



Heavy-Light Wavefunctions in Lattice QCD *

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Using a multistate smearing method, Coulomb gauge wave functions of heavy-light mesons are studied in lattice QCD. Wave functions for the ground state, the first radially excited S-wave state, and the lowest P-wave states of a heavy-light meson are calculated in quenched approximation. The results are found to be in remarkably good agreement with the predictions of a simple relativistic quark model.

1. Introduction

The evolution of lattice gauge theory techniques has greatly enhanced our understanding of quark-gluon dynamics in QCD. Heavy-light mesons provide an ideal laboratory for lattice QCD studies. The static approximation ($m_Q \rightarrow \infty$) in which the heavy quark propagator is replaced by a straight time-like Wilson line provides a framework which allows a quantitative study of masses, decay constants, mixing amplitudes, and electroweak form factors.[1] Since heavy-light mesons have only one dynamical light (valence) quark, these systems are also well suited to the study of constituent quark ideas [2] and the chiral quark model [3].

In view of the success of the nonrelativistic (NR) potential model for heavy $Q\bar{Q}$ mesons, one interesting question for heavy-light systems is the nature and extent of the deviation from the NR potential picture as one of the quarks becomes light. Here we present results of a numerical lattice study of this question. Our findings support a surprisingly simple answer. The Coulomb gauge wave functions obtained in lattice QCD agree, within the accuracy of our calculations, with the results of a simple relativistic generalization of the NR quarkonium potential model. It is only necessary to replace the NR kinetic energy term in the Hamiltonian by its relativistic form, leaving the NR potential unchanged. The only adjustable parameter is the quark mass parameter μ . This

description holds down to fairly small values of the current quark mass, corresponding to a pion mass of approximately $300 MeV/c^2$, well into the region where the NR description fails.

2. Wavefunctions in Lattice QCD

In lattice QCD, the properties of hadronic states are studied using correlation functions of operators which couple to the state. Originally local operators were used. More recently smeared (non-local) operators have been found to improve the ability to extract the masses of meson and baryon ground states [4]. Many of the present studies have been done with configurations and propagators fixed in Coulomb gauge and operators which smear the position of the quark field uniformly over a spatial cube of variable size. However a constant cube of any size is a very crude approximation to the ground state wave function [5]. Hence, the propagator generally has significant contamination from higher states out to times large compared to the inverse of the energy splitting between the ground state and the lowest excited state.

This is a particular problem in the study of heavy-light correlators because they become noisy rather rapidly in time. Unfortunately, this is an unavoidable feature of heavy-light systems [6,7]. Recently a multistate smearing technique has been proposed [6] which allows the extraction of the properties of heavy-light states from relatively short times.

*Talk presented by H. Thacker



The details of the multistate smearing method have been presented elsewhere[6]. By choosing an appropriate orthonormal set of smearing functions and diagonalizing the corresponding matrix of correlators, one obtains the wave functions of not only the lowest lying state in a given channel, but also of radially excited states. Here we define the wave function to be the vacuum-to-one-particle matrix element,

$$\Psi(\vec{r}) = \sum_a \langle 0 | q_a(\vec{r}, 0) Q_a^\dagger(0, 0) | B \rangle \quad (1)$$

where $|B\rangle$ is the state of interest. The sum is over color, and spin labels are suppressed.

Here we discuss the wave functions obtained for the 1S and 2S levels of the S-wave pseudoscalar meson as well as a preliminary study of the 1P state.[8] The main emphasis will be on the remarkable quantitative agreement between the lattice QCD wave functions and those obtained from a simple relativistic quark model Hamiltonian. The results for heavy-light decay constants and spectroscopy will be presented at this conference by Eichten.[9]

The investigation used an existing set of 50 configurations (each separated by 2000 sweeps) generated by ACPMAPS on a $16^3 \times 32$ lattice at $\beta = 5.9$. The configurations were fixed to Coulomb gauge and light quark propagators with $\kappa = .158$ were used. Only the four lowest energy smearing functions were included ($N = 4$).

3. Relativistic Quark Model

The optimized wave functions obtained from our lattice data by the multistate smearing method turn out to be, within errors, the same as the eigenfunctions of a lattice version of the spinless, relativistic quark model Hamiltonian, which we will now define. In the absence of gauge fields, the free quark Hamiltonian can be exactly diagonalized by introducing momentum space creation and annihilation operators for quarks and antiquarks. In the continuum,

$$H_0 = \int \frac{d^3p}{(2\pi)^3} \sqrt{p^2 + \mu^2} \sum_i [\alpha_i^\dagger(p) \alpha_i(p) + \beta_i^\dagger(p) \beta_i(p)] \quad (2)$$

where the sum is over spin and color labels. In terms of the covariant quark propagator, the particle and antiparticle operators are associated with propagation forward and backward in time, respectively. Since H_0 contains no pair creation ($\alpha^\dagger \beta^\dagger$) terms, it is possible to formulate the eigenvalue problem as that of a one-body operator, $H_0 \rightarrow \sqrt{\mu^2 - \nabla^2}$. If we now turn on the gauge interaction and introduce a heavy-quark, static color source, the description of the bound light quark becomes, in principle, drastically more complicated. We know that, in the limit $\mu \gg \Lambda_{QCD}$ where the dynamical quark becomes heavy, the primary effect of the color source is to introduce a static, confining potential $V(r)$ whose form is well-measured and consistently given by both $Q\bar{Q}$ phenomenology and lattice QCD,

$$H_0 \rightarrow H = H_0 + V(r) \quad (3)$$

At this stage, the Hamiltonian can still be regarded as a one-body operator[10]. As the mass of the quark becomes light, one expects more complicated effects arising from the gauge interaction which render the Hamiltonian eigenvalue problem intractable. These effects include the creation of gluons and light $q\bar{q}$ pairs, as well as the exchange of transverse and non-instantaneous gluons with the static source. From the numerical results presented in the next section, we conclude that these effects are relatively small, and that the heavy-light meson system is well-described by the Hamiltonian (6), which we will refer to as the spinless relativistic quark model (SRQM).

The construction of explicit eigenfunctions of the SRQM Hamiltonian is easily accomplished by a numerical procedure. First the operator H is discretized on a 3D lattice by replacing the spatial derivatives with finite differences. The potential energy $V(r)$ is just the static energy measured on the same configurations used to study the heavy-light spectrum. Then the resolvent operator $(E - H)^{-1}$ acting on a source vector χ is computed by a numerical matrix inversion (conjugate gradient) algorithm. Finally, the parameter E is varied to find the poles in the output vector $(E - H)^{-1}\chi$. The location of the pole is an eigenvalue of H , and its residue is the corresponding eigenfunction. In

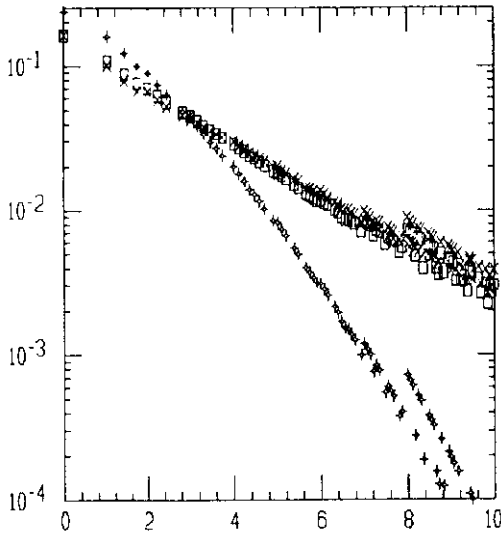


Figure 1. Comparison of the 1S state in LQCD (\times 's) with the NRQM (+'s) and the SRQM (boxes).

the next section we compare the wave functions obtained in this way from the SRQM Hamiltonian with the lattice QCD results.

4. Comparison of Wavefunctions

Using the four state smeared correlator described in section 2 an initial study for the S-wave channel was carried out. After some iterative improvement of the smearing functions, it was found that the value $\mu = .23$ for the dimensionless mass parameter in the SRQM Hamiltonian gave the best agreement with the lattice QCD wave functions with $\beta = 5.9, \kappa = .158$. In Fig. 1 the LQCD wave function is plotted with the SRQM wave function. For comparison, the non-relativistic (NR) Schrodinger wave function (obtained by replacing the relativistic kinetic term by $p^2/2m$) is also plotted. The mass parameter in the NR Hamiltonian was adjusted to give the same slope at the origin in the ground state wave function. Notice that, for large r , the QCD and

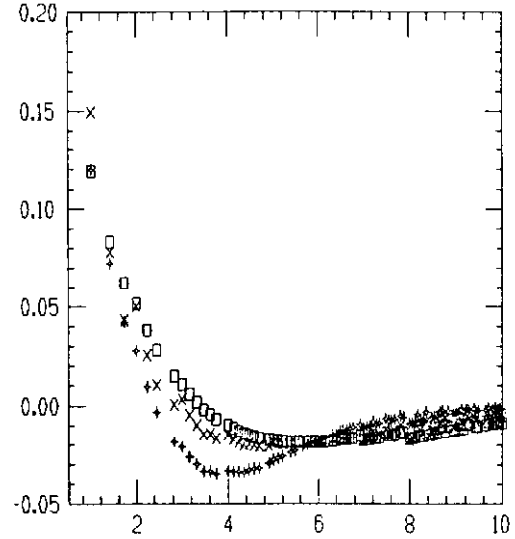


Figure 2. Comparison of the 2S state in LQCD (\times 's) with the NRQM (+'s) and the SRQM (boxes).

SRQM wave functions both fall exponentially. On the other hand, the NR wave function falls faster than exponentially ($\exp(-ar^{\frac{3}{2}})$), as expected from the behavior of the analytic solution in a pure linear potential (Airy function). Remarkably, by including the relativistic kinetic term, the SRQM wave functions are brought into excellent agreement with those of lattice QCD, without changing the potential from its nonrelativistic form.

In Fig. 2 we plot the excited 2S state from LQCD along with the corresponding wave functions from the SRQM and the NR model. The QCD wave function is somewhat more peaked at the origin, however, the overall agreement between QCD and the SRQM is excellent. Here, there are no adjustable parameters, m being already fixed from the 1S state fit. Finally, in Fig. 3 we show some preliminary results of a study of the 1P state. Here the solid line is the 1P wave function from the SRQM. The data points depict the evolution of the P-wave LQCD radial wave-function extracted from time slices $T = 2$ (+'s),

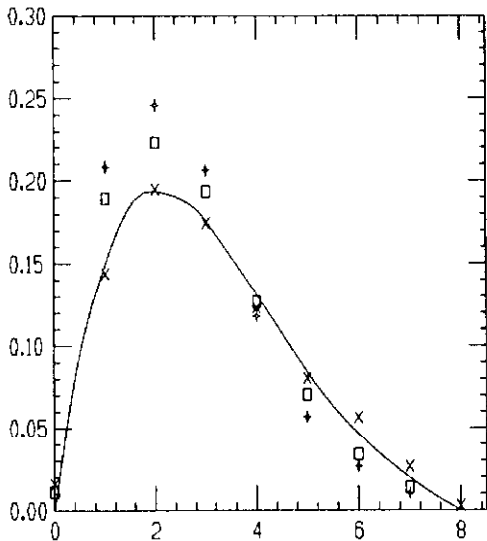


Figure 3. The 1P state in LQCD extracted from $T=2$ (+'s), $T=4$ (boxes) and $T=6$ (x's).

$T=4$ (boxes), and $T=6$ (x's), starting with an approximate guess for the initial smearing function. The ansatz for the initial smearing function used here was a simple re^{-ar} form. As the LQCD wave function evolves in Euclidean time, it appears to approach a true eigenstate whose wavefunction again agrees remarkably well with the SRQM result, with no adjustable parameters.

5. Discussion

Additional studies are in progress using a variety of lattice sizes, gauge coupling strengths, and light quark masses. Preliminary results of these studies are fully consistent with the conclusions presented here. The agreement of lattice QCD with the SRQM wave functions suggests that the relativistic propagation of the light valence quark is the most important effect which must be included in a description of heavy-light mesons. Other field theoretic effects such as the presence of multibody components of the wave-

function (containing gluons along with light $q\bar{q}$ pairs arising, in quenched approximation, from the propagation of the valence quark backward in time) are of less quantitative importance in determining the shape of the valence quark wave function. Further numerical studies of the connection between lattice QCD and the relativistic quark model are planned.

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REFERENCES

- 1 For a review of heavy-light systems on the lattice see: E. Eichten, in *Lattice 90*, Nucl. Phys. B(Proc.Suppl.) 20 (1991) 475.
- 2 J. D. Bjorken, SLAC preprint SLAC-PUB-5278 (1990).
- 3 A. Manohar and H. Georgi, Nucl. Phys. B234 (1984) 189.
- 4 P. Bacilieri et. al., Nucl. Phys. B317 (1989) 509.
- 5 S. Hashimoto and Y. Saeki, Mod. Phys. Lett. A7 (1992) 387.
- 6 A. Duncan, E. Eichten, and H. Thacker, Nucl. Phys. B (Proc. Suppl) 26 (1992) 391; 26 (1992) 394.
- 7 A. Kronfeld, in *Lattice 89*, Nucl. Phys. B (Proc.Suppl.) 17 (1990) 313.
- 8 For a recent study of light hadron wave functions, see M. W. Hecht and T. A. DeGrand, Phys. Rev. D46 (1992) 2155, and references therein.
- 9 E. Eichten, talk presented at this conference.
- 10 E. E. Salpeter, Phys. Rev. 87, (1952) 328.