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THE IMPLICATIONS OF THE COBE-DMR RESULTS FOR COSMIC STRINGS

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ABSTRACT

We compare the anisotropies in the cosmic microwave background radiation measured by the COBE experiment to the predictions of cosmic strings. We use an analytic model for the $\Delta T/T$ power spectrum that is based on our previous numerical simulations to show that the COBE results imply a value for the string mass per unit length, μ under the assumption that cosmic strings are the source of the measured anisotropy. We find $G\mu = 1.5 \pm 0.5 \times 10^{-6}$ which is consistent with the value of μ thought to be required for cosmic strings to seed galaxy formation.

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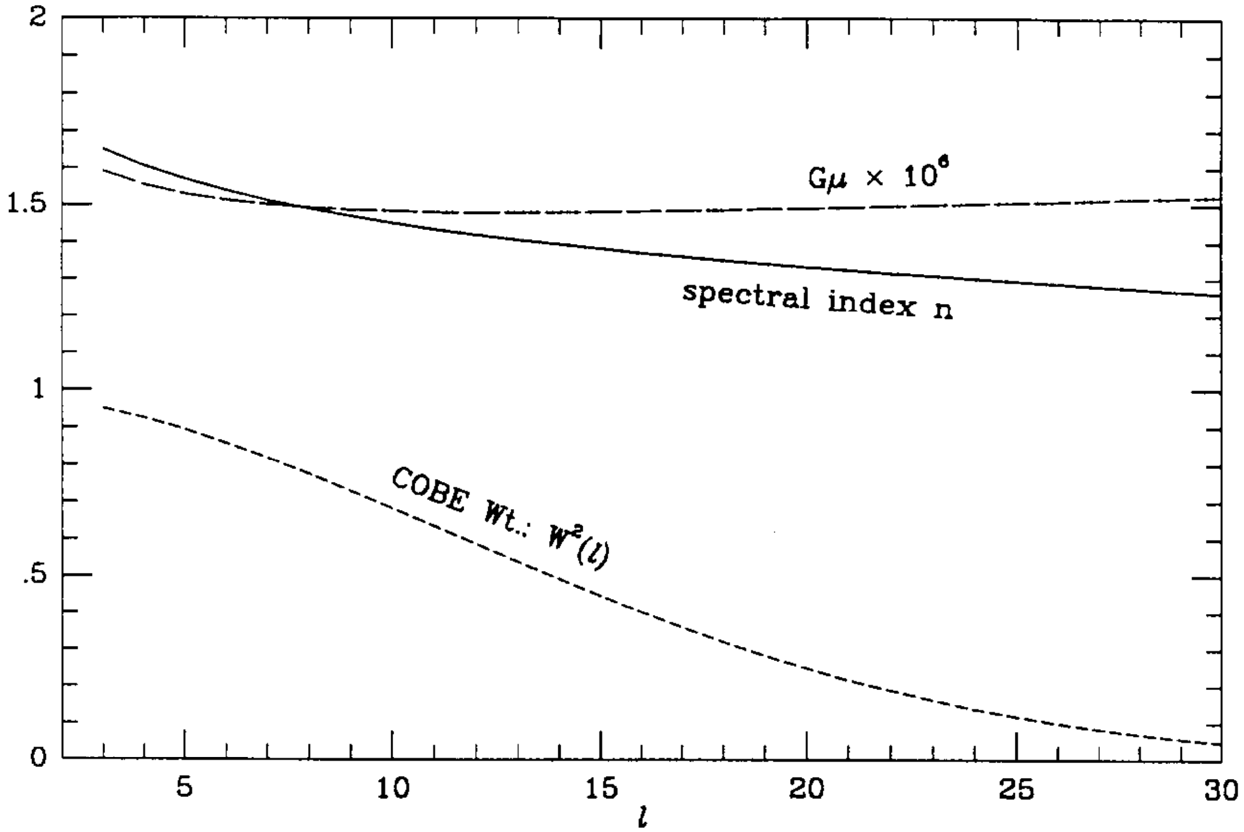

FIGURE 1 The spectral index $n = \frac{d \ln C_l}{dl} + 3$, the Gaussian weight function due to the COBE-DMR beam smearing and the best fit value of $G\mu$ are plotted as a function of l assuming the COBE best fit power spectrum for $n = 1.4$.

I. Introduction

Recently the COBE (COsmic Background Explorer) collaboration has announced brightness fluctuations in the sky at centimeter wavelengths (Smoot *et. al.*, 1992) as measured by the DMR experiment. These fluctuations do not have the characteristics expected of emission from foreground objects at low redshifts and are believed to represent intrinsic temperature fluctuations in the cosmic microwave background radiation (MBR). Henceforth we will assume that the brightness fluctuations are intrinsic temperature fluctuations (i.e. MBR anisotropies). The result is very exciting as these intrinsic fluctuations are perhaps the only available probe of cosmological structures on the scale of thousands of megaparsecs. Determining the nature of structures on these large scales can provide important hints as to the primordial fluctuations on smaller scales which grew to form objects from stars and planets to superclusters of galaxies. The large scale fluctuations also give important hints as to the very early history of our universe.

At present only details of the spectrum ($\equiv$ angular correlation function) of temperature perturbations have been announced. This is not enough information to determine the detailed nature of the production of the temperature perturbations. In the usual picture of primordial adiabatic perturbations, the temperature fluctuations arise from the "gravitational redshift" effects associated with primordial potential hills and valleys. If this is the case then the COBE results indicate that the initial spectrum of perturbations is approximately of the Harrison-Zel'dovich form (Harrison, 1970, and Zel'dovich, 1972). Another possibility is that the temperature fluctuations arose from the time varying gravitational field associated with the motion of seeds, such as cosmic strings, global monopoles, or cosmic textures (Kibble, 1976, Vilenkin, 1980, Zel'dovich, 1980, Turok, 1989, Bennett and Rhie, 1990). The interpretation of the COBE results in these two types of models are very different, although the method for calculating the anisotropies is essentially the same (Sachs and Wolfe, 1967). For primordial perturbations COBE is seeing density fluctuation on the surface of last scattering which are correlated on scales much greater than the "apparent" causal horizon at that time. In the case of seeded perturbations, the temperature fluctuations on large scales are induced at times long after last scattering, and no "acausal" correlations are required. In spite of this difference the two types of theories can produce very similar temperature fluctuations. For example, all of the seed models mentioned above lead to a final spectrum of density and temperature fluctuations which are at least approximately of the Harrison-Zel'dovich form on the scales observed by COBE (Albrecht and Stebbins, 1992a and 1992b, Park, Spergel and Turok, 1991, and Bennett, Rhie and Weinberg, 1992).

Thus, the COBE results should be viewed as encouraging from the point of view of proponents of seeded perturbations. However COBE not only measures the shape of the spectrum of perturbations, but also the amplitude. For the theory to be acceptable the amplitude that is measured should be the same as is needed to form the observed structures in the universe. While our ignorance of galaxy formation may leave us a little fuzzy as to the exact amplitude that is required, there is still only a range of amplitudes which might be considered acceptable. In this *Letter* we will consider the cosmic string scenario, and find that our best estimate of the amplitude of perturbations indicated by COBE is within the acceptable range. Textures and global monopoles are considered elsewhere (Turok and Spergel, 1990, Bennett and Rhie, 1992). Another issue which we do not address in this paper is whether the pattern of anisotropy is consistent with the predictions of cosmic strings. The COBE collaboration has not yet released enough information to determine this, and if they had, it seems likely that the signal to noise would be too low to distinguish the non-Gaussian character of cosmic string induced anisotropies.

2. Constraints of String Model from First Year of COBE DMR

What does COBE say about strings, assuming they are the cause of the anisotropy? The amplitude of the perturbations produced by cosmic strings is proportional to the mass per unit length μ of the strings. So from the COBE results we should be able to normalize this parameter. As with other models, we must rely on the existence of a preponderance of non-baryonic dark matter at the time of last scattering in order that the small scale perturbations not be washed out by sound waves. The nature of the dark matter is an important parameter of the string model. It may be hot dark matter (HDM) such as a light massive neutrino, or cold dark matter (CDM) such as an axion. The shape of the COBE fluctuation spectrum will not tell us anything about which type of dark matter there is, because on the scales that COBE probes, both HDM and CDM act essentially the same. However the value of μ we obtain may give us some clue as to the nature of the dark matter.

The best available estimates of MBR anisotropy from strings are those of Bouchet, Bennett, and Stebbins (1988, hereafter BBS). Several groups are working to improve on these calculations but no results are yet available. The problem with the BBS calculation is they have used a formalism appropriate for the small angle and for strings in Minkowski space (Stebbins 1988). Clearly neither is completely appropriate for the COBE experiment. However the results are not completely inappropriate either. For photons coming much closer than a horizon distance from a piece of string BBS should be accurate. We will make some adjustments of the results of BBS for the deviations from the small angle approximation, but corrections for expansion are much more complicated and will not be attempted. Thus, the reader should keep in mind that there remain significant systematic errors in the results we will present.

The results in BBS for the power spectrum of temperature fluctuation is given by the fitting BBS equation (4) which may be rewritten:

$$\int_0^{\lambda} d\lambda \int_{a_i}^{2a_i} da \, \frac{dC(0)}{d\lambda \, da} = A^2 \, F\left(\frac{\lambda}{\Theta_{\mathrm{H_i}}}\right) \quad A = 6\frac{G\mu}{c^2} \quad F(x) = \left(\frac{x^{1.7}}{(0.6)^{1.7} + x^{1.7}}\right)^{0.7} , \tag{1}$$

where $C(0)$ gives the mean-square $\frac{\Delta T}{T}$, a is the scale factor when these temperature fluctuations are produced, Θ_{Hi} is the tangent of the angle subtended, perpendicular to the line-of-sight, by the horizon when $a = a_i$, and λ refers to the angular wavelength in radians of a Fourier decomposition of the temperature pattern in the small angle approximation. We may approximate equation (1) by

$$\frac{dC(0)}{d\ln\lambda \, d\ln a} = \frac{A^2}{\ln 2} \frac{\lambda}{\Theta_{\mathrm{H_i}}} F'\left(\frac{\lambda}{\Theta_{\mathrm{H_i}}}\right) \tag{2}$$

where $C(\theta)$ is the angular correlation function of the temperature anisotropy.

If we work in units where $a_{\mathrm{now}} = 1$, then we may use $\Theta_{\mathrm{H_i}} = \sqrt{a}/(1 - \sqrt{a})$ and we find

$$\begin{aligned} \frac{dC(0)}{d\ln\lambda} &= \frac{A^2}{\ln 2} \int_{a_{\mathrm{ls}}}^{1} \frac{\lambda}{\Theta_{\mathrm{H_i}}} F'\left(\frac{\lambda}{\Theta_{\mathrm{H_i}}}\right) \frac{da}{a} \\ &= \frac{2A^2}{\ln 2} \left[(1 - \sqrt{a_{\mathrm{ls}}}) \, F\left(\frac{\lambda}{\Theta_{\mathrm{H_{ls}}}}\right) - \int_{\sqrt{a_{\mathrm{ls}}}}^{1} F\left(\frac{\lambda}{\Theta_{\mathrm{H_i}}}\right) d\sqrt{a} \right] , \end{aligned} \tag{3}$$

where "ls" refers to the last-scattering surface. When $\Theta_{\mathrm{H_{ls}}} \ll \lambda \ll 1$ the $[\cdots]$ in eq. (3) goes to unity and we can see that the temperature power spectrum is "scale-invariant". Physically, this limit corresponds to the case where all the contributions to $\Delta T/T$ come from strings at $\sqrt{a_{ls}} \ll \sqrt{a} \ll 1$. The effects due to non-zero a_{ls} are negligible on the COBE scales, however deviations from scale invariance as one goes beyond $\lambda \ll 1$ are not. An important deviation from scale invariance which is due to the contribution of low redshift strings to small angle anisotropies is properly included in Eq. (3) .

A Fourier decomposition on small scales corresponds to a decomposition into spherical harmonics on the sphere of the sky:

$$\frac{\Delta T}{T}(\hat{\mathbf{n}}) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} a_{(l,m)} Y_{(l,m)}(\hat{\mathbf{n}}) \tag{4}$$

where since $\nabla^2 Y_{(l,m)} = -l(l+1)Y_{(l,m)}$ we see that λ becomes $2\pi/\sqrt{l(l+1)}$ and only a discrete set of wavelengths are allowed. The analog of the power spectra is

$$C_l = \sum_{m=-l}^{l} |a_{(l,m)}|^2 \tag{5}$$

which gives the mean square temperature fluctuation is

$$C(0) = \sum_{l=0}^{\infty} \frac{2l+1}{4\pi} C_l. \tag{6}$$

For large l we make take the sum to an integral obtaining

$$\frac{dC(0)}{d\ln l} = \frac{dC(0)}{d\ln \lambda} = \frac{l(l+1)}{2\pi} C_l \qquad l \gg 1 \tag{7}$$

which may be compared to Eq. (3).

Smoot *et. al.*(1992) have fit their results to "power law" correlation functions of the form

$$C_l = (Q_{rms-PS})^2 \frac{4\pi}{5} \frac{\Gamma(l + \frac{n-1}{2})\,\Gamma(\frac{9-n}{2})}{\Gamma(l + \frac{5-n}{2})\,\Gamma(\frac{3+n}{2})} \ . \tag{8}$$

When they take into account the cosmic variance, and do not include the measured quadrupole in their fit, they obtain best fit values of $n = 1.15^{+0.45}_{-0.65}$ and $Q_{rms-PS} = 5.96 \pm 1.68 \times 10^{-6}$. When they include the quadrupole (which they believe is more susceptible to systematic errors than the larger l modes), they obtain $n = 1.5$ and $Q_{rms-PS} = 5.1 \times 10^{-6}$.

In the extreme small angle approximation, cosmic strings predict a spectral index of $n = 1$, but for the range of l that COBE is sensitive to and for which we expect our calculations ($5 \lesssim l \lesssim 20$) to be valid, we find a spectral index closer to $n = 1.4$ (see Fig. 1). These can be compared to the COBE fits for fixed n: $Q_{rms-PS} = 6.11 \pm 1.68 \times 10^{-6}$ for $n = 1$ (Smoot *et. al.*, 1992), and $Q_{rms-PS} = 4.75 \pm 1.30 \times 10^{-6}$ for $n = 1.4$ (G. Smoot, private communication). For the $n = 1$ case and the extreme small angle approximation, we can neglect the integral in eq. (3), and then use eqs. (1), (7), and (8) to find

$$G\mu = \frac{1}{6}\sqrt{\frac{6\ln 2}{5}} Q_{rms-PS} = 0.152\, Q_{rms-PS} = 9.3 \pm 2.6 \times 10^{-7} . \qquad (9)$$

A more accurate value can be obtained by using the $n = 1.4$ fit values to compare with eq. (3) evaluated numerically. The results of this calculation are displayed in Fig. 1 which shows the results of the integral in eq. (3), and the resulting values of $G\mu$. Our best fit value of $G\mu \simeq 1.49 \times 10^{-6}$ is shown as a function of l because the $n = 1.4$ power law is not a perfect fit to our results. Also plotted in Fig. 1 is the Gaussian weight function $W^2(l) = e^{-l(l+1)/17.8^2}$ which describes the smoothing due to the COBE-DMR 7° beam.

Now let us attempt to estimate the errors in our calculation. These errors come from two main sources: the small angle approximation, and the Minkowski space approximation. We expect that the errors due to the small angle approximation are small if we restrict ourselves to moderately large l values (say $l > 5$). The errors due to the Minkowski space approximation are not so easy to quantify, but one obvious symptom of this approximation is the power spectrum given in eq. (1) which extends outside the horizon. In a complete treatment of this problem, one would expect that the power spectrum might get cut off at around $\lambda = \Theta_{H_i}$ due to the required compensation of the string perturbations by the matter fields. Inserting this cutoff in our calculation gives $G\mu \simeq 1.85 \times 10^{-6}$. Although one might expect that adding the effect of compensation would always tend to decrease $\Delta T/T$ and therefore increase our estimate of $G\mu$, there are cases in which adding in the compensation actually serves to increase $\Delta T/T$ (Stebbins and Veeraraghavan, 1992). Therefore, we will take $(1.85 - 1.49) \times 10^{-6} = 0.36 \times 10^{-6}$ to be our (symmetric) 1-σ error bars. Adding these in quadrature with COBE's experimental error bars yields $G\mu = 1.49 \pm 0.52 \times 10^{-6}$ which is our prediction.

How does this prediction for $G\mu$ compare with smaller angular scale experiments? The answer to this question is muddied somewhat by the possibility that the universe underwent reionization, but some conclusions are still possible. A detailed comparison with the OVRO experiment (Readhead, *et. al.*, 1989) indicates a rather weak limit, $G\mu \lesssim 4 \times 10^{-6}$ due in part to the non-Gaussian character of the anisotropies on arc minute scales (Bennett, Bouchet, and Stebbins, 1992). In a reionized universe, the OVRO limit would be much weaker. More intriguing is a comparison with the MAX experiment which has likely detected anisotropy on a 1° scale (Devlin, *et. al.*., 1992). (They have a strong signal which is consistent with CMB anisotropy, but they do not make a definitive claim that this is what they have detected.) A preliminary analysis of their data indicates that it is consistent with our value for $G\mu$ in models both with and without early reionization.

Another observational constraint on μ comes from the gravitational waves they produce, which can cause "jitter" in the timing of rapidly rotating pulsars. Our predicted value of μ is more than an order of magnitude below the upper limits set by Bennett and Bouchet (1991). The Bennett-Bouchet limit has the advantage that it has very little dependence on poorly understood details of cosmic string evolution, but Caldwell and Allen (1991) have shown that a considerably more stringent limit is possible when one assumes a specific model for cosmic string evolution. The most stringent limits on μ found by Caldwell and Allen should be regarded with some skepticism, however, since they rely on an unpublished analysis of the pulsar timing data (Ryba, 1991). (Previous unpublished analyses of the pulsar timing data (Taylor, 1989) have resulted in "limits" that have subsequently been revised upward by an order of magnitude (Stinebring, *et. al.*, 1990).) In addition, Caldwell and Allen have ignored the possibility that infinitely long cosmic strings might radiate a significant amount energy directly into gravity waves as claimed by Allen and Shellard (1992). If true, this would serve to weaken the Caldwell and Allen bounds on μ by a factor of ~ 2. If we accept the unpublished pulsar timing analysis and revise Allen and Caldwell's limits upward by a factor of 2 to account for the radiation from long strings, then we find that the Allen and Caldwell limits are $G\mu \leq 6 \times 10^{-7}$ for $h = 1$ and $G\mu \leq 1.6 \times 10^{-6}$ for $h = 0.75$. Thus, if $h = 1$ and one accepts the unpublished analysis, then our fit to the COBE data is almost inconsistent with the pulsar timing data at the 2σ level. If we consider only the published pulsar timing analysis or smaller values of h, then there is good agreement between our fit to COBE and the pulsar timing data.

3. Conclusions

In this *Letter* we have shown that the recent COBE results are quite consistent with the idea that inhomogeneities in our universe were induced by cosmic strings in an flat FRW cosmology, which is predominantly dark matter. If this is so then we estimate that the mass per unit length of the strings is $1.49 \pm 0.52 \times 10^{-6}$. This can be compared to the value of $G\mu$ thought to be required in order to seed galaxy formation. The most sophisticated calculation to date of the values of $G\mu$ required for string seeded galaxy formation scenarios have been done by Albrecht and Stebbins (1992a and 1992b). They estimate $G\mu \approx 2.0 \times 10^{-6}/b_8$ for the $h = 1$ hot dark matter (HDM) model, $G\mu \approx 4.0 \times 10^{-6}/b_8$ for the $h = 0.5$ HDM model, $G\mu \approx 1.8 \times 10^{-6}/b_8$ for the $h = 1$ cold dark matter (CDM) model, and $G\mu \approx 2.8 \times 10^{-6}/b_8$ for the $h = 0.5$ CDM model. $h = H_0/(100\text{km}/\text{sec}\,\text{Mpc}^{-1})$ and b_8 is the bias factor which gives the normalization of the density field: the RMS density fluctuation in a sphere of radius $8\,h^{-1}$ Mpc is $1/b_8$. Thus, values of b_8 between 1 and 3 seem to be consistent with the COBE-DMR results. This is roughly the range of values that are considered to be plausible from considerations of galaxy formation. In contrast, the standard CDM model for galaxy formation seems to require much smaller values of b_8. The COBE measurement implies $b_8 = 0.90 \pm 0.25$ for $h = 0.5$, $\Omega_b = 0.03$; $b_8 = 0.59 \pm 0.16$ for $h = 0.75$, $\Omega_b = 0.03$; and $b_8 = 1.03 \pm 0.28$ for $h = 0.5$, $\Omega_b = 0.1$ for the standard CDM model according to the calculations of Bond and Efstathiou (1987). Since $1.5 \lesssim b_8 \lesssim 2.5$ is generally thought to be required for standard CDM, the COBE data is only marginally consistent with this model. Thus, if the calculations presented here and the calculations of Albrecht and Stebbins (1992a and 1992b) are confirmed by more detailed work, we can conclude that the COBE-DMR anisotropy measurements favor cosmic string models over standard CDM.

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