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Final-State Interaction of Longitudinal Vector Bosons

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Abstract

In the standard Higgs model of electroweak symmetry breaking, the Higgs boson is associated with both vector-boson and fermion mass generation. In contrast, we discuss a two-Higgs-doublet model in which these masses are associated with two different scalar bosons. We show that the Higgs boson associated with vector-boson mass generation produces a *dip* in the cross section for $t\bar{t} \to ZZ$ via a final-state interaction. Such a dip will be difficult to observe at the LHC/SSC.

1. Introduction

The most direct probe of the electroweak-symmetry-breaking mechanism is longitudinal-vector-boson scattering [1]. For example, in the standard Higgs model, the Higgs boson appears as a resonance in this process [2].

The LHC/SSC will provide the first opportunity to study longitudinal-vector-boson scattering. There are other sources of longitudinal-vector-boson pairs at these machines, and one might ask if we could study longitudinal-vector-boson scattering indirectly via the final-state interaction (rescattering) of the vector bosons.

One difficulty with posing such a question is how to separate the effects of the final-state interaction from direct effects. For example, in the process $gg \to V_L V_L$ ($V = W, Z$; L denotes longitudinal polarization), which proceeds via a top-quark loop, the standard-model Higgs boson couples directly to the top quark as an s-channel resonance [3]. This direct effect is much larger than any effect due to a final-state interaction.

In this paper we study a model in which the Higgs boson which appears as a resonance in longitudinal-vector-boson scattering does not couple directly to the top quark, so we can study its effect on $gg \to V_L V_L$ via a final-state interaction. For simplicity we actually study the process $t\bar{t} \to V_L V_L$ (t = top quark), which one may regard as a subprocess of $gg \to V_L V_L$.

We have chosen the process $gg \to V_L V_L$ because it is a large source of longitudinal vector bosons at the LHC/SSC [4,5]. The process $q\bar{q} \to V_L V_L$ is also a copious source of longitudinal vector bosons (except for $Z_L Z_L$), but almost entirely in the $J = 1$ partial wave [6]. The model we study has only $J = 0$ resonances, so it does not produce large effects in the latter process. The process $V_T V_{T,L} \to V_L V_L$ (T denotes transverse polarization) has been shown not to be a large source of longitudinal vector bosons [7].

In the next section we discuss the model we use, which is based on two Higgs doublets. This model is theoretically well motivated and interesting in its own right. In section 3 we study the process $t\bar{t} \to Z_L Z_L$, including the final-state interaction of the longitudinal vector bosons. Section 4 discusses the results and draws conclusions.

2. Two-Higgs-doublet model

In the standard model of the electroweak interaction, both the weak vector bosons and the fermions acquire mass via the Higgs mechanism, which breaks the SU(2) × U(1) symmetry down to electromagnetism. The symmetry is broken by an SU(2) scalar doublet which acquires a vacuum-expectation value. A scalar particle, dubbed the Higgs boson, becomes part of the physical spectrum. Thus the Higgs boson is associated with both vector-boson and fermion mass generation.

In the absence of the Higgs boson or, more generally, a model for the symmetry-breaking mechanism, the tree amplitude for longitudinal-vector-boson scattering is proportional to $g^2 s/M_W^2$, and violates the unitarity bound at $s \approx 4\pi\sqrt{2}/G_F \approx (1.2\ \mathrm{TeV})^2$ [1,8]. The Higgs boson, exchanged in the s, t, and u channels, cancels the terms in the amplitude which grow with energy, leaving an amplitude proportional to $g^2 m_H^2/M_W^2$ [9,10]. Thus we may say that the Higgs boson is responsible for "unitarizing" the longitudinal-vector-boson scattering amplitude, in the sense that the unitarity bound is respected at all energies.

Similarly, the amplitude for $t\bar{t} \to V_L V_L$, with the t and $\bar{t}$ of the same helicity, is proportional to $g^2 m_t \sqrt{s}/M_W^2$ [11]. The unitarity bound is violated at an energy which depends on m_t [11,8] [1],

$$\sqrt{s} \approx \frac{4\pi\sqrt{2}}{3G_F m_t}. \tag{2.1}$$

The Higgs boson, exchanged in the s channel, again cancels the terms which grow with energy, leaving terms proportional to $g^2(m_H^2/M_W^2)m_t/\sqrt{s}$ and $g^2(m_t^2/M_W^2)m_t/\sqrt{s}$ [12]. Thus the Higgs boson also "unitarizes" $t\bar{t} \to V_L V_L$.

One may regard the standard Higgs model as economical, in that it generates both vector-boson and fermion masses via the same mechanism. However, we have no guarantee that nature has chosen this model, so we should keep an open mind regarding the possible manifestations of the vector-boson and fermion mass-generating mechanisms. These two mechanisms could be quite distinct and have very different experimental signatures. This point has been appreciated at least since the development of technicolor [13] and extended technicolor [14], and it has recently been emphasized in Ref. [15]. Note, however, that in technicolor/extended technicolor

[1] The unitarity bound of Ref. [11] is considerably strengthened in Ref. [8] by considering the $I = 0$, $J = 0$, spin zero, color-singlet amplitude.

both $V_LV_L \to V_LV_L$ and $t\bar{t} \to V_LV_L$ are unitarized by the same condensate, at a scale $\mathcal{O}(4\pi\sqrt{2}/G_F)$ [11].

In this paper we consider what is possibly the simplest model in which the vector-boson and fermion masses are generated by separate mechanisms: a two-Higgs-doublet model. One may introduce a discrete symmetry to ensure that only one doublet couples to fermions [16]. If this doublet has a small vacuum-expectation value, it couples only weakly to vector bosons, but with correspondingly enhanced strength to fermions. The neutral scalar boson associated with this doublet unitarizes $t\bar{t} \to V_LV_L$, but it contributes very little to the unitarization of $V_LV_L \to V_LV_L$, which is unitarized almost entirely by the neutral scalar boson associated with the other doublet. This model has been considered previously in Ref. [17] with a rather different motivation. We do not regard this model as a candidate for a fundamental theory, but as the simplest model which embodies the philosophy described above, and useful for suggesting signatures for the vector-boson and fermion mass-generating mechanisms.

3. $t\bar{t} \to Z_LZ_L$

For calculational expediency, we consider only terms of enhanced electroweak strength, $\mathcal{O}(g^2m^2/M_W^2)$, where m is a Higgs-boson or top-quark mass. Formally, this choice corresponds to the limit $g \to 0$ with v fixed ($M_W = \frac{1}{2}gv$). The polarization vectors of the longitudinal vector bosons can be approximated by $\epsilon_L^\mu(p) \approx p^\mu/M_V$ in this limit.

The tree-level Feynman diagrams for $t\bar{t} \to Z_LZ_L$ are shown in Fig. 1. The necessary Feynman rules can be found in Refs. [17,18] [2]. The amplitude, with the t and $\bar{t}$ of the same helicity and opposite color, is

$$A_0 = -\frac{g^2m_t}{4M_W^2}\bar{v}(p_2)\left[1 + m_t\left(\frac{1}{\not{p}_3 - \not{p}_1 - m_t} + \frac{1}{\not{p}_2 - \not{p}_3 - m_t}\right)\right. \\ \left. - \frac{1}{\sin\beta}\left(\sin\alpha\cos(\beta-\alpha)\frac{s}{s - m_1^2 + im_1\Gamma_1} + \cos\alpha\sin(\beta-\alpha)\frac{s}{s - m_2^2 + im_2\Gamma_2}\right)\right]u(p_1) \tag{3.1}$$

where m_t is the top-quark mass and $m_{1,2}$ are the masses of the Higgs bosons. The parameter β is related to the ratio of the vacuum-expectation values of the two Higgs

[2] Although the appendix of Ref. [18] specializes to the supersymmetric two-Higgs-doublet model, the HVV and $Ht\bar{t}$ couplings are valid for the model considered here, with $H^0 = H_1$, $h^0 = H_2$.

doublets by $\tan\beta = v_2/v_1$; we consider $\tan\beta << 1$. The parameter α mixes the neutral scalar Higgs bosons from the two doublets. The case under consideration corresponds to no mixing, so we set $\alpha = 0$. One then sees that the H_1-exchange diagram vanishes, and the H_2-exchange diagram alone is responsible for "unitarizing" the amplitude, i.e., canceling the first term in Eq. (3.1) at high energy.

We now wish to calculate the effect of a final-state interaction (rescattering) of the longitudinal vector bosons. Away from the H_1 resonance, the longitudinal vector bosons are weakly coupled, and the effect of rescattering is small. We therefore need only consider diagrams involving an s-channel H_1 boson. The contributing Feynman diagrams, in 't Hooft-Feynman gauge, are shown in Fig. 2. An H_2H_1 counterterm is necessary to absorb the ultraviolet divergence in the second diagram. This counterterm renormalizes the mixing parameter α from its bare value [17]. We set the renormalized α to zero. The necessity to retune the renormalized α to zero at one loop is a consequence of the fact that $\alpha = 0$ (or any other value) is not enforced by any symmetry in this model. In effect we are taking the short-distance part of the loop integral, canceling it against the bare α, and regarding the remaining finite part as a long-distance final-state interaction.

The one-loop amplitude for $t\bar{t} \to Z_L Z_L$ is [3]

$$\begin{aligned} A_1 = & -\frac{1}{(4\pi)^2}\frac{g^4 m_1^2 m_t}{16 M_W^4}\cos^2(\beta-\alpha)\frac{s}{s-m_1^2+im_1\Gamma_1} \\ & \bar{v}(p_2)\Big[m_t^2\left(C_{11}^Z - C_{12}^Z + (1-\gamma_5)(C_{11}^W + C_0^W) - (1+\gamma_5)C_{12}^W\right) \\ & + \frac{\cos\alpha\sin(\beta-\alpha)}{\sin\beta}\frac{m_2^2}{s-m_2^2+im_2\Gamma_2}\left(B_0^W + \frac{1}{2}B_0^Z\right)\Big]u(p_1) \end{aligned} \tag{3.2}$$

where

$$\begin{aligned} \frac{1}{i\pi^2}\int d^4k \quad & \frac{k^\mu}{[k^2-M_Z^2][(k+p_1)^2-m_t^2][(k+p_1+p_2)^2-M_Z^2]} \\ & = C_{11}^Z p_1^\mu + C_{12}^Z p_2^\mu \\ \frac{1}{i\pi^2}\int d^4k \quad & \frac{\{1; k^\mu\}}{[k^2-M_W^2][(k+p_1)^2-m_b^2][(k+p_1+p_2)^2-M_W^2]} \\ & = \{C_0^W;\ C_{11}^W p_1^\mu + C_{12}^W p_2^\mu\} \end{aligned}$$

[3] The coupling of the Goldstone bosons to the Higgs bosons is obtained by multiplying the corresponding H_iVV coupling by $-m_i^2/2M_V^2$ and dropping the $g^{\mu\nu}$. Note that the couplings in the appendix of Ref. [18] pertain specifically to the supersymmetric two-Higgs-doublet model.

and

$$\begin{aligned} B_0^V &= \frac{1}{i\pi^2}\int d^4k \frac{1}{[k^2 - M_V^2][(k+p_1+p_2)^2 - M_V^2]} + \text{counterterm} \\ &= -(1-4M_V^2/s)^{1/2}\ln\left(\frac{(1-4M_V^2/s)^{1/2}+1}{(1-4M_V^2/s)^{1/2}-1}\right) + 2\,. \end{aligned}$$

The counterterm has been chosen such that $B_0^V = 0$ at $s = 0$. Note that B_0^V has a (positive) imaginary part for $s > 4M_V^2$. The loop integrals were evaluated with the codes FF [19] and LOOP [20] [4].

Although any value of β is theoretically acceptable, a very small value enhances the top-quark Yukawa coupling such that it becomes strong, and perturbation theory breaks down. Unitarity of $t\bar{t} \to t\bar{t}$ suggests that this occurs for [12,8]

$$\sin^2\beta \approx \frac{3G_F m_t^2}{4\pi\sqrt{2}}\cos^2\alpha. \tag{3.3}$$

Since we are interested in performing a perturbative calculation, we choose a value of β greater than that given by Eq. (3.3).

The two-Higgs-doublet model also contains a charged scalar, $H^\pm$, and a neutral pseudoscalar, A. These particles, as well as H_2, contribute to H_1 production at one loop via diagrams in which they replace one or both of the Goldstone bosons in Fig. 2. The cubic scalar interactions are model dependent, so we have not included these contributions. We argue in the next section that they do not change the qualitative results of our calculation.

4. Results and conclusions

As a typical example of what one would expect to observe, we show in Fig. 3 the square of the zeroth partial wave of $t\bar{t} \to Z_L Z_L$, with the t and $\bar{t}$ of the same helicity and opposite color, for $m_1 = 500$ GeV, $m_2 = 300$ GeV, $m_t = 130$ GeV, $\beta = 0.3$, and $\alpha = 0$. The H_2 resonance, which couples directly to the top quark, produces the expected peak. The H_1, which does not couple directly to the top quark, produces a *dip*. Such dips are generic for a final-state interaction which proceeds via a resonance, and are known in hadronic physics [23,24,25]. This phenomenon is reviewed in an appendix, both diagrammatically and via the Omnès-Muskhelishvili

[4] LOOP is a completely FORTRAN version of Veltman's FORMFactor [21].

formalism [26,27]. A simple understanding of this phenomenon is gained by noting that the absorptive part of the loop diagrams, obtained via the Cutkosky rules by "cutting" the Goldstone-boson propagators, is proportional to the product of the tree amplitude $t\bar{t} \to V_L V_L$ and the vector-boson-scattering amplitude $V_L V_L \to V_L V_L$ (using the equivalence of the Goldstone bosons and V_L in the limit $g \to 0$). At the peak of the H_1 resonance, the latter amplitude is purely imaginary; thus the absorptive part of the loop diagrams interferes destructively with the tree amplitude, producing a dip. Including $H_2, H^{\pm}$, and A in the loop diagrams does not change this result qualitatively, as is made clear in the appendix.

Unfortunately, a dip in the $V_L V_L$ invariant mass spectrum will be very difficult to observe at the LHC/SSC. There is a large continuum background from $q\bar{q} \to V_T V_T$ [6] and $gg \to V_T V_T$ [4,5] which is typically an order of magnitude larger than the $gg \to V_L V_L$ continuum. At the H_2 resonance, $gg \to V_L V_L$ is enhanced such that the signal is comparable to or greater than the background. However, a dip in the $gg \to V_L V_L$ process near the H_1 mass will be difficult to distinguish on the large continuum background. Furthermore, the H_1 will appear as a resonance in longitudinal-vector-boson scattering, so one would have to separate this process from $gg \to V_L V_L$ in order to have any chance of observing the dip in the latter. Such a separation may be possible by tagging the forward jets associated with longitudinal-vector-boson scattering, but it is not one-hundred percent efficient [28].

We conclude that while $t\bar{t} \to V_L V_L$ (via $gg \to V_L V_L$ at the LHC/SSC) is likely to tell us about the mass-generating mechanism for the top quark, it is unlikely to reveal information on the mass-generating mechanism for the vector bosons via the final-state interaction of the longitudinal vector bosons. If the particles associated with vector-boson mass generation couple to top quarks (as in the standard Higgs model) or to gluons [29], they could manifest themselves directly in the process $gg \to V_L V_L$.

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Appendix A

We show that a final-state interaction which proceeds via a resonance produces a dip near the resonance mass. The argument we give is a generalization of that given in the appendix of the first paper of Ref. [24]. We also show that the same result can be obtained via the Omnès-Muskhelishvili formalism [26,27].

If we denote by g the coupling of the vector bosons, of mass M, to the spin-zero resonance, the zeroth partial wave of the elastic scattering amplitude of the vector bosons is

$$\frac{1}{\beta}e^{i\delta}\sin\delta = -\frac{g^2}{16\pi}\,\frac{1}{s-m^2+g^2\Pi} \tag{A.1}$$

where $\beta = (1-4M^2/s)^{1/2}$, δ is the phase shift, and $\Pi = B_0^V/(4\pi)^2$ is the renormalized two-point function. Let us denote by a_B the zeroth partial wave of the weak production amplitude of the strongly-interacting vector bosons. The zeroth partial wave of the full amplitude, given by the sum of a_B and the one-loop amplitude formed by the rescattering of the vector bosons through the resonance (see Fig. 4), is

$$\begin{aligned} a &= a_B - g^2 L\frac{1}{s-m^2+g^2\Pi} \\ &= \frac{1}{s-m^2+g^2\Pi}\,\left[a_B\left(s-m^2+g^2\Pi\right)-g^2L\right] \end{aligned} \tag{A.2}$$

where L is the loop integral associated with the loop diagram. Unitarity, via the Cutkosky rules, tells us

$$\mathrm{Im}\,\Pi = \frac{1}{16\pi}\beta \tag{A.3}$$

and

$$\mathrm{Im}\,L = \frac{1}{16\pi}\beta a_B \tag{A.4}$$

so the numerator of the amplitude is real, with a zero near $s=m^2$. The real parts of the loop integrals shift the position of the zero.

The Omnès-Muskhelishvili formalism provides a means of implementing unitarity and analyticity [26,27]. Writing and solving a dispersion relation for $a - a_B$, one obtains [23,24,25,26,27]

$$a = a_B - \Omega \frac{1}{\pi} \int_{4M^2}^{\infty} \frac{ds'}{s' - s} [\mathrm{Im}\, \Omega^{-1}] a_B \tag{A.5}$$

where

$$\begin{aligned} \Omega &= \exp\left[\frac{s}{\pi} \int_{4M^2}^{\infty} \frac{ds'}{(s' - s)s'} \delta(s') \right] \\ &= -\frac{m^2}{s - m^2 + g^2 \Pi} \end{aligned} \tag{A.6}$$

is the Omnès function for the phase shift given by Eq. (A.1) [24]. Eq. (A.5) may thus be written

$$a = a_B - g^2 \frac{1}{s - m^2 + g^2 \Pi} \left[\frac{1}{\pi} \int_{4M^2}^{\infty} \frac{ds'}{s' - s} \beta a_B \right] . \tag{A.7}$$

The factor in the bracket is the integral representation of L, which proves the equivalence of Eqs. (A.7) and (A.2). A counterterm, or subtraction constant, is necessary if L is ultraviolet divergent.

One may add an additional term, $P\Omega$, to Eq. (A.5), where P is an arbitrary real polynomial, and still satisfy the unitarity and analyticity requirements of the Omnès-Muskhelishvili formalism. For $P =$ constant, this term represents a direct coupling of the resonance to the initial state. In Ref. [30] the effect of a final-state interaction via a technirho resonance on $e^+e^- \to W^+W^-$ is represented by $a_B\Omega$, resulting in a resonant enhancement near the technirho mass. Our arguments show that this term corresponds to a direct coupling of the technirho, not to a final-state interaction. In Ref. [31] the technirho is introduced via the Gounaris-Sakurai model, which is based on vector-meson dominance. This also corresponds to a direct coupling, not to a final-state interaction, as claimed in that work.

The phase variation of $e^+e^- \to W^+W^-$ due to a final-state interaction is discussed in Ref. [32].

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Figure captions

Fig. 1 - Feynman diagrams for $t\bar{t} \to Z_L Z_L$ in a two-Higgs-doublet model. For $\alpha = 0$, only H_2 contributes to the last diagram.

Fig. 2 - Feynman diagrams of enhanced electroweak strength for $t\bar{t} \to Z_L Z_L$ near the H_1 resonance, in 't Hooft-Feynman gauge. The Goldstone bosons are denoted by $w^{\pm}, z$. One may regard these diagrams as representing a final-state interaction of the longitudinal vector bosons via the H_1 resonance.

Fig. 3 - Square of the zeroth partial wave of $t\bar{t} \to Z_L Z_L$, with the t and $\bar{t}$ of the same helicity and opposite color, versus center-of-mass energy, for $m_1 = 500$ GeV, $m_2 = 300$ GeV, $m_t = 130$ GeV, $\beta = 0.3$, and $\alpha = 0$. Both the tree amplitude (dashed) and the amplitude including the final-state interaction of the longitudinal vector bosons (solid) are shown. The H_2 resonance produces the expected peak, while the H_1, which does not couple directly to the top quark, produces a dip.

Fig. 4 - Weak production amplitude plus the one-loop amplitude formed by the rescattering of the final particles through a resonance.

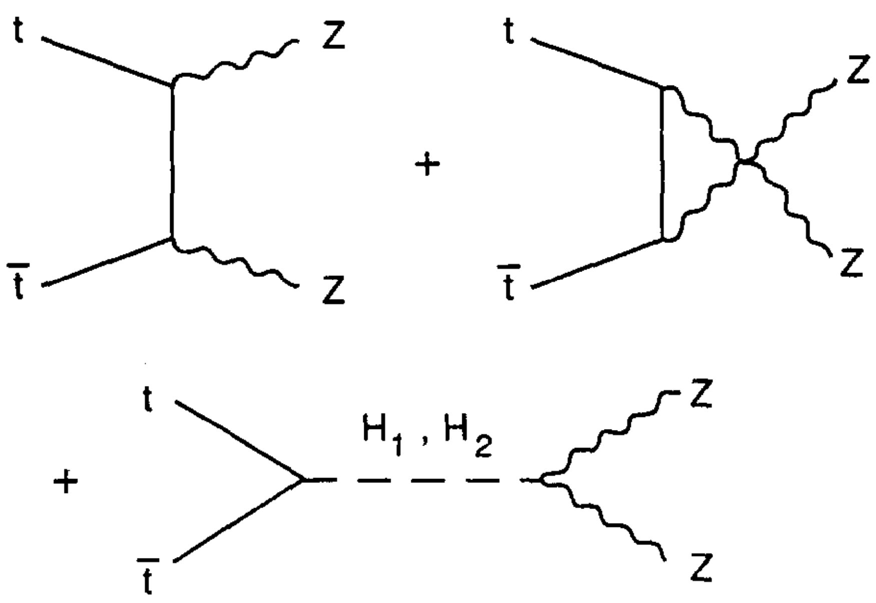


Figure 1

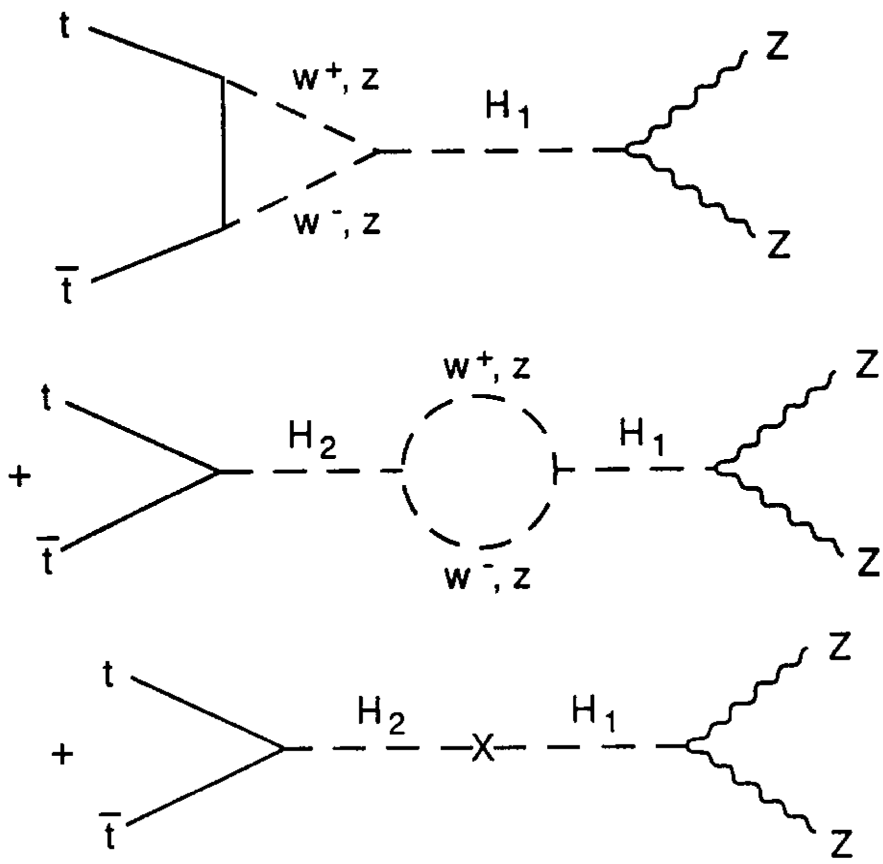
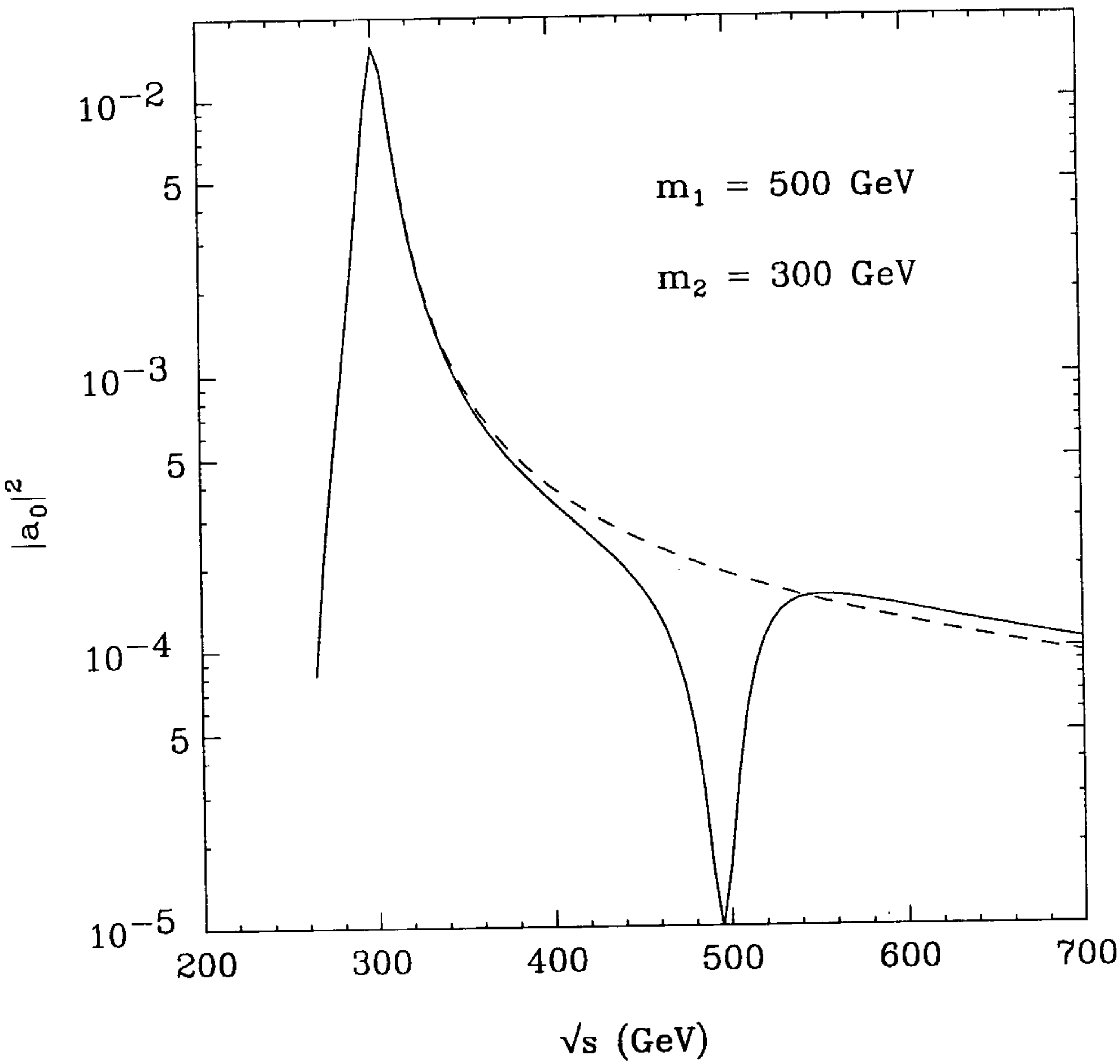


Figure 2



m_1 = 500 GeV
m_2 = 300 GeV
|a_0|^2
10^-2
5
10^-3
5
10^-4
5
10^-5
200
300
400
500
600
700
√s (GeV)

Figure 3

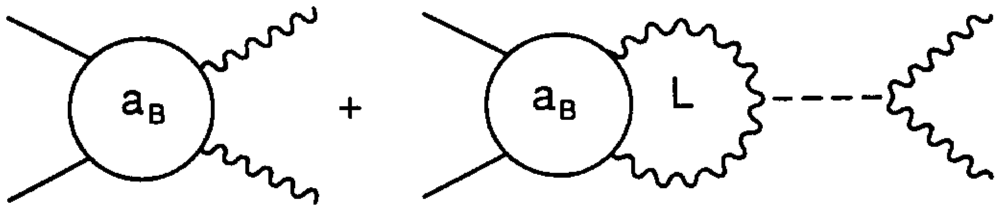


Figure 4