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Corrections to trilinear gauge vertices and $e^+e^- \Rightarrow W^+W^-$ in technicolor theories

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We use an effective Lagrangian to study the leading corrections to $WW\gamma$ and WWZ vertices from new, weak isospin conserving, heavy physics. The corrections occur in $g_1^Z - 1$, $\kappa_Z - 1$, and $\kappa_A - 1$ and input from low energy QCD is used to estimate their size in technicolor theories. We then study the enhancement of these corrections in the process $e^+e^- \Rightarrow W^+W^-$ at high energies.

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There has been recent interest[1-4] in the electroweak corrections due to heavy particles with a characteristic mass Λ greater than m_Z. If the heavy particles participate in SU(2)×U(1) symmetry breaking then they, when integrated out, will generate additional effective interactions involving the electroweak gauge bosons and the triplet of Goldstone bosons. These interactions are very conveniently described by a gauged chiral Lagrangian.

All parameters in this chiral Lagrangian are finite quantities renormalized at the Z mass scale. We consider terms in this effective theory at order p^2 and p^4 in the low energy expansion. The electroweak corrections induced at these orders are not suppressed by powers of m_Z/Λ, unlike terms of higher order in the energy expansion. Thus the chiral Lagrangian approach immediately focuses our attention on the finite parameters most important to electroweak corrections.

In this note we will consider the possible weak isospin and CP conserving corrections to trilinear gauge vertices. From the analysis in [5] one finds that there are only two such terms at O(p^4) other than the kinetic terms. We adopt the notation in [5] and write

$$\begin{aligned} L_{VVV} = {} & \frac{F^2}{4} Tr\{ \nabla_\mu U^\dagger \nabla^\mu U\} - {}^1\!/_2 Tr\{W_{\mu\upsilon} W^{\mu\upsilon}\} - {}^1\!/_2 Tr\{B_{\mu\upsilon} B^{\mu\upsilon}\} \\ & - iL_9\, Tr\{g' B_{\mu\upsilon} \nabla^\mu U \nabla^\upsilon U^\dagger + g W_{\mu\upsilon} \nabla^\mu U^\dagger \nabla^\upsilon U\} \\ & + L_{10} g g'\, Tr\{U^\dagger B_{\mu\upsilon} U W^{\mu\upsilon}\} \end{aligned} \quad (1)$$

$\nabla_\mu U \equiv \partial_\mu U - igUW_\mu + ig' B_\mu U,\ W \equiv W_a(x)\tau_a\ ,\ B \equiv B(x)\tau_3\ ,\ U \equiv exp(-2i\pi_a(x)\tau_a/F)$
$\pi_a(x)$ is the Goldstone boson triplet. We retain only the τ_3 part of the hypercharge generator in $B_{\mu\upsilon}$ since we are assuming that the sum over the hypercharges of the heavy fermions vanishes. Also for our purposes we should not apply equations of motion to eliminate terms. But there is only one additional independent term which is usually eliminated by the equations of motion, $Tr\{(\nabla_\mu \nabla^\mu U) \nabla_\upsilon \nabla^\upsilon U^\dagger\}$,[5] and this term contributes neither to a trilinear gauge vertex or to a gauge boson self energy.[1]

The L_{10} term does contribute to gauge boson self energies via the term

$$\tfrac{1}{2} L_{10} g g' B_{\mu\upsilon} W_3{}^{\mu\upsilon} \quad (2)$$

This "oblique" correction will end up correcting the Z and A couplings to the charged W. This correction will be in addition to the explicit trilinear vertices in the L_9 and L_{10} terms.

We start by making the conventional transformation to the mass eigenstate basis.

$$W_{3\mu} = c_\theta Z'_\mu + s_\theta A'_\mu \qquad B_\mu = -s_\theta Z'_\mu + c_\theta A'_\mu \tag{3}$$

$$c_\theta \equiv \frac{g}{\sqrt{g^2 + g'^2}} \qquad s_\theta \equiv \frac{g'}{\sqrt{g^2 + g'^2}} \tag{4}$$

In this primed basis the kinetic terms for the neutral fields are:

$$L_{\rm kin} = -\tfrac{1}{4}[1-2\Delta_Z][Z'^{\mu\upsilon}]^2 - \tfrac{1}{4}[1-2\Delta_A][A'^{\mu\upsilon}]^2 + \tfrac{1}{2}\Delta_{AZ}A'_{\mu\upsilon}Z'^{\mu\upsilon} \tag{5}$$

where

$$\Delta_{AZ} = \frac{(c_\theta^2 - s_\theta^2)e^2}{c_\theta s_\theta} L_{10} \tag{6}$$

$$\Delta_A = e^2 L_{10} \tag{7}$$

$$\Delta_Z = -e^2 L_{10} \tag{8}$$

An additional transformation is necessary to obtain standard kinetic terms. This transformation is uniquely defined if we are to remain in a mass eigenstate basis. We treat the Δ's as small quantities; our results henceforth are true to lowest order in Δ's. The transformation is

$$Z'_\mu = [1 + \Delta_Z]Z_\mu \tag{9}$$

$$A'_\mu = [1 + \Delta_A]A_\mu + \Delta_{AZ}Z_\mu \tag{10}$$

In the unprimed basis the kinetic terms have conventional form. The Z mass is

$$m_Z = (1 + \tilde{\Delta}_Z)\, m_Z^{\rm o} \tag{11}$$

where $m_Z^{\rm o}$ is the Z mass in the absence of corrections, and

$$\tilde{\Delta}_Z = -e^2 L_{10} \tag{12}$$

The transformation (9-10) modifies the Z and A couplings to everything else. The trilinear gauge vertices from the $Tr\{W_{\mu\upsilon}W^{\mu\upsilon}\}$ and $Tr\{B_{\mu\upsilon}B^{\mu\upsilon}\}$ terms are, in terms of the primed fields,

$$L^{\rm o}_{WWV} = -ie\frac{c_\theta}{s_\theta}\left(W_{\mu\upsilon}{}^\dagger W^\mu Z'^\upsilon - W_{\mu\upsilon}W^{\mu\dagger}Z'^\upsilon\right) - ie\frac{c_\theta}{s_\theta}W_\mu{}^\dagger W_\upsilon Z'^{\mu\upsilon} - ie\, W_\mu{}^\dagger W_\upsilon A'^{\mu\upsilon} \tag{13}$$

The transformation to the unprimed fields redefines the charge

$$e_* = [1 + \Delta_A]e \tag{14}$$

But the s_θ/c_θ factors also require attention. It is convenient to define the following weak mixing angle[6] in terms of well measured quantities. ($\alpha_*^{-1} = 128.8 \pm 0.1$)

$$s_Z = \sin(\theta_Z) \qquad c_Z = \cos(\theta_Z) \tag{15}$$

$$s_Z^2 c_Z^2 \equiv \frac{\pi\alpha_*}{\sqrt{2}G_F m_Z^2} \tag{16}$$

The relation between s_Z and s_θ is found from (11), (14), (16) and the following.

$$s_\theta^2 c_\theta^2 = \frac{\pi\alpha}{\sqrt{2}G_F m_Z^{o2}} \tag{17}$$

By converting s_θ/c_θ to s_Z/c_Z and by transforming to the unprimed fields we obtain

$$\begin{aligned} L^o_{WWV} &= -ie_*\frac{c_Z}{s_Z}g_1^{Zo}\left(W_{\mu\nu}{}^\dagger W^\mu Z^\nu - W_{\mu\nu}W^{\mu\dagger}Z^\nu\right) \\ &\quad - ie_*\frac{c_Z}{s_Z}\kappa_Z^o W_\mu{}^\dagger W_\nu Z^{\mu\nu} - ie_* W_\mu{}^\dagger W_\nu A^{\mu\nu} \end{aligned} \tag{18}$$

$$g_1^{Zo} - 1 = \kappa_Z^o - 1 = \Delta_Z - \Delta_A + \Delta_{AZ}\frac{s_\theta}{c_\theta} + \frac{\Delta_A - \tilde{\Delta}_Z}{c_\theta^2 - s_\theta^2} \tag{19}$$

[With (19) it is easy to incorporate other sources of oblique corrections, such as isospin violating effects or the effects of Z and A mixing with a new U(1) gauge boson. The appropriate expressions for the quantities Δ_A, Δ_Z, Δ_{AZ}, and $\tilde{\Delta}_Z$ may be found in ref. [7].]

We now add to (18) the corrections coming from the explicit trilinear gauge vertices in L_9 and L_{10}. These terms only correct the vertices already present. Other possible trilinear vertices can only originate in terms in the chiral Lagrangian which violate weak isospin or CP or are higher order in the energy expansion. Combining the corrections and expressing things in terms of L_9 and L_{10} we obtain

$$\begin{aligned} L_{WWV} &= -ie_*\frac{c_Z}{s_Z}g_1^Z\left(W_{\mu\nu}{}^\dagger W^\mu Z^\nu - W_{\mu\nu}W^{\mu\dagger}Z^\nu\right) \\ &\quad - ie_*\frac{c_Z}{s_Z}\kappa_Z W_\mu{}^\dagger W_\nu Z^{\mu\nu} - ie_*\kappa_A W_\mu{}^\dagger W_\nu A^{\mu\nu} \end{aligned} \tag{20}$$

$$g_1^Z - 1 = -\frac{1}{2}\frac{e^2}{c_\theta^2 s_\theta^2}L_9 + \frac{e^2}{c_\theta^2(c_\theta^2 - s_\theta^2)}L_{10} \tag{21}$$

$$\kappa_Z - 1 = -\frac{1}{2}\frac{(c_\theta^2 - s_\theta^2)e^2}{c_\theta^2 s_\theta^2}L_9 + \frac{2e^2}{c_\theta^2 - s_\theta^2}L_{10} \tag{22}$$

$$\kappa_A - 1 = -\frac{e^2}{s_\theta^2}(L_9 + L_{10}) \tag{23}$$

For illustration we may choose the typical value $L_{10} = -0.045$ for a one family technicolor model, as estimated in ref. [1]. L_9 may be estimated in a similar way; in low energy QCD L_9 is similar in magnitude and is opposite in sign to L_{10}.[5] The chiral log contributions from Goldstone boson and technipion loops to L_9 and L_{10} are also equal in magnitude and opposite in sign. If we choose $L_9 = -L_{10}$ then we find that $g_1^Z - 1 = \kappa_Z - 1 = -0.023$ and $\kappa_A - 1 = 0$. Of course the zero is not to be taken seriously, but some cancellation in the contributions to κ_A can be expected. This is unfortunate, as is the rather small corrections to the Z couplings. Clearly, these effects will be difficult to see in the near future.

On the other hand at high energies (but below Λ) these corrections may be enhanced by a factor $(E/m_Z)^2$, by an effect known as "delayed unitarity".[8] We shall discuss this for the process $e^+e^- \Rightarrow W^+W^-$. This process will provide a good example of the chiral Lagrangian approach to electroweak corrections. We shall find that certain cancellations between oblique corrections and vertex corrections are made readily apparent in this approach.

The *AWW* and *ZWW* vertices are probed in the second diagram of Fig. (1). The uncorrected vertices are such that there is a cancellation between parts of the two diagrams, thus avoiding a total cross section growing with s. The corrections to these vertices disrupts this cancellation, thus implying that there is a piece of the cross section which grows like s. But since the corrections are small, unitarity would only be violated at energies above the scale Λ for which the effective Lagrangian does not apply. Unitarity is recovered in the underlying theory.

The full amplitude arising from the second diagram of Fig. (1) may be written as follows (we follow the notation of ref. [8] as much as possible):

$$M = \frac{-ie^2}{s}\bar{\mathrm{v}}\gamma_\mu \mathrm{u}\Gamma^{\mu\alpha\beta}(q, \bar{q}, P)\varepsilon_\alpha(q)\varepsilon_\beta^*(\bar{q}) \tag{24}$$

$$\Gamma^{\mu\alpha\beta}(q, \bar{q}, P) = F_1(q - \bar{q})^\mu \delta^{\alpha\beta} + F_3\left[P^\alpha \delta^{\mu\beta} - P^\beta \delta^{\mu\alpha}\right] \tag{25}$$

$$F_i = Qf_i^A + \frac{I_3 - s_\theta^2 Q}{s_\theta^2}\frac{s}{s - m_Z^2}f_i^Z \qquad i = 1 \text{ or } 3 \tag{26}$$

$I_3 = -{}^1/_2, 0$ for e_L, e_R. Of the seven possible terms[8-9] in the most general $\Gamma^{\mu\alpha\beta}(q, \bar{q}, P)$, only the F_1 and F_3 terms receive corrections from L_9 and L_{10}. The object is to decompose these corrections into contributions to f_1^A, f_1^Z, f_3^A, and f_3^Z. The uncorrected values are:

$$\left(f_1^A = \tfrac{1}{2}f_3^A\right)_0 = 1$$
$$\left(f_1^Z = \tfrac{1}{2}f_3^Z\right)_0 = 1 \tag{27}$$

The corrections are shown in the three diagrams a, b, and c in Fig. (2). The oblique correction in a is transferred to the vertices according to the transformation (9-10). We obtain the following.

$$\left(f_1^A = \tfrac{1}{2}f_3^A\right)_a = 2\Delta_A + \Delta_{AZ}\frac{c_\theta}{s_\theta}\frac{s}{s - m_Z^2}$$
$$\left(f_1^Z = \tfrac{1}{2}f_3^Z\right)_a = 2\Delta_Z + \Delta_{AZ}\frac{s_\theta}{c_\theta} \tag{28}$$

Factors of Δ_A and Δ_Z originate at each vertex in the photon and Z exchange diagrams respectively. The A-Z mixing means that the Z has an additional coupling to the Q of the electron; this gives the second term in the first equation. The c_θ/s_θ factor is present since the Z couples this much stronger to the W's. The A-Z mixing also means that the Z has an additional coupling to the W proportional to the photon coupling to the W; this gives the second term in the second equation. We may write (28) in terms of L_{10}.

$$\left(f_1^A = \tfrac{1}{2}f_3^A\right)_a = \left(2 + \frac{c_\theta^2 - s_\theta^2}{s_\theta^2}\frac{s}{s - m_Z^2}\right)e^2 L_{10}$$
$$\left(f_1^Z = \tfrac{1}{2}f_3^Z\right)_a = \left(-2 + \frac{c_\theta^2 - s_\theta^2}{c_\theta^2}\right)e^2 L_{10} \tag{29}$$

Another effect of the oblique correction is to shift the mass of the Z (see (11)).

$$\left(f_1^Z = \tfrac{1}{2}f_3^Z\right)_a = \frac{\delta m_Z^2}{s - m_Z^2} = -2\frac{m_Z^2}{s - m_Z^2}e^2 L_{10} \tag{30}$$

The corrections in b and c are simply read off from the appropriate vertices in the L_9 and L_{10} terms.

$$(f_3^A)_b = -\frac{e^2}{s_\theta^2}L_{10} \qquad (f_3^Z)_b = \frac{e^2}{c_\theta^2}L_{10} \tag{31}$$

$$(f_1^Z)_c = -\frac{1}{2}\frac{e^2}{c_\theta^2 s_\theta^2}L_9 \qquad (f_3^A)_c = -\frac{e^2}{s_\theta^2}L_9 \qquad (f_3^Z)_c = -\frac{e^2}{s_\theta^2}L_9 \tag{32}$$

We now may add up the corrections *a-c* and add them to the uncorrected values.

$$f_1^A = 1 + \left(2 + \frac{c_\theta^2 - s_\theta^2}{s_\theta^2}\frac{s}{s - m_Z^2}\right)e^2 L_{10} \tag{33}$$

$$f_1^Z = 1 - \frac{1}{2}\frac{e^2}{c_\theta^2 s_\theta^2}L_9 - \left(\frac{1}{c_\theta^2} + 2\frac{m_Z^2}{s - m_Z^2}\right)e^2 L_{10} \tag{34}$$

$$f_3^A = 2 - \frac{e^2}{s_\theta^2}L_9 + \left(\frac{4s_\theta^2 - 1}{s_\theta^2} + 2\frac{c_\theta^2 - s_\theta^2}{s_\theta^2}\frac{s}{s - m_Z^2}\right)e^2 L_{10} \tag{35}$$

$$f_3^Z = 2 - \frac{e^2}{s_\theta^2}L_9 - \left(\frac{1}{c_\theta^2} + 4\frac{m_Z^2}{s - m_Z^2}\right)e^2 L_{10} \tag{36}$$

The dominant correction at high energies is the correction which grows like s/m_Z^2, and this occurs only in the production of longitudinal W's. The amplitude for this correction is proportional to $F_3 - F_1$. Ignoring terms suppressed by m_Z^2/s we find

$$(F_3 - F_1)_{m_Z = 0} = \frac{I_3}{s_\theta^2} - \frac{1}{2}\frac{([c_\theta^2 - s_\theta^2]I_3 + Q s_\theta^2)e^2}{c_\theta^2 s_\theta^4}L_9 \tag{37}$$

The first term cancels against the neutrino exchange diagram. Thus the leading (in s) correction is due entirely to L_9. The leading L_{10} contributions cancel between the diagrams *a* and *b* in Fig. (2). The chiral Lagrangian approach makes this cancellation clear as follows. The L_{10} term has no coupling of a photon or Z to two Goldstone bosons. Then it cannot contribute a piece to the Goldstone boson production cross section which grows like s. And thus by the equivalence theorem[10] it cannot contribute a piece to the longitudinal W production cross section which grows like s.

We show results for $L_9 = 0.045$ and $L_{10} = -0.045$ in Fig. (3). Corrected and uncorrected quantities are indicated by solid and dashed lines respectively. The top two curves give the total cross section at a scattering angle θ of 90 degrees. The next two curves are for the production of longitudinal W's. This is clearly the source of most of correction to the total cross section. These curves will of course peak and turn over when mass thresholds of the new physics are reached.

The Q term in (37) means that the cross section for incoming $e^+_L + e^-_R$ (for which $I_3 = 0$) will also have a correction piece growing with s. From the lowest two curves in Fig. (3) we see that the total cross section grows rapidly in this case. Thus polarizing the electrons can serve to enhance that part of the total cross section which grows like s.

We remark here that $O(p^6)$ terms in the chiral Lagrangian also will give contributions to the cross section which grow like s, and this may occur in the production of transverse W's. But the coefficient of such a term has an explicit factor of $1/\Lambda^2$, and in the end this contribution to the total cross section will be suppressed by a factor of order $(m_W/\Lambda)^2$ relative to the production of longitudinal W's.

The results of this paper may be easily applied to the case of a new heavy family of weakly interacting fermions. The appropriate values of L_9 and L_{10} follow from a one-loop calculation in a chiral quark model.[11]

$$L_9 = -2L_{10} = \frac{N_d}{48\pi^2} \qquad (38)$$

$N_d = 4$ is the number of doublets. This leads to results for the one-loop corrections to $e^+e^- \Rightarrow W^+W^-$ in disagreement with those given in [8], in particular with eq. (4.3) of that reference. On comparing results for $(F_3 - F_1)\big|_{m_Z = 0}$ we find that a factor of $([c_\theta^2 - s_\theta^2]I_3 + Q s_\theta^2)/c_\theta^2$ in our result replaces a factor of I_3 in their result.

We have found that the isospin preserving corrections to trilinear gauge vertices in a typical technicolor theory are rather small. These corrections are determined by the two parameters L_9 and L_{10}, and both parameters may be estimated by comparing to low energy QCD. These corrections are enhanced in $e^+e^- \Rightarrow W^+W^-$ at energies well above threshold, and we found that the leading s/m_Z^2 effects are determined solely by L_9. By calculating or estimating L_9 and L_{10} the effects of any new isospin preserving physics may easily accounted be for. Other effects are also easily introduced into the chiral Lagrangian.

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Figure Captions

Fig. (1) The two graphs contributing to $e^+e^- \Rightarrow W^+W^-$

Fig. (2) Oblique and vertex corrections arising from the L_9 and L_{10} terms in the Lagrangian in equation (1)

Fig. (3) Solid curves include corrections and dashed lines exclude corrections. The various cross sections are plotted in units of the point cross section $1\ R = 4\pi\alpha^2/3s$.

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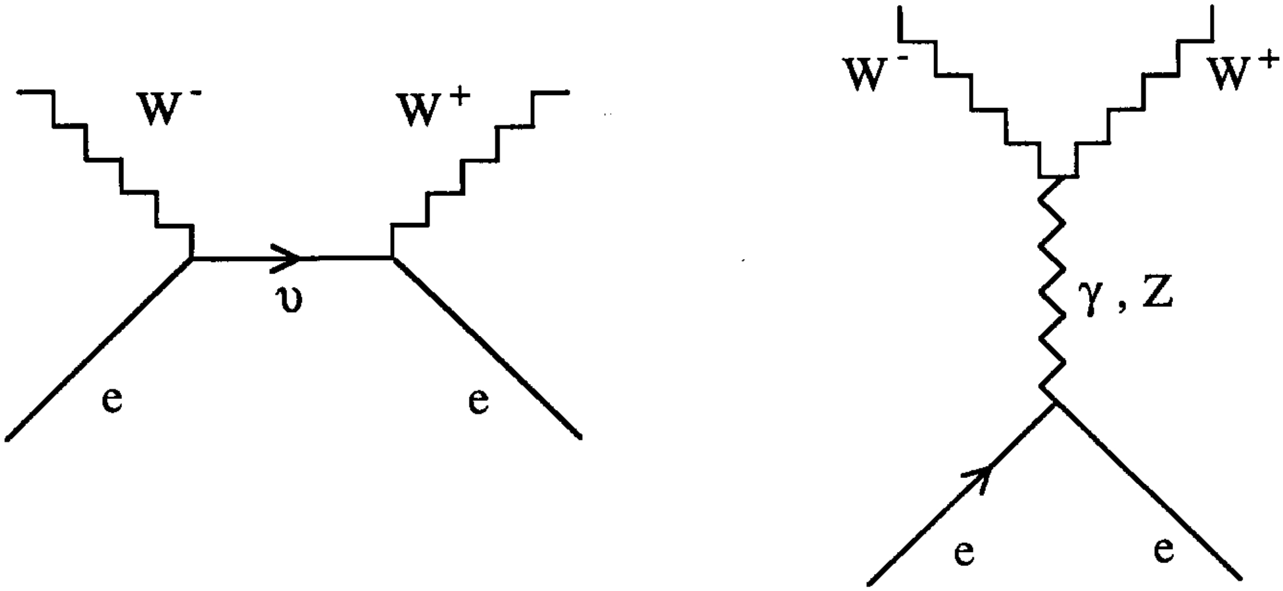


Figure (1)

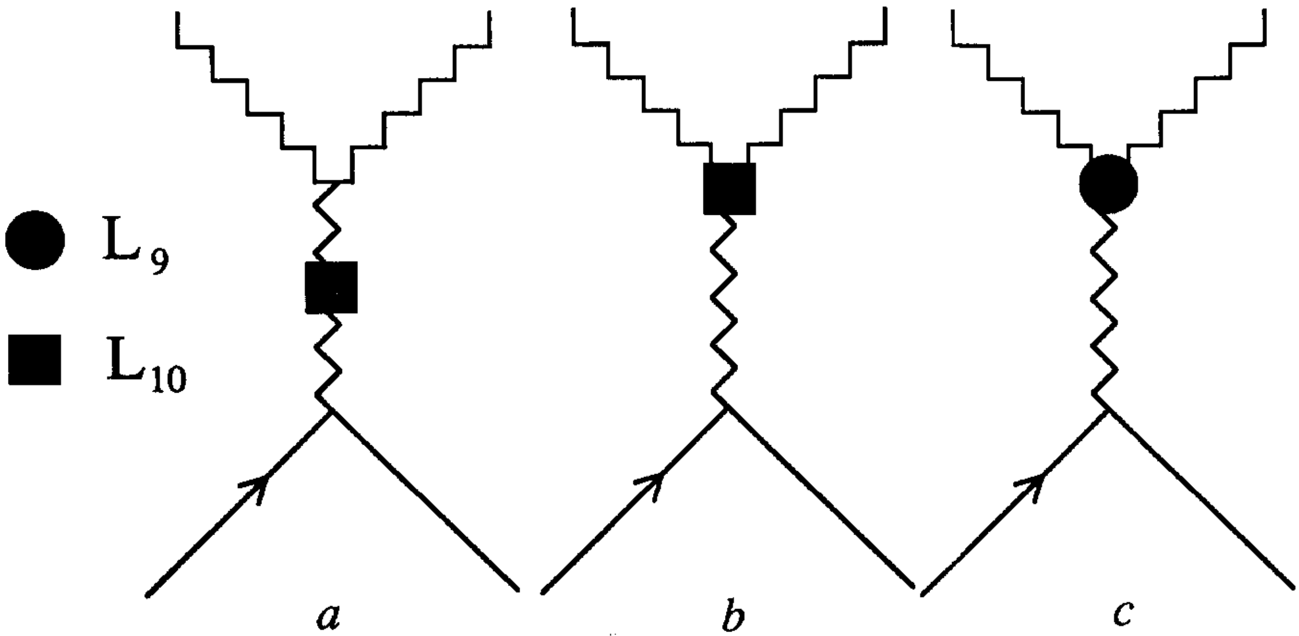
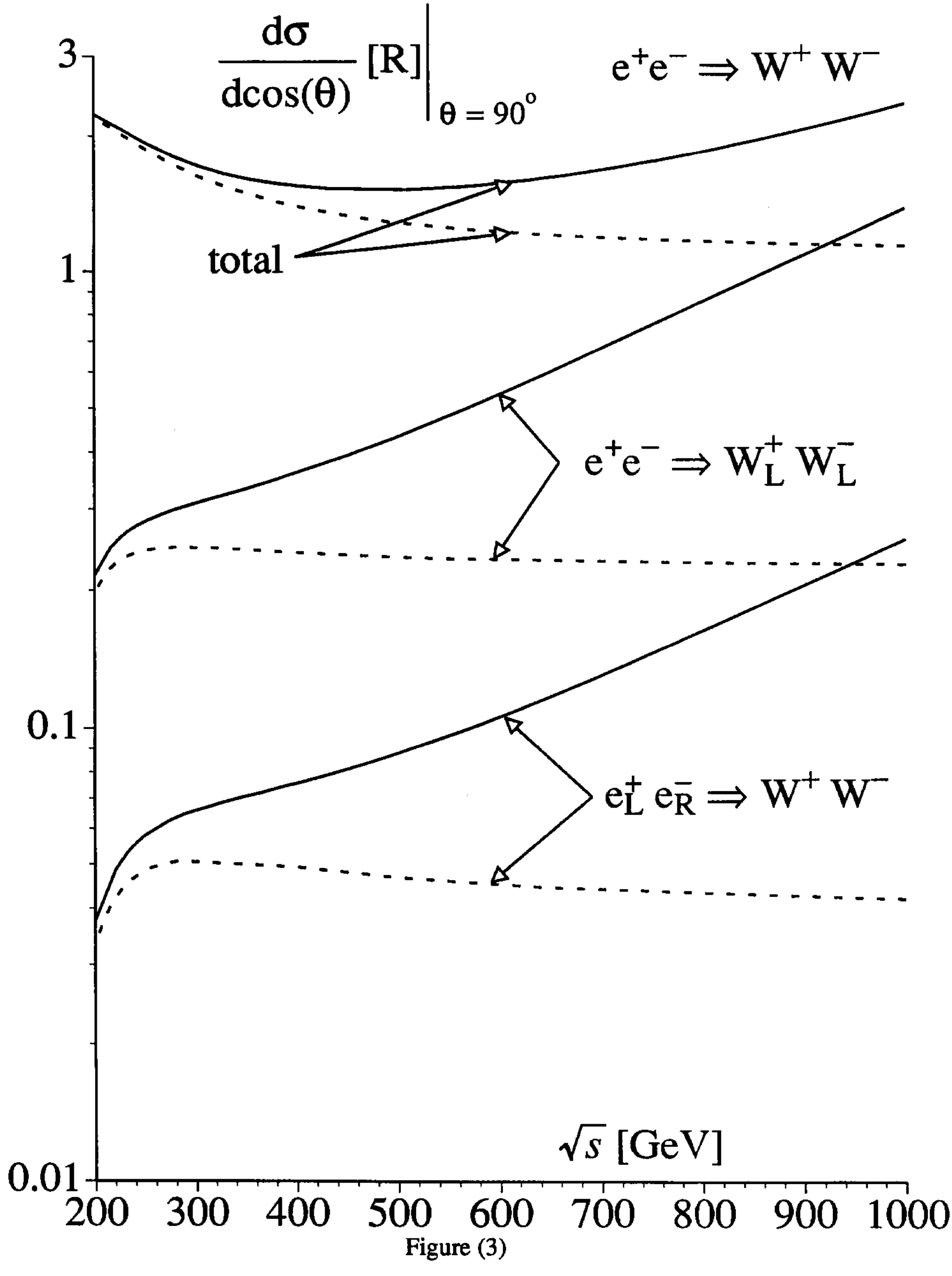


Figure (2)



$\frac{d\sigma}{d\cos(\theta)}[R]\Big|_{\theta = 90^o}$
$e^+e^- \Rightarrow W^+ W^-$
total
$e^+e^- \Rightarrow W_L^+ W_L^-$
$e_L^+ e_R^- \Rightarrow W^+ W^-$
$\sqrt{s}$ [GeV]
3
1
0.1
0.01
200
300
400
500
600
700
800
900
1000

Figure (3)