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A Class of Models with Large Neutrino Magnetic Moment

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We show how a list of symmetry principles can be used to generate a class of models in which the neutrino masses are suppressed while relatively large magnetic moments are allowed. The simplest example is a model for neutrino mass proposed by Zee some time ago. We show how the model can be improved based on the symmetry principle and we also demonstrate other models in this class which avoid some of the weakness in the simplest model.

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I. INTRODUCTION

There has been a lot of interest in models or mechanisms which can produce large magnetic moment for neutrinos because of the anticorrelation of the solar neutrino flux with sun spot activity[1]. One of the problems of having a large magnetic moment is that, on general grounds, the model will induce a large neutrino mass as well. Therefore, the existing strong constraint on neutrino mass translates into a stringent upper bound on the magnetic moment. The magnetic moment that is required to produce sun spot anticorrelation is about $10^{-11}\mu_B$[2]. Without looking into the details of any model, one expects the generic value for the magnetic moment, μ_ν, to be roughly $e\frac{m_\nu}{M^2} \times (loopfactor)$ where m_ν is neutrino mass and M is a typical mass scale in the loop. The loop factor can be estimated to be $(\frac{g_2^2}{16\pi^2})^n$ where n is the difference between the number of loops needed to obtain μ_ν minus the number of loops needed to obtain m_ν. Ignoring the loop factor, for $M = M_W = 80$ GeV and $m_\nu = 10eV$, μ_ν is $10^{-15}\mu_B$, too small for our purpose.

Clearly one has to arrange some magic to overcome the general argument above. That is, one needs a model in which the ratio $r = \frac{M}{M_W}$ is smaller as 10^{-2}. If one prefers to have the neutrino masses in the range that will give rise to resonant oscillation, this number will have to be even smaller as we shall discuss later. In field theory, one natural way of producing a small number is to make use of a custodial symmetry. The symmetry has to suppress neutrino mass and allow magnetic moment. A very general scheme using an $SU(2)$ symmetry was proposed by Voloshin[3]. Implementations of such symmetry in realistic models[3–6] achieve only limited success. A class of them breaks this custodial symmetry spontaneously. Clearly, the central scheme of this approach is to avoid excessive fine tuning. One usually considers the model (or its fine tuning) more natural when the small ratio r can be related to the ratio of the custodial symmetry breaking scale v_c over the electroweak breaking scale V_2. However most of the models in the literature in this class fail to achieve this because various experimental or astrophysical constraints require the scale v_c to be higher than V_2. A variation of Voloshin's scheme using discrete groups for custodial symmetry[7, 8] makes it easier to achieve the above requirement via "minimal fine tuning". However, such models have yet to face up to the potential domain wall problem.

A more modest approach is to introduce an explicit soft breaking term for the custodial symmetry. The dimensionful coupling constant of this soft breaking term is then fine tuned to be small. This approach may avoid some of the physical constraints related to spontaneous breaking of symmetry, and therefore requires only a lesser fine tuning. However, it does not avoid the basic fine tuning questions mentioned above.

Still another approach, which leads to the class of models that we wish to concentrate on here, is to recognize the fact that in the standard model itself there are already some parameters which are experimentally magically small numbers in the theory. They include the electron and muon masses. No satisfactory theory has yet been constructed to explain their small values. Even though some intricate schemes have been constructed to generate small values for them, they are generally considered too complicated. Here we shall not concern ourselves with such questions. Instead, we wish to ask if one can relate the small number that we need for r to the smallness of electron and muon masses. Even though there is no need for custodial symmetry in general in such a scheme, the concept of such a symmetry as an approximate one can still be very useful in decoding the necessary ingredients for achieving such a goal.

Clearly, one needs a theory in which, when the electron and muon masses are set to zero as a first approximation, the ratio r is zero automatically. That is, in that limit, one recovers a theory in which neutrino masses are zero but magnetic moments are not. This can be achieved most easily by requiring the existence of a custodial symmetry which emerges *automatically* in the limit of vanishing electron and muon masses. This has the added advantage that one does not need to *impose* any artifical, spontaneously broken, global symmetry from the outset. In fact one does not have to look far for such a symmetry. As we shall see, the Voloshin symmetry can serve very well for this purpose. In the standard model, when the Yukawa couplings associated with electron and muon masses are set to zero, the theory automatically has leptonic chiral $SU(2)\times SU(2)$ symmetry. Therefore, the Voloshin symmetry seems very natural as an accidental approximate symmetry. Theoretically, it is certainly appealing to link the smallness of the neutrino masses relative to the magnetic moments to the smallness of the charged lepton masses relative to the gauge boson masses. This is the central scheme of the class of models which we wish to explore and exploit. Though, the custodial symmetry in such a scheme is not broken softly, but by the Yukawa couplings of charged leptons, it is still useful as a tool to analyze the model.

We have chosen not to introduce any extra leptons or gauge bosons in our scheme. Clearly models that implement this scheme with extra leptons or extended gauge bosons are possible. However they are not necessary for the present purposes of illustrating the full complexity and subtlety the scheme. It also should be emphasized that while the scheme takes advantage of the known facts that the electron and muon masses are small it does not purport their explanation. It may be possible to extend the models to provide also an explanation of their smallness. However that will be beyond the scope of this paper and may confuse the issue at hand to attempt it here.

In order to generate magnetic moments for the neutrinos, one has to break lepton number. However, to suppress neutrino masses one has to make sure this breaking preserves the custodial symmetry. In some models even this suppression factor is not enough to suppress the neutrino mass difference which has to be as small as $\Delta m^2 = 10^{-7} eV^2$[9]. In that case, additional symmetry such as ZKM[10] symmetry may be needed to obtain a realistic model. Here, by ZKM symmetry, we mean electron number minus muon number, $L_e - L_\mu$. The symmetry will only allow flavor off-diagonal entries in the neutrino majorana mass matrix. Therefore, the mass difference is zero to all orders. To summarize, the above arguments indicate that one may generate a reasonably successful model by keeping the following principles. (a) One must make sure the breaking of custodial symmetry is confined to the coupling constants which are either known to be small experimentally, such as the Yukawa couplings that give masses to the electron and muon, or, is in a sector which is remote from the leptonic sector. An example of the latter are the quartic self couplings of the Higgs fields. Such symmetry breaking couplings generally have negligible effects because it usually takes multi-loop diagrams for the these couplings to be effective in the leptonic sector. (b) The lepton number symmetry breaking must be confined to the sector in which the custodial symmetry is preserved. (c) One must either impose ZKM symmetry or implement the spectrum such that ZKM symmetry is automatic. The ZKM symmetry may be an exact symmetry, or, it can be broken softly. In the spirit of avoiding imposing global symmetry mentioned earlier, one certainly prefers the symmetry to be automatic.

In the following we shall first illustrate these ideas using the model of Zee [11] and then discuss a number of other models which may be simpler or may contain some significant phenomenological improvements. In section II, we show that the Zee model belongs to this class of models, and how the general principles above are realized in the model. The Zee model example also exposes important technical issues associated with the mechanism. Some simple variations of the model including the one proposed by Barr, Freire and Zee [12] are also discussed. In section III, we discuss another source of contribution to the magnetic moment in the model which can dominate over the previous ones. In Section IV, we discuss another model, the Triplet model, which exhibits automatic ZKM symmetry in the two generation case.

The ZKM symmetry can in general be kept unbroken so that the neutrino mass difference will be zero to all orders. However, aspects of neutrino propagation in solar matter may make the soft breaking case more attractive. Nevertheless, since the unbroken case is not definitely ruled out by the experiments, we shall simplify the issues by assuming that the ZKM symmetry is unbroken until Section V. The implications for the different implementations of ZKM symmetry including soft breaking are discussed in section V.

Unless we specify otherwise, we will incorporate only two generations of leptons in these models. The third generation can be decoupled easily by imposing the corresponding lepton number symmetry. Complications that can arise if the third generation does not decouple will be discussed in section V or briefly commented on wherever else it is appropriate.

Our aim is not to advocate any specific model, but to emphasize the effectiveness of the principles listed above.

II. THE ZEE MODEL

The Zee model[11] is a simple extension of the Standard Model Higgs sector by the addition of a charged $SU(2)_L$ singlet, h^+. The singlet can couple to the leptons as follows:

$$f_{ab} L_{aL}^T C L_{bL} h^+ \tag{1}$$

where

$$L_{1L} = \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix}, L_{2L} = \begin{pmatrix} \nu_{\mu L} \\ \mu_L \end{pmatrix} \tag{2}$$

(a and b are lepton flavor indices). The coupling matrix f_{ab} must be antisymmetric. Adhering to our recipe given in the introduction, the singlet scalar is the most promising way to transfer lepton number to the Higgs sector while preserving the custodial symmetry. If the two lepton doublets themselves form an $SU(2)_H$ doublet, Eq. (1) remains invariant because the antisymmetric combination of fields transforms as a singlet. We shall use this $SU(2)_H$ Voloshin type symmetry as the custodial symmetry discussed in the introduction[13].

Without introducing new gauge interactions or new leptons, the only other way to introduce a lepton number carrier is to couple the leptons to an $SU(2)_L$ triplet

$$T = \begin{pmatrix} T^{++} \\ T^{+} \\ T^{0} \end{pmatrix} \tag{3}$$

with a flavor symmetric coupling. This coupling does not respect the $SU(2)_H$ global symmetry mentioned above, and such models of neutrino magnetic moment can be troublesome[14]. Therefore we shall not pursue this line of inquiry.

A model which adds only the singlet h^+ to the usual doublet in the Higgs scalar spectrum conserves lepton number. Therefore, additional structures should be added to produce neutrino mass or magnetic moment. One such structure, proposed by Zee[11], is an additional Higgs doublet which permits the coupling

$$M_{ij} h^- (\phi_i \phi_j) \tag{4}$$

where

$$\phi_i = \begin{pmatrix} \phi_i^+ \\ \phi_i^0 \end{pmatrix} \tag{5}$$

and i is a Higgs doublet generation index. For this interaction to be gauge invariant, it will require that the coupling M_{ij} be antisymmetric. When no discrete symmetry is imposed on the Higgs fields, without loss of generality one can choose a basis in which $\langle\phi_1\rangle \neq 0$ and $\langle\phi_2\rangle = 0$. We shall analyze the model for this case and comment on the case with discrete symmetry later.

Following the general arguments in the previous section, we should first analyze the model in the approximation that the small ordinary Yukawa couplings associated with ϕ_i are negligible. Since lepton number breaking is necessary to generate magnetic moment one should examine the flow of lepton number in the Lagrangian in that limit in order to identify the essential elements of the model. To understand why the magnetic moment is suppressed it is helpful to define an approximate lepton number symmetry so that the breaking effects are as far removed from the neutrinos as possible. In that spirit, two units of lepton number can be assigned to h^+. Since $\langle\phi_2\rangle = 0$, the cubic coupling M_{12} in Eq(4) suggests that two units of lepton number should be assigned to ϕ_2 and none to ϕ_1. This pushes the potential lepton number breaking interactions further away from the neutrino sector. In fact, lepton number is broken by the Higgs quartic self-interaction of the form $\lambda\phi_2^\dagger\phi_2\phi_2^\dagger\phi_1$ where λ is the coupling constant. Note that this quartic coupling is consistent with the assumption about vacuum expectation values made above. Alternative sources of lepton number breaking are the couplings of ϕ_2 to the quark sector, or to exotic fermions. This mechanism will be explored in the next section. In addition, one should be reminded that, in the neutrino sector of the whole theory, the leptonic Yukawa couplings which we proposed to ignore earlier also break this particular version of lepton number. However they are numerically small as commented before.

One can now proceed to evaluate the ratio of neutrino mass to magnetic moment. Figure 1 shows one loop $\Delta L = 2$ processes which contribute to neutrino mass and magnetic moment. Note that these graphs are proportional to the empirically small lepton masses and small Yukawa couplings of ϕ_2. For the magnetic moment the presence of these factors are one loop artifacts. We can escape these suppressive factors by going to two loops as shown in Figure 2. In these diagrams, one can blame the the lepton number breaking on the quartic couplings, λ, of the Higgs fields which generate a cubic term after symmetry breaking. However, the crucial point is that, in the case of the contributions to the neutrino masses, the suppressive factors of Yukawa couplings and masses can not be avoided by going to higher loop diagrams. To understand this one has to analyze the approximate custodial symmetry mentioned in the introduction.

In the limit that the ordinary leptonic Yukawa couplings are ignored, the two generation theory has an automatic $SU(2)_H$ symmetry which transforms only the two leptonic $SU(2)_L$ multiplets as a doublet. Therefore, the remaining lepton number breaking interactions mentioned above are all $SU(2)_H$ invariant. Since the $SU(2)_H$ forbids neutrino mass, one concludes that the small Yukawa couplings have to be invoked in the diagram to generate nonzero masses . Since the f_{ab} coupling is also needed to have lepton number breaking in the diagram, the chirality of the neutrino mass operator implies that additional mass insertion has to be present in the diagram too. We therefore conclude that, to generate neutrino mass, the one loop suppressive factors mentioned earlier can not be avoided by using higher loop diagrams. Note that our argument is independent of the specific diagrams that can contribute. For the subset of diagrams that has the general structure shown in Fig. 3, this effect was interpreted as the result of a "spin-polarization mechanism" in Ref. [12]. In the Zee model and the extension provided in Ref. [12] it is easy to show that the other two loop diagrams that do not have the structure of Fig. 3 are also suppressed by charged lepton masses. However, such a mechanism won't be sufficient for diagrams at even higher loop level. In Section IV, we will show a model in which the spin-polarization argument is not sufficient even at the two loop level.

The one-loop contribution to neutrino masses estimated from Fig. 1 is

$$m_\nu^{1-loop} = \frac{1}{16\pi^2}\frac{f_{e\mu}Y_{\mu\mu}M_{12}m_\mu^2}{M_2^2} ln(M_h/M_2). \tag{6}$$

Here, $Y_{\mu\mu}\frac{m_\mu}{V_\phi}$ is the Yukawa coupling of ϕ_2 to the muon and its neutrino. If one ignores the potential source of lepton number breaking due to the scalar couplings to the quarks, the two loop contributions to the masses and magnetic moments from diagrams in Fig. 2 can conservatively be estimated[12] (for $M_h > M_W$)to be

$$m_\nu^{2-loop} = \frac{1}{120}\frac{1}{(16\pi^2)^2}\frac{f_{e\mu}g^2V_\phi^2\lambda M_{12}m_\mu^2}{M_h^2M_2^2} ln(\frac{M_2^2}{M_h^2}) \tag{7}$$

and

$$\mu_\nu^{2-loop} \simeq \frac{1}{60}\frac{1}{(16\pi^2)^2}\frac{f_{e\mu}g^2eV_\phi^2\lambda M_{12}}{M_h^2M_2^2} ln(\frac{M_2^2}{M_h^2}) \sim \frac{1}{2}\frac{e}{m_\mu^2}m_\nu^{2-loop} \tag{8}$$

where $V_\phi \equiv \langle\phi_1\rangle$ and M_2 is the mass of the second Higgs doublet (we assume the mass of the charged and neutral components to be roughly equal). The surprising factors of $\frac{1}{120}$ and $\frac{1}{60}$ are the result of cancellations between various diagrams[15]. To obtain a numerical value for these quantities we assume the following natural values of $V_\phi \sim 250$ GeV, $M_h \simeq M_2 \simeq M_{12} \sim 100$ GeV, $f_{e\mu} \simeq \lambda \sim 0.1$, $g^2 \simeq 0.41$. With these inputs we find $m_\nu^{1-loop} \sim Y_{\mu\mu}\times 60eV$, $m_\nu^{2-loop} \sim 9\times10^{-4}eV$, and $\mu_\nu^{2-loop} \sim 5\times10^{-14}\mu_B$.

We notice that the one-loop contribution to the neutrino mass is much larger than the two-loop contribution as expected. In fact, the natural value for m_ν^{1-loop} exceeds the experimental bound on the electron neutrino. However, since the magnetic moment is insensitive to the Yukawa coupling, $Y_{\mu\mu}$, of ϕ_2, we can suppress the one loop contribution to neutrino mass by making this coupling one order of magnitude smaller than its natural value, $Y_{\mu\mu} \sim 1$. This is of course due to the fact that the coupling is both lepton number and $SU(2)_H$ breaking as required for neutrino mass.

More seriously, since Y_{ij} are in general not flavor diagonal, the natural mass square difference for the two neutrino flavors should be about the same as the mass square. The large mass difference creates a barrier that is too great for the magnetic spin-flip oscillation to be effective. It can also give rise to dangerous flavor changing effects such as $\mu \to e + \gamma$. The easiest way to avoid both problems is to impose the ZKM-type [10] symmetry $L_e - L_\mu$. (When tau is included in the analysis the ZKM symmetry can involve L_τ as well). The triplet model described in the next section provides an example in which the ZKM symmetry arises automatically.

Alternatively, one can try to use additional symmetry to suppress the one loop contributions to masses. The model of Barr, Freire, and Zee (BFZ) with three Higgs doublets allows one to get rid of the one loop contribution by discrete symmetry. In the basis in which only one of the Higgs doublets has a VEV, a discrete symmetry is imposed on the other two Higgs doublets to totally remove their couplings to the leptons. As a result, the lepton number violating effect can be localized in the quartic self coupling of Higgs such as $\lambda_{23}\phi_1^\dagger\phi_2\phi_1^\dagger\phi_3$. This removes the one-loop processes such as those in Fig. 1. In this case the leading contribution to neutrino mass is coming from the two loop diagrams as in Fig. 2. BFZ choose the value of the product λM_{12} to be large enough to get a magnetic moment of $10^{-11}\mu_B$. However, even in this case the neutrino masses (and their difference) , as estimated earlier, are still not small enough. Fortunately the two-loop masses and the two-loop magnetic moments have different dependence on Higgs boson masses. If the Higgs boson masses are chosen so that the mass difference between the charged and neutral components of the Higgs doublets are small, the two-loop neutrino mass can be further suppressed.

As in the BFZ case, the two doublet Zee model also requires some adjustment to increase the size of the magnetic moment. Following Ref. [12], to reach $\mu_\nu \sim 10^{-11}\mu_B$, one can raise the value of the product λM_{12} by one to two orders of magnitude. The side effect of this fine tuning will be an increase in the two-loop neutrino mass contribution to about $10^{-2}eV$. Lim, Marciano, and Akhmedov[9] have shown that a neutrino mass square difference of about $10^{-7}eV^2$ is required to permit resonance in the convection layer of the Sun. We can in principle arrange the masses of the Higgs bosons such that the neutrino mass square difference is suppressed, as proposed by BFZ. Alternatively one can impose the ZKM symmetry and give up the possibility of resonance in the solar interior (see section V).

With ZKM symmetry to remove neutrino mass difference, and discrete symmetry such as the one used by BFZ to remove the one-loop mass, this model has a natural ratio $R \equiv m_\nu^{2-loop}/\mu_\nu^{2-loop} \sim m_\mu^2/e$. This implies an automatic suppression of mass to magnetic moment at two loops, so that for $\mu_\nu \sim 10^{-11}\mu_B$, $m_\nu \sim 10^{-1}eV$. This relation is a general result of this class of models when one succeeds in eliminating the one loop contribution to masses.

III. MODELS WITH FERMION LOOPS

There is another important source of contribution which can arise in the Zee model when the Higgs doublets are allowed to couple to the quark sector or to an exotic fermion sector. In that case these Yukawa couplings become the source of lepton number breaking. With no exotic fermions, a typical two-loop contribution to magnetic moment with a quark loop is shown in Figure 4. The advantage of this contribution is that the accidental suppressive combinatorial/cancellation factors of $\frac{1}{60}$ or $\frac{1}{120}$ in equations (7) and (8) can be avoided, giving them one or two orders of magnitude advantage. However, *both* mass *and* magnetic moment are enhanced.

For the contribution due to the quark loop[16], the two-loop mass and magnetic moment can be estimated to be

$$m_\nu^{2-loop} \sim \frac{1}{(16\pi^2)^2} \frac{f_{e\mu} g^2 Y_{tb} m_t m_\mu^2 V_\phi M_{12}}{M_h^2 M_\phi^2} \ln(\frac{m_t}{M_W}) \tag{9}$$

$$\mu_\nu^{2-loop} \sim \frac{1}{(16\pi^2)^2} \frac{f_{e\mu} e g^2 Y_{tb} m_t V_\phi M_{12}}{M_h^2 M_\phi^2} \ln(\frac{m_t}{M_W}) \sim \frac{e}{m_\mu^2} m_\nu^{2-loop} \tag{10}$$

For $m_t \sim M_h \sim M_\phi \sim M_{12} \sim 100$ GeV and the charged Higgs Yukawa coupling $Y_{tb} \sim 1$, these estimates give $m_\nu^{2-loop} \sim 0.1eV$ and $\mu_\nu^{2-loop} \sim 10^{-10}(m_\nu^{2-loop}/eV)\mu_B \sim 10^{-11}\mu_B$. These values can be further suppressed by choosing a smaller Y_{tb} which is not directly related to quark masses. These estimates show that the two-loop $\Delta L = 2$ processes proceeding through a fermion loop can be the leading contribution, and reach $\mu_\nu \sim 10^{-11}\mu_B$ with no fine tuning.

Of course the two loop contribution to neutrino mass is significant only when the one-loop contribution can be suppressed either by some mild fine tuning or by adding an extra Higgs doublet á la BFZ. The potentially dangerous tree level flavor changing neutral current can easily be avoided by imposing discrete symmetry[16].

The role played by the quark in the inner loop can be replaced by exotic vector-like quarks with a few advantages. One of such choices can be readily found in most of the superstring inspired E_6 models with vectorial fermions which have the same charge as the down quark. The singlet Higgs h^+ also appears naturally as the E_6 partner of the usual Higgs bosons in such models. The vectorial fermions have the same gauge quantum numbers as right-handed down quarks. The fermionic couplings can be written as

$$\begin{gathered} f_D\overline{Q_L}D_R\Phi + f_d\overline{Q_L}d_R\Phi + f_h\overline{D_L}t_Rh^- \\ M_D\overline{D_L}D_R + M_d\overline{D_L}d_R \end{gathered} \tag{11}$$

where Q_L is the doublet of left-handed quarks, $D_{L,R}$ are the vectorial fermions, f_d is the usual Yukawa coupling, f_D is the exotic Yukawa coupling, and M_d and M_D are bare masses. For simplicity we used only one pair of $D_{L,R}$. Since D_R and d_R have the same quantum numbers, in general they will mix. Without loss of generality, we can choose the basis such that the bare mass M_d is zero. If one assumes that the D quarks couple with roughly the same strength to all three d quarks, then the strongest constraint on the mixing parameter $\alpha = \frac{f_D\langle\Phi\rangle}{M_D}$ can be obtained from the values of $K - \bar{K}$ or $B - \bar{B}$ mixing due to the Z boson mediated flavor changing neutral current. In this case the limit on α is about 10^{-3}.[17] However if one assumes the D quarks in question couple mainly or exclusively to the third generation, then the best bound, for relatively heavy D quarks, can be obtained from the precision measurement of the ratio g_V/g_A in Z decay. It places only a mild constraint on α to be less than 0.3[18]. Since the lighter d quark generations have nothing to do with our mechanism, we shall take the last assumption for numerical estimates later.

Since the scalar singlet h^+ couples to the quarks directly and breaks lepton number, one no longer needs the second doublet. This is the first advantage. A consequence of this is that is that these models have *automatic* ZKM symmetry because the Yukawa couplings of a single Higgs doublet can be made lepton flavor diagonal without loss of generality. As a result, many dangerous flavor changing leptonic processes are suppressed. A second advantage is that lepton number violation will not occur until the two-loop level, so there is no one-loop neutrino mass. A third advantage is that the suppressive combinatorial/cancellation factors in equations (7) and (8) may be avoided just as in the case of quark loops.

In this "one-doublet Zee model", a two loop contribution to the magnetic moment is shown in Fig. 5. In such models, the lepton number violation can be blamed on the coupling f_D. From Fig. 5 we estimate the two-loop mass and magnetic moment to be

$$m_\nu^{2-loop} \sim \frac{1}{(16\pi^2)^2} \frac{f_{e\mu} f_D f_h g^2 m_t m_\mu^2 V_\phi L}{M_D M_h^2} \tag{12}$$

$$\mu_\nu^{2-loop} \sim \frac{1}{(16\pi^2)^2} \frac{f_{e\mu} f_D e f_h g^2 m_t V_\phi L'}{M_D M_h^2} \tag{13}$$

where L, L' are logarithmic functions of ratios of masses which we take to be of order unity. Given $f_{e\mu} \sim f_D \sim f_h \sim 0.1$, $M_D \sim 200$ GeV and $m_t \sim M_h \sim 100$ GeV we find $m_\nu^{2-loop} \sim 10^{-3}eV$ and $\mu_\nu^{2-loop} \sim 10^{-13}\mu_B$. Though this magnetic moment is too small, a larger value can be achieved by choosing a larger values for the three couplings f_D, $f_{e\mu}$ and f_h. Since this model has automatic ZKM symmetry, the neutrino mass difference is zero to all orders.

Before moving on to discuss the three generation schemes, we describe another variant of the Zee model to further illustrate our arguments.

IV. THE TRIPLET MODEL

In place of the second Higgs doublet in Zee Model, we can use a real Higgs triplet,

$$T = \begin{pmatrix} T^+ \\ T^0 \\ T^- \end{pmatrix} \tag{14}$$

with hypercharge zero. The triplet allows the coupling of the doublet to the scalar singlet:

$$C_T h^-(T^\dagger(\phi \otimes \phi)) + h.c. \tag{15}$$

An immediate consequence of the model, when the third neutrino is decoupled, is an automatic ZKM symmetry just as the one-doublet model in Section III. The ZKM symmetry remains intact even after spontaneous symmetry breaking.

The size of the VEV $\langle T^0 \rangle$ is constrained by the measured ratio of the charged and neutral gauge boson masses[19]

$$\rho = \frac{M_W}{M_Z cos\theta_W} = 1 + 8\frac{V_T^2}{V_\phi^2} \tag{16}$$

where $V_T \equiv \langle T^0 \rangle$ and $V_\phi \equiv \sqrt{2}\langle \phi^0 \rangle$. The experimental limit, $\rho = 1.003 \pm 0.004$[20] means that $V_T \leq 8$ GeV. Unfortunately, this small V_T will suppress lepton number violating amplitudes.

If $\langle T^0 \rangle$ is nonzero, one immediately obtains a one-loop contribution to mass as shown in Figure 6. The interaction in Eq. (15) induces a mixing between h^+, and ϕ^+ or T^+. Since T does not couple to fermions we are forced to use the mixing between h^+ with ϕ^+ and pick up a factor of V_T. The contribution to the mass can be estimated to be (for $M_h > M_\phi$)

$$M_{one-loop} \simeq \frac{1}{16\pi^2} \frac{f_{\mu e} C_T V_T m_\mu^2}{M_h^2} \ln(\frac{M_h}{M_\phi}) \tag{17}$$

For the natural values of $M_h \sim M_\phi \sim 100$ GeV and $C_T \sim f_{\mu e} \sim 0.1$, one obtains the Dirac mass of the ZKM neutrinos to be $M_{one-loop} \sim 1eV$ which, unlike the Zee model, escapes the experimental bound on neutrino mass without fine tuning. As expected, it is proportional to the Yukawa coupling which provides three orders of magnitude suppression.

One may ask if it is possible to suppress the neutrino mass further by eliminating the one loop contribution. This can be achieved if one can manage to have $V_T = 0$ naturally. However it turns out to be very difficult in this model. The Higgs potential contains the term

$$M_{T\phi} T^\dagger (\phi \otimes \tilde{\phi}) + h.c. \tag{18}$$

which can induce a nonzero VEV for T of order $\langle T^0 \rangle = \frac{M_{T\phi} V_\phi^2}{M_T^2}$ even if the tree level mass of T, M_T^2, is chosen to be positive. Thus, if $M_{T\phi} \neq 0$, we must have $\langle T \rangle \neq 0$. If we remove the trilinear coupling, $M_{T\phi}$, by some discrete symmetry, it is easy to see that all the remaining couplings, except the small leptonic Yukawa coupling which we wish to avoid, break the lepton number symmetry by four units and therefore can not give rise to magnetic moment or mass. One could replace the $M_{T\phi}$ interaction with a cubic coupling of the triplet, but, to do this, more than three real (or, two complex) triplet fields are required. We shall not complicate the model further here. From this point on we consider only the case $\langle T \rangle \neq 0$. The size of $\langle T \rangle$ however can be made smaller than the previous estimate if the tree level mass for T becomes negative.

We now assign lepton numbers to the fields to study the lepton number breaking. Choosing $L(h^+) = 2$, $L(\phi) = 2$, and $L(T) = 0$ one observes that the lepton number breaking can be blamed on the quartic self coupling of the doublet. After gauge symmetry breaking the quartic coupling will become an effective cubic coupling for the doublet. This allows lepton number violation even when the Yukawa couplings are ignored. The resulting two-loop contributions to magnetic moment are shown in Fig. 7. The magnetic moments are estimated to be (for $M_h > M_\phi$)

$$\mu_\nu^{2-loop} \sim \frac{1}{(16\pi^2)^2} \frac{f_{e\mu} e g \lambda C_T V_T V_\phi^2}{M_h^2 M_\phi^2} \ln(\frac{M_h}{M_\phi}) \tag{19}$$

where λ is the doublet quartic self coupling of the doublet. For $f_{e\mu} = 0.1$, $\lambda \sim 1$, and $M_h \sim M_\phi \sim 100$ GeV this gives $\mu_\nu \sim 10^{-12}\mu_B$. The similar two loop contributions to the neutrino mass give

$$m_\nu^{2-loop} \sim \frac{1}{(16\pi^2)^2} \frac{f_{e\mu} g \lambda C_T V_T V_\phi^2 m_\mu^2}{M_h^2 M_\phi^2} \ln(\frac{M_h}{M_\phi}) \tag{20}$$

which has the numerical value $m_\nu^{2-loop} \sim 10^{-2} eV$.

Just as the two-doublet Zee model, the natural value for magnetic moment in the triplet model is slightly too small. The value however can be increased in size by using larger value for C_T or λ so that $\mu_\nu \sim 10^{-11}\mu_B$ and $m_\nu^{2-loop} \sim 0.1 eV$.

This model also provides a very good illustration of the insufficiency of the "spin-polarization mechanism" argument which we mentioned earlier. In addition to the two-loop contribution in Figure 7, there are other two- loop contributions to magnetic moment such as the one in Figure 8. The estimate of this contribution to magnetic moment gives the same order of magnitude as in Eq. (19). However, in this case, the spin-polarization argument is ineffective in detecting that the contributions to neutrino mass still will carry the usual suppression factor of multiple Yukawa couplings (or light lepton masses). The fact that such suppression factors are not avoidable for neutrino masses can be easily implied from the general symmetry argument that we described in the introduction. In fact in Ref.[12], such arguments were resorted to in order to assert such factors in three or higher loop diagrams.

V. ZKM SYMMETRY AND THREE GENERATION MODELS

When exploring models with lepton number violation one should keep a watchful eye on the process $\mu \to e\gamma$ which is strongly constrained experimentally. The Zee model can have a large contribution to this decay through the h^+-mediated graph in Fig. 9[11]. Due to the antisymmetry of f_{ab}, all three generations are required for this process to take place, i.e., the amplitude is proportional to $f_{\mu\tau}f_{\tau e}$. Therefore the experimental bound on $\mu \to e\gamma$ gives a very strong constraint on $f_{\mu\tau}f_{\tau e}$. To suppress these couplings we can either impose a symmetry such as tau number conservation, or fine tune the h^+ couplings to the tau to be tiny ($f_{\mu\tau}f_{\tau e} < 10^{-6}(M_h/150\mathrm{GeV})^2$)[11]. A natural way to satisfy this constraint is to have a ZKM symmetry so that at least one of the two couplings, $f_{\mu\tau}$ or $f_{\tau e}$, is naturally small or vanishing. The ZKM can be broken softly so as to control the size of the effect of the breaking. Therefore we shall discuss the three generation case only in that context.

To study different choices of ZKM symmetry in the three generation case, we shall first recall some aspects of the theory of neutrino oscillations. The neutrino oscillation problem has been studied by several authors[9, 21, 22].

In the ultrarelativistic approximation to the Dirac equation, the Hamiltonian has the form M_ν^2/E where E is the energy of the neutrino. For the purposes of oscillation dynamics, one can subtract a multiple of the unit matrix from the Hamiltonian, and hence, one can remove the mass matrix if M_ν^2/E is proportional to the unit matrix. In a magnetic field B, the Hamiltonian has additional off-diagonal elements due to terms of the form $-\mu_\nu B$ which will cause a "precession" between neutrinos of different flavors. The exact amount of neutrino flux reduction is difficult to predict. It depends on the details of the variation of the magnetic field in the sun. The magnetic moment is required to be large enough to bring the oscillation length down to the size of the solar convection zone where the strength of the magnetic field is believed to be correlated with the sunspot cycle. Thus we require $\mu_\nu Bx \sim 1$, which for $x_{convection} \sim 10^{10} cm$ and $B = 1$ Tesla gives the previously quoted necessary magnetic moment $\mu_\nu \sim 10^{-11}\mu_B$.

Note that the ZKM symmetry typically gives rise to a neutrino mass matrix which is traceless and singular in the Zee model and its variants. This implies that the mass eigenstates in the three generation case are one massless Majorana neutrino and one massive Dirac neutrino. If the ZKM symmetry is such that Majorana eigenstate decouples, there will be no transition magnetic moment between the massless neutrino and the Dirac neutrino. Generally, the spin precession is most important for the Dirac neutrino, since the mass degeneracy between left and right handed components presents the no barrier to magnetic precession. Unless the Dirac mass is very small, the massless Majorana neutrino will not spin-precess in vacuum.

In the medium, matter effects will induce masses for all of the neutrino flavors, and give an additional charged current contribution to the electron neutrino mass. The matter induced masses will contribute to the diagonal elements of the mass matrix in the flavor basis, so there need not be a massless eigenstate nor a pure Dirac state in the solar medium. In stars, the neutral current interactions are flavor-independent and therefore not relevant for oscillation. However, the charged current interactions differ due to the net electron density in the medium. The ordinary weak interaction gives rise to a net positive effective mass to the electron neutrino only. In the Zee model, additional interactions mediated by the h^+ can have interesting implications as we will see later.

There are several choices of ZKM symmetry for the three generation case. The simplest ones are $L_e - L_\mu$ and $L_e - L_\tau$. In these cases, ν_τ or ν_μ automatically decouples and we need to focus on only two of the generations. Here we consider the $L_e - L_\mu$ symmetric system. The result for the case of $L_e - L_\tau$ is similar. Assuming the one-loop contribution to neutrino mass is eliminated, one can write

$$M_\nu = \begin{pmatrix} 0 & f_{e\mu} m_\mu^2 \mathcal{G} & 0 \\ f_{e\mu} m_\mu^2 \mathcal{G} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{21}$$

$$\mu_\nu = \begin{pmatrix} 0 & f_{e\mu} e\mathcal{G} & 0 \\ -f_{e\mu} e\mathcal{G} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{22}$$

where $\mathcal{G}$, with dimension $1/M$, represents other factors appearing in the two-loop calculation.

Here we have a pure Dirac neutrino in vacuum, so there is no barrier to precession. In the solar medium, a mass splitting will arise, and this will damp the oscillation. As long as the matter induced mass splitting is smaller than μB the oscillation will still be effective. The two energies happen to be comparable in the convection zone of the Sun, where $G_F n_e \sim \mu B$[22]. Given the uncertainty of our understanding of the convection zone, it is possible that the precession is still effective even with the damping. Such damping can be avoided if a soft breaking of the ZKM symmetry can be arranged. In that case, the mass of the muon neutrino can be made slightly heavier than the electron neutrino (say, by $10^{-4} eV$). This allows for the possibility of resonant mixing inside a non-zero electron density[9]. For models discussed earlier, it is easy to implement soft breaking of ZKM symmetry with minimal extensions of

Higgs sectors without altering the main features we were exploring. While one may need to fine tune the soft breaking parameter to arrange a resonant solution, such fine tuning is usually considered technically natural due to the ZKM symmetry.

The other choices of ZKM symmetry which forbid $\mu \to e\gamma$ are $L_e - L_\mu \pm L_\tau$. In the case of $L_e - L_\mu + L_\tau$, $f_{\tau e}$ is zero. The neutrino mass and magnetic moment matrices become

$$M_\nu = \begin{pmatrix} 0 & f_{e\mu} m_\mu^2 \mathcal{G} & 0 \\ f_{e\mu} m_\mu^2 \mathcal{G} & 0 & f_{\mu\tau} m_\tau^2 \mathcal{G} \\ 0 & f_{\mu\tau} m_\tau^2 \mathcal{G} & 0 \end{pmatrix} \tag{23}$$

$$\mu_\nu = \begin{pmatrix} 0 & f_{e\mu} e \mathcal{G} & 0 \\ -f_{e\mu} e \mathcal{G} & 0 & f_{\mu\tau} e \mathcal{G} \\ 0 & -f_{\mu\tau} e \mathcal{G} & 0 \end{pmatrix} \tag{24}$$

In principle, both the electron-muon and the muon-tau oscillations can occur in the presence of a magnetic field. For natural values of the model parameters, $M_{\mu\tau}$ will be two orders of magnitude larger than $M_{e\mu}$. This implies that the massless neutrino is mostly electron flavor. In this case, the oscillation in vacuum would be mainly between muon and tau neutrinos.

Within the solar medium, the electron neutrino will increase in mass due to the net electron density in the medium. One may wonder whether this charged current induced mass that contributes to the $(M_\nu^2)_{ee}$ component can compensate for the vacuum mass difference $(M_\nu^2)_{\mu\mu} - (M_\nu^2)_{ee}$ and lead to a resonance precession involving the electron neutrino. Unfortunately, the small electron density in the convection zone can only overcome small mass gaps of order $10^{-4} eV$[22]. The mass gap associated with Eq. (23) is $((M_\nu^2)_{\mu\mu} - (M_\nu^2)_{ee})/E = (M_{\mu\tau})^2/E$, which has a natural value of $(f_{\mu\tau}/0.1)^2 \times 10^{-14}$ GeV, far greater than $\mu_\nu B = (\mu_\nu/10^{-11}\mu_B) \times (B/\text{Tesla}) \times 6 \times 10^{-25}$

GeV or the typical induced mass $G_F(n_e - (1/6)n_n) \sim 10^{-25}$ GeV (convection zone). Thus, to achieve the resonance behaviour will require much higher densities, such as those in supernova cores. Alternatively, fine tuning $f_{\mu\tau}$ will reduce the mass of the Dirac neutrino and permit a resonance in the solar medium. One may wish consider such fine tuning to be due to a soft breaking of $L_e - L_\mu$ symmetry.

If instead we choose the symmetry $L_e - L_\mu - L_\tau$, and again take natural values of the model parameters, the massless neutrino is mostly ν_μ flavor with a small mixture of ν_τ. ν_e pairs up with the remainder of ν_τ and ν_μ to form a Dirac neutrino. Matter effects will split the degeneracy of the Dirac neutrino components. As with the $L_e - L_\mu$ symmetry, the induced mass splitting becomes comparable to the magnetic moment energy in the convection zone of the Sun. Therefore, the phenomenology will be very similar to the case with $L_e - L_\mu$ symmetry.

To summarize, it appears that in the absence of fine tuning or soft breaking, the extended ZKM symmetry, $L_e - L_\mu + L_\tau$, is not effective to generate solar neutrino depletion, while the cases of $L_e - L_\mu$, $L_e - L_\tau$ and $L_e - L_\mu - L_\tau$ symmetries all can give sizable depletions.

There is still another intriguing new interaction we have not explored. The h^+ mediated flavor changing currents which exist in the Zee model can lead to new neutrino scattering processes such as the one depicted in Fig. 10. One may ask whether these processes can serve as surrogate mass differences for the purposes of an MSW oscillation or simply for cancellation of the matter effects which could damp the magnetic oscillations within the Sun. The possibility of using new interactions for this purpose has been discussed by Guzzo, Masiero, and Petcov[23].

There are two questions to be answered on this track. The first concerns the sign of the effective mass generated by the h^+ mediated neutrino scattering compared to the positive effective mass of ν_e by the ordinary weak contribution in the solar medium[21, 24]. Secondly, is the size of the contribution large enough to affect the resonance phenomenology inside the Sun? Note that the h^+ mediated scattering depends only on the electron density N_e, but not on the smaller neutron density N_n. The four fermion operator generated from the h^+ mediated diagram is similar to the one generated by the W^+. However the gauge boson propagator and Higgs boson propagator differ by a minus sign, and the anti-particle nature of $\bar{\nu}_\mu$ picks up another minus sign. Therefore we find out that, interestingly, the h^+ mediated interaction in Fig. 10 also contributes a *positive* effective mass to $\bar{\nu}_\mu$ (or $\bar{\nu}_\tau$, in the case of $L_e - L_\mu - L_\tau$ symmetry).

In the the Standard Model, the W^+ and Z interactions give positive effective masses a_{ν_e} and $a_{\bar{\nu}_\mu}$ to ν_e and $\bar{\nu}_\mu$ respectively, as defined in Ref. [9]. The h^+ interaction adds a positive $\epsilon \sim (8/g^2) f_{e\mu}^2 M_W^2 / M_h^2$ contribution to $a_{\bar{\nu}_\mu}$,

$$a_{\nu_e} = (G_F/\sqrt{2})(2N_e - N_n), \quad a_{\bar{\nu}_\mu} = (G_F/\sqrt{2})(\epsilon N_e + N_n).$$

The nontrivial resonance occurs at $N_n/N_e = (1 - \epsilon/2)$. However, the ratio N_n/N_e inside the sun varies from 0.15 at the convective zone to 0.5 at the core. So, unfortunately, the condition requires ϵ larger than one. Such a large value of ϵ has been ruled out by the experimental constraints from μ and τ lepton decays. Therefore, the h^+ mediated interaction is not likely to emulate the MSW resonance or remove the matter induced damping of magnetic precession in practice.

VI. CONCLUSION

We have defined a general class of models that can give relatively large magnetic moments to neutrinos while keeping their masses within experimental limits. We have also illustrated the way one can go about constructing such a theory. We are not trying to pinpoint one particularly successful model. Instead, we tried to bring out the simple symmetry principles behind success of such a class of models. In this class of model the small numbers are blamed on the smallness of the Yukawa couplings associated with electron or muon masses. No attempt is made to explain the size of electron or muon masses. If the anticorrelation of the solar neutrino defficiency with the sunspot cycle persists in future data, one may attempt a satisfactory theory that can explain the smallness of electron and muon masses *and* large magnetic moments of their neutrinos at one stroke.

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FIGURES

FIG. 1. The one-loop contribution to the neutrino mass in the minimal Zee Model, which has a pair of Higgs doublets $\phi_{1,2}$ and a charged singlet h^+. The same diagram also gives rise to the transitional magnetic moment of the neutrino with the understanding that the external photon is attached to any charged line.

FIG. 2. The two-loop contribution to the neutrino magnetic moment in the minimal Zee Model. The lepton number nonconservation occurs via the Higgs quartic coupling.

FIG. 3. A generic class of diagrams which give a large contribution to magnetic moment through the "spin–polarization mechanism".

FIG. 4. The two-loop contribution to the magnetic moment in a model where the lepton number violation occurs via the quark coupling.

FIG. 5. The two-loop contribution to the magnetic moment in a model with only one Higgs doublet ϕ, and an additional D vector quark.

FIG. 6. The one-loop contribution to the neutrino mass in a model with a triplet $T(Y=0)$.

FIG. 7. The two-loop contribution to the neutrino magnetic moment in a model with a triplet $T(Y=0)$. These diagrams are typical ones which fall into the class of "spin-polarization mechanism" enhancement.

FIG. 8. The two-loop contribution to the neutrino magnetic moment in a model with a triplet $T(Y=0)$. These diagrams are important, but do not fall into the class of "spin-polarization" enhancement.

FIG. 9. A diagram for the process $\mu \to e\gamma$.

FIG. 10. The h^+ mediated four fermion process $\nu_{\mu,\tau} e \to \nu_{\mu,\tau} e$.

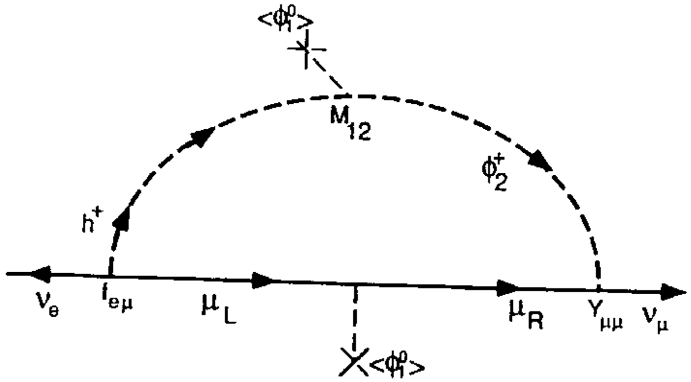


Figure 1

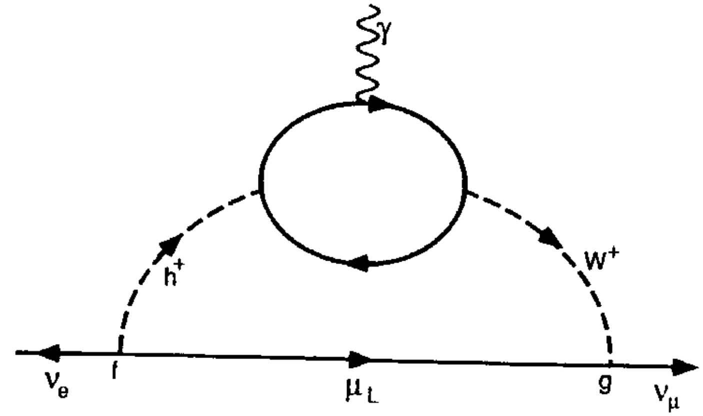


Figure 3

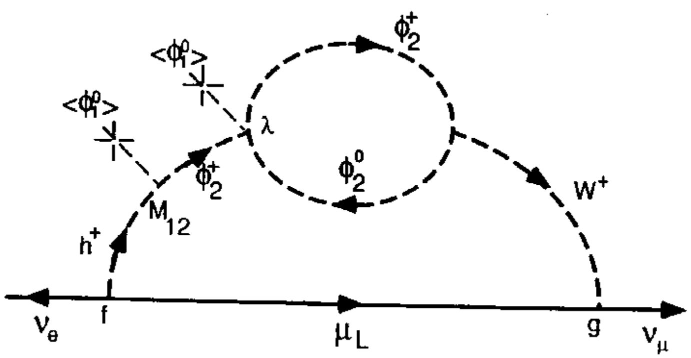
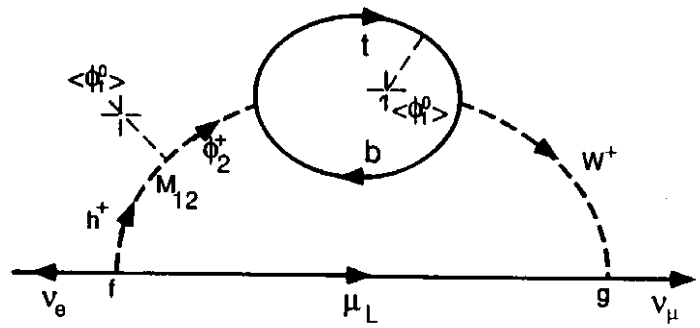


Figure 2



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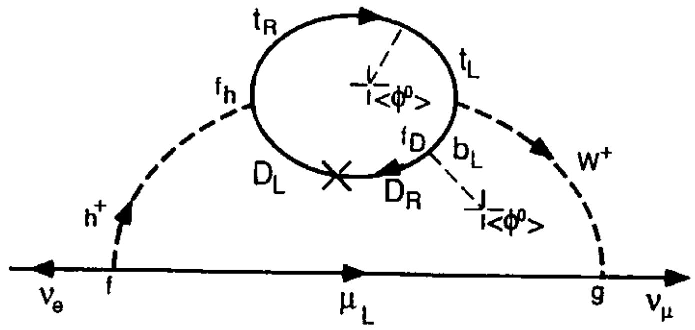

Figure 5

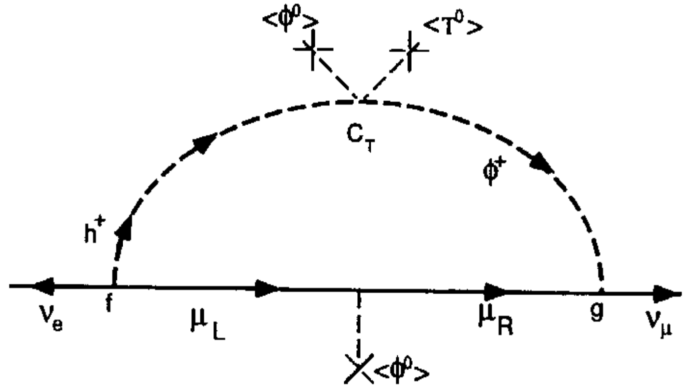

Figure 6

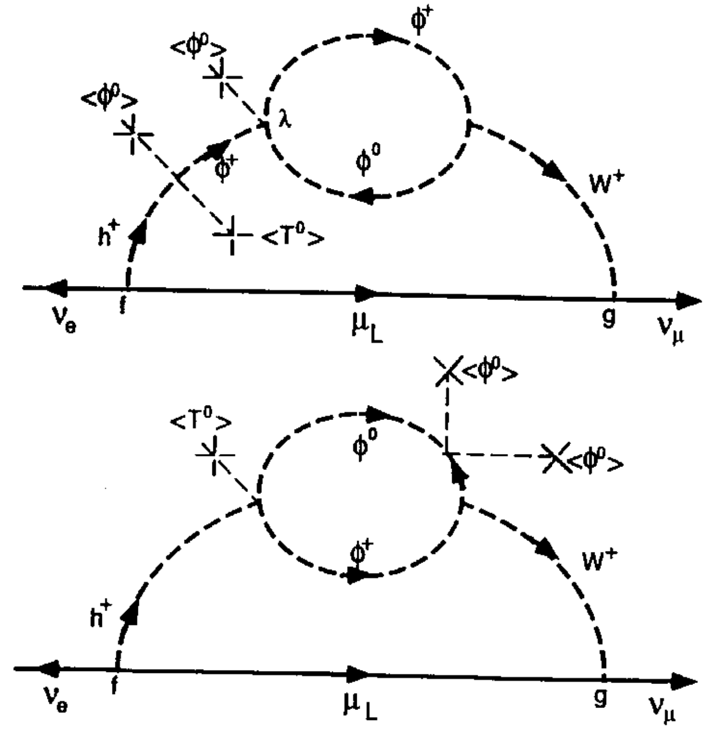

Figure 7

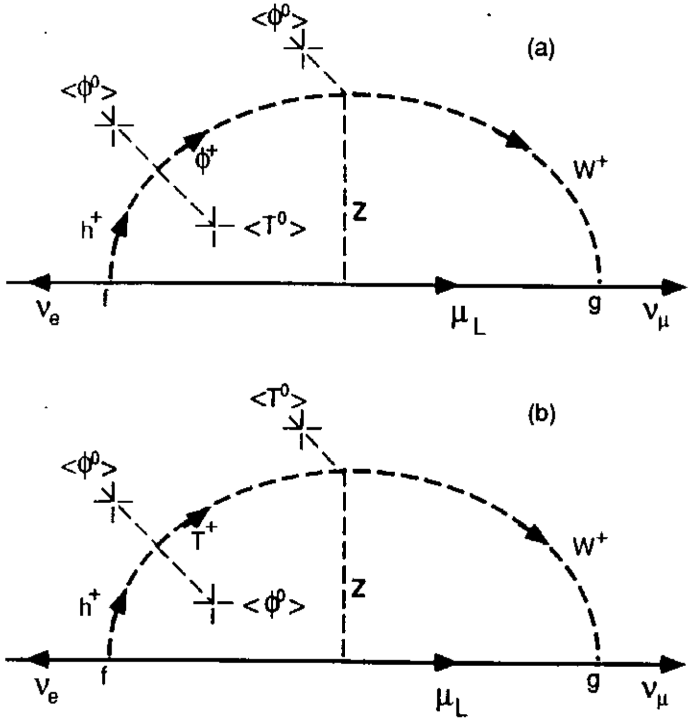

Figure 8

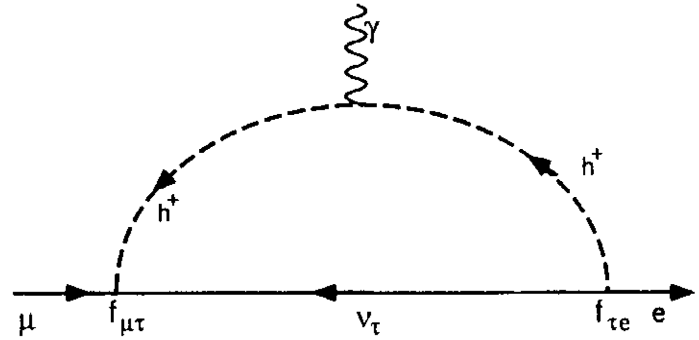

Figure 9

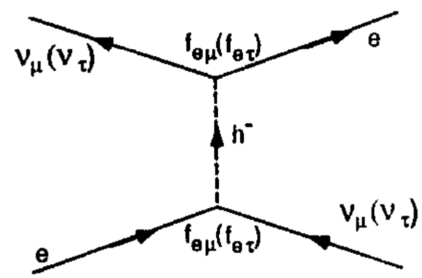

Figure 10