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Non scale-invariant density perturbations from chaotic extended inflation

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Chaotic inflation is analysed in the frame of scalar-tensor theories of gravity. Fluctuations in the energy density arise from quantum fluctuations of the Brans-Dicke field and of the inflaton field. The spectrum of perturbations is studied for a class of models: it is non scale-invariant and, for certain values of the parameters, it has a peak. If the peak appears at astrophysically interesting scales it may help to reconcile the Cold Dark Matter scenario for structure formation with large-scale observations.

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I. INTRODUCTION

One of the most successful models for the origin of the primordial energy density fluctuations which gave rise to the observed structures in the universe is given by inflation. Quantum fluctuations of scalar fields present during the inflationary era give rise to classical fluctuations in the energy density. In the simplest model of one inflaton field driving the universe expansion, as it slowly rolls down a smooth potential, the resulting fluctuations are adiabatic, with scale invariant spectrum and Gaussian distributed. These are also the kind of primordial fluctuations usually considered in one of the most accepted scenarios for galaxy formation, the standard Cold Dark Matter (hereafter CDM) one. This scenario successfully explains many of the observed properties of galaxy clustering. However, it has shown to have difficulties in explaining some large-scale observations such as the galaxy peculiar motions,[1] the clustering of clusters,[2] the angular correlation function of galaxies[3] and some peculiar features in the very large-scale galaxy distribution.[4]

Different models in which the inflationary paradigm can be realized have been proposed. These are generally identified as old inflation, new inflation, chaotic inflation and extended inflation. In the old inflation picture the universe undergoes a first order phase transition during which the regions in the false vacuum phase expand exponentially, it has the problem that, due to the speed of the false vacuum expansion, the true vacuum bubbles fail to percolate. In the new and chaotic inflationary scenarios this problem is solved, as inflation occurs during the slow rolling of the inflaton field to the minimum of its potential. The extended inflation model has recently been proposed as an alternative solution to the problems of old inflation.[5] A first order phase transition for the inflaton field is considered in the frame of the Brans-Dicke theory of gravity. This makes the universe expand as a power law (instead than exponentially) during the phase transition, so that the percolation of false vacuum bubbles can occur. However, this model has the problem that it is not possible to simultaneously satisfy the bounds coming from the observed smoothness of the Cosmic Microwave Background Radiation (hereafter CMBR) and from time-delay experiments. Modified versions of this model have been proposed to overcome this difficulty.[6]

Later on it has been pointed out by Linde[7] that the Brans-Dicke gravity theory also has interesting consequences for chaotic inflationary scenarios due to the fact that the effective gravitational constant can take different values in different places of the universe. One of the modifications of the original extended inflation model proposed[8] considers more general scalar-tensor theories of gravity in which the Einstein gravity theory is an attractor for the solutions, but that can considerably differ from it in the past. The main purpose of this paper is to explore the consequences of chaotic type inflation taking place in these gravitational theories. We show that the fact that curvature fluctuations are determined by both the fluctuations in the inflaton and in the Brans-Dicke field gives rise to perturbations with non scale-invariant spectrum. In particular, it is possible that the spectrum has a peak at a given wavelength. This kind of spectrum can help to reconcile CDM predictions with large-scale observations without running into conflict with the CMBR anisotropy limits, provided that the peak in the spectrum appears at an appropriate wavelength. Other models in which this kind of spectrum can arise have been discussed in the literature.[9] For example, a mountain on the top of an underlying scale invariant spectrum arises when there is a hill on the scalar field potential.[10] Another possibility is to consider two interacting scalar fields. In this case the interaction between the fields can give rise to variations in the effective masses of the fields during their evolution and can make m^2_{eff} change sign, thus originating a peak in the resulting density perturbations. (Other kinds of non scale-invariant fluctuations in inflationary models have been discussed in Refs. 11.) The mechanism considered here is quite different. The peak in the spectrum originates when the effect of the inflaton fluctuations becomes dominant over the Brans-Dicke field fluctuations in determining the total curvature fluctuations. A problem of these models is that, in order to make the features in the spectrum appear at scales of astrophysical interest, it is necessary to impose rather precise conditions on the inflationary model, either on the initial conditions of the fields or on the potential parameters. This problem is also present in the model discussed here, as the initial value of the Brans-Dicke field determines the scale at which the peak in the spectrum appears. Another model where non scale-invariant perturbation are generated has been recently proposed,[12] in which a period of extended inflation is followed by a period of slowroll inflation.

The organization of the paper is as follows. In section 2 the chaotic extended inflation model is introduced; the conformal transformation to the Einstein frame, the equations of motion in that frame and the inflationary solutions are discussed. In section 3 the generation of curvature perturbations in this model is studied and the possibility that their spectrum is non scale-invariant at observable scales is discussed. The resulting spectrum of perturbations for some particular values of the model parameters is presented. In section 4 the evolution of the inflaton and of the Brans-Dicke field is analysed in the frame of the stochastic approach to inflation. In section 5 we discuss the results.

II. CHAOTIC EXTENDED INFLATION MODEL

A. The model

We will explore the realization of chaotic inflation in the frame of a class of scalar-tensor theories of gravity which are described by the action

$$S = \int d^4x\sqrt{-g}\left(-\phi R + \frac{\omega(\phi)}{\phi} g^{ab}\partial_a\phi\partial_b\phi + \frac{1}{2}g^{ab}\partial_a\sigma\partial_b\sigma - V(\sigma)\right), \qquad (1)$$

where ϕ is the Brans-Dicke field and σ the inflaton field. The scalar field ϕ is normalized to be $(2\mathcal{K}^2)^{-1}$ at present, with $\mathcal{K}^2 \equiv 8\pi G_N$ (G_N being the Newton constant). In the standard Brans-Dicke theory $\omega(\phi)$ is a constant, and it is constrained by present observations to be large, $\omega > 500$. A more general scalar-tensor theory is the Bergmann[13]-Wagoner[14]-Nordtvedt[15] one, where $\omega(\phi)$ is an arbitrary function. Special cases are Bekenstein's[16] "variable mass theory" and Barker's[17] "constant G_N theory". In these theories the function $\omega(\phi)$ can have the property that, for the present value of the scalar field ϕ, the theory is indistinguishable from General Relativity, but, for past values of ϕ, it could lead to significant differences in cosmological models. Particularly interesting is the case in which the system dynamically evolves to an attractor solution indistinguishable from General Relativity. This kind of theories has recently been considered in order to overcome the problems associated with the original extended inflationary model proposed by La and Steinhardt.[5] With this scope a particular class of models, based on the function $\omega(\phi)$ defined by

$$\omega(\phi) + \frac{3}{2} = \frac{\beta}{(1 - 2\mathcal{K}^2\phi)^{\mu}}, \tag{2}$$

with β and μ constants, has been analysed by García-Bellido and Quirós.[8] We will analyse the consequences of chaotic inflation in the frame of this gravity theory.

The action in eq. (1) is written in the Jordan frame, where the gravitational coupling depends on ϕ. However, it shows to be more convenient to work in the conformally transformed Einstein frame in which the gravitational part of the action takes the usual form. The conformal transformation connecting the two frames is given by

$$g_{ab} = \Omega^{-2}\tilde{g}_{ab}, \qquad \text{with} \quad \Omega = \sqrt{2\mathcal{K}^2\phi}. \tag{3}$$

It is also useful to redefine the scalar field as[18]

$$\varphi \equiv \int \frac{d\phi}{\phi}\sqrt{\frac{\omega(\phi)+\frac{3}{2}}{2\mathcal{K}^2}}. \tag{4}$$

In this frame the action (1) transforms to

$$S = \int d^4x\sqrt{-\tilde{g}}\left(-\frac{R(\tilde{g})}{2\mathcal{K}^2} + \frac{1}{2}\tilde{g}^{ab}\partial_a\varphi\partial_b\varphi + \frac{1}{2}\Omega^{-2}(\varphi)\tilde{g}^{ab}\partial_a\sigma\partial_b\sigma - \Omega^{-4}(\varphi)V(\sigma)\right). \tag{5}$$

We will call $F(\varphi) \equiv \Omega^{-2}(\varphi) = (2\mathcal{K}^2\phi)^{-1}$.

B. Evolution of the fields

The equations of motion for the Brans-Dicke field φ and the inflaton σ in the Einstein frame are given by

$$\ddot{\varphi} + 3H\dot{\varphi} = -2F(\varphi)\partial_\varphi F\ V(\sigma) + \partial_\varphi F\frac{\dot{\sigma}^2}{2}, \tag{6}$$

$$\ddot{\sigma} + 3H\dot{\sigma} + \frac{\dot{F}}{F}\dot{\sigma} = -F(\varphi)\partial_\sigma V, \tag{7}$$

having restricted our analysis to a homogeneous and isotropic universe. Here a dot denotes differentiation with respect to t and $H = \dot{a}/a$. The Friedmann equation reads

$$H^2 = \frac{\mathcal{K}^2}{3}\left(\frac{\dot{\varphi}^2}{2} + F(\varphi)\frac{\dot{\sigma}^2}{2} + F^2(\varphi)V(\sigma)\right). \tag{8}$$

To deal with this coupled system of equations it is convenient to change the time variable from t to $\alpha = \ln(a/a_0)$, where a_0 can be taken as the value of the scale factor a at the beginning of inflation. The system transforms to

$$H^2\varphi'' + \mathcal{K}^2F^2V\varphi' = -2F\partial_\varphi F\ V + H^2\partial_\varphi F\frac{\sigma'^2}{2}, \tag{9}$$

$$H^2\sigma'' + \mathcal{K}^2F^2V\sigma' + H^2\frac{\partial_\varphi F}{F}\varphi'\sigma' = -F(\varphi)\partial_\sigma V, \tag{10}$$

and

$$H^2 = \frac{\mathcal{K}^2F^2(\varphi)V(\sigma)}{3 - \frac{\mathcal{K}^2}{2}\left(\varphi'^2 + F(\varphi)\sigma'^2\right)}, \tag{11}$$

where primes denote derivatives with respect to α.

In what follows we will consider the particular function $\omega(\phi)$ quoted in equation (2). Two particular constant μ values make it possible to handle the problem analytically: these are $\mu = 1$ and $\mu = 2$. The corresponding $F(\varphi)$ is given by

$$F_{\mu=1}(\varphi) = \cosh^2(\mathcal{K}\varphi\sqrt{2/\beta}), \tag{12}$$

$$F_{\mu=2}(\varphi) = 1 + e^{-\mathcal{K}\varphi\sqrt{2/\beta}}. \tag{13}$$

In the first case the field φ rolls down its potential and asymptotically approaches $\varphi = 0$ at late times. In the second case the field rolls down asymptotically approaching infinity. All the results obtained are qualitatively similar in the two models, so for definiteness we will develop the computations for the $\mu = 2$ case.

We will consider for the inflaton a potential $V(\sigma) = M^4 \exp(-\gamma\mathcal{K}\sigma)$, with γ a positive constant, which gives interesting possibilities for the spectrum of the resulting energy density perturbations.[19]

It can be seen that the condition to have an inflationary expansion ($\ddot{a} > 0$) corresponds to

$$2 > \mathcal{K}^2(\varphi'^2 + F(\varphi)\sigma'^2). \tag{14}$$

In this regime we can neglect the second term in the right hand side of eq. (9) and the second derivative term, so that the equation decouples from the one for σ and can be integrated, leading to

$$\mathcal{K}\sqrt{2/\beta}(\varphi - \varphi_f) + \exp(\varphi\mathcal{K}\sqrt{2/\beta}) - \exp(\varphi_f\mathcal{K}\sqrt{2/\beta}) = \frac{4}{\beta}(\alpha - \alpha_f). \tag{15}$$

It can be seen that neglecting the second derivative term is a good approximation for $4/3 \ll \beta(1 + \exp(\mathcal{K}\varphi\sqrt{2/\beta}))^2$ and that neglecting the inflaton kinetic term is a good approximation for $\gamma^2 \ll 4(1 + e^{-\mathcal{K}\varphi\sqrt{2/\beta}})$. Note that for large negative values of φ eq. (15) reduces to the solution corresponding to a scalar field rolling down an exponential potential, i.e. $\varphi = \varphi_f + 2\sqrt{2/\beta\mathcal{K}^2}(\alpha - \alpha_f)$.

At late times $\varphi \gg 1$ and $F(\varphi)$ approaches unity. The effective Newton constant tends to its present value as $\phi \to (2\mathcal{K}^2)^{-1}$ and from eq. (2) we see that ω grows up making the theory indistinguishable from General Relativity.

The evolution equation for σ can be solved in the slow-rolling approximation neglecting the second derivative term. It is convenient to solve it in terms of φ instead of α. Combining eqs. (10) and (9), in the approximation discussed above, we get

$$\frac{d\varphi}{d\sigma} = -\frac{\partial_\varphi F}{\gamma\mathcal{K}}, \tag{16}$$

which can be directly integrated. The solution is given by

$$\sigma = \sigma_f + \frac{\gamma\beta}{4}\left(e^{\mathcal{K}\varphi\sqrt{2/\beta}} - e^{\mathcal{K}\varphi_f\sqrt{2/\beta}}\right). \tag{17}$$

The approximations performed hold for $8/3 \ll \beta(1+\exp(\mathcal{K}\varphi\sqrt{2/\beta}))^2$ and $\gamma^2 \ll 6(1+\exp(-\mathcal{K}\varphi\sqrt{2/\beta}))$.

As we will see in the following, the relevant quantities for the study of density perturbations are φ'^2 and $F\sigma'^2$, which in this case are given by

$$\varphi'^2 = \frac{8}{\mathcal{K}^2\beta}\frac{1}{(1+e^{\mathcal{K}\varphi\sqrt{2/\beta}})^2}, \tag{18}$$

$$F\sigma'^2 = \frac{\gamma^2}{\mathcal{K}^2(1+e^{-\mathcal{K}\varphi\sqrt{2/\beta}})}. \tag{19}$$

From eqs. (18) and (19) we see that φ'^2 is a decreasing function of φ (and hence of time), while $F\sigma'^2$ is an increasing one. Thus, for early times φ'^2 will be the dominant one, while $F\sigma'^2$ will dominate at late times.

The exponential potential for the inflaton, as opposed to other chaotic inflation potentials, does not lead to a phase where the inflaton field oscillates starting the reheating process. The end of inflation in this case must be determined by some external reheating mechanism. Without loss of generality we can consider that inflation ends at $\sigma_f = 0$, in which case M^4 corresponds to the potential energy at the end of inflation. On the other hand, we know that at the end of inflation the value of the Brans-Dicke field ϕ in our patch of the universe must be such that the gravity theory is now very close to General Relativity. As the strongest variation of the φ field occurs during the inflationary era, we will impose this constraint at the end of inflation, $F(\varphi_f) \approx 1$.

To conclude this section let us briefly discuss the relation between quantities in the Einstein frame and in the Jordan (physical) frame. If we write the line element in the Einstein frame as

$$ds^2 = dt^2 - a^2(t)d\ell^2$$

and the one in the Jordan frame as

$$ds^2 = d\tau^2 - b^2(\tau)d\ell^2,$$

they are related by

$$a(t) = F^{-1/2}b(\tau), \tag{20}$$

$$dt = F^{-1/2}d\tau. \tag{21}$$

At late times, when F approaches unity, the two frames practically coincide.

From eq. (20), we see that the number of e-foldings of inflation in the two frames is related by

$$\ln(a/a_0) = \frac{1}{2}\ln(F_0/F_f) + \ln(b_f/b_o), \tag{22}$$

where quantities at the beginning of inflation have been denoted by a subscript 0 and quantities at the end of inflation by an f.

Furthermore, the condition for inflation in the Einstein frame ($d^2a/dt^2 > 0$) is fulfilled when condition (14) holds. It can be seen that under this condition also the Jordan frame inflates ($d^2b/d\tau^2 > 0$).

C. Constraints from Post-Newtonian experiments

There are different constraints that an alternative theory to General Relativity must fulfill in order to agree with post-Newtonian experiments.[20] They constrain essentially the present value of ω and the present rate of change of the Newton constant.

As the theory that we are considering evolves approaching General Relativity, if we impose these constraints at the end of the inflationary era, they will be also fulfilled at the present time. In the last subsection we have seen that the theory approaches General Relativity during inflation. It is easy to see that this is also the case during the radiation and matter dominated eras. The equation of motion for φ during the radiation dominated period can be written in the Einstein frame as

$$\frac{1}{a^3}(a^3\dot{\varphi})^{\cdot} = -\frac{\partial_\varphi F}{2F}(\rho - 3p) = 0, \tag{23}$$

where we have denoted d/dt by a dot. Changing the time variable to α, this equation can be solved as

$$\varphi = \varphi_f + \varphi'_f(1 - e^{-(\alpha-\alpha_f)}), \tag{24}$$

where φ_f and φ'_f denote the Brans-Dicke field and its derivative at the end of inflation. As φ'_f is positive, we see that φ continues to increase during the radiation dominated era and consequently $F(\varphi) \to 1$, $\phi \to (2\mathcal{K}^2)^{-1}$ and $\omega \to \infty$. It can be seen that φ also grows during the matter dominated era. The equation of motion (23) in this case can be written as

$$\varphi'' + \frac{3}{2}\varphi' = \frac{3\sqrt{2/\beta}}{2\mathcal{K}}\frac{1}{1 + e^{\mathcal{K}\varphi\sqrt{2/\beta}}}, \tag{25}$$

which is difficult to integrate; from it, however, it is easy to see that φ' is always positive as

$$\varphi' e^{3\alpha/2} = \varphi'_r e^{3\alpha_r/2} + \frac{3\sqrt{2/\beta}}{2\mathcal{K}} \int_{\alpha_r}^{\alpha} d\alpha' \frac{e^{3\alpha'/2}}{1 + e^{\mathcal{K}\varphi(\alpha')\sqrt{2/\beta}}}, \tag{26}$$

where the subscript r denotes the values at the end of the radiation dominated era. The last term is always positive, thus $\varphi' > \varphi'_r e^{3(\alpha_r - \alpha)/2} > 0$. (For a demonstration of the convergence to General Relativity starting from the Jordan frame, see Ref. 8.)

The first constraint to be considered is the one coming from the present value of ω determined from radar time-delay measurements, $\omega > 500$.[21] The value of ω at the end of inflation is determined through the relation

$$\sqrt{\frac{\omega + 3/2}{\beta}} = 1 + e^{\mathcal{K}\varphi\sqrt{2/\beta}}.$$

Thus, the constraint on ω determines a lower limit on the final value of the Brans-Dicke field at the end of inflation, corresponding to

$$\mathcal{K}\varphi_f\sqrt{2/\beta} > \ln\left(\sqrt{\frac{500}{\beta}} - 1\right). \tag{27}$$

Another constraint comes from the present rate of change of the Newton constant. The observable gravitational constant is given in terms of ϕ by[20]

$$G = \frac{1}{\phi}\left(1 + \frac{1}{2\omega + 3}\right). \tag{28}$$

Its rate of change per expansion time in the Jordan (observable) frame is given by

$$\frac{dG}{d\tau}\frac{1}{G\,H} = \frac{\varphi'\,\partial_\varphi F}{F^{3/2}}\left(\frac{2\beta + 1 - 1/F^2}{2\beta + (1 - 1/F)^2}\right) \equiv \eta. \tag{29}$$

Experimental limits coming from the Viking lander ranging data[22] constrain it to $\eta = 0.04 \pm 0.08$ (assuming $H_0 = (2 \times 10^{10}\text{yr})^{-1}$). From the analysis of the φ evolution, we see that η evolves towards zero in all the periods of interest. Thus, if at the end of inflation η was inside that interval, it will be there even more now. In order to fulfill this condition at the end of inflation it is necessary that

$$\mathcal{K}\varphi_f\sqrt{2/\beta} > \ln\left(\frac{10}{\sqrt{\beta}} - 1\right), \tag{30}$$

which always holds if condition (27) is satisfied. Big Bang nucleosynthesis imposes a stronger limit on the variability of G,[23] $|\eta| < \pm 0.018$. This bound amounts to changing the factor 10 by a factor 15 in eq. (30); this condition is also fulfilled whenever condition (27) is.

Finally, let us consider the constraint coming from the lower limit on $H_0\tau_0$, where H_0 is the present Hubble constant and τ_0 the cosmic time, $H_0\tau_0 > 0.4$. Making use of the Einstein equations

$$\begin{aligned} H^2 &= \frac{\mathcal{K}^2}{3}\left(\rho + \frac{\dot{\varphi}^2}{2}\right), \\ \frac{\ddot{a}}{a} &= -\frac{\mathcal{K}^2}{3}\left(\frac{\rho}{2} + \dot{\varphi}^2\right), \end{aligned} \tag{31}$$

it can be seen that, for values of φ at the end of inflation like those allowed by eq. (27), this constraint is easily fulfilled.

In the next section the generation of density perturbations is studied working in the Einstein frame, in which the treatment is closely related to the usual chaotic inflation case.[24] The fact that the two frames coincide at late times, when all the wavelengths of interest enter the Hubble radius, implies that our results apply without changes to the observable perturbations in the Jordan frame. An alternative method to study density perturbations has been developed, in which the Jordan frame is used.[25]

III. GENERATION OF PERTURBATIONS

A. Curvature perturbations

In this model the energy density perturbations arise as a consequence of fluctuations in the inflaton field and in the Brans-Dicke field.

We will work in the longitudinal gauge where the metric takes the form

$$ds^2 = (1 - 2\Phi)dt^2 + a^2(t)(1 + 2\Phi)d\ell^2, \tag{32}$$

where Φ coincides with the gauge invariant potential Φ_H defined by Bardeen[26] and corresponds to the usual gravitational potential. It can be expanded in a superposition of modes of wavenumber $\vec{k}$, $\Phi_{\vec{k}}$ (we will not include the subindex $\vec{k}$ in what follows).

Curvature perturbations are described by the amplitude of the variable ζ,[27] given by

$$\zeta = \frac{2}{3}\frac{1}{1+w}(\Phi + H^{-1}\dot{\Phi}) + \Phi\left(1 + \frac{2}{9}\frac{1}{(1+w)}\frac{k^2}{a^2H^2}\right), \tag{33}$$

which is constant during the evolution for wavelengths much larger than the Hubble radius; w is the ratio between the pressure and the energy density. The power spectrum of ζ (variance per $\ln k$) is defined as

$$P_\zeta(k) = \frac{k^3}{2\pi^2} < |\zeta_k|^2 > . \tag{34}$$

For scales that are just entering the Hubble radius during the matter dominated era the amplitude of P_ζ and P_Φ are related by (see, e.g., Refs. 27,10)

$$P_\zeta^{1/2}\Big|_{HC} = \frac{5}{3}\, P_\Phi^{1/2}\Big|_{HC} = \frac{5}{2}\,\frac{\delta\rho}{\rho}\Big|_{HC} = \frac{15}{2}\,\frac{\delta T}{T}\Big|_{HC}, \tag{35}$$

where the subscript HC refers to the Hubble radius crossing time. The amplitude of ζ modes during inflation in terms of the perturbations of φ and σ can be computed as follows. From the $0-j$ component of the Einstein equations we have

$$\dot{\Phi} + H\Phi = -\frac{\mathcal{K}^2}{2}\left(\dot{\varphi}\delta\varphi + F(\varphi)\dot{\sigma}\delta\sigma\right). \tag{36}$$

Thus, a good estimate of ζ for large wavelengths during the inflationary period is given by

$$\zeta \simeq \frac{2}{3}\frac{1}{1+w}(\Phi + H^{-1}\dot{\Phi}) = -\frac{H(\dot{\varphi}\delta\varphi + F(\varphi)\dot{\sigma}\delta\sigma)}{\dot{\varphi}^2 + F(\varphi)\dot{\sigma}^2}, \tag{37}$$

or in terms of the variable α

$$\zeta = -\frac{(\varphi'\delta\varphi + F(\varphi)\sigma'\delta\sigma)}{\varphi'^2 + F(\varphi)\sigma'^2}. \tag{38}$$

This treatment is a good approximation in our case, as both the fields and their derivatives are smoothly varying during the entire period of inflation. A more careful treatment, integrating the evolution equation for the fields and metric perturbations, is instead necessary in other cases.

B. Power spectrum

The amplitude of the field fluctuations $\delta\varphi$ and $\delta\sigma$ is given by the quantum fluctuations of the short wavelength modes of these fields. The power spectrum per $\ln k$ of scalar field fluctuations can be computed from the two-point function of the field. For a power-law inflation it is given by[28]

$$P_\varphi(k) = \frac{k^3}{(2\pi)^3}\frac{p}{(p-1)}\frac{|\Gamma(\nu)|^2}{Ha^3}\left|\frac{k}{aH}\frac{p/2}{p-1}\right|^{-2\nu}, \tag{39}$$

where p is the power of the expansion ($a \propto t^p$ and $p = 2/\gamma^2$ for the exponential potential $V \propto \exp(-\gamma\mathcal{K}\varphi)$ case) and $\nu \equiv (3p-1)/(2p-2)$. For $p \to \infty$ it reduces to the well-known value $H^2/(2\pi)^2$, corresponding to a de Sitter space. At Hubble radius crossing time the power spectrum is given by

$$P_{\varphi_H}(k) = \frac{2|\Gamma(\nu)|^2}{(2\pi)^3}\left(\frac{p/2}{(p-1)}\right)^{1-2\nu} H^2(\varphi)\Bigg|_{HC}, \tag{40}$$

where the Hubble constant is evaluated at the value that φ gets when the wavenumber k crosses the Hubble radius. This amplitude applies only to minimally coupled scalar fields. Thus, as it has been argued in Ref. 24, it can be applied to the Brans-Dicke field fluctuations $\delta\varphi$ in the Einstein frame but not in the Jordan one. For the fluctuations of the inflaton field $\delta\sigma$ we cannot take that amplitude, due to the fact that its kinetic term does not have the canonical form, because of the factor $F(\varphi)$ in its Lagrangian. However, for the inflaton field it is easier to work in the Jordan frame, where it is an ordinary minimally coupled field. Thus, in this frame we can estimate the power spectrum of $\delta_J\sigma$ by eq. (39). Transforming to the Einstein frame we get from eq. (40) that, at Hubble radius crossing,

$$P_{\sigma_H}(k) = \frac{2|\Gamma(\nu)|^2}{(2\pi)^3} \left(\frac{p/2}{(p-1)}\right)^{1-2\nu} \frac{H^2}{F}\bigg|_{HC} \tag{41}$$

(see also Ref. 29).[30] In the model that we are studying the universe expansion is not given by a single power law for the whole evolution but it can be well approximated by different power laws for different periods with slightly different values of p. The value of p can be determined from the Friedmann equations $1/p = 3(1+w)/2 = \mathcal{K}^2(\varphi'^2 + F\sigma'^2)/2$.

From eqs. (38) and (41) it can be seen that the curvature perturbations will be dominated by the inflaton fluctuations in the case that $\varphi' < \sqrt{F}\sigma'$ and by Brans-Dicke field fluctuations in the opposite case. In the last section we have seen that the kinetic contribution of the Brans-Dicke field is the largest one at early times while the inflaton dominates at later times. Hence, curvature perturbation at larger scales are determined by Brans-Dicke field fluctuations $\zeta_\varphi \approx -\delta\varphi/\varphi'$, while shorter scale ones are determined by inflaton field fluctuations $\zeta_\sigma \approx -\delta\sigma/\sigma'$.

1. Qualitative analysis

The qualitative shape of the spectrum at Hubble radius crossing can be determined as follows. For the larger wavelengths

$$P_{\zeta}^{1/2}(k) \approx \frac{P_{\varphi}^{1/2}}{\varphi'} \approx C(p)\frac{\mathcal{K}^2 M^2}{4\pi}\sqrt{\frac{\beta}{6}}(1+e^{-\mathcal{K}\varphi\sqrt{2/\beta}})$$
$$(1+e^{\mathcal{K}\varphi\sqrt{2/\beta}})\exp\left(\frac{\gamma^2\beta}{8}(e^{\mathcal{K}\varphi_f\sqrt{2/\beta}}-e^{\mathcal{K}\varphi\sqrt{2/\beta}})\right), \tag{42}$$

where $C(p) \equiv (2-2/p)^{2\nu-1}|\Gamma(\nu)|^2/\pi$ is a factor that goes to one for $p \gg 1$ and is smaller than one for values of p closer to one. The spectrum (42) is a decreasing function of φ, it has a minimum near φ equal to zero, then it increases again and it eventually becomes decreasing for large values of φ. (For some values of γ and β another minimum in P_ζ can appear at a negative value of φ, however this is not the case in the examples that we will consider). The Hubble radius crossing condition for increasing wavenumbers k corresponds to decreasing values of φ. Thus, the spectrum of perturbations first decreases with k, it has a minimum and then increases again. However, when the kinetic term of the inflaton becomes dominant the curvature perturbations are no longer determined by eq. (42), but by

$$P_{\zeta}^{1/2}(k) \approx \frac{P_{\sigma}^{1/2}}{\sqrt{F}\sigma'} \approx C(p)\frac{\mathcal{K}^2 M^2}{2\pi\sqrt{3}\gamma}\left(1+e^{-\mathcal{K}\varphi\sqrt{2/\beta}}\right)^{3/2}$$
$$\exp\left(\frac{\gamma^2\beta}{8}(e^{\mathcal{K}\varphi_f\sqrt{2/\beta}}-e^{\mathcal{K}\varphi\sqrt{2/\beta}})\right), \tag{43}$$

which is a decreasing function of φ. Thus, for the shortest wavelengths leaving the Hubble radius during the last part of inflation, the spectrum of perturbations again decreases with k. Combining the two behaviours we see that a peak in the spectrum appears at the wavelengths that are leaving the Hubble radius when the inflaton kinetic term becomes dominant. The peak will appear provided that the inflaton kinetic energy becomes dominant for a value of φ larger than that corresponding to the minimum of P_{ζ_φ} (eq. (42)). In order to make this happen there is a condition to be satisfied by the parameters appearing in $F(\varphi)$ and $V(\sigma)$, namely $\beta\gamma^2 \lesssim 4/3$. On the other hand, the condition that the peak in the spectrum appears at astrophysically interesting scales requires a particular initial condition for the Brans-Dicke field, or equivalently, a particular final value of φ at the end of inflation.

For large wavenumber perturbations, which cross the Hubble radius when the inflaton kinetic term dominates, we can explicitly compute the spectrum of ζ perturbations at Hubble radius crossing as a function of k from eq. (43) and the crossing condition $k = aH$. It is given by

$$P_{\zeta}^{1/2}(k) = \frac{C(p)\mathcal{K}^2 M^2}{2\pi\sqrt{3}\gamma} \left(\frac{k\sqrt{3}}{\mathcal{K}M^2 a_f}\right)^{\frac{\gamma^2}{\gamma^2-2}}, \tag{44}$$

with a_f the value of the scale factor at the end of inflation.

The spectral index of ζ is related to the spectral index of the energy density perturbations n by $P_{\zeta}^{1/2} \propto k^{(n-1)/2}$ (for $n = 1$ the spectrum is scale invariant and $P_{\zeta}^{1/2}$ is independent of k). Thus, we see that the spectral index corresponding to the long wavenumber region is approximately given by $n \simeq (1 - 3\gamma^2/2)/(1 - \gamma^2/2)$, which is the well-known result for power-law inflation with an exponential potential.[19]

It has been shown in Ref. 31 that a power spectrum with a primordial spectral index n smaller than one, like the one we obtain for the large wavenumber perturbations, can help to avoid some difficulties of the CDM scenario, as it has more power on large scales than the usual scale invariant spectrum. However, lowering n increases the amplitude of fluctuations at large scales, running into possible conflict with the CMBR anisotropy limits. This problem may be avoided in the model discussed here if the peak in the power spectrum occurs at a scale somewhat larger than 100 Mpc so that the amplitude of perturbations decreases for the larger scales relevant for the quadrupole anisotropy. The amplitude of the perturbations is determined by the value of M. It must be fixed with the help of eq. (35) in such a way that the limits on the CMBR anisotropy are satisfied.

2. Examples

The shape of the spectrum for the large wavelengths does not have a simple analytical expression. Fig. 1 shows the power spectrum of ζ at horizon crossing for the case $\beta = 4$ and $\gamma = 0.38$. As these values of the parameters are very close to the limits under which the analytical solutions of the equations of motion for φ and σ presented in section II-B are no more valid, the exact numerical solution of the system of eqs. (9) and (10) has been used. The value of the Brans-Dicke field at the beginning of the period of inflation relevant for our patch of the universe needs to be in the range between $-3.5/\mathcal{K}$ and $-1.5/\mathcal{K}$ in order to make the peak in the spectrum appear at interesting scales. The corresponding value of φ_f is $\varphi_f\mathcal{K} \sim 5.7$, which satisfies eq. (27). We have taken $M = 3.4 \times 10^{-3} m_P$ that makes the amplitude of perturbations consistent with CMBR anisotropy limits. The value of γ determines the spectral index for the shorter wavelength region of the spectrum. The considered value $\gamma = 0.38$ corresponds approximately to $n = 0.85$. To decrease the amplitude of perturbations at larger wavelengths, small values of β are preferable. However, in order to make the universe have an inflationary expansion for large negative values of φ, there is a constraint on β, i.e. $\beta \gtrsim 4$. When this constraint is fulfilled, as in the case considered in Fig. 1, the inflationary expansion can start at the Planck energy.

On the other hand, when values of β smaller than 4 are considered, this does not mean that inflation does not happen at all, but just that it starts when the field φ becomes larger than a certain value, making the number of e-foldings smaller. In this case, the evolution of φ for large and negative values is too fast, making the kinetic energy contribution the dominant one and the expansion non-inflationary. As φ evolves, its kinetic term decreases and, when it becomes subdominant, inflation starts. Fig. 2 shows the spectrum of perturbations arising for a case like this. The parameters are chosen to be $\beta = 3$, $\gamma = 0.43$. The value of the Brans-Dicke field at the beginning of the last 63 e-foldings of inflation needs to be in the range $-1.5/\mathcal{K}$ and $-0.5/\mathcal{K}$. The corresponding value at the end of inflation is given by $\mathcal{K}\varphi_f \sim 5.4$, which satisfies the bound coming from eq. (27). The value $M = 2\times 10^{-3} m_P$ satisfies CMBR anisotropy limits. The spectral index corresponding to the short wavelength part of the spectrum in this case is given by $n \approx 0.8$. The smallest wavenumber appearing in the graph corresponds to the first wavelength leaving the Hubble radius when inflation starts. A necessary condition for the model to be consistent with observations is that such a wavelength is larger than our present horizon.

IV. STOCHASTIC EVOLUTION

The dynamical evolution of the inflaton and of the Brans-Dicke field, taking into account the effects of quantum fluctuations is most clearly studied in the frame of the stochastic approach to inflation.[32] This is also the best tool to obtain quantitative information about the statistics of curvature perturbations.[33,34]

In this frame the dynamics of the system is described by two coupled Langevin equations for long wavelength modes ($k << aH$) of φ and σ. These equations can be easily written in the Einstein frame by adding to the classical equations of motion a noise term whose amplitude is fixed by the r.m.s. fluctuation of the fields at Hubble radius crossing, given by eqs. (40) and (41). They read

$$\varphi' = \frac{2}{\mathcal{K}}\sqrt{\frac{2}{\beta}}\frac{1}{(1+e^{\mathcal{K}\varphi\sqrt{2/\beta}})} + \frac{\mathcal{K}M^2\Gamma(\nu)}{2\pi^{3/2}}\left(\frac{p/2}{p-1}\right)^{\frac{1}{2}-\nu}\frac{e^{-\gamma\mathcal{K}\sigma}}{\sqrt{3-\frac{1}{p}}}(1+e^{-\mathcal{K}\varphi\sqrt{2/\beta}})\eta_\varphi(\alpha),$$

$$\sigma' = -\frac{\gamma}{\mathcal{K}}\frac{1}{(1+e^{-\mathcal{K}\varphi\sqrt{2/\beta}})} + \frac{\mathcal{K}M^2\Gamma(\nu)}{2\pi^{3/2}}\left(\frac{p/2}{p-1}\right)^{\frac{1}{2}-\nu}\frac{e^{-\gamma\mathcal{K}\sigma}}{\sqrt{3-\frac{1}{p}}}(1+e^{-\mathcal{K}\varphi\sqrt{2/\beta}})^{\frac{1}{2}}\eta_\sigma(\alpha), \tag{45}$$

where η_φ and η_σ are Gaussian noises with zero mean and correlation function $\langle\eta_\varphi(\alpha)\eta_\varphi(\alpha')\rangle = \langle\eta_\sigma(\alpha)\eta_\sigma(\alpha')\rangle = \delta(\alpha-\alpha')$. In the approximation leading to these stochastic equations the fine-grained components of the fields are treated as free even when φ and σ interact, thus we assume $\langle\eta_\varphi(\alpha)\eta_\sigma(\alpha')\rangle = 0$.

By integrating these Langevin equations it is possible to compute the statistical distribution of the field fluctuations and of the density perturbations.[34] Non-Gaussian fluctuations in the energy density are expected to arise whenever the effect of short wavelength quantum fluctuations of a field becomes important compared to the classical force in the evolution of the long wavelength part of the field. In the case of a single scalar field this criterion implies that non-Gaussian fluctuations are always associated to large amplitude fluctuations.[35] This can be easily seen by the analog of eq. (38) for one field, $\zeta \simeq -\mathcal{K}^2\delta\varphi/\varphi'$. In order to have ζ small enough to be consistent with CMBR anisotropy limits ($\mathcal{O}(10^{-4})$), it is necessary that the change in φ due to quantum fluctuations in one expansion time $\delta\varphi \sim H/2\pi$ is much smaller than the change φ' due to the classical force in the same interval. This implies that fluctuations of reasonable amplitude are Gaussian distributed. A similar argument applies also to our case. As it has been discussed above, curvature perturbations are dominated by the fluctuations in that field whose kinetic term gives the largest contribution at the time when the associated wavelength leaves the Hubble radius. (For the larger scales $\zeta \approx -\delta\varphi/\varphi'$ and for the shorter ones $\zeta \approx -\delta\sigma/\sigma'$.) For each of these regimes the same analysis applies. At the transition between the two regimes (when the peak in the spectrum appears) we do not expect non-Gaussian features to appear in the curvature perturbation distribution since all variables change smoothly there. These qualitative arguments are fully confirmed by the numerical integration of eqs. (45). Using the technique described in Ref. 34 we have numerically integrated the system of equations for the two choices of parameters corresponding to Fig. 1 and Fig. 2. Initial conditions have been chosen such that only fluctuations that are now inside a patch of the universe of the observable size are considered. From the distribution of the inflaton and the Brans-Dicke field the resulting distribution of ζ was computed using eq. (38). In both cases the distribution of curvature perturbations is pretty Gaussian.

V. CONCLUSIONS

Chaotic inflation in the frame of scalar-tensor theories of gravity has been analysed. We have considered a class of models in which the gravity theory has General Relativity as an attractor but that differs from it significantly during inflation. In particular, the generation of curvature perturbations in these models was studied. These perturbations get contributions from both fluctuations of the inflaton and of the Brans-Dicke fields. This makes it possible for the power spectrum to be quite different from the scale-invariant one. The quantum fluctuations of the field that has the largest kinetic contribution when a given wavelength leaves the Hubble radius during inflation are the dominant ones in determining the amplitude of curvature perturbations. As a result, the long wavelength part of the spectrum is dominated by the Brans-Dicke field fluctuations, while the short wavelength part by the inflaton ones. The spectrum of the curvature perturbations associated to the Brans-Dicke field has a dip for the models considered. The spectrum associated to the inflaton fluctuations instead is always decreasing with k. Thus, it is possible that a combination of these two behaviours results in a curvature spectrum that is peaked at a certain wavelength, which might help to reconcile the CDM model with observations. In fact, a peak in the spectrum is easily obtained. However, the scale at which the peak appears depends quite strongly on the value of the Brans-Dicke field at the end of inflation and the height and width are determined by the potential parameters. In order to make the peak occur at astrophysically interesting scales particular initial values for φ have to be chosen. The final value of the Brans-Dicke field is constrained by post-Newtonian experiments as shown by eq. (27). This constraint is automatically fulfilled by the values of φ_f coming from the initial values required to have the peak at an interesting scale. However, apart from that constraint there is no other restriction on the value of φ_f. Thus, the reason why the initial value for the Brans-Dicke field should be in the allowed range cannot be explained within the model. Finally, we have analysed the evolution of the system in the stochastic inflation approach and we have studied the statistical distribution of curvature perturbations. They are very nearly Gaussian distributed: unlike other models where non scale-invariant fluctuations are produced, the feature in the spectrum of curvature perturbations does not imply a deviation from the Gaussian statistics. The class of scalar-tensor theory of gravity analysed is just one of the possible alternative theories to General Relativity that is consistent with observations. The results obtained show that, if gravity is given by one of this theories, the spectrum of curvature perturbations originated during chaotic inflation may be quite different from the usual one and may help to reconcile CDM models with large-scale observations.

VI. ACKNOWLEDGEMENTS

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[30]Note that the index p appearing in eq. (41) should be that characterizing the power of the expansion of the Jordan frame scale factor $b(\tau)$, which differs in general from that in the Einstein frame. For example in the pure Brans-Dicke theory $p_J = 2p_E - 1$, where p_J denotes the expansion index in the Jordan frame and p_E that in the Einstein frame. For values of φ large and negative the model studied here closely resembles the Brans-Dicke theory, thus the same relation applies; when φ grows to positive values the two frames tend to coincide and $p_E \sim p_J$. This correction, however, is not very significative for the computations performed here. It would apply to the smaller wavelengths, that leave the Hubble radius when the inflaton kinetic term dominates. It can be seen that, for the values of β and γ to be considered, the value of p_E, at the time when the inflaton kinetic term becomes the dominant one, is $p_E \sim 6$ and increases afterwards. For these values of p_E, a change to p_J of the order just discussed makes a negligible correction to the factor $|\Gamma(\nu)|(p/(2p-2))^{\frac{1}{2}-\nu}$ appearing in eq. (41).

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FIG. 1. Power spectrum of ζ for the case $\beta = 4$ and $\gamma = 0.38$.

FIG. 2. Power spectrum of ζ for the case $\beta = 3$ and $\gamma = 0.43$.

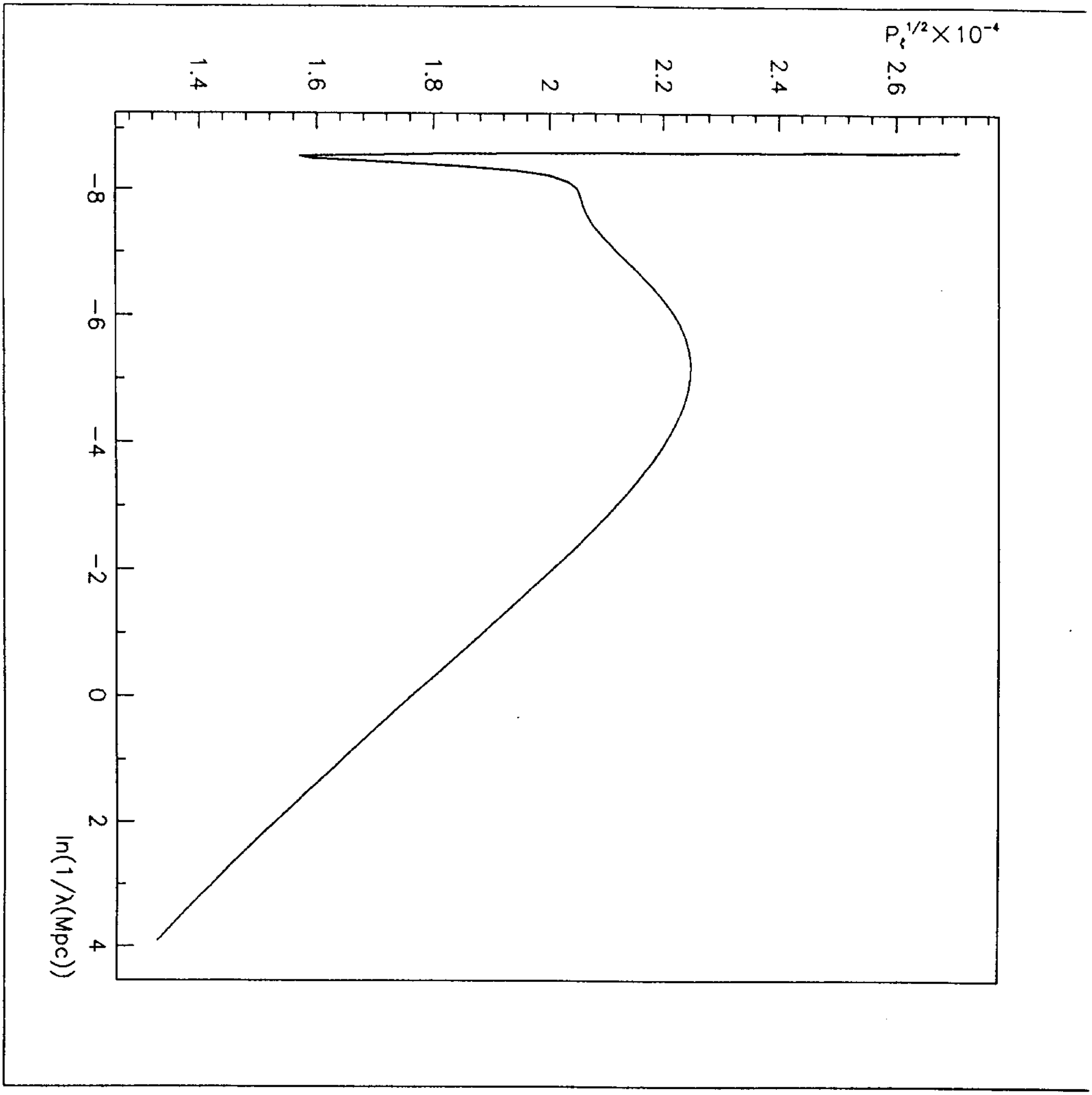

FIG. 1

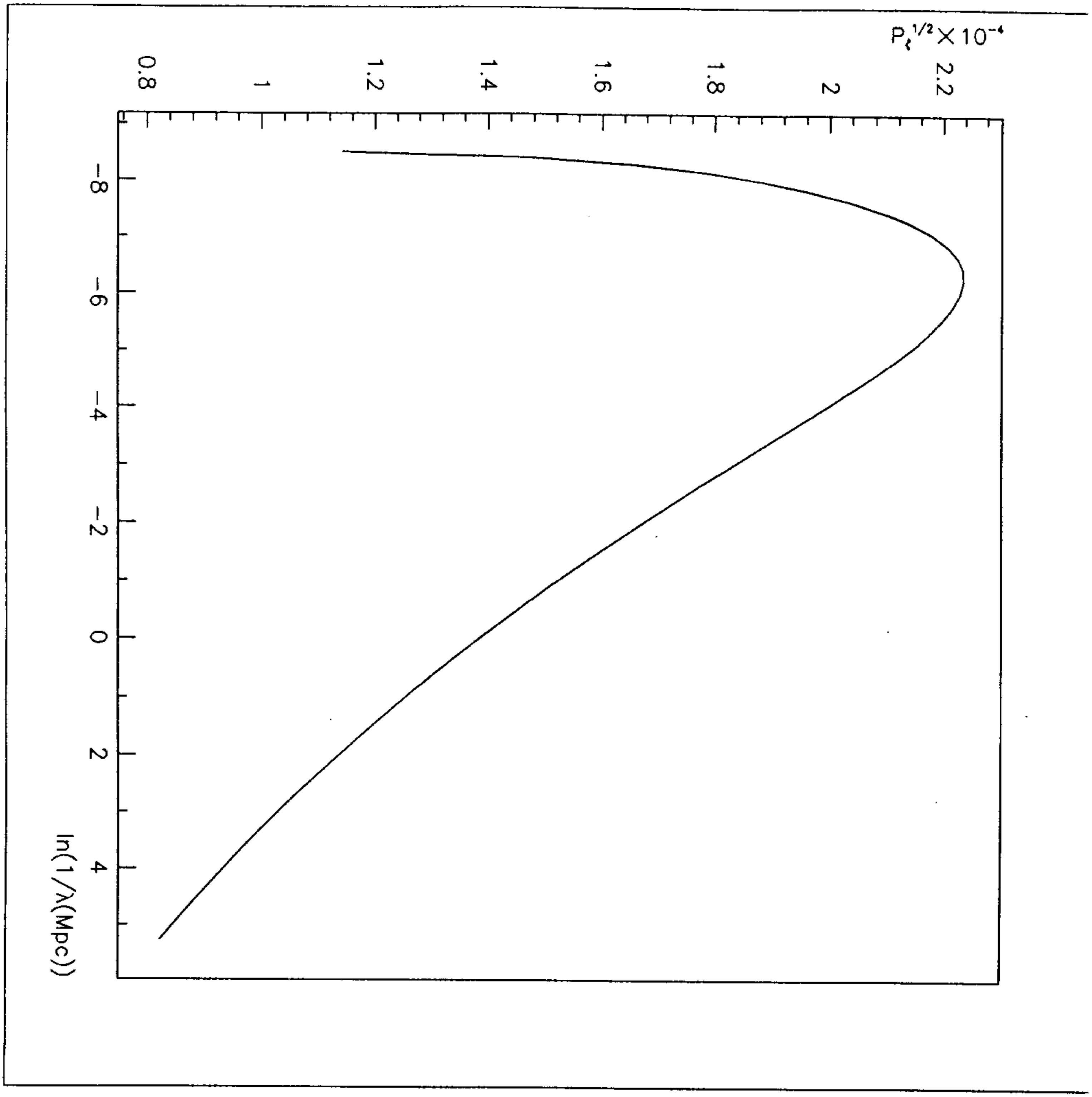

FIG. 2