Fermi National Accelerator Laboratory

NSF-ITP-92-06
CMU-HEP92-05
FNAL-PUB-92/34-A
HUTP-92A011
VAND-TH-92-2
January, 1992

Solutions to the Strong-CP Problem in a World with Gravity

Richard Holman,[a,b] Stephen D. H. Hsu,[a,c] Thomas W. Kephart,[d]

Edward W. Kolb,[a,e] Richard Watkins,[e] Lawrence M. Widrow[a,f]

[a] *Institute for Theoretical Physics, University of California, Santa Barbara, CA 93106*

[b] *Physics Department, Carnegie Mellon University, Pittsburgh, PA 15213*

[c] *Lyman Laboratory of Physics, Harvard University, Cambridge, MA 02138*

[d] *Department of Physics and Astronomy, Vanderbilt University, Nashville, TN 37235*

[e] *NASA/Fermilab Astrophysics Center*
Fermi National Accelerator Laboratory, Batavia, IL 60510
and Department of Astronomy and Astrophysics, Enrico Fermi Institute
The University of Chicago, Chicago, IL 60637

[f] *Canadian Institute for Theoretical Astrophysics*
University of Toronto, Toronto, Ontario, M5S 1A1, CANADA

Abstract

We examine various solutions of the strong-CP problem to determine their sensitivity to possible violations of global symmetries by Planck scale physics. While some solutions remain viable even in the face of such effects, violations of the Peccei–Quinn (PQ) symmetry by non-renormalizable operators of dimension less than 10 will generally shift the value of $\overline{\theta}$ to values inconsistent with the experimental bound $\overline{\theta} \lesssim 10^{-9}$. We show that it is possible to construct axion models where *gauge* symmetries protect PQ symmetry to the requisite level.

Operated by Universities Research Association Inc. under contract with the United States Department of Energy

It is well known that there are two contributions to CP violation in the standard model. First, QCD instantons induce a term $\mathcal{L}_{\mathrm{QCD}} = \theta \,\mathrm{tr}\, G\tilde{G}$ in the effective Lagrangian, which violates both P and CP [1]. Here, θ is a dimensionless coupling constant, which one might naively expect to be of order unity. Second, the quark mass matrix can be complex, leading to a CP-violating phase in the Kobayashi-Maskawa mixing matrix. The phase of the quark mass matrix gives rise to an additional contribution $\theta_{\mathrm{QFD}} = \arg\det \mathcal{M}_{\mathrm{q}}$ to the coeffiecient of $\mathrm{tr}\, G\tilde{G}$. The degree of strong-CP violation is controlled by the parameter $\overline{\theta} = \theta + \arg\det \mathcal{M}_{\mathrm{q}}$, which is constrained by measurements of the electric dipole moment of the neutron to be less than 10^{-9} [2]. The strong-CP problem is that there is no reason for these two contributions, which arise from entirely different sectors of the standard model, to sum to zero to such high accuracy.

The solutions that have been proposed for the strong-CP problem fall into three general classes. First, there are those that rely on the existence of an extra global $U(1)_A$ symmetry. This symmetry arises naturally if one or more of the quark masses are zero [3]. In this case, it can be shown that the QCD θ parameter becomes unobservable. This solution is considered unattractive, since experimental evidence implies that it is unlikely that any of the quarks are massless. Peccei and Quinn [4] (PQ) proposed a solution to the strong-CP problem in which they introduced an auxiliary, chiral $U(1)_{\mathrm{PQ}}$ symmetry that is spontaneously broken at a scale f_a, giving rise to a Nambu–Goldstone boson a known as the axion [5]. This symmetry is explicitly broken by instanton effects. This explicit breaking generates a mass for the axion of order $m_a \sim \Lambda^2/f_a$, where Λ is the QCD scale. The important point is that the effective potential for the axion has its minimum at $\langle a/f_a \rangle = -\overline{\theta}$. It follows that when the axion field relaxes to its minimum, the coefficient of $\mathrm{tr}\, G\tilde{G}$ is driven to zero. This solution has received the most attention and has been explored by many authors.

A second class of solutions involve models where an otherwise exact CP is either softly or spontaneously broken. Specific models have been proposed where θ is calculably small and within the experimental limits [6].

A third class of solutions involve the action of wormholes [7]. As we will argue below, wormholes can break global symmetries explicitly, thus giving rise to potentially large contributions to $\overline{\theta}$. However, under certain assumptions, it can be shown that wormholes actually have the effect of setting $\overline{\theta} = 0$ [7].

In this letter, we address the question of whether these solutions to the strong-CP problem can remain viable if Planck scale effects break global symmetries explicitly. There are many arguments suggesting that all global symmetries are violated at some level by gravity. First, no-hair theorems tell us that black holes are able to swallow global charge. This allows for a gedanken experiment in which a quanta with global charge "scatters" with a black hole, leaving only a slightly more massive black hole, but one with indeterminate global charge as dictated by the no-hair theorem. Heuristically, if one considers "virtual" black hole states of mass M arising from quantum gravity, one can integrate them out to yield global charge violating operators suppressed by powers of M, where M might be as small as M_{Pl}, the Planck mass.

Another indication that gravity might not respect global symmetries comes from wormhole physics [8]. Wormholes are classical solutions to Euclidean gravity that describe changes in topology. Integrating over all wormholes (with a cutoff on their size) yields a low-energy effective action that contains operators of *all* dimensions that violate global symmetries [9]. The natural scale of violation in this case is the wormhole scale, usually thought to be very near (within an order of magnitude or so) M_{Pl}.

Without explicit calculations of these effects, we are left with the following prescription: Due to our lack of understanding of physics at the Planck scale, we have no choice but to interpret theories that do not include gravity in a quantum mechanically consistent way as *effective* field theories with a cutoff at M_{Pl}. If we adhere rigorously to this principle, we are then required to add all higher dimension operators (suppressed by powers of M_{Pl}) consistent with the symmetries of the full theory at M_{Pl}. As discussed above, it seems very unlikely that the full theory respects global symmetries. We note that it would be particularly surprising if the entire theory respects $U(1)_{\rm PQ}$, since this symmetry is already explicitly broken by instanton effects. We should note that similar ideas were noted briefly in the prescient paper of Georgi, Hall, and Wise [10]; however, we are now in a position to be somewhat more specific about the nature of the Planck scale effects in question and to explore their consequences.

We consider first the implications for the axion model. To be specific, we consider a generic invisible axion model [11] in which an electroweak singlet ϕ, charged under $U(1)_{\rm PQ}$, is responsible for spontaneous breaking of the PQ symmetry. We may parametrize ϕ on the vacuum manifold as $\phi = (f_a/\sqrt{2})\ \exp(ia/f_a)$, where a is the axion field. The effects of the QCD anomaly are to generate a mass for the axion of order $m_a \sim \Lambda^2/f_a$, where Λ is the QCD scale. A variety of astrophysical and cosmological constraints on the axion force f_a into a narrow range of $10^9\text{GeV} \lesssim f_a \lesssim 10^{12}\text{GeV}$ for standard axions, or in a still narrower range around 10^7GeV for hadronic axions [12].

The instanton induced potential for a takes the form [4]:

$$V(a) = \Lambda^4 \cos(a/f_a + \theta). \tag{1}$$

where θ is the QCD theta angle in a basis where the quark mass matrix is real. While dominating the path integral with instantons is probably a bad approximation in an unbroken gauge theory like QCD, there are rigorous results [13] showing that the minimum of $V(a)$ occurs at strong-CP conserving values.

One possibility is that gravity does not respect $U(1)_{\rm PQ}$ at all, as is the case if wormhole effects are large. In this case, one should include renormalizable operators such as

$$\Delta V(\phi) \sim M_W^2 \phi^2 + \text{h.c.} \tag{2}$$

Here M_W is the wormhole scale, which is expected to be of the order of the Planck mass. With the addition of these operators, the PQ symmetry is strongly broken and axions never arise at all.

A second possibility is the $U(1)_{\rm PQ}$ is only broken through non-renormalizable operators of higher dimension. This can occur if either wormhole effects are suppressed or if the PQ symmetry is automatic, i.e., it is present "automatically" when one includes all renormalizable terms consistent with a given gauge group. As we shall see below, higher dimension operators will also spoil the axion solution to the strong-CP problem except possibly in the case of an automatic PQ symmetry, where gauge symmetries can eliminate operators up to some required high dimension.

We now explore the effect upon the axion potential of dimension D operators such as

$$O_D = \frac{\alpha_D}{{M_{Pl}}^{D-4}}\, \phi^{*a}\phi^b + \text{h.c.} \qquad (a \neq b;\;\; a+b=D), \tag{3}$$

which explicitly break $U(1)_{PQ}$. Operators of dimension D will modify the axion potential of Eq. (1):

$$V(a) = \Lambda^4 \cos(\ /f_a+\theta) + \sum \Delta_n \cos(na/f_a + \delta_n) \qquad (n = D,\ D-2,\ D-4, \ldots), \tag{4}$$

where $\Delta_n \sim \alpha_D f_a^D/{M_{Pl}}^{D-4}$, and δ_n is a phase angle. Let us simply analyze the $n = 1$ contribution. The extra contribution will shift the minimum of the axion potential away from the strong-CP conserving minimum of $\langle a/f_a\rangle = -\theta$. Unless $\epsilon = \langle a/f_a\rangle + \theta$ is less than 10^{-9} the amount of CP violation obtained will be in conflict with experiment. The minimum of the axion potential is now determined by $f_a V'(a) \simeq \Lambda^4\epsilon + \Delta_1 \sin(\epsilon-\theta+\delta_1) = 0$. The magnitude of $\sin(\epsilon - \theta + \delta_1)$ will not, in general, be small, and $\epsilon \sim \Delta_1/\Lambda^4$.

Since we know $\epsilon < 10^{-9}$, $\Delta_1 < 10^{-9}\Lambda^4$. For dimension D operators, we expect $\Delta_1 \sim \alpha_D f_a^D / {M_{Pl}}^{D-4}$. Using $\Lambda = 10^{-1}$GeV, the limit on ϵ translates into the following limit on the dimension D of the operator as a function of f_a and α_D:

$$D \lesssim \frac{89 + \log \alpha_D}{9 - \log (f_a/10^{10}\text{GeV})}. \tag{5}$$

If Eq. (5) is satisfied, it is very simple to show that the higher-dimension operators will have an insignificant effect on the axion mass. In fact, the zero temperature axion mass is just $m_a \sim \Lambda^2(1+\epsilon)/f$. However, we should note that the temperature dependence of the axion mass is quite different in the presence of higher dimensional operators. In particular, the mass induced by the higher dimension operators is *always* "turned on." This may affect axion cosmology in interesting ways. We are currently investigating this topic, as well as such effects on other theories (such as Majoron models) relying upon Nambu–Goldstone boson physics [14].

These results at first seem puzzling, since low-energy physics is not in general sensitive to physics at the Planck scale. However, Nambu–Goldstone bosons have the peculiar property that although they are massless (or very light in the case of pseudo-Nambu–Goldstone bosons such as the axion), they are not, properly speaking, part of the low-energy theory as evidenced by the fact that self-couplings, and couplings to light fields are suppressed by a power of a large mass scale. The fact that a light particle such as the axion is part of the high-energy sector accounts for its interesting properties, but also renders it susceptible to high-energy corrections.

In a generic invisible-axion model, there is no reason why a term such as ϕ^5/M_{Pl} could not be generated (here ϕ is a gauge-singlet field). This term would give rise to unacceptable shifts in $\overline{\theta}$ unless $\alpha_D \lesssim 10^{-44-\log(f_a/10^{10}\text{GeV})}$, which is remarkably small. Is there any to avoid this problem?

There are, in fact, ways to construct axion models which suppress higher dimensional operators as needed. This construction is based on the notion of automatic PQ symmetries [10], as described above. We first consider a supersymmetric automatic model based on the gauge group $E_6 \times U(1)_X$ [15]. The superfield content of the model is some number of **27**'s with X charges ± 1 and a $\overline{\mathbf{351}}$ with X charge 0. The most general renormalizable, gauge-invariant superpotential will only contain terms of the form $\mathbf{27}_1 \cdot \mathbf{27}_{-1} \cdot \overline{\mathbf{351}}_0$, where the subscripts denote the $U(1)_X$ charges. This automatically gives rise to a PQ symmetry in which the **27**'s have PQ charge +1 and the $\overline{\mathbf{351}}$ has PQ charge −2. The lowest dimension operators consistent with gauge invariance in the superpotential that break the PQ symmetry are terms like $\mathbf{27}^6$, $\overline{\mathbf{351}}^6$, and $(\mathbf{27} \cdot \mathbf{27} \cdot \overline{\mathbf{351}})^2$. These will then give rise to dimension 10 operators in the effective Lagrangian. Furthermore, it is relatively easy to see that we can break the gauge symmetries and the PQ symmetry spontaneously in such a way so that the final PQ symmetry (a linear combination of the original PQ symmetry and some broken gauge symmetries) is broken around 10^{10} GeV.

It is also possible to construct automatic PQ models based on supersymmetric $SU(N)$ GUT's that suppress higher dimension operators to any desired level for sufficiently large N. Models of this type without exotic fermions must all have at least four different chiral matter irreducible representations whose Young tableaux consist of a single column. Needless to say, these are exceedingly unattractive models. They will tend to have many extra families, which in addition to a host of phenomenological problems, will possibly destroy the asymptotic freedom of QCD.

Planck scale physics may also significantly affect the other solutions for the strong-CP problem [16]. As described above, the second class of solutions are based upon models where CP is softly or spontaneously broken. How they fare under Planck scale physics depends on whether dimension four operators are generated, or whether only higher dimension operators appear. If renormalizable operators can be generated, then the violation of CP by Planck scale effects will give rise to a $\mathrm{tr}\, G\tilde{G}$ term, thus regenerating the strong-CP problem (we should note, however, that the coefficient of such a term could be exponentially suppressed if it appeared in some controlled semiclassical expansion about some classical configuration [9]).

Let us next consider the case in which only non-renormalizable operators are generated by Planckian physics. In this case, all models with fields that acquire vacuum expectation values well below the Planck scale (typically the weak scale), will generate corrections to $\overline{\theta}$ that are highly suppressed by powers of M_{Pl}. In essence, this is nothing more than a restatement of the effective field theory philosophy: as long as we consider physics at energies below the cutoff of our theory, the dominant effects come from the renormalizable operators in the theory. This way of thinking about effective field theories explains why the PQ solution is so susceptible to possible effects of gravity. The problem is that the PQ scale is too close to M_{Pl} while the constraints on $\overline{\theta}$ are too tight.

Although we have seen that wormholes are troublesome for models that claim to solve the strong-CP problem, there is some indication that wormhole effects themselves might drive the QCD $\overline{\theta}$ parameter to a CP conserving value [7]. Within the framework of Coleman's wormhole calculus [17] (which has since been shown to be naive in some respects [18]), $\overline{\theta}$ became a function of the wormhole parameters. The implementation of Coleman's prescription for determining the value of these parameters was then shown to set $\overline{\theta}$ to a CP conserving value. It is not impossible that a more sophisticated approach to the wormhole calculus would still lead to a similar situation. However, until a better understanding of wormholes and quantum gravity in general is reached, this will remain a conjecture.

In conclusion, we see that Planck-scale physics can have dramatic effects on axion physics. If one wants to pursue the axion solution to the strong-CP problem, automatic models such as those presented here are probably the only consistent approach that can be taken. We have also argued that the other known solutions are essentially unaffected by gravity. The essential difference between the PQ and the non-axionic solutions is due to the sensitivity of the Nambu–Goldstone boson to physics at energies near the scale of spontaneous symmetry breaking. It remains to be seen whether other facets of the axion scenario, such as the axion energy density crisis [19] will be modified by the effects considered here.

In the course of this work we learned that the effect of gravity on the Peccei–Quinn mechanism is also being considered by Kamionkowski and March-Russell [20], and by Barr and Seckel [21]. We would like to thank them for calling their work to our attention.

It is a pleasure to thank S. Giddings, S. J. Rey and A. Strominger for useful discussions. This research was supported in part by the National Science Foundation under grant No. PHY89–04035. SDH acknowledges support from the National Science Foundation under grant NSF–PHY–87–14654, the state of Texas under grant TNRLC–RGFY106, and from the Harvard Society of Fellows. EWK and RW were supported by the NASA (through grant NAGW–2381 at Fermilab) and by the DOE (at Chicago and Fermilab), RH was supported in part by DOE grant DE–AC02–76ER3066. TWK was supported by the DOE (grant DE–FG05–85ER40226).

References

1. C. G. Callan Jr., R. F. Dashen, and D. J. Gross, *Phys. Lett.* **63B** (1976) 334;
   R. Jackiw and C. Rebbi, *Phys. Rev. Lett.* **37** (1976) 172;
   G. 't Hooft *Phys. Rev. D* **14** (1976) 3432.

2. K. F. Smith, et. al., *Nucl. Phys* **B234** (1990) 191.

3. See, e.g., D. B. Kaplan and A. Manohar, *Phys. Rev. Lett.* **56** (1986) 2004.

4. R. D. Peccei and H. R. Quinn, *Phys. Rev. Lett.* **38** (1977) 1440; *Phys. Rev. D* **16** (1977) 1791.

5. S. Weinberg, *Phys. Rev. Lett.* **40** (1978) 223;
F. Wilczek, *Phys. Rev. Lett.* **40** (1978) 279.

6. A. Nelson, *Phys. Lett.* **136B** (1984) 387;
S. M. Barr, *Phys. Rev. Lett.* **53** (1984) 329.

7. H. B. Nielsen and M. Ninomiya, *Phys. Rev. Lett.* **62** (1989) 1429;
K. Choi and R. Holman, *Phys. Rev. Lett.* **62** (1989) 2575;
J. Preskill, S. P. Trivedi, and M. Wise, *Phys. Lett.* **223B** (1989) 26.

8. S. Giddings and A. Strominger, *Nucl. Phys* **B307** (1988) 854;
S. Coleman, *Nucl. Phys* **B310** (1988) 643;
G. Gilbert, *Nucl. Phys* **B328** (1988) 159.

9. S. J. Rey, *Phys. Rev. D* **39** (1989) 3185.

10. H. Georgi, L. J. Hall, and M. B. Wise, *Nucl. Phys* **B192** (1981) 409.

11. M. Dine, W. Fischler, and M. Srednicki, *Phys. Lett.* **109B** (1981) 199;
A. P. Zhitnitskii, *Sov. J. Nucl. Phys.* **31** (1980) 260.

12. E. W. Kolb and M. S. Turner, *The Early Universe* (Addison–Wesley, Redwood City, 1990).

13. C. Vafa and E. Witten, *Phys. Rev. Lett.* **53** (1984) 535.

14. R. Holman, S. D. H. Hsu, E. W. Kolb, R. Watkins, and L. M. Widrow, work in progress.

15. P. H. Frampton and T. W. Kephart, *Phys. Rev. D* **25** (1982) 1459;
R. Holman and T. W. Kephart, *Phys. Lett.* **167B** (1986) 169.

16. see J. E. Kim, *Phys Rep.* **150** (1987) 1, and references therein.

17. E. Baum, *Phys. Lett.* **133B** (1983) 185;
S. W. Hawking, *Phys. Lett.* **134B** (1984) 403;
S. Coleman, *Nucl. Phys* **B310** (1984) 643.

18. W. Fischler, I. Klebanov, J. Polchinski, and L. Susskind, *Nucl. Phys* **B327** (1989) 157.

19. J. Preskill, M. Wise, and F. Wilczek, *Phys. Lett.* **120B** (1983) 127;
L. Abbott and P. Sikivie, *ibid*, (1983) 133;
M. Dine and W. Fischler, *ibid*, (1983) 137.

20. M. Kamionkowski, J. M. Russell, IASSNS-HEP-92/6.

21. S. Barr and D. Seckel, Bartol preprint.