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Constraints from Primordial Nucleosynthesis on the Mass of the τ Neutrino

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The familiar nucleosynthesis constraint on the number of neutrino species, $N_\nu \lesssim 3.4$, applies to *massless* neutrino species. An MeV-mass neutrino can have even greater impact, and we show that primordial nucleosynthesis excludes a τ-neutrino mass from 0.3 to 25 MeV (Dirac) and 0.5 to 25 MeV (Majorana) provided that its lifetime $\tau_\nu \gtrsim 1$ sec, and from 0.3 to 30 MeV (Dirac) and 0.5 to 32 MeV (Majorana) for $\tau_\nu \gtrsim 10^3$ sec. A modest improvement in the laboratory mass limit—from 35 to 25 MeV—would imply that the τ-neutrino mass must be less than 0.5 MeV (provided $\tau_\nu \gtrsim 1$ sec).

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The agreement of the predictions of primordial nucleosynthesis for the abundances of the light isotopes D, ^{3}He, ^{4}He, with ^{7}Li with their measured abundances is one of the great triumphs of the hot big-bang cosmology. Because of this success primordial nucleosynthesis has been used as a probe of both cosmology and particle physics [1]. In particular, primordial nucleosynthesis provides a stringent limit to the number of "light" (mass $\ll$MeV) degrees of freedom, which, expressed in terms of the equivalent number of light neutrino species, is $N_\nu \lesssim 3.4$ [2]. Recent precision measurements of the properties of the Z^0 have confirmed this in spectacular fashion: $N_\nu = 3.0 \pm 0.1$ [3].

In the context of nucleosynthesis, "light" refers to neutrinos of mass much less than 1 MeV, while "heavy" refers to neutrinos of mass of an MeV or greater, a division that traces to the temperature of the Universe when the weak interactions freeze out: $T_F \sim 1$ MeV. The present laboratory limit to the mass of the τ neutrino is $m_\nu < 35$ MeV [4]; if the τ-neutrino mass is greater than order 1 MeV, it will not be a light degree of freedom during nucleosynthesis and the above limit does not apply. Since the *equilibrium* energy density of a massive neutrino species is less than that of a "massless" neutrino species it would seem that the nucleosynthesis limit is irrelevant for the τ neutrino.

Neutrinos do not stay in thermal equilibrium; they decouple at temperature of order a few MeV [5]. After a massive neutrino species decouples and becomes nonrelativistic, its energy density grows relative to a massless neutrino species: $\rho(\nu_\tau)/\rho_\nu(m_\nu=0) \simeq (m_\nu/3.15T_\nu)r \propto t^{1/2}$, where $\rho_\nu(m_\nu=0) = 7\pi^2 T_\nu^4/120$, r is the ratio of the number density of massive neutrinos to massless neutrinos after freeze-out, and T_ν is the neutrino temperature. For an MeV-mass neutrino species r is order unity, and when the neutron-to-proton ratio freezes out ($T_F \sim 1$ MeV) its energy density is comparable to or even greater than a massless species. Since the ^{4}He yield is very sensitive to the value at which the neutron-to-proton ratio freezes out, which depends upon the energy density of the Universe, an MeV-mass neutrino species can have a significant effect on ^{4}He production.

Later, when $T \ll 1$ MeV, the energy density of a massive τ neutrino is even more significant: When the actual synthesis of the light elements begins in earnest ($t \sim 200$ sec, $T \sim 0.1$ MeV), it can be 10 times that of a massless species, comparable to the *total* energy density. Around this epoch D (and ^{7}Li for $\eta \lesssim 3\times10^{-10}$) have become overabundant (relative to nuclear statistical equilibrium) and are being burnt to ^{4}He. Speeding up the expansion leads to the earlier quenching of the reactions that burn D (and ^{7}Li) and significantly increases their primordial abundances. Thus an MeV-mass neutrino can have a greater effect on the yields of D and ^{7}Li than a massless neutrino species.

The impact of a massive neutrino on nucleosynthesis has been studied previously [6]. The purpose of our Letter is to update and extend the previous work and obtain a very noteworthy bound to the mass of the τ neutrino: Our bound, together with a modest improvement in the laboratory limit from 35 to 25 MeV, would imply that the mass of ν_τ must be less than 0.5 MeV, provided its lifetime is greater than 1 sec.

The τ neutrino affects nucleosynthesis through its effect on the expansion rate of the Universe H: $H^2 = 8\pi G\rho_{\rm tot}/3$, where the total energy density $\rho_{\rm tot}$ is that of a thermal bath comprised of photons, $e^\pm$ pairs, and three neutrino species. Usually the three neutrino species are taken to be massless (well justified for ν_e and ν_μ). We have modified the primordial nucleosynthesis code [7] to allow for a massive τ neutrino. Specifically, its energy density has been changed to

$$\rho(\nu_\tau) = r\left(\frac{[(3.151T_\nu)^2 + m_\nu^2]^{1/2}}{3.151T_\nu}\right)\rho_\nu(m_\nu=0)\,, \qquad (1)$$

where the $(3.151T_\nu)^2$ term accounts for its kinetic energy. Note, for $T_\nu \ll m_\nu/3$ the ratio $\rho(\nu_\tau)/\rho_\nu(m_\nu=0)$ depends only upon rm_ν. The relic abundance r is determined by standard techniques: numerical integration of the Boltzmann equation [5],

$$\frac{dn_\nu}{dt} = -3Hn_\nu - \langle\sigma|v|\rangle_{\rm ann}[n_\nu^2 - (n_\nu^{\rm eq})^2]\,, \qquad (2)$$

where n_ν is the number density of τ neutrinos, $n_\nu^{\rm eq}$ is their equilibrium number density, $r = n_\nu / n_\nu(m_\nu = 0)$, $\langle \sigma |v| \rangle_{\rm ann}$ is the thermally averaged, total annihilation cross section (for $m_\nu \lesssim 100$ MeV, $\nu_\tau \bar{\nu}_\tau \to \nu_e \bar{\nu}_e, \nu_\mu \bar{\nu}_\mu, e^- e^+$).

Several points should be noted. First, we assume that the interactions of the τ neutrino are described by the standard $SU(2)_L \otimes U(1)_Y$ electroweak gauge theory. If it were to have additional interactions, equilibrium would be tracked longer, its relic abundance r would be smaller, and our limits would be weakened. Second, since MeV-mass neutrinos freeze out when they are still semirelativistic, care must be taken in computing their annihilation cross section: Neither the nonrelativistic nor ultrarelativistic forms are appropriate. We have computed the exact annihilation cross section and performed the thermal average numerically [8]. Finally, in the case of a Dirac neutrino, one must determine whether or not all four helicity states ($\nu_\pm$ and their CP conjugates $\bar{\nu}_\mp$) are populated. (A Majorana neutrino has but two helicity states.) The helicity eigenstates ($\nu_\pm$), the eigenstates of a freely propagating neutrino, do not coincide with the chirality eigenstates ($\nu_{L,R}$), the eigenstates of the electroweak interactions. A freely propagating $\nu_\mp$ has projection of order $0.5[1 \pm |\mathbf{p}|/(E+m)]$ onto ν_L and $0.5[1 \mp |\mathbf{p}|/(E+m)]$ onto ν_R. In the relativistic regime, a ν_- (ν_+) is a ν_L (ν_R) with a ν_R (ν_L) admixture of order $m_\nu/2E_\nu$. Thus ν_-'s and $\bar{\nu}_+$'s interact with ordinary weak interactions, while the interactions of ν_+'s and $\bar{\nu}_-$'s are suppressed by a factor of $(m_\nu/2E_\nu)^2$. However, if the "wrong-helicity" states ($\nu_+, \bar{\nu}_-$) interact sufficiently to thermalize, the number of spin degrees of freedom doubles.

There are two types of reactions that can thermalize the wrong-helicity states: (i) spin-flip interactions that arise due to electroweak interactions [9], and (ii) new interactions for the wrong-helicity states. Because a ν_+ state has an order $m_\nu/2E_\nu$ projection onto ν_L, ordinary electroweak interactions mediate ν_- to ν_+ transitions at a rate given roughly by $m_\nu^2/4E_\nu^2$ times the rate for proper-helicity neutrinos to interact. Taking all processes into account, the production rate of ν_+'s is $\Gamma_+ \sim 0.1 G_F^2 m_\nu^2 T^3$; comparing this to the expansion rate one finds that the spin-flip interactions become ineffective at thermalizing ν_+ and $\bar{\nu}_-$ at temperatures below $T_* \simeq (300\ {\rm MeV})[m_\nu/(300\ {\rm keV})]^{-2}$. A decoupling temperature $T_* \sim 300$ MeV corresponds to the quark-hadron transition temperature. If the wrong-helicity states decouple before the quark-hadron transition—the case for $m_\nu \lesssim 300$ keV—their number density relative to the negative-helicity states will be greatly diminished when the entropy in the quark-gluon plasma is transferred to the hadronic degrees of freedom. On the other hand, if they decouple at a temperature below that of the quark-hadron transition—$m_\nu \gtrsim 300$ keV—their number density will be comparable to the negative-helicity states [10].

New interactions can also populate the wrong-helicity ν_τ states; e.g., $e_L^+ e_R^- \to \nu_+ \bar{\nu}_-$ mediated by a new gauge boson. However, we can be confident that they do not: If

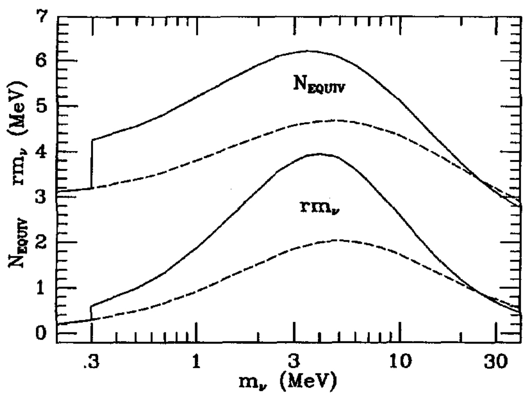

FIG. 1. The relic abundance times neutrino mass rm_ν and the ^{4}He yield expressed in equivalent number of massless neutrino species [13] as a function of the τ-neutrino mass (Dirac mass, solid curves; Majorana mass, broken curves). The ^{4}He abundance constrains the number of massless neutrinos to be less than 3.4. The discontinuity for a Dirac mass of 0.3 MeV arises because we assume that the wrong-helicity states are fully populated for $m_\nu \gtrsim 0.3$ MeV and unpopulated for $m_\nu \lesssim 0.3$ MeV [11].

they did, e and μ neutrinos would contribute as four light-neutrino species, which violates the nucleosynthesis bound. Thus, new interactions—unless for some reason flavor or mass dependent—cannot keep the wrong-helicity ν_τ states in equilibrium. On the basis of spin-flip interactions we assume that for $m_\nu \gtrsim 300$ keV the wrong-helicity states are fully populated, and for $m_\nu \lesssim 300$ keV they are not populated [11].

The measured primordial abundances of the light elements are the following: $Y_P = 0.23 \pm 0.01$; ${\rm D/H} \gtrsim 10^{-5}$; $({\rm D} + {}^3{\rm He})/{\rm H} \lesssim 1.1 \times 10^{-4}$; and ${}^7{\rm Li/H} = (1.2 \pm 0.3) \times 10^{-10}$ (for further discussion see Refs. [1] and [12]). For the purpose of setting the τ-neutrino mass limit we need upper limits to Y_P, $({\rm D} + {}^3{\rm He})/{\rm H}$, and ${}^7{\rm Li/H}$. Following the authors of Ref. [12] we adopt $Y_P \le 0.24$, $({\rm D} + {}^3{\rm He})/{\rm H} \le 1.1 \times 10^{-4}$, and ${}^7{\rm Li/H} \le 1.7 \times 10^{-10}$.

In Fig. 1 we show the effect of a massive τ neutrino on ^{4}He production expressed as the equivalent number of massless neutrino species [13]. In the extreme, a massive Dirac τ neutrino can have the effect of more than four light-neutrino species. The limit to the equivalent number of light-neutrino species, $N_\nu \le 3.4$ (based upon $Y_P \le 0.24$), is exceeded if the τ-neutrino mass is between 0.3 and 25 MeV (Dirac) and 0.5 and 25 MeV (Majorana). Since the ^{4}He abundance is affected by the increased expansion rate when the neutron-to-proton ratio freezes out, which occurs at $t \sim$ sec and $T \sim$ MeV, this mass constraint applies provided $\tau_\nu \gtrsim 1$ sec.

Next, consider $({\rm D} + {}^3{\rm He})/{\rm H}$, in Fig. 2 we see that a massive τ-neutrino species increases ${\rm D} + {}^3{\rm He}$ in a similar—but much more significant—way than additional massless neutrino species. While we have not shown D or ^{3}He individually, it is the D abundance that is most

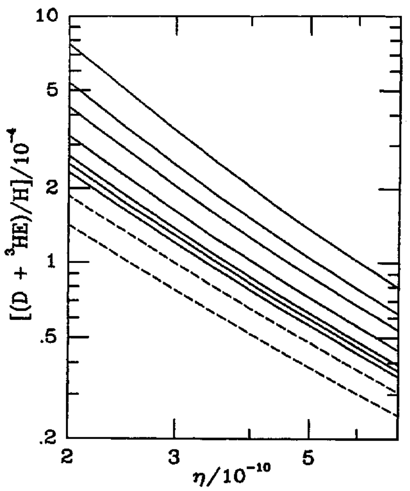

FIG. 2. The yield of $D+{}^3He$ as a function of rm_ν for two massless neutrino species and a massive τ neutrino as a function of the baryon-to-photon ratio η. The solid curves from bottom to top correspond to rm_ν=0.5, 0.6, 0.7, 1.0, 1.5, 2.0, and 3.0 MeV. For reference, results are also shown for two and four massless neutrinos alone (broken curves).

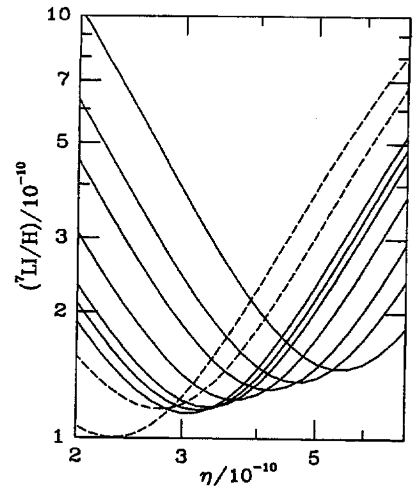

FIG. 3. Same as Fig. 2 except for ^{7}Li. The solid curves from left to right correspond to rm_ν=0.5, 0.6, 0.7, 1.0, 1.5, 2.0, and 3.0 MeV.

affected. The reason is simple: When nucleosynthesis commences ($t\sim200$ sec) D/H has reached a value of almost 10^{-2}. The final abundance depends upon how much D is left unburnt and an increase in the expansion rate when nucleosynthesis commences quenches these reactions earlier leaving more D unburnt.

Finally, consider ^{7}Li/H; the yield of ^{7}Li is not monotonic (Fig. 3). This is because for small η ^{7}Li is overabundant when nucleosynthesis commences and its final abundance is determined by how much ^{7}Li is left unburnt [by $^7Li(p,\alpha)^4He$], while for large η, ^{7}Li is produced as ^{7}Be [by $^4He(^3He,\gamma)^7Be$] which later decays (through electron capture) to ^{7}Li. That two production mechanisms are at work results in the "^{7}Li trough." Increasing the expansion rate around the time nucleosynthesis commences leads to increased ^{7}Li production for small η (more ^{7}Li remains unburnt) and decreased ^{7}Li production for large η (the production of ^{7}Be quenches earlier), which shifts the trough to larger η. Because the energy density contributed by a massive τ neutrino can be many times that of a massless neutrino when nucleosynthesis commences, its impact on the yields of D and ^{7}Li can be much greater. Also note that the effect of a massive τ neutrino on $D+{}^3He$ and ^{7}Li depends only upon rm_ν, since $\rho(\nu_\tau)/\rho_\nu(m_\nu=0)$ depends only upon rm_ν for $T\ll1$ MeV.

If the massive τ-neutrino species has a lifetime of at least 10^3 sec it will affect all the light-element abundances. The constraints based upon the overproduction of ^{4}He, and ^{7}Li and $D+{}^3He$ are complementary: Avoiding the overproduction of ^{4}He is most constraining at large η, while avoiding overproduction of ^{7}Li and $D+{}^3He$ is most constraining at small η. We used the following algorithm to obtain our mass constraint: Choose m_ν and calculate r and rm_ν; use the abundances of ^{7}Li and $D+{}^3He$ to find the *minimum* value of η permitted; check to see if the predicted value of Y_P is less than 0.24; if not, the chosen value of m_ν is excluded. Following this procedure we find the excluded mass range: 0.3 to 30 MeV (Dirac) and 0.5 to 32 MeV (Majorana).

What are the theoretical expectations for the τ-neutrino lifetime? Electroweak interactions allow a massive τ neutrino to decay to $\nu_{e,\mu}+\gamma$, and if it is heavier than 1 MeV to $e^\pm+\nu_e$ [14]. The lifetime for the first process is very long,

$$\tau_\nu=512\pi^4\alpha^{-1}G_F^{-2}I^{-2}m_\nu^{-5}\sin^{-2}2\theta$$
$$\simeq(2\times10^{12}\,\text{sec})[m_\nu/(1\text{ MeV})]^{-5}/\sin^2 2\theta\,,$$

where $I\sim m_\tau^2/m_W^2$ is the Glashow-Iliopoulos-Maiani (GIM) suppression factor and θ is the ν_τ-$\nu_{e,\mu}$ mixing angle. For the later process, which is not GIM suppressed, the lifetime $\tau_\nu\simeq(1.2\times10^5\text{ sec})[m_\nu/(1\text{ MeV})]^{-5}/\sin^2 2\theta$, where θ is the ν_e-ν_τ mixing angle. Neutrino interactions beyond the electroweak theory could lead to additional decay modes. For example, if there are interfamily interactions characterized by a symmetry-breaking scale f, the lifetime for the process $\nu_\tau\to\nu_{e,\mu}+\phi$ (ϕ is the massless Goldstone boson associated with family-symmetry breaking) is

$$\tau_\nu\sim8\pi f^2/m_\nu^3$$
$$\sim(2\times10^6\,\text{sec})[m_\nu/(1\text{ MeV})]^{-3}[f/(10^{10}\text{ GeV})]^2$$

(Ref. [15]). In any case it seems reasonable to expect the lifetime of an MeV-mass τ neutrino to be greater than 1 sec.

In conclusion, an MeV-mass ν_τ can have a significant effect on primordial nucleosynthesis, not only increasing the ^{4}He yield, but also that of D and ^{7}Li [6]. Based upon the overproduction of ^{4}He alone we exclude the interval 0.3 to 25 MeV (Dirac) or 0.5 to 25 MeV (Majorana) provided that $\tau_\nu \gtrsim 1$ sec; considering D+^{3}He, ^{4}He, and ^{7}Li together, we exclude a τ-neutrino mass in the interval 0.3 to 30 MeV (Dirac) or 0.5 to 32 MeV (Majorana) provided that $\tau_\nu \gtrsim 10^3$ sec [16]. Since a Dirac mass for the τ neutrino of 14 keV to about 1 MeV has already been excluded based upon the cooling of SN 1987A [17], the lower boundary of the excluded range in the Dirac case is actually about 14 keV.

Our constraint from nucleosynthesis is particularly timely as the CLEO Collaboration hopes to improve their ν_τ mass sensitivity to about 20 MeV in the near future [18]. If they succeed and find no evidence for a τ-neutrino mass, their mass limit together with our nucleosynthesis bound will constrain m_{ν_τ} to be less than 0.5 MeV, provided that $\tau_\nu \gtrsim 1$ sec. Further, if and when our constraint is tested by laboratory experiment, it will afford another test of the standard cosmology at this very early epoch.

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[2] G. Steigman, D. N. Schramm, and J. Gunn, Phys. Lett. **66B**, 202 (1979); K. A. Olive *et al.*, Phys. Lett. B **236**, 454 (1990).

[3] F. Dydak reported an average for the four LEP experiments of $N_\nu = 3.0 \pm 0.1$; in *Proceedings of the Twenty-Fifth International Conference on High Energy Physics*, edited by K. K. Phua (World Scientific, Singapore, 1991). It should be noted that the two "experiments" measure slightly different quantities; nucleosynthesis constrains the number of light degrees of freedom in thermal equilibrium at a temperature of 1 MeV, while measurements of the properties of the Z^0 determine the number of species that couple to the Z^0 with mass less than 45 GeV.

[4] Particle Data Group, M. Aguilar-Benitez *et al.*, Phys. Lett. B **239** (1990).

[5] For a textbook discussion of neutrino decoupling see, e.g., E. W. Kolb and M. S. Turner, *The Early Universe* (Addison-Wesley, Redwood City, CA, 1990), Chap. 5.

[6] D. A. Dicus *et al.*, Phys. Rev. D **17**, 1529 (1978); E. W. Kolb and R. J. Scherrer, *ibid.* **25**, 1481 (1982); R. J. Scherrer and M. S. Turner, Astrophys. J. **331**, 19 (1988); **331**, 33 (1988).

[7] The code we used is due to L. Kawano, Fermilab Report No. FERMILAB-PUB-88/34-A (to be published). It is a modernized version of the code written by R. V. Wagoner, Astrophys. J. **179**, 343 (1973), with cross sections and reaction rates from W. A. Fowler, G. R. Caughlin, and B. A. Zimmerman, Annu. Rev. Astron. Astrophys. **5**, 525 (1967); **13**, 69 (1975); M. J. Harris *et al.*, *ibid.* **21**, 165 (1983). We have taken the mean neutron lifetime to be $\tau_n = 881$ sec (8.8 sec, or 2σ, below the currently adopted value [4]). Only the predicted abundance of ^{4}He depends significantly upon τ_n; it increases monotonically: $\Delta Y_P \simeq 2.1 \times 10^{-4} \Delta\tau_n$(sec). We lower τ_n by 2σ because we use the *overproduction* of ^{4}He to constrain the τ-neutrino mass. We only consider the effect of a massive ν_τ on nucleosynthesis through its contribution to the energy density. There are other, smaller effects: slight heating of ν_e's, ν_μ's, γ's, and $e^\pm$'s through residual annihilations, and the fact that during nucleosynthesis r is slightly larger than its asymptotic value. We estimate that in sum these *increase* the impact of a massive ν_τ on ^{4}He by about 5%.

[8] A detailed discussion of $\langle\sigma|v|\rangle_{\rm ann}$ and the relic neutrino abundance will be given by A. Chakravorty, E. W. Kolb, and M. S. Turner (to be published); also see P. Gondolo and G. Gelmini, Nucl. Phys. (to be published).

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[10] The effectiveness of spin-flip interactions at populating the wrong-helicity states has also been considered by G. M. Fuller and R. A. Malaney (to be published).

[11] The precise value of the neutrino mass above which spin-flip interactions thermalize the wrong-helicity states after the QCD transition depends upon $T_{\rm QCD}^{1/2}$, and the transition temperature $T_{\rm QCD}$ is only known to be between 100 and 400 MeV. This introduces the dominant uncertainty in this mass. Our constraints only depend upon this mass to the extent that the lower mass limit is different for Dirac and Majorana neutrinos—0.3 vs 0.5 MeV.

[12] Olive *et al.*, Ref. [2]; B. E. J. Pagel, Phys. Scr. (to be published).

[13] We have converted the predicted ^{4}He abundance to equivalent number of massless neutrino species using

$$N_{\rm equiv} = [Y_P(\eta = 4\times 10^{-10}, r, m_\nu) - 0.2395]/0.0133 + 3.0\,.$$

To obtain an approximate value for Y_P from the curves in Fig. 1, the reader can use $Y_P \simeq 0.2243 + 0.011 \times \ln(\eta/10^{-10}) + 0.0133(N_{\rm equiv} - 3)$.

[14] A. de Rujula and S. L. Glashow, Phys. Rev. Lett. **45**, 942 (1980).

[15] Y. Chikashige *et al.*, Phys. Rev. Lett. **45**, 1926 (1980); F. Wilczek, *ibid.* **49**, 1549 (1982); D. Reiss, Phys. Lett. **115B**, 217 (1982).

[16] The reader who is interested in a more precise description of the lifetime dependence of the mass limit is referred to Scherrer and Turner, Ref. [6]. There are other astrophysical limits to the mass and lifetime of MeV-mass neutrinos. In particular, massive neutrinos with lifetimes greater than about 10^6 sec can photodissociate ^{4}He and ^{7}Li, either by their decays or their annihilations, which rules out masses above about 8 MeV. See, e.g., J. A. Frieman *et al.*, Phys. Rev. D **41**, 3080 (1990); or D. Lindley, Astrophys. J. **294**, 1 (1985).

[17] R. Gandhi and A. Burrows, Phys. Lett. B **246**, 149 (1990); *ibid.* erratum (to be published); M. S. Turner, Fermilab Report No. FERMILAB-Pub-91/136-A, 1991 (to be published).

[18] B. C. Barish and M. Perl (private communication).