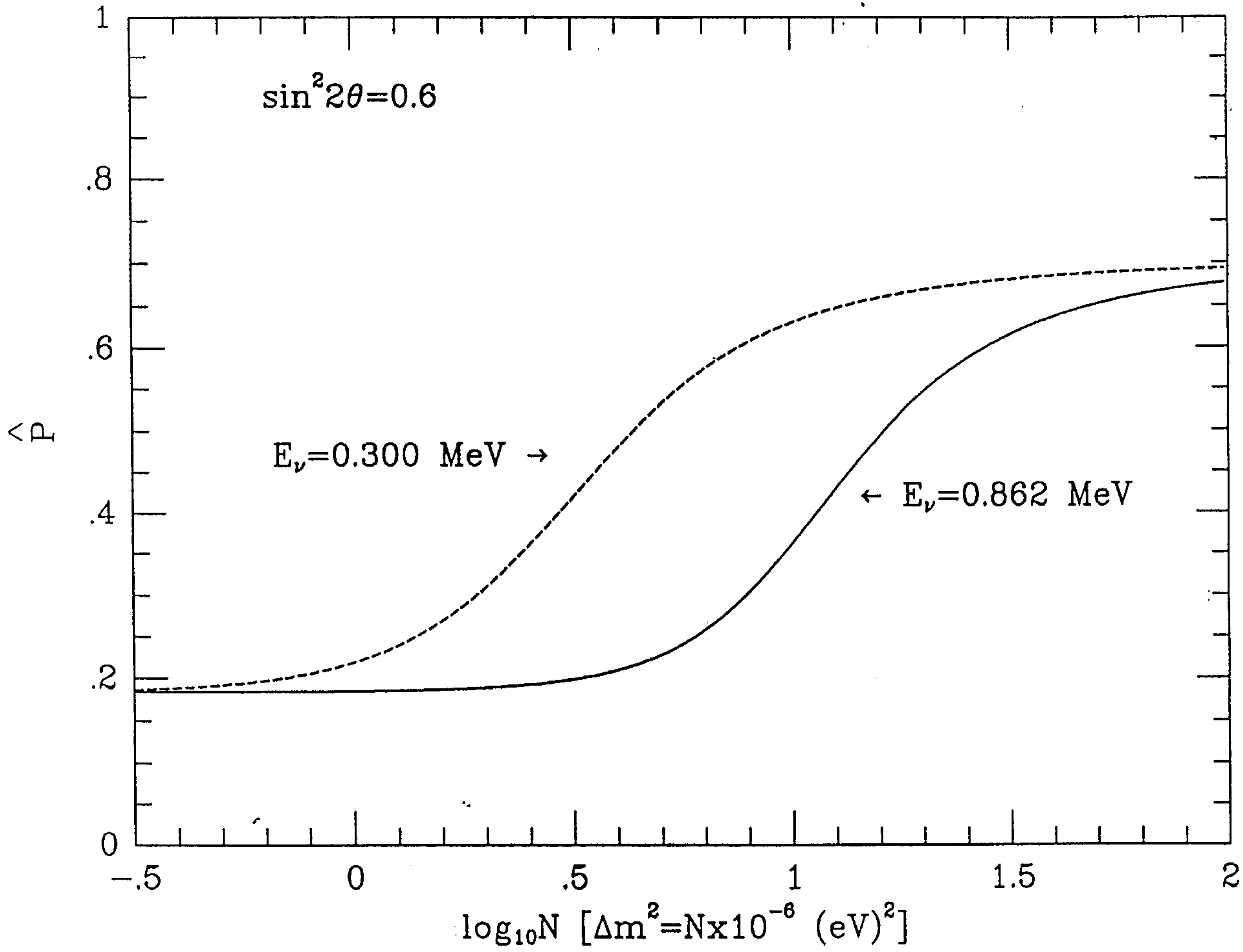
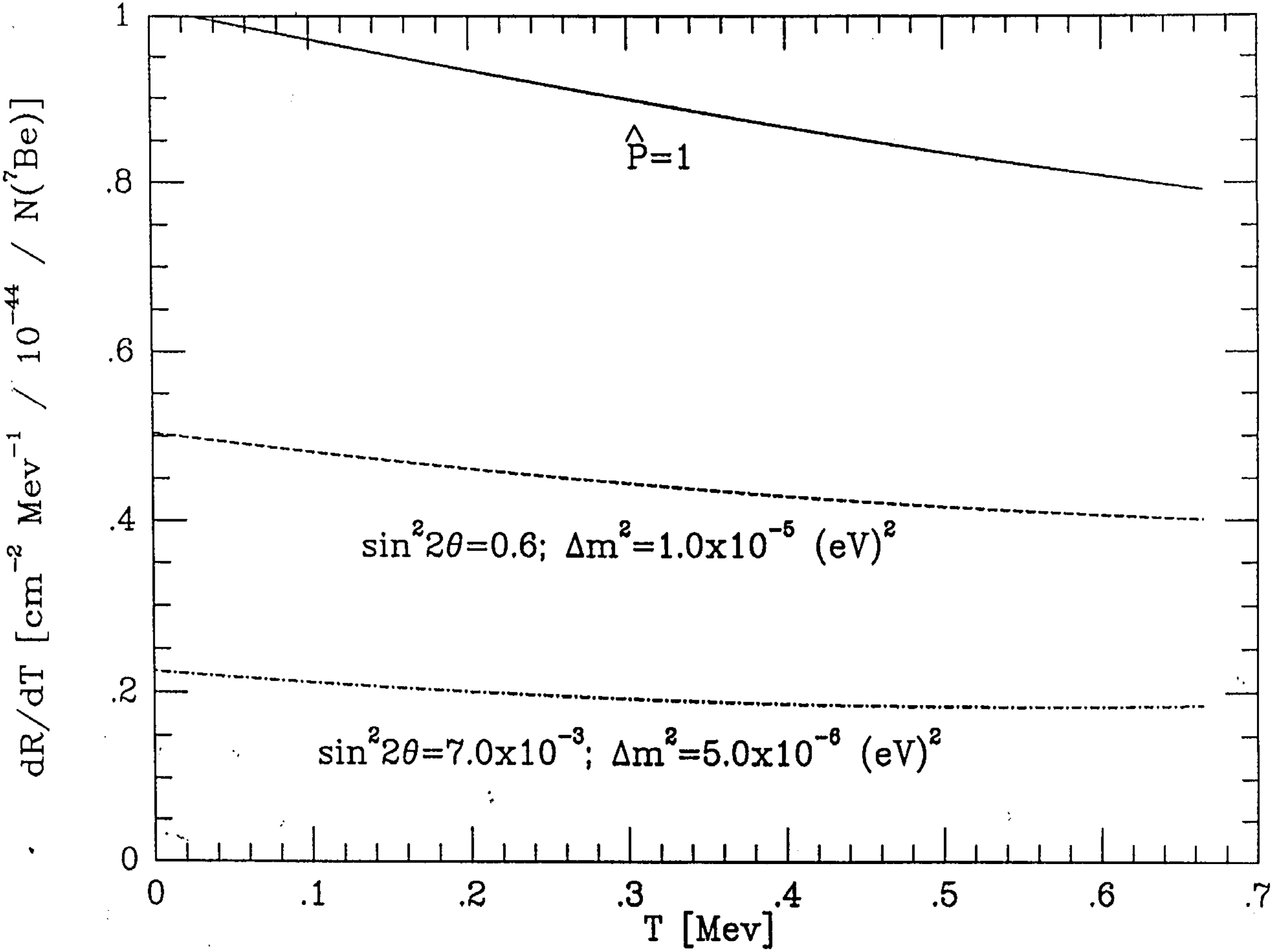

$\hat{P}$
1
.8
.6
.4
.2
0
$\sin^2 2\theta = 0.6$
$E_\nu = 0.300$ MeV →
← $E_\nu = 0.862$ MeV
−.5
0
.5
1
1.5
2
$\log_{10} N\ [\Delta m^2 = N \times 10^{-6}\ (\mathrm{eV})^2]$

FIGURE 3


dR/dT [cm^{-2} Mev^{-1} / 10^{-44} / N(^{7}Be)]
T [Mev]
0
.2
.4
.6
.8
1
0
.1
.2
.3
.4
.5
.6
.7
$\hat{P}=1$
$\sin^2 2\theta=0.6$; $\Delta m^2=1.0\times10^{-5}$ (eV)^2
$\sin^2 2\theta=7.0\times10^{-3}$; $\Delta m^2=5.0\times10^{-6}$ (eV)^2