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Gravitational Waves from First-order Cosmological Phase Transitions

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Abstract. A first-order cosmological phase transition that proceeds through the nucleation and collision of true-vacuum bubbles is a potent source of gravitational radiation. Possibilities for such include first-order inflation, GUT-symmetry breaking, and electroweak-symmetry breaking. We have calculated gravity-wave production from the collision of two scalar-field vacuum bubbles, and, using an approximation based upon these results, from the collision of 20 to 30 vacuum bubbles. We present estimates of the relic background of gravitational waves produced by a first-order phase transition; in general, $\Omega_{\rm GW} \sim 10^{-9}$ and $f \sim 10^{-6}\,{\rm Hz}\,(T/\,{\rm GeV})$.

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Introduction. Gravitational radiation provides a powerful probe of some of the most spectacular astrophysical events (coalescing neutron-star binaries, black hole collisions, supernovae, etc.) as well as the earliest moments of the Universe. Further, a new generation of wide-band detectors—laser interferometric devices or LIGO's—with the sensitivity to detect gravity waves from both astrophysical and cosmological sources is currently under development [1]. Since the Universe is transparent to gravitational waves back to the Planck epoch ($T \sim 10^{19}\,\mathrm{GeV}$, $t \sim 10^{-43}\,\mathrm{sec}$), the stochastic gravitational-wave background should contain an unrivaled fossil record of cosmological events, including the 0.9 K gravitational analog of the microwave background and gravitational radiation from cosmic strings [2], soliton stars [3], inflation [4, 5], and cosmological phase transitions [6].

Our focus in this *Letter* is the production of gravitational waves in a first-order cosmological phase transition that proceeds through the nucleation of true-vacuum bubbles [7]; candidates for such a phase transition include GUT-symmetry breaking/first-order inflation [8] ($T \sim 10^{15\pm1}\,\mathrm{GeV}$, $t \sim 10^{-36\pm2}\,\mathrm{sec}$), electroweak symmetry breaking ($T \sim 200\,\mathrm{GeV}$, $t \sim 10^{-11}\,\mathrm{sec}$), and phase transitions yet to be discussed. Since the present frequency of gravity waves produced in a phase transition depends upon the temperature at which it occurs, $f \sim 10^{-6}\mathrm{Hz}(T/\mathrm{GeV})$, across its spectrum the gravity-wave background could well contain a record of the thermodynamic history of the Universe. Our predictions are based upon a detailed numerical study of gravitational radiation produced by the collision of two scalar-field vacuum bubbles [9] and an extension of this work, designed to simulate a realistic phase transition, involving the collision of more than 100 bubbles [10].

In a first-order cosmological phase transition the Universe gets "hung up" in a metastable, false-vacuum state; if the energy barrier between the false- and true-vacuum states is sufficiently large, significant supercooling occurs and the transition to the true vacuum proceeds through the nucleation [7] and percolation of bubbles of true vacuum [11]. Once nucleated, a vacuum bubble expands outward with constant acceleration and quickly approaches the speed of light, being driven by the pressure difference between its true-vacuum interior and its false-vacuum exterior. The false-vacuum energy liberated as the bubble expands ($E_{\rm vac} = 4\pi\rho_{\rm vac}t^3/3$) exists in the form of bubble-wall kinetic energy and is eventually transformed into particles (and thermalized) when bubbles collide [12]. The false-vacuum energy released is substantial: much greater than that in the ambient thermal bath of radiation, so that the entropy of the Universe is increased significantly. For all these reasons, a strongly first-order cosmological phase transition is expected to be a potent source of gravity waves [13].

We first describe our numerical calculations of the amount of gravitational radiation produced by the collisions of true-vacuum bubbles, and then use these results to estimate the amount of gravitational radiation produced in a "generic" first-order cosmological phase transition. The fact that we are able make such an estimate is because the spectrum and the amount of gravitational radiation produced by the collision of two vacuum bubbles depends only upon the grossest features of the collision, bubble size at collision and the false-vacuum energy. Our predictions depend only upon: (1) the ratio of the rate of change of the bubble nucleation rate to the expansion rate of the Universe (which sets the bubble size); and (2) the temperature at which the phase transition takes place.

When bubbles collide. In Ref. [9] we computed numerically the gravitational radiation produced by the collision of two scalar-field vacuum bubbles. To be more specific, we used the field equations to evolve the scalar-field configuration corresponding to two vacuum bubbles separated by a distance d for a time $\tau \sim d$. We found that the fraction of the vacuum energy ($E_{\rm vac} \simeq 4\pi\rho_{\rm vac}\tau^3/3$) released into gravitational waves is

$$\frac{E_{\rm GW}}{E_{\rm vac}} \simeq 1.3 \times 10^{-3} \left(\frac{\tau}{H_{\rm vac}^{-1}}\right)^2 . \qquad (1)$$

Here τ is the duration of the phase transition and $H_{\rm vac}^2 \equiv 8\pi G\rho_{\rm vac}/3$ is the value of the Hubble parameter associated with the false-vacuum energy. The spectrum of gravitational radiation scales, $dE_{\rm GW}/df \propto \rho_{\rm vac}^2\tau^6$, and peaks at a frequency $f \simeq 0.6/\tau$; see Fig. 1. In these calculations we made the following assumptions: (1) the separation of the bubbles is much larger than their initial size (valid in all cases of interest); (2) the duration of the phase transition is comparable to the separation of the bubbles ($0.5 \lesssim \tau/d \lesssim few$, which is what is expected, see below); (3) the gravitational effects on the evolution of the bubbles can be neglected (valid for $\tau \lesssim H_{\rm vac}^{-1}$), though the calculation was carried out to all orders in v/c; and (4) the expansion of the Universe can be neglected (valid for $\tau \lesssim H_{\rm vac}^{-1}$).

The fact that our results depend only upon the grossest features of the collision, and can thus be characterized by simple scaling expressions like Eq. (1), comes about because the production of gravitational radiation is insensitive to the internal structure of the colliding bubbles. By using the quadrupole radiation formula, $E_{\rm GW} \sim GQ_3^2\Delta t$ (where Q_3 is the triple time derivative of the quadrupole moment of the energy distribution and Δt is the duration of the collision), we can motivate our remarkable result [5]. Because the problem is relativistic and only depends upon the gross features of the collision there is but one time/distance scale: $d \sim \tau$. Thus, $Q_3 \sim (\rho_{\rm vac}\tau^3 \cdot \tau^2)/\tau^3$ and $E_{\rm GW} \sim G\rho_{\rm vac}^2\tau^5$. By comparing $E_{\rm GW}$ to the vacuum energy liberated by the two bubbles, $E_{\rm vac} \sim \rho_{\rm vac}\tau^3$, and using the fact that $H_{\rm vac}^2 \sim G\rho_{\rm vac}$, Eq. (1) follows immediately.

The simplicity of our results suggested that we might develop an approximation to handle the collision of many bubbles and more realistically simulate a phase transition. In Ref. [9] we showed that we could reproduce our scalar-field results by an "envelope approximation:" Each bubble is treated as a very thin shell expanding at the speed of light with its total energy increasing as t^3; when two bubbles collide, their overlap region is ignored, and they are described by the envelope of their uncollided portions. This approximation reproduces our results for the collision of two scalar-field bubbles accurately (e.g., energy spectrum to 5%); moreover, since we are not following the detailed evolution of a relativistic scalar field we arrive at these results in a fraction of the time.

To simulate a realistic phase transition we nucleated bubbles in a spherical volume according to the nucleation rate (per unit volume) $\Gamma \propto e^{\beta t}$ until the entire volume was in the true vacuum. We carried out five such simulations, 20 to 30 bubbles in each and 127 bubbles in all. The spectrum of gravitational radiation, averaged over all five simulations and six directions per simulation, is shown in Fig. 1. The spectrum is very similar to that for two bubbles, indicating that our original idea of computing the gravitational radiation from a phase transition as a sum of that from binary collisions was a reasonable one.

To match the multi-bubble spectrum with that from two bubbles we must take $\tau \simeq 3/\beta$. This is consistent with expectations: Below we show that with $\Gamma \propto e^{\beta t}$, the duration of the phase transition and typical bubble size are $\tau \sim d \sim few/\beta$. At low frequencies, $f \lesssim f_{\rm max} \simeq 0.2\beta$, the two spectra have the same slope; at high frequencies, the spectrum from our multi-bubble simulations falls off more slowly, presumably due to the high-frequency radiation produced by small bubbles. The fraction of vacuum energy liberated into gravitational waves in our multi-bubble simulations is about a factor of five larger,

$$\frac{E_{\rm GW}}{E_{\rm vac}} \simeq 6.0 \times 10^{-2} \left(\frac{H_{\rm vac}}{\beta}\right)^2; \tag{2}$$

where in making the comparison we have used $\tau = 3/\beta$.

Bubble nucleation. The bubble nucleation rate (per unit volume) is generally of the form $\Gamma = \mathcal{M}^4 \exp[-A(t)]$, where the tunneling action A depends upon time through its dependence on the temperature (or in first-order inflation through its dependence upon the evolution of another field [7, 11]). The prefactor is more difficult to compute and is less important [7]: All the "action" is the action; we assume that $\mathcal{M}$ is an energy scale characteristic of the phase transition, of the order of the fourth-root of the false-vacuum energy density (and phase-transition temperature). The completion of the phase transition occurs roughly when $\Gamma \sim H^4$, which corresponds to a nucleation rate of the order of one bubble per Hubble time per Hubble volume.

Given $A(t)$, it is easy to describe the kinematics of the phase transition in detail: duration, distribution of bubble sizes, etc. (see Ref. [11]). As is discussed in Ref. [11], it usually suffices to expand the action around $t = t_*$, the end of the phase transition,

$$A(t) = A_* - \beta(t - t_*) + \cdots; \tag{3a}$$

where $\beta \equiv -[dA/dt]|_{t=t_*} > 0$. This expansion is general enough to describe most first-order phase transitions, as the rate of change of the action determines essentially all quantities of interest. We can solve for A_* by equating $\Gamma(t_*)$ to $H_*^4 \sim \mathcal{M}^8/{m_{\rm Pl}}^4$:

$$A_* \simeq 4\ln(m_{\rm Pl}/\mathcal{M}); \tag{3b}$$

where $m_{\rm Pl} = G^{-1/2} = 1.22 \times 10^{19}\,{\rm GeV}$, and, for a strongly first-order phase transition, the value of the Hubble parameter at the end of the transition $H_* \simeq H_{\rm vac}$.

Bubble nucleation and percolation depend upon $\Gamma(t)$ and $I(t) \equiv \int_0^t \Gamma(t')V(t,t')dt'$, where $V(t,t') = 4\pi[R(t)\int_{t'}^t du/R(u)]^3/3$ is the physical volume at time t of a bubble nucleated at time t' and $p(t) = e^{-I(t)}$ is the probability that a point still remains in the false vacuum at time t (R is the cosmic-scale factor). For our model, $I(t) = I(t_*)\exp[\beta(t-t_*)]$. The end of the phase transition occurs when only a tiny fraction of space remains in the false vacuum, $p(t_*) = e^{-M}$ with $I(t_*) = M \gg 1$. Likewise, the beginning of the phase transition corresponds to $p(t) = e^{-m}$ with $I(t) = m \ll 1$. Using these definitions, the duration of the phase transition $\tau = \ln(M/m)/\beta \sim few/\beta$.

The distribution of bubble sizes r (per unit volume) at the end of the transition is

$$\left.\frac{dn}{dr}\right|_{t_*} = \left[R(t)^4\Gamma(t)e^{-I(t)}\right]\Big|_{t(r)}; \tag{3c}$$

where $t(r)$ is the time that a bubble of size r was nucleated, defined by $r = R(t)\int_{t(r)}^{t_*} du/R(u)$. Matters simplify if we assume that the transition lasts a Hubble time or less ($\beta \gg H$), so that expansion of the Universe can be neglected; for most cases of interest, this is a good approximation [11]. Then the distribution of bubble sizes is

$$\left.\frac{dn}{dr}\right|_{t_*} = \frac{\beta^4 I(t_*)}{8\pi}\exp\left[-\beta r - I(t_*)e^{-\beta r}\right]. \tag{3d}$$

For $I(t_*) \gg 1$, the mean bubble size $\bar{r} = (C + \ln I)/\beta \sim few/\beta$ ($C = 0.577$ is Euler's constant), and the total number of bubbles (per unit volume) $N = \int_0^\infty (dn/dr)dr = \beta^3/8\pi$, so that the typical separation between bubble centers $d \sim N^{-1/3} = (8\pi)^{1/3}/\beta \simeq few/\beta$.

Our model for bubble nucleation shows that the duration of the phase transition, and the typical bubble size and separation are all comparable and determined by $\beta \equiv d\ln\Gamma/dt$. Finally, let us relate β to A_*, and thereby to T_*. The tunneling action varies with time because of its temperature dependence; unless the parameters of the model are "tuned," the timescale for change in the action should be comparable to the timescale on which the temperature changes—the Hubble time—and thus one expects $\beta \sim A_*/H_*^{-1}$. Based on this, we anticipate that up to a factor of order unity $H_*/\beta \sim 1/A_* \sim \mathcal{O}[1/\ln(m_{\rm Pl}/T_*)]$. For the temperatures of interest, say 1 GeV to 10^{16} GeV, H_*/β is of order a few percent, so that the fraction of vacuum energy liberated into gravitational waves is expected to be 10^{-2} to 10^{-4}. (In first-order inflation [8], H_*/β should be closer to unity [11], so that the fraction of vacuum energy liberated in gravity waves may be as large as 10%.)

Relic gravity waves. We now use our numerical results for the production of gravitational waves from bubble collisions to estimate the stochastic background resulting from a "generic" strongly first-order phase transition. Because the energy released in the course of the phase transition is much greater than the thermal energy present before the transition, the ratio of the energy density in gravitational waves to that in radiation after the transition is just given by Eq. (2), $\rho_{\rm GW}/\rho_{\rm rad} \simeq 5.2\times 10^{-2}(H_*/\beta)^2$, where $H_* \simeq H_{\rm vac}$,

$H_*^2 = 8\pi G\rho_{\rm rad}/3$, $\rho_{\rm rad} = g_*\pi^2 T_*^4/30$, T_* is the temperature just have the phase transition, and g_* counts the total number of relativistic degrees of freedom at temperature T_*. Thereafter the energy density in gravitational radiation decreases as R^{-4}, and the frequency of the gravitational waves red shifts as R^{-1}. If we assume the expansion has been adiabatic since the phase transition [i.e., entropy per comoving volume $S \propto R^3 g(T)T^3 = \text{const}$], it follows that ratio of the value of scale factor at the end of the phase transition ($= R_*$) to its present value ($= R_0$) is: $R_*/R_0 \simeq 8.0 \times 10^{-14}(100/g_*)^{1/3}(\text{GeV}/T_*)$, and we find that

$$f_{\max} \simeq 3.0 \times 10^{-8}\,\text{Hz}\left(\frac{\beta}{H_*}\right)\left(\frac{g_*}{100}\right)^{1/6}\left(\frac{T_*}{\text{GeV}}\right); \tag{4a}$$

$$\Omega_{\rm GW} h^2 \simeq 1.0 \times 10^{-6}\left(\frac{H_*}{\beta}\right)^2\left(\frac{100}{g_*}\right)^{1/3}; \tag{4b}$$

where $\Omega_{\rm GW}$ is the fraction of critical density contributed by gravitational waves, h is the present value of the Hubble parameter in units of $100\,\text{km sec}^{-1}\ \text{Mpc}^{-1}$, and $f_{\max}$ is the frequency where the spectrum of gravitational radiation peaks today. Since waves of all frequencies are red shifted by the same factor, the spectrum today has the same shape as it did originally (see Fig. 1). For orientation, the contribution of the microwave background to the present density is $\Omega_\gamma h^2 \simeq 2.5 \times 10^{-5}$, and the dimensionless strain $\bar{h}$ produced by gravity waves of frequency f is related to their (spectral) energy density by

$$\bar{h}(f) \equiv \sqrt{4G\rho_{\rm GW}(f)}/\pi f \simeq 1.3 \times 10^{-18}\sqrt{\Omega_{\rm GW}(f)h^2}/(f/\text{Hz}); \tag{5}$$

where $\Omega_{\rm GW}(f) \propto k^5|h(k)|^2$ and $\bar{h}(f) \propto k^{3/2}|h(k)|$ refer to the contributions per octave to the energy density and strain, and $h(k = 2\pi f)$ is the Fourier transform of the transverse-traceless metric perturbation (see Ref. [1] for more details).

Previous estimates for the amount of gravitational radiation produced by the electroweak and QCD phase transitions were $\Omega_{\rm GW} \sim \mathcal{O}(10^{-13})$ [6]; with $\beta/H_* = \ln(m_{\rm Pl}/T_*) \sim 10^2$ our estimate is larger by a factor of about 1000—but of course depends upon the precise value of H_*/β and the transition being strongly first order (which seems very unlikely for the QCD transition). The comparison to the results of Refs. [6] is not really a fair one, as the authors there assumed that these phase transitions were only weakly first-order and proceeded through the nucleation of thermal bubbles.

Conclusions. Based upon numerical simulations of vacuum-bubble collisions we have estimated the spectrum of gravitational radiation that arises in a strongly first-order phase transition. Generically, we predict a present energy density, $\Omega_{\rm GW} h^2 \simeq 1.0 \times 10^{-6}(H_*/\beta)^2$, and characteristic frequency, $f_{\max} \simeq 3.0 \times 10^{-8}\,\text{Hz}(\beta/H_*)(T_*/\,\text{GeV})$, where $\beta \equiv d\ln\Gamma/dt$ parameterizes the rate of change of the bubble nucleation rate. The quantity H_*/β sets the typical bubble size and duration of the phase transition (in Hubble units), and is expected to be $\mathcal{O}[1/\ln(m_{\rm Pl}/T_*)] \sim few \times 10^{-2}$ for a typical phase transition, and perhaps as large as order unity for an inflationary phase transition. Current detectors—resonant bars—and the LIGO's planned for the future are sensitive to frequencies in the $10 - 10^4$ Hz range; thus, the most "promising" cosmological phase transition would be one that occurred at a temperature around 10^7 GeV (for which we have no compelling candidate). At these frequencies the first-generation LIGO detectors are expected to be able to detect stochastic gravity waves with energy density $\Omega_{\rm GW} h^2 \gtrsim 10^{-7}$, and second-generation detectors are expected to improve to $\Omega_{\rm GW} h^2 \gtrsim 10^{-11}$ [1], making the detection of gravity waves from such a phase transition within the realm of possibility.

Our numerical results for the generation of gravitational waves in a cosmological phase transition and our estimates for their contribution to the stochastic background today strictly only apply to a strongly first-order phase transition, i.e., one that proceeds through true-vacuum bubbles and produces significant entropy [13]. Whether or not our results are applicable to, or can be easily modified to treat, a first-order phase transition that proceeds through the nucleation and percolation of thermal bubbles remains to be seen.

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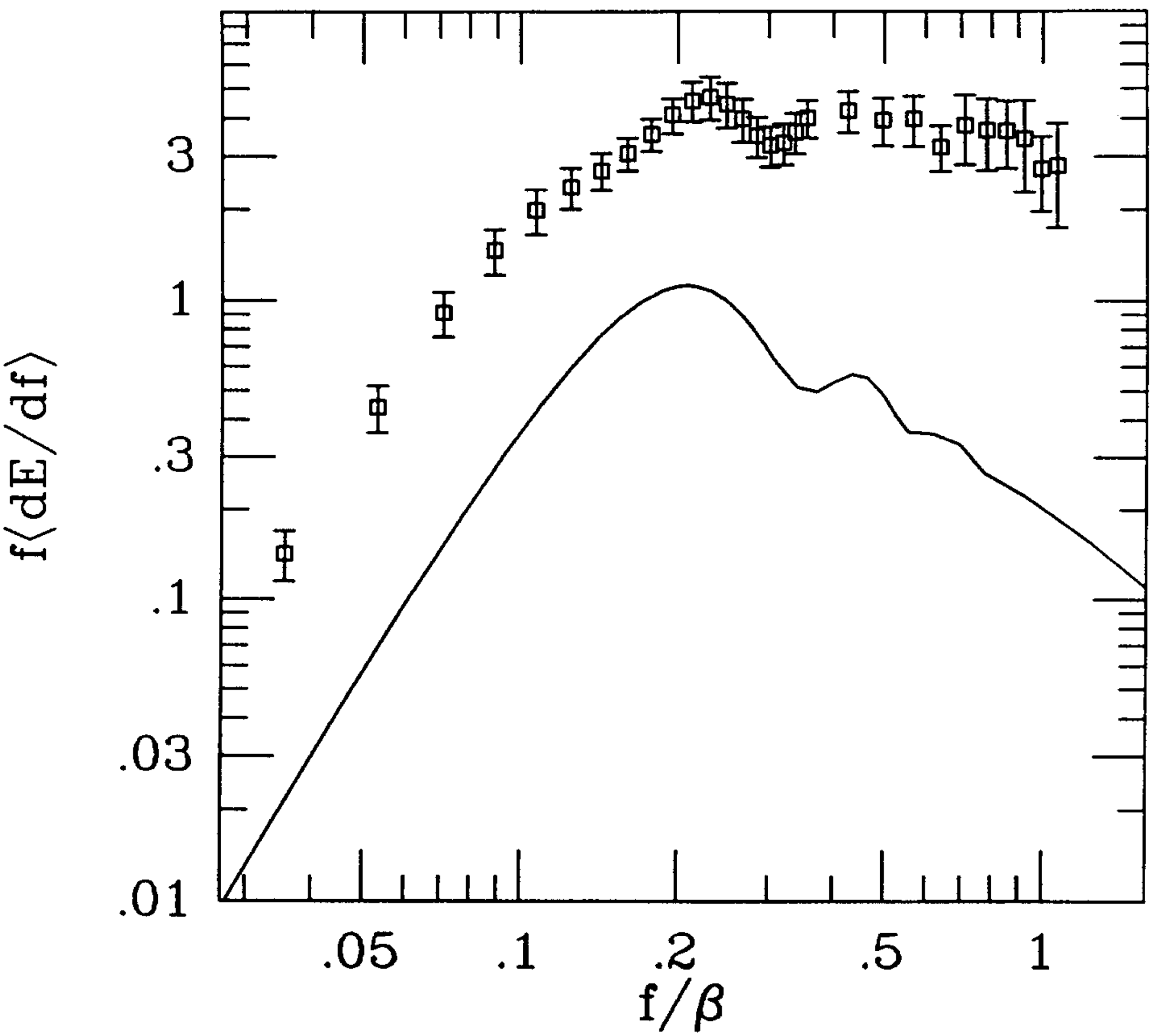
[13] All of our results only strictly apply to vacuum bubbles—bubbles nucleated at zero temperature—as opposed to thermal bubbles—bubbles nucleated at finite temperature. The crucial difference is that for a vacuum bubble all the vacuum energy released goes into the kinetic energy of the bubble wall, whereas a thermal bubble interacts with the ambient plasma and much of the energy released goes into the plasma. Precisely how much supercooling is required so that bubbles nucleated in a first-order phase transition behave like vacuum bubbles is not well quantified. We use the term "strongly first order" to mean that the bubbles associated with the phase transition can be treated as vacuum bubbles.

Figure Caption

Figure 1: The spectrum of gravitational radiation (energy per octave) produced by the collision of two vacuum bubbles (solid curve; from [9]). At low frequencies the energy per octave increases as $(f/\beta)^{2.8}$ and at high frequencies it decreases as $(f/\beta)^{-1.8}$. The points with error flags show the spectrum of gravitational radiation produced in simulations of a realistic phase transition with bubble nucleation rate $\Gamma \propto e^{\beta t}$. This spectrum was averaged over five simulations and six directions per simulation [10]. Error flags indicate the deviation of the mean: $\sigma = 4\pi f\, [\sum_{i=1}^{N}(dE/df d\Omega - \langle dE/df d\Omega\rangle)^2]^{1/2}/N$. Although the units on the energy axis are arbitrary, they are the same for both the two-bubble and multi-bubble simulations; for the two-bubble simulation $\tau = 3/\beta$ (see text).



f⟨dE/df⟩
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