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Asymptotic Behavior of 2-d Black Holes

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We consider solutions of the field equations for the large N dilaton gravity model in $1+1$ dimensions of Callan, Giddings, Harvey, and Strominger (CGHS). We find time dependant solutions in the weak coupling region with finite mass and vanishing flux, as well as solutions with lie entirely in the Liouville region.

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1. Introduction

In the years following the discovery of Hawking radiation and the associated evaporation of black holes [1], there have been many efforts to either prove or refute the resulting implication that an initially pure state can collapse into a black hole and evaporate into a mixed state. The fact that such efforts have not proven successful is due to a combination of complications, including principally those of the backreaction of the Hawking radiation on the metric, and of the regions of large curvature (and hence strongly coupled quantum gravity effects) which are expected in gravitational collapse.

Recently, Callan et. al. (CGHS) proposed a model which seemed to avoid some of these difficulties [2]. It consistts of gravity coupled to a dilaton and conformal matter in $1+1$ dimensions. For a single matter field it was found the backscatter (i.e., the Hawking radiation) occured in a region of strong coupling. By proliferating the number N of matter fields, it was believed that the essential physics was occur in a region of small coupling and hence be amenable to a systematic $1/N$ semiclassical expansion.

These initial hopes were dashed [7][5] by the observation that the dilaton develops a singlurity at a finite value, dependent on N, precisely in the region where quantum fluctuations begin to become large. As a result, a number of groups [4][6][3] have recently tried to explore, both numerically and analytically, the solutions of the large N field equations. In particular, one is interested in the final "endpoint" of the Hawking radiation. Therefore, in [4][6][3]the fields were assumed to depend only on a "spatial" coordinate (of which there are a few natural choices). For example, in [3], a series of solutions with finite ADM mass and vanishing incoming and outgoing flux were found. Starting at weak coupling at spatial infinity, they were found to "bounce" back to weak coupling in the region of the singularity mentioned above.

The static approximation used to derive these results is a significant simplification, but makes it difficult to consider the approach to the endpoint of the Hawking process. In the following, we will consider time-dependent (approximate) solutions to the CGHS equations. We will find solutions which still have finite ADM mass and vanishing flux, as well as regions with a time-dependent singular event horizon. In a later section, we will also discuss a series of perturbative, time dependant solutions which lie entirely in the Liouville region, followed by some concluding remarks.

2. The CGHS Model

The CGHS model of dilaton gravity coupled to N conformal matter fields in $1+1$ dimensions with coordinates σ and τ is defined by the action

$$S = \frac{1}{2\pi}\int d^2\sigma\sqrt{-g}[e^{-2\phi}(R + 4(\partial\phi)^2 + 4\lambda^2) - \frac{1}{2}\sum_{i=1}^{N}(\partial f_i)^2], \tag{2.1}$$

where g,ϕ, and f_i represent the metric, dilaton, and matter fields, respectively, and λ^2 is the cosmological constant. Integrating out the matter fields and going to conformal gauge, where

$$g_{+-} = -\frac{1}{2}e^{2\rho}, \tag{2.2}$$

$$g_{++} = g_{--} = 0, \tag{2.3}$$

($x_\pm = \tau \pm \sigma$), the resulting action is

$$S = \frac{1}{\pi}\int d^2\sigma[e^{-2\phi}(\partial_+(2\phi-\rho)\partial_-(2\phi-\rho) - \lambda^2 e^{2\rho}) + (\frac{N}{12} - e^{-2\phi})\partial_+\rho\partial_-\rho]. \tag{2.4}$$

The equations of motion for ρ and ϕ are

$$T_{+-} = e^{-2\phi}(2\partial_+\partial_-\phi - 4\partial_+\phi\partial_-\phi - \lambda^2 e^{2\rho}) - \frac{N}{12}\partial_+\partial_-\rho = 0, \tag{2.5}$$

$$-4\partial_+\partial_-\phi + 4\partial_+\phi\partial_-\phi + 2\partial_+\partial_-\rho + \lambda^2 e^{2\rho} = 0. \tag{2.6}$$

Since the gauge has been fixed as in (2.3), there are two constraint conditions, namely,

$$T_{\pm\pm} = e^{-2\phi}(4\partial_\pm\phi\partial_\pm\rho - 2\partial_\pm^2\phi) - \frac{N}{12}(\partial_\pm\rho\partial_\pm - \partial_\pm^2\rho + t_\pm(\sigma_\pm)) = 0, \tag{2.7}$$

where the functions $t_\pm$ are fixed by boundary conditions.

The simplest and most important nontrivial solution of (2.5)and (2.6)is the linear dilaton vacuum

$$\rho = 0,\ \phi = -\frac{\lambda}{2}(\sigma^+ - \sigma^-). \tag{2.8}$$

This vacuum has a singulrity at

$$\phi = \phi_{cr} = -\frac{1}{2}ln\frac{N}{12}, \tag{2.9}$$

as seen by calculating the sign of the kinetic operator in (2.4). As in previous papers, we will call the region of $\phi < \phi_{cr}$ the dilaton region, and $\phi > \phi_{cr}$ the Liouville, or strong coupling region.

3. Finite Mass Solutions

Solutions to (2.5) and (2.6) with finite ADM mass were first found in [3]by assuming that both ϕ and ρ are time independant. In that case, (2.5) and (2.6) become

$$T_{+-} = e^{-2\phi}(-\frac{1}{2}\phi'' + \phi'2 - \lambda^2 e^{2\rho}) + \frac{N}{48}\rho'' = 0, \tag{3.1}$$

$$\phi'' - \phi'^2 - \frac{1}{2}\rho'' + \lambda^2 e^{2\rho}, \tag{3.2}$$

where the primes denote $d/d\sigma$. Linearizing about the linear dilaton vacuum solution (2.8), for vanishing incoming and outgoing flux $t_{\pm}$, asymptotically the resulting equations can be expressed as

$$2\lambda\delta\phi' + 2\lambda^2\delta\rho - \lambda\delta\rho' = 0, \tag{3.3}$$

$$\delta\phi'' = \delta\rho''(1 - \frac{N}{24}e^{-2\lambda\sigma}). \tag{3.4}$$

The asymptotic form of the solutions of these equations is

$$\delta\phi = -\frac{M}{2\lambda}e^{-2\lambda\sigma} + \dots, \tag{3.5}$$

$$\delta\rho = -\frac{M}{2\lambda}e^{-2\lambda\sigma} + \dots, \tag{3.6}$$

where the parameter M is the ADM mass, given by evaluating

$$M = 2e^{2\lambda\sigma}(\lambda\delta\rho + \delta\phi'). \tag{3.7}$$

at spatial infinity.

Before going beyond the static case, it should be noted that one can expand $\delta\phi$ and $\delta\lambda$ in powers of $\epsilon = e^{-2\lambda\sigma}$,

$$\delta\phi = \sum_{n=1}^{\infty} a_n \epsilon^n, \ \delta\rho = \sum_{n=1}^{\infty} b_n \epsilon^n, \tag{3.8}$$

with $a_1 = b_1 = -\frac{M}{2\lambda}$. Substituing into the full linearized equations, one finds the relations

$$(1 - n^2)a_{n+1} - b_{n+1} + \frac{N}{24}n^2 b_n = 0, \tag{3.9}$$

$$(n+1)b_n = 2na_n, \tag{3.10}$$

from which one easily finds

$$a_{n+1} = \frac{N}{12}\frac{n^3}{n^3+n^2-n-1}a_n. \tag{3.11}$$

For large n, this suggests that we must have $\frac{N}{12} \leq 1$ for the series to converge. For example, for $\frac{N}{12} = 1$, the resulting series for $\delta\phi$ is roughly

$$\delta\phi \sim \frac{M}{2\lambda}\frac{1}{e^{2\lambda\sigma}-1}, \tag{3.12}$$

thus implying that our linearizing approximation is breaking down for small σ. It is perhaps of interest that the requirement $\frac{N}{12} \leq 1$ implies (2.9) that $\phi_{cr} > 0$ so that the effective critical coupling constant $e^{2\phi_{cr}} > 1$.

Let us now proceed beyond the static limit, but continue to require a finite ADM mass. Including time derivatives, in the σ, τ coordinate system the linearized equations read

$$e^{2\rho}(-\frac{1}{2}\delta\phi'' - 2\lambda\delta\phi' - 2\lambda^2\delta\rho + \frac{1}{2}\delta\ddot{\phi}) + \frac{N}{48}(\delta\rho'' - \delta\ddot{\rho}) = 0, \tag{3.13}$$

$$\delta\phi'' - \delta\ddot{\phi} + 2\lambda\delta\phi' + \frac{1}{2}\delta\ddot{\rho} - \frac{1}{2}\delta\rho'' + 2\lambda^2\delta\rho = 0. \tag{3.14}$$

From (3.7), we see that finite ADM mass requires both $\delta\rho$ and $\delta\phi$ vary asymptotically as $e^{-2\lambda\sigma}$, as in (3.5)and (3.6). If we express the perturbations about the linear dilaton vacuum as

$$\delta\phi = x(\tau)e^{-2\lambda\sigma}, \tag{3.15}$$

$$\delta\rho = y(\tau)e^{-2\lambda\sigma}, \tag{3.16}$$

then to leading order (3.13) and (3.14) become, respectively,

$$2\lambda^2 x - 2\lambda^2 y + \frac{1}{2}\ddot{x} = 0, \tag{3.17}$$

$$\frac{1}{2}\ddot{y} - \ddot{x} = 0. \tag{3.18}$$

It is a simple matter now to assume that x and y both vary as $e^{\omega\tau}$ and solve for ω and the relative amplitudes. Of course, one solution is just

$$x = y = \frac{M}{2\lambda} + a\tau \tag{3.19}$$

as in (3.5)(where $a = 0$). The other solution is easily seen to be

$$x(\tau) = \frac{1}{2}y(\tau) = ae^{-2\lambda\tau} + be^{2\lambda\tau}. \tag{3.20}$$

Substituting (3.20)into (3.7), we seen that the time dependancy of $\delta\phi$ and of $\delta\rho$ cancel, and the ADM mass is constant, even thought the metric and the dilaton are certainly not. Presumably, we should set the coefficient b in (3.20) to zero, so that the solution is well behaved as $\tau \to \infty$, as should the coefficient of the linear term in the $\omega = 0$ solution.

The behavior of these solutions can be understood in much the same manner as in the static case [3]. Let us concentrate on the $M = 0$ solution, as it has been suggested that it represents the true quantum vacuum of the theory [3]. In any case, for τ sufficiently negative, the time dependant terms dominate over the static terms. As one integrates the equation of motion in from spatial infinity, the solution may approach the singularity at ϕ_{cr} (in the static case, this approach was guaranteed). In this region, we can essentially set $\rho = 0$, and $\phi = \phi_{cr} + \varphi$. The resulting equation of motion is

$$-\varphi(\varphi'' - \ddot{\varphi}) = \frac{1}{2}(\varphi'^2 - \dot{\varphi}^2 - \lambda^2). \tag{3.21}$$

If we continue to assume that $\dot{\varphi} = -2\lambda\varphi$, then (3.21)can be integrated, yielding

$$\varphi'^2 - \frac{A}{\varphi} - \frac{10}{3}\lambda^2\varphi^2 = \lambda^2, \tag{3.22}$$

where A is an integration constant. As long as $A \neq 0$, this is the equation for a particle in a potential with an infinite barrier at the origin, so φ will bounce back to the weak coupling regime.

We can also discuss the behavior of the solutions for any region where $\rho \to -\infty$, in particular as $\sigma \to -\infty$, assuming that $ae^{-2\lambda\tau} < \frac{M}{2\lambda}$, as was discussed in the static case in [3], by dropping terms proportional to $e^{2\rho}$ which become irrelevant for $\rho \to -\infty$. For in that case we have

$$e^{-2\phi} = -\frac{N}{12}\rho + a_+\sigma_+ + a_-\sigma_- + b, \tag{3.23}$$

$$e^{-\phi}\sqrt{e^{-2\phi} - \frac{N}{12}} - \frac{N}{12}ln[\sqrt{e^{-2\phi} - \frac{N}{12}} + e^{-\phi}] = f(\sigma_-) + g(\sigma_+), \tag{3.24}$$

where $a_\pm$ and b are constants, and f and g are arbitrary functions of their arguments (in the static case [3], one has $f + g = -a\sigma + c$), the only priviso being that f must be smooth (i.e., $f(\sigma_-)$ is the integral of a completely arbitrary function). Concentrating on a region where $f+g \to \infty$, we have

$$e^{-2\phi} \sim f+g+\frac{N}{24}ln(f+g), \tag{3.25}$$

$$\rho \sim -\frac{12}{N}(f+g)-\frac{1}{2}ln(f+g)-a_+\sigma_+ - a_-\sigma_- - b. \tag{3.26}$$

Using the formula for the curvature,

$$R = 8e^{-2\rho}\partial_+\partial_-\rho, \tag{3.27}$$

we have

$$R \sim \frac{\partial_+ g \partial_- f}{f+g} e^{\frac{24}{N}(f+g-a_+\sigma_+-a_-\sigma_--b)}, \tag{3.28}$$

(where we have redefined the constants $a_\pm$ and b). Taking, for example, $g(\sigma_+) \sim (\sigma_+ - \sigma_+^0)^{-\alpha}, \alpha > 0$, we see that σ_+^0 is a singular event horizon. Since $\sigma_+ = \tau + \sigma$, the location of the horizon is not constant in time τ. Furthermore, the fact that $\delta\rho$ grows more rapidly than $\delta\phi$, as seen in (3.20), suggests that such regions might be of greater importance in understanding the full evolution of the system, particularly for the $M=0$ solution, which has been proposed to be the true vacuum of the theory. In fact, in the original, unperturbed field equation (2.5) , we see that if $\phi' = \dot{\phi} = -2\lambda\phi$, then ρ is forced to approach $-\infty$, unless $e^{2\phi} \sim 24/N$. Of course, at this point, depending on N, we may no longer be in the weak coupling regime which we have been discussing, but rather in the strong coupling, or Liouville region, which we consider below.

Of course, for large τ, the time dependant terms are small, and the solution behaves as in the static case, where ϕ penetrates closer and closer to ϕ_{cr} before bouncing back to weak coupling [3]. But for τ sufficiently large and negative, we are effectively dealing with the $M=0$ solution, in which ρ will tend to grow faster than ϕ and singular event horizons should appear. It is questionable whether or not this is a reasonable condition for the true vacuum of the theory. Actually, it seems more reasonable that the final state of the system, in response to some incoming matter, would have a potentially complicated causal structure. Of course, our solutions are nonsingular at ϕ_{cr} whereas the incoming matter is singular there, so the interpretation of these solutions remains unclear.

To complement these solutions, we should in prinicple search for time dependant solutions with regular horizons, as was done in [6], [3], and [4], generally by using the "spatial" variable $s = x_+x_-$ and then imposing continuity conditions at the horizon at $s=0$. Including time dependant terms, of course, will affect the location of the horizon in general, and we have not yet made a determined effort to analyze the range of possibilities. Work on this problem is in progress.

4. The Liouville Region

As argued in [7], [8], solutions which lie entirely in the Liouville region contain important information concerning the behavior of extremal four-dimensional dilaton black holes. Secondly, it might be possible that a configuration in the Liouville region might evolve into the weak coupling region, even if the reverse is impossible.

To analyze this region, we introduce the new dependant variable [3]

$$\psi = e^{-\phi}, \tag{4.1}$$

in terms of which the action is just

$$S = \frac{1}{\pi}\int d^2\sigma(4\partial_+\psi\partial_-\psi + 4\psi\partial_+\psi\partial_-\rho - \lambda^2\psi^2 e^{2\rho} + \frac{N}{12}\partial_+\rho\partial_-\rho). \tag{4.2}$$

The resulting field equations (which can just as easily be derived from the original field equations upon substituting (4.1)) are

$$T_{+-} = -2\partial_+\psi\partial_-\psi - 2\psi\partial_+\partial_-\psi - \lambda^2\psi^2 e^{2\rho} - \frac{N}{12}\partial_+\partial_-\rho = 0, \tag{4.3}$$

$$4\partial_+\partial_-\psi + 2\psi\partial_+\partial_-\rho + \lambda^2\psi e^{2\rho} = 0. \tag{4.4}$$

The simplest solution to these equations is the trivial solution

$$\psi = 0, \rho = 0. \tag{4.5}$$

If we now perturb these equations about (4.5), we see that every term in (4.3) is quadratic except the last term, so we just have

$$\delta\rho = f_+(\sigma_+) + f_-(\sigma_-), \tag{4.6}$$

where $f_\pm$ are arbitraty functions. Similarly, the linearization of (4.4) yields simply the Klein Gordon equation

$$\partial_+\partial_-\delta\psi + \frac{\lambda^2}{4}\delta\psi = 0 \tag{4.7}$$

for a particle with $m^2 = \lambda^2/4$.

Another solution of (4.3) (4.4) is [3]

$$\psi^2 = \frac{N}{24}, \tag{4.8}$$

$$\rho = -\ln(\sqrt{2}\lambda\sigma), \tag{4.9}$$

which is an example of anti-deSitter space, as the curvature turns out to be $R = -4\lambda^2$. Linearizing again, we find

$$-2\sqrt{\frac{N}{24}}\partial_-\partial_+\delta\psi - \frac{N}{12}\partial_-\partial_+\delta\psi - \frac{N}{24}\frac{1}{\sigma^2}\delta\rho - \sqrt{\frac{N}{24}}\frac{\delta\psi}{\sigma^2} = 0, \tag{4.10}$$

and

$$4\sqrt{\frac{N}{24}}\partial_-\partial_+\delta\psi + \frac{N}{12}\partial_-\partial_+\delta\rho + \frac{N}{24}\frac{1}{\sigma^2}\delta\rho = 0. \tag{4.11}$$

Adding the equations, we have

$$2\partial_-\partial_+\delta\psi - \frac{\delta\psi}{\sigma^2} = 0, \tag{4.12}$$

which is just the equation for a particle in a $1/r^2$ potential. For example, going to the static limit, we have

$$\delta\psi'' = -\frac{2}{\sigma^2}\delta\psi, \tag{4.13}$$

with solutions

$$\delta\psi = a_1\sigma^{\beta_1} + a_2\sigma^{\beta_2}, \tag{4.14}$$

where the a_i are constants and the β_i are the solutions of the quadratic equation $x^2 - x + 2 = 0$. Since the β_i are therefore complex, whereas ψ should be real, it would seem that this is an inappropriate background for such a perturbative analysis.

5. Discussion

Spurred on in part by recent advances in string theory [9] , we have witnessed a great increase in the number of toy models, particularly in low dimensions, made available for the study of phenomena such as Hawking radiation and the final state of black holes which involve fundamental issues surrounding quantum gravity. The CGHS model is an especially simple yet sufficiently rich example of such a model. Unfortunately, there remain significant barriers which interfere with our greater understanding of quantum gravity. Of the various groups who have studied the CGHS system, there are adherents of a variety of scenarios, including naked singularities [4] , macroscopic objects [10] , the "bounce" scenario [3] , and so on.

In this letter, we have tried to begin the program of going beyond the static limit applied earlier [3] [4] [6]. We know that the classical no-hair theorems, which essentially say that a black hole is characterized by the quantum numbers of long range fields, such as mass, charge, angular momentum, cannot contain quantum mechanical information. What we have found is that specifying the mass of the black hole does not fully specify the metric or dilaton, even to leading order asymptotically. There is active research underway on a variety of quantum-mechanical effects on black holes, see [11]for example for a thorough discussion of quantum hair and Aharonov-Bohm type interactions of black holes.

In the present case, in the original CGHS model (i.e., $N = 1$), the picture of the black hole was of an asymptotically flat plane connected via a throat-like horizon to a semi-infinite cylinder-like region. When matter impinges on this system, one might imagine, for example, that while the asymptotically flat region would eventually see a constant mass, the matter might be hurtling down the cylinder behind the event horizon in a complicated and possibly singular fashion. Even the horizon itself need not be fixed, though of course that would be measureable to an asymptotic observer.

Another important factor which we have come across is the problem of the crossover between weak coupling and Liouville regions. In spite of the initial hopes, it appears that the important physics is occuring precisely in this region, where we cannot ignore futher quantum corrections. This region is small (of order λ^{-1}) in the large N limit, so the model may yet be viable for questions regarding longer range phenomena. Furthermore, because of this great uncertainty, we cannot say for certain that propagation through the apparant singularity is in fact forbidden. Perhaps a further exploration of the appropriate boundary conditions or additional terms in the $1/N$ expansion will suggest a way out of our present dilemnas.

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