Fermi National Accelerator Laboratory

CMU-HEP90-15
FERMILAB-Pub-90/218-A
October 1990

Scale-invariant extended inflation

Richard Holman,[1] Edward W. Kolb,[2] Sharon L. Vadas,[3] Yun Wang[1]

[1] *Physics Department, Carnegie Mellon University, Pittsburgh PA 15213*

[2] *NASA/Fermilab Astrophysics Center*
Fermi National Accelerator Laboratory, Batavia, IL 60510
and
Department of Astronomy and Astrophysics and Enrico Fermi Institute
The University of Chicago, Chicago, IL 60637

[3] *Physics Department, The University of Chicago, Chicago, IL 60637*

Abstract

We propose a model of extended inflation which makes use of the non-linear realization of scale invariance involving the dilaton coupled to an inflaton field whose potential admits a metastable ground state. The resulting theory resembles the Jordan–Brans–Dicke version of extended inflation. However, quantum effects, in the form of the conformal anomaly, generate a mass for the dilaton, thus allowing our model to evade the problems of the original version of extended inflation. We show that extended inflation can occur for a wide range of inflaton potentials with no fine-tuning of dimensionless parameters required. Furthermore, we also find that it is quite natural for the extended inflation period to be followed by an epoch of *slow-rollover* inflation as the dilaton settles down to the minimum of its induced potential.

I. INTRODUCTION

Until recently, the usual methods for implementing inflation involved either "new"[1] or "chaotic"[2] models. In both models inflation occured during an epoch in which a scalar field known as the "inflaton" slowly rolled down a flat potential. The problem with this approach is that it typically involves fine tuning of microphysical parameters entering the scalar potentials of the fields whose slow rolling drives the inflationary transition. Since inflation is supposed to solve fine tuning problems, the advantages of these models are not altogether clear in this respect.

Such considerations led La and Steinhardt[3] to return to the original version of inflation proposed by Guth,[4] i.e., where inflation was driven by a first-order phase transition in which a scalar field, initially trapped in a metastable vacuum, escapes via bubble nucleation.

As is well known, the problem with first-order transitions for inflation is that values of the bubble percolation parameter $\epsilon \equiv \lambda/H^4$ (where λ is the bubble nucleation rate per volume and H is the Hubble parameter during inflation) that allow for sufficient inflation, do not allow for completion of the phase transition. That is to say, the true vacuum never percolates.[5] La and Steinhardt were able to avoid this problem by using a Jordan–Brans–Dicke[6] theory of gravity which allows power-law solutions for the scale factor $a(t)$ in a false-vacuum dominated epoch. This makes the percolation parameter ϵ a function of time, so that it can start small enough to allow for sufficient inflation, but then grow large enough so that the true vacuum percolates. This version of inflation is known as "extended" inflation.

Unfortunately, it was soon pointed out[7] that extended inflation also ran into problems. The size distribution of the bubbles formed in extended inflation was found to be nearly scale invariant. This gave rise to an overabundance of large bubbles which could not have thermalized their wall energy by the time of nucleosynthesis or recombination unless the Brans–Dicke parameter ω ($\omega \to \infty$ is the general relativity limit) was smaller than $\mathcal{O}(20)$. Since experimentally[8] it is known that $\omega \gtrsim 500$, it is clear that there was a problem!

Many models of extended inflation have been constructed since;[9] some of them meet all of the observational requirements. However, it is safe to say that none of them are simple and natural. In particular, none of them are compelling from the point of view of either particle physics or general relativity. For example, Brans–Dicke theory is not regarded as a fundamental theory of itself, but rather, it is usually considered from the point of view that it represents an approximation of some unified theory.

It is our aim in this work to provide a class of models of extended inflation that comes closer than previous ones in achieving the goals of simplicity and motivation. We will use the dilaton of hidden scale invariance to play the role of the Jordan–Brans–Dicke field. The previous exploration of these models have been motivated by an attempt to solve the cosmological constant problem[10] or to construct models for slow-rollover inflation.[11] Here we take a different approach and see whether these models can lead to a natural extended inflation scenario. We will be able to avoid the bounds on the (effective) ω parameter coming from the solar system experiments by using the conformal anomaly to give the dilaton a mass. We will also show that under some (relatively generic) circumstances the extended inflationary phase can set up initial conditions in the dilaton field and generate a second phase of *slow-rollover* inflation! The number of e-folds in this second inflationary phase can be controlled by varying the bounce action S_B for the "inflaton" field ϕ that generates the extended inflationary phase.

In the next section, we discuss some basic facts about scale and Weyl invariance; in particular, how to construct theories embodying these symmetries. We also discuss the conformal anomaly and how it can be made to give the dilaton a mass. In Section III we employ the non-linear realization of scale invariance, made possible by use of the dilaton, to construct a scale-invariant theory containing an inflaton field whose potential admits a metastable ground state (together with a stable one). We then show that this theory naturally admits extended inflationary solutions, and find restrictions on the so-called dilaton decay constant from the demands that there be no large bubble problem or excessive density fluctuations. Section IV addresses the question of whether the dilaton itself can give rise to second round of inflation (slow rollover this time). We find that it can and this can be used to change the density fluctuation spectrum in interesting ways. We conclude in Section V. There are two technical appendices.

II. HIDDEN SCALE INVARIANCE

As pointed out by Buchmüller and Dragon,[10] the standard model is *almost* scale invariant, except for the mass parameter that determines the Higgs doublet expectation value. This feature is also present in many unified theories. Thus, it may well profit us to study unified theories like the standard model, but with a non-linear realization of scale invariance added to the theory. It is this procedure that we describe in this section, mostly for the sake of completeness of the exposition (see also Ref. [12]).

Recall the scaling transformation law for a scalar field ϕ: under the transformation $x \rightarrow e^{\alpha}x$, ϕ transforms as

$$\phi(x) \rightarrow e^{\alpha}\phi(e^{\alpha}x). \tag{2.1}$$

If α is taken to be infinitesimal, this transformation becomes:

$$\delta_S[\phi] = \alpha(1 + x \cdot \partial)\phi. \tag{2.2}$$

Consider a monomial such as $m_0^{4-d}\phi^d$ that might appear in the potential $V_0(\phi)$ for ϕ. Applying our transformation law to it we find:

$$\delta_S[m_0^{4-d}\phi^d] = (d-4)m_0^{4-d}\phi^d + \partial_\nu(x^\nu m_0^{4-d}\phi^d). \tag{2.3}$$

Thus, the non-invariance of this monomial under scale transformations is clearly due to the fact that the mass dimension of the coefficient m_0^{4-d} of the monomial differs from 0, which is no surprise.

We may realize scale invariance in a *non-linear* fashion by making use of the *dilaton* σ, the Nambu-Goldstone boson of spontaneously broken scale invariance, to make the monomial appear to have dimension 4. This is done by having σ transform as follows under a scale transformation:

$$\delta_S[\sigma] = f + x\cdot\partial\sigma. \tag{2.4}$$

As is usual for Nambu-Goldstone bosons, this transformation law is inhomogeneous. The parameter f is the dilaton "decay" constant, in analogy with the case of the pion. We will take it to be near the Planck scale.

We may now use the dilaton to make $m_0^{4-d}\phi^d$ scale invariant. Consider the quantity $\exp(\sigma/f)$. Under a scale transformation, we have

$$\delta_S[\exp(\sigma/f)] = (1 + x\cdot\partial)\exp(\sigma/f). \tag{2.5}$$

Thus $\exp(\sigma/f)$ transforms as a field of scaling dimension 1, i.e., as a regular scalar field. It is now easy to see that $m_0^{4-d}\phi^d\exp[(4-d)\sigma/f]$ is scale invariant (up to a total derivative):

$$\begin{aligned}\delta_S[m_0^{4-d}\phi^d\exp\{(4-d)\sigma/f\}] &= (4 + x\cdot\partial)\{m_0^{4-d}\phi^d\exp[(4-d)\sigma/f]\} \\ &= \partial_\lambda\left\{x^\lambda m_0^{4-d}\phi^d\exp[(4-d)\sigma/f]\right\}.\end{aligned} \tag{2.6}$$

We see then that if $V_0(\phi)$ is a polynomial potential for ϕ, a scale-invariant potential $V_{SI}(\phi,\sigma)$ can be constructed from it by taking every parameter of non-zero mass dimension d, and multiplying it by $\exp[d\sigma/f]$. This is equivalent to the potential

$$V_{SI}(\phi,\sigma) = \exp(4\sigma/f)\ V_0[\exp(-\sigma/f)\phi]. \tag{2.7}$$

We define a new scalar field $\tilde{\phi} \equiv \exp(-\sigma/f)\phi$ with scaling dimension zero such that the scale-invariant potential can be written as

$$V_{SI}(\tilde{\phi}, \sigma) = \exp(4\sigma/f)\ V_0(\tilde{\phi}). \tag{2.8}$$

In order to construct a scale-invariant theory, we must also deal with the kinetic term of ϕ, as well as that of the dilaton. It is easy to see that the canonical ϕ kinetic term, proportional to $(\partial_\mu\phi)^2$, is scale invariant as it stands (formally, this may be seen from the transformation law: $\delta_S[\partial_\mu\phi] = (2 + x \cdot \partial)(\partial_\mu\phi)$, which can be easily be proven). However, since the scale-invariant version of the potential for ϕ naturally involves $\tilde{\phi}$, one can ask if there is a kinetic term for ϕ that also involves $\tilde{\phi}$. Such a term does in fact exist and is associated with Weyl transformations.

If we embed our theory in a background spacetime described by a metric $g_{\mu\nu}$, we can demand that the resulting theory be invariant under local rescalings of the metric: $g_{\mu\nu} \to g'_{\mu\nu} = \exp[2\gamma(x)]\ g_{\mu\nu}$. In order that the potential term $\sqrt{-g}\ V_{SI}(\sigma, \phi)$ be invariant, we demand that σ and ϕ transform as

$$\begin{aligned} \phi' &= \exp[-\gamma(x)]\ \phi \\ \sigma' &= \sigma - f\gamma(x). \end{aligned} \tag{2.9}$$

The ϕ kinetic term is *not* Weyl invariant. However, if we make use of $\partial_\mu(\sigma/f)$ as a gauge field for Weyl symmetry we can write down a covariant derivative for ϕ:

$$D_\mu\phi \equiv \partial_\mu\phi - \partial_\mu(\sigma/f)\phi. \tag{2.10}$$

This equation can also be written as $\exp(\sigma/f)\partial_\mu\tilde{\phi}$.

It is clear that $D_\mu\phi$ transforms covariantly under a Weyl transformation:

$$D'_\mu\phi' = \exp(-\gamma)D_\mu\phi. \tag{2.11}$$

Thus, the kinetic term

$$\mathcal{L}_{\rm kin} = \frac{1}{2}\sqrt{-g}\, g^{\mu\nu} D_\mu \phi D_\nu \phi \tag{2.12}$$

is manifestly Weyl invariant and clearly involves ϕ only in the combination $\tilde{\phi}$. This is the term used by Buchmüller and Dragon[10] in their work. Therefore we may make the choice of two scalar fields, ϕ and $\tilde{\phi}$, for the dynamical degree of freedom. Both choices reduce to the original theory in the limit that $\sigma \to \infty$. The Lagrangians for the two possibilities are

$$\begin{aligned} \mathcal{L}_{\tilde{\phi}} &= \sqrt{-g}\left\{ g^{\mu\nu}\frac{1}{2}\exp(2\sigma/f)\partial_\mu\tilde{\phi}\partial_\nu\tilde{\phi} - \exp(4\sigma/f)V_0(\tilde{\phi})\right\} \\ \mathcal{L}_{\phi} &= \sqrt{-g}\left\{ g^{\mu\nu}\frac{1}{2}\partial_\mu\phi\partial_\nu\phi - \exp(4\sigma/f)V_0[\exp(-\sigma/f)\phi)]\right\}. \end{aligned} \tag{2.13}$$

Although these Lagrangians in general describe different dynamics, either choice will result in the same cosmological model for extended inflation as we show in Appendix A. In the following we use $\mathcal{L}_{\tilde{\phi}}$ as it more closely resembles previous models of extended inflation. The form of $V_0(\tilde{\phi})$ we use will be discussed below.

So far, we have concentrated on making the ϕ sector scale invariant. It is not clear, however, that the dilaton sector of the theory or the gravitational sector of the theory need also be scale invariant. It could well be that the scale invariance is an "accidental" symmetry of the matter part of the action but *not* of the full action. If we were to choose a scale-invariant kinetic term for the dilaton and for gravity, it would take the form

$$\mathcal{L}'_{\rm gravity} = \sqrt{-g}\left\{ -\exp(-2\sigma/f)\frac{R}{16\pi G_N} - g^{\mu\nu}\frac{f^2}{2}\partial_\mu[\exp(\sigma/f)]\partial_\nu[\exp(\sigma/f)]\right\}. \tag{2.14}$$

In Appendix B we show that this choice of a scale-invariant $\mathcal{L}_{\rm gravity}$ will not lead to extended inflation.

It turns out that our purposes are better suited by the choice of a *non*-scale-invariant dilaton kinetic term, as well as a *non*-scale-invariant coupling to gravity. The combined gravity-dilaton part of the Lagrangian will therefore read:

$$\mathcal{L}_{\text{gravity}} = \sqrt{-g}\left\{-\frac{R}{16\pi G_N} + \frac{1}{2}g^{\mu\nu}\partial_\mu\sigma\partial_\nu\sigma\right\}. \tag{2.15}$$

The total Lagrangian $\mathcal{L}_{\text{total}} = \mathcal{L}_{\tilde{\phi}} + \mathcal{L}_{\text{gravity}}$ can be rewritten via a Weyl rescaling of the metric to yield a Jordan-Brans-Dicke model.

We should note the possibility that quantum corrections from the non-scale-invariant sector could feed in to the scale invariant matter sector, breaking the scale invariance explicitly. This can, however, be avoided by keeping both the dilaton and the gravity sectors as *classical* backrounds, thus preventing their quantum effects from being felt by the matter sector. In order to do this, we must restrict our use of this description of scale invariance to scales far below the Planck scale (we thank D. Kosower for this observation).

This is all very well for a classically scale-invariant theory. However, it is well known[13] that quantum effects break scale invariance by the explicit use of some form of a regulator mass. The effect of this anomaly is to induce a mass for the dilaton, in much the same way as QCD instanton effects together with the chiral anomaly induce a mass for the axion.

In order to calculate the dilaton mass generated by quantum effects, we will construct a *local* term involving ϕ, and σ, whose scale variation yields the conformal anomaly. We concern ourselves only with the non-derivative part of the anomaly, partly since these are the terms that induce a potential for σ and partly since the other terms consist of higher derivative and/or curvature terms that are irrelevant to our discussion.

Let us compute the one-loop effective potential for ϕ coupled to the dilaton. As is usual in theories involving non-linear realizations of broken symmetries, we treat the Nambu-Goldstone boson field as a *classical* background in any computation involving loops of other fields. Now in order to compute the one-loop effective potential for ϕ, we must compute the determinant of the second functional derivative of the action with respect to ϕ:[14]

$$\begin{aligned}\mathcal{O}(x,y) &\equiv \frac{\delta^2 S[\phi,\sigma]}{\delta\phi(x)\delta\phi(y)} \\ &= \left[-\Box_x + \exp(4\sigma/f)\,\frac{\partial^2}{\partial\phi^2}V_0(\tilde{\phi})\right]\delta^4(x-y). \end{aligned} \tag{2.16}$$

Note that this operator is the same regardless of which kinetic term (Weyl or just scale invariant) is chosen. This is because the dilaton field is being taken as a *constant* background field here. Note that we are also only considering a flat background, since the terms involving the curvature tensor contain only higher derivatives.[15]

Converting the ϕ derivatives to $\tilde{\phi}$ ones, the operator $\mathcal{O}(x,y)$ becomes:

$$\mathcal{O}(x,y) = \left[-\Box_x + \exp(2\sigma/f)\; V_0''(\tilde{\phi})\right]\delta^4(x-y). \tag{2.17}$$

From this, the one-loop correction to the potential can be read off:[14]

$$\Delta V_{1-\mathrm{loop}}(\tilde{\phi},\ \sigma) = \exp(4\sigma/f)\left\{\frac{[V_0''(\tilde{\phi})]^2}{(8\pi)^2}\left[\ln\frac{V_0''(\tilde{\phi})}{\mu^2} - \frac{3}{2}\right] + \frac{2[V_0''(\tilde{\phi})]^2}{(8\pi)^2}\frac{\sigma}{f}\right\}, \tag{2.18}$$

where μ is a regulator mass scale. In order to complete our calculation of the effective potential, various renormalization conditions must be imposed on the parameters of the potential. We will assume that this has been done in the sequel so that all parameters are the renormalized ones. With the following definitions:

$$\begin{aligned}\Delta(\tilde{\phi}) &\equiv [2/(8\pi)^2][V_0''(\tilde{\phi})]^2 \\ V(\tilde{\phi}) &\equiv V_0(\tilde{\phi}) + \Delta V_{1-\mathrm{loop}}(\tilde{\phi},\ \sigma=0), \end{aligned} \tag{2.19}$$

the potential including the relevant part from the anomaly reads

$$\hat{V}(\tilde{\phi},\sigma) = \exp(4\sigma/f)\left[V(\tilde{\phi}) + \Delta(\tilde{\phi})\frac{\sigma}{f}\right] + \Lambda, \tag{2.20}$$

where Λ will be adjusted to give $\hat{V}=0$ at the true minimum of the potential. Note that $V(\tilde{\phi})$ is simply the 1-loop potential ignoring the dilaton.

As we are interested in a potential for first-order phase transitions, we will assume that there are two extrema for this potential, a false-vacuum value assumed to be $\tilde{\phi} = 0$, and a true vacuum value, denoted as $\langle\tilde{\phi}\rangle$. Given the potential of Eq. (2.20) for σ, it is easy to compute the dilaton vacuum expectation value $\langle\sigma\rangle$, mass m_σ, and Λ.[10] Defining $m_{\tilde{\phi}}^2 \equiv V_0''(\langle\tilde{\phi}\rangle)$, we have then:

$$\begin{aligned}
\left.\frac{d\hat{V}(\langle\tilde{\phi}\rangle,\sigma)}{d\sigma}\right|_{\sigma=\langle\sigma\rangle} = 0 \quad &\Rightarrow \quad \langle\sigma\rangle = -\frac{f}{4} - \frac{\rho_T}{\Delta(\langle\tilde{\phi}\rangle)} \\
\left.\frac{d^2\hat{V}(\langle\tilde{\phi}\rangle,\sigma)}{d\sigma^2}\right|_{\sigma=\langle\sigma\rangle} = m_\sigma^2 \quad &\Rightarrow \quad m_\sigma^2 = \exp\left[-1 - 4\rho_T/\Delta(\langle\tilde{\phi}\rangle)\right]\frac{m_{\tilde{\phi}}^4}{8\pi^2 f^2} \\
\hat{V}(\langle\tilde{\phi}\rangle,\langle\sigma\rangle) = 0 \quad &\Rightarrow \quad \Lambda = \exp\left[-1 - 4\rho_T/\Delta(\langle\tilde{\phi}\rangle)\right]\frac{m_{\tilde{\phi}}^4}{128\pi^2},
\end{aligned} \tag{2.21}$$

where we have used the fact that $\Delta(\langle\tilde{\phi}\rangle) = 2m_{\tilde{\phi}}^4/(8\pi)^2$, and we have defined $\rho_T \equiv V(\langle\tilde{\phi}\rangle)$.

We will be interested in the potential at three stages. The first stage is during extended inflation, when $\tilde{\phi}$ is anchored in the false vacuum, $\tilde{\phi} = 0$, and $V(\tilde{\phi} = 0) = \rho_V$. In this stage the potential is

$$\hat{V}(0,\sigma) = \exp(4\sigma/f)\left[\rho_V + \frac{2}{(8\pi)^2}[V''(0)]^2\frac{\sigma}{f}\right] + \exp\left[-1 - 4\rho_T/\Delta(\langle\tilde{\phi}\rangle)\right]\frac{m_{\tilde{\phi}}^4}{128\pi^2}. \tag{2.22}$$

The next era is when the $\tilde{\phi}$ field tunnels from its false-vacuum value to $\tilde{\phi} = \langle\tilde{\phi}\rangle$. We will assume that during this era σ is constant but has not reached its true-vacuum value.

The final epoch in our analysis will be after the phase transition has completed but σ is still evolving to its ground state. This can be an epoch of slow-rollover inflation. The relevant potential during this period is

$$\begin{aligned}
V_{\text{eff}}(\sigma) \;=\; & \hat{V}(\langle\tilde{\phi}\rangle,\sigma) = \frac{m_{\tilde{\phi}}^4}{128\pi^2}\exp\left[-1 - 4\rho_T/\Delta(\langle\tilde{\phi}\rangle)\right] \\
& \times\left[4\exp\left[4\sigma/f + 4\rho_T/\Delta(\langle\tilde{\phi}\rangle) + 1\right]\left(\frac{\sigma}{f} + \frac{\rho_T}{\Delta(\langle\tilde{\phi}\rangle)}\right) + 1\right].
\end{aligned} \tag{2.23}$$

We now turn to the study of the two periods of inflation examining the extended inflation phase in Section III, and then turning to the slow-rollover phase in Section IV.

III. THE EXTENDED INFLATION ERA

Consider the epoch of extended inflation when $\tilde{\phi}$ is anchored in its false-vacuum value $\tilde{\phi} = 0$, and σ is evolving. Recall that the action for the theory is

$$\begin{aligned} S[g,\tilde{\phi},\sigma] &= \int d^4x \sqrt{-g}\left\{-\frac{R}{16\pi G_N} + \frac{1}{2}g^{\mu\nu}\partial_\mu\sigma\partial_\nu\sigma + \frac{1}{2}\exp(2\sigma/f)g^{\mu\nu}\partial_\mu\tilde{\phi}\partial_\nu\tilde{\phi}\right. \\ &\left. -\exp(4\sigma/f)\left[V(\tilde{\phi}) + \Delta(\tilde{\phi})\frac{\sigma}{f}\right] - \exp\left(-1 - 4\rho_T/\Delta(\langle\tilde{\phi}\rangle)\right)\frac{m_\phi^4}{f^2}\right\}. \end{aligned} \quad (3.1)$$

We will now show that this action can give rise to extended inflation.

The cosmological equations of motion coming from this action can be written as

$$\begin{aligned} \left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} &= \frac{8\pi G_N}{3}\left[\frac{1}{2}\dot{\sigma}^2 + \frac{1}{2}\exp\left(2\sigma/f\right)\dot{\tilde{\phi}}^2 + \hat{V}(\tilde{\phi},\sigma)\right] \\ \ddot{\tilde{\phi}} + 3\frac{\dot{a}}{a}\dot{\tilde{\phi}} + 2\frac{\dot{\sigma}}{f}\dot{\tilde{\phi}} &= -\exp\left(-2\sigma/f\right)\frac{\partial\hat{V}(\tilde{\phi},\sigma)}{\partial\tilde{\phi}} \\ \ddot{\sigma} + 3\frac{\dot{a}}{a}\dot{\sigma} &= \frac{1}{f}\left\{\exp\left(2\sigma/f\right)\dot{\tilde{\phi}}^2 - 4\hat{V}(\tilde{\phi},\sigma) - \exp\left(4\sigma/f\right)\Delta(\tilde{\phi})\right\}. \end{aligned} \quad (3.2)$$

We take the initial state to be such that $\tilde{\phi}$ is trapped in the metastable minimum at $\tilde{\phi} = 0$. It can be checked that the high temperature corrections will allow this.[16] Then $\dot{\tilde{\phi}} = \ddot{\tilde{\phi}} = 0$ and $\hat{V}(0,\sigma)$ is as given in Eq. (2.22). Then our equations of motion reduce to

$$\begin{aligned} \left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} &= \frac{8\pi G_N}{3}\left\{\frac{1}{2}\dot{\sigma}^2 + \exp(4\sigma/f)\left[\rho_V + \Delta(0)\frac{\sigma}{f}\right] + \Lambda\right\} \\ \ddot{\sigma} + 3\frac{\dot{a}}{a}\dot{\sigma} &= -\frac{4}{f}\exp(4\sigma/f)\left\{\rho_V + \Delta(0)\left[\frac{\sigma}{f} + \frac{1}{4}\right]\right\}. \end{aligned} \quad (3.3)$$

Now, these equations with $\Delta(0) = \Lambda = 0$ will give rise to extended inflationary solutions[17] for $k = 0$:

$$\begin{aligned} a(t) &= a(0)(1+Bt)^{\pi\hat{f}^2} \\ \sigma(t) &= \sigma_0 - \frac{f}{2}\ln(1+Bt) \\ B^2 &= \frac{8}{\hat{f}^2(3\pi\hat{f}^2 - 1)}\exp(4\sigma_0/f)\frac{\rho_V}{M_P^2} \end{aligned} \quad (3.4)$$

where we have defined $\hat{f} \equiv f/M_P$, where M_P is the Planck mass. Thus, we see that as long as

$$\begin{aligned} \left|\Delta(0)\frac{\sigma}{f}\right| &< \rho_V \\ |\Delta(0)| &< 4\rho_V \\ \left|\ln\left(\frac{\Lambda}{\rho_V}\right)\right| &< \left|4\frac{\sigma}{f}\right|. \end{aligned} \tag{3.5}$$

we will have extended inflation with the above solutions. Since $\Delta(0)$ is the result of the one-loop potential, it is naturally supressed by a factor $32\pi^2$ relative to tree-level quantities such as ρ_V. Therefore the necessary inequalities are expected to be satisfied naturally.

Given that our theory can induce an extended inflationary epoch, we must now check that (i) this epoch lasts long enough to solve the cosmological problems, (ii) that the large-bubble problem is avoided, and (iii) that there be no excessive density fluctuations.[18] Since the results obtained in Refs. (7, 18) were presented in terms of an effective Jordan–Brans–Dicke theory, we convert our theory into such a model. The main result we will need is that the effective JBD ω parameter in our theory is given by: $\omega_{\text{eff}} \equiv 2\pi\hat{f}^2 - 3/2$. The constraints on our model can be written either in terms of ω_{eff} or $\hat{f}$.

First, we demand that the theory actually inflate. This then requires that $\ddot{a}/a > 0$, which in turn implies that $\pi\hat{f}^2 > 1$. We will defer to later the enforcement of the constraint of having sufficient inflation. In order to solve the large bubbles problem, the analysis of Ref. (7) requires that we take $\omega_{\text{eff}} \lesssim 20$, or $\pi\hat{f}^2 \lesssim 10$. The density fluctuations analysis of Ref. (18) demands that if these density fluctuations have anything to do with galaxy formation, and that there be no excessive density fluctuations, then $\omega_{\text{eff}} \gtrsim 10$, or $\pi\hat{f}^2 \gtrsim 6$. This constraint may actually be weaker than it seems, since if the second slow roll over period of inflation (to be discussed in the next section) occurs, these density fluctuations may be inflated away. It may also be weaker if $\rho_V^{1/4}/M_P$ is smaller than typical

GUT scales of 10^{-5}. However, since this need not occur, we include the constraint here.

To get sufficient inflation, we must find the time t_{end} at which inflation can be said to be over. In the extended inflation scenario, inflation ends when the percolation parameter $\epsilon = \lambda/H^4$ attains its critical value, which we take to be of order unity (see Ref. (5) for a more careful assessment of this value). Using the results in Ref. (17), we find that

$$\epsilon(t) = \frac{\lambda_0}{(\pi \tilde{f}^2 B)^4} \exp(4\sigma_0/f)(1 + Bt)^2, \tag{3.6}$$

where λ_0 is the bubble nucleation rate computed when $\sigma = \text{const} = 0$. The above equation needs some explanation. In Ref. (17), we calculated the nucleation rate assuming that the tunnelling happened only in the $\tilde{\phi}$ direction, i.e., $\sigma = \text{const}$. We have also assumed this here. However, here we are *not* taking the Brans–Dicke theory to be the fundamental one. Thus, this assumption cannot be justified by use of the approximations of Ref. (17). There, the Brans-Dicke kinetic term could be neglected along with the Ricci scalar term since both terms were proportional to $1/G_N$ and thus decoupled from the action in the zero-gravity limit. Here we must also take the limit $f \to \infty$. With the rescaling $\tilde{\sigma} = \sigma/f$, the dilaton kinetic term becomes $f^2 g^{\mu\nu}\partial_\mu\tilde{\sigma}\partial_\nu\tilde{\sigma}$, which decouples from the action in the $f \to \infty$ limit. It is in this limit that the dilaton will remain constant during tunnelling. This approximation is necessary as the two-field tunnelling problem is currently beyond our capabilities to solve.

Another comment that needs to be made here concerns the choice of the kinetic term for the ϕ field. Since we again must compute various functional determinants of operators involving this kinetic term, it would seem that it might make a difference which one (scale or Weyl invariant) was used. However, we have already assumed that tunnelling proceeds only in the $\tilde{\phi}$ direction, i.e., that the dilaton field is taken to be constant during tunneling. Thus $\exp(\sigma/f)\partial\tilde{\phi} = \partial\phi$ in this limit. Therefore, there is no difference between the two kinetic terms in the Euclidean bounce; the nucleation rates will be the same.

Setting $\epsilon(t_{\rm end}) \simeq \mathcal{O}(1)$, we find:

$$1 + Bt_{\rm end} = 8\pi^2 \exp(2\sigma_0/f) \left(\frac{\widehat{f}^2}{3\pi\widehat{f}^2 - 1}\right) \frac{\rho_V}{\lambda_0{}^{1/2} M_P^2}. \tag{3.7}$$

This immediately yields the number of e-folds in extended inflation:

$$N_{\rm ext-infl} = \ln\left(\frac{a(t_{\rm end})}{a(0)}\right) = \pi\widehat{f}^2 \left[\ln\left(\frac{8\pi^2\widehat{f}^2}{3\pi\widehat{f}^2 - 1}\right) + 2\ln\left(\frac{\mu}{M_P}\right) + \frac{S_B}{2} + \frac{2\sigma_0}{f}\right] \tag{3.8}$$

where we have written $\rho_V = \mu^4$ (μ is the mass scale of the inflaton), and $\lambda_0 \simeq \rho_V \exp(-S_B)$. A plot of the number of e-folds of inflation during the extended inflation era is shown in Fig. 1 as a function of $S_B + 4\sigma_0/f$ for $\widehat{f} = 1/2$, 1, and 2. We can see that it depends linearly on $S_B + 4\sigma_0/f$, and approximately quadratically on $\widehat{f}$.

The sufficient inflation constraint can be written as:

$$(1 + Bt_{\rm end})^{\pi\widehat{f}^2} \gtrsim e^{65}. \tag{3.9}$$

Taking the log of this yields the relation

$$\widehat{f}^{-2} \lesssim \frac{\pi}{65}\left[\ln(8\pi^2) - \ln(3\pi - \widehat{f}^{-2}) + 2\frac{\sigma_0}{f} + \ln\left(\frac{\rho_V}{\lambda_0{}^{1/2} M_P^2}\right)\right]. \tag{3.10}$$

Given the bounds on $\widehat{f}$ and initial values for σ, this gives a relation between the vacuum energy density and the bounce action S_B, which can be satisfied without fine tuning.

We see then, that under the assumption that the anomaly term can be neglected during this epoch, extended inflation can occur for $\pi\widehat{f}^2$ in the range from 6 to 10. This implies that f must be almost equal to M_P (actually, slightly larger). We do *not* take this as evidence of a fine-tuning problem for the following reason. The theory we have written down is an effective theory incorporating the effects of the breaking of scale invariance by use of the dilaton. The correct theory will have a particular value of f. If it is in the appropriate range, extended inflation will occur; if not it won't. Since we do not know how to construct the more detailed theory our model is supposed to represent, we content ourselves with finding the conditions under which extended inflation can occur.

For completeness sake, and for use in the next section, we will restate the results from Ref. (18) concerning the adiabatic density perturbations generated by fluctuations in the dilaton field during extended inflation. The density perturbations generated from bubbles are assumed to be small, since most bubbles are nucleated near the end of extended inflation. The slow-roll approximation is also used. In Ref. (18), it was stated that this approximation is valid in the large ω limit. However, since $\omega_{\rm eff} \equiv 2\pi\hat{f}^2 - 3/2$, and $\hat{f}$ can be as small as 1/2, we must be more precise. We can check to see when the slow-roll conditions are satisfied in extended inflation. These are $\ddot{\sigma} \ll 3\dot{\sigma}\dot{a}/a$ and $\dot{\sigma}^2/2 \ll V$. Using the Eqs. (3.4) for σ, a, and B, we find that these conditions become $|\ddot{\sigma}/(3\dot{\sigma}\dot{a}/a)| = 1/(3\pi\hat{f}^2) \ll 1$ and $|(\dot{\sigma}^2/2)/(\rho_V \exp(4\sigma/f))| = 1/(3\pi\hat{f}^2 - 1) \ll 1$. The smallest $\hat{f}$ can be is the minimal amount which is needed for inflation to occur: $\pi\hat{f}^2 > 1$. Using $\hat{f}^2 = 1/\pi$, we see that the slow-roll conditions are only marginally satisfied. For $\hat{f} \gtrsim 1$, however, they are clearly satisfied.

Assuming that the slow-roll conditions do hold, we can calculate the density perturbations generated. As it has already been done in Ref. (18), we will only outline the relevant calculations. At horizon crossing, the density contrast is[19]

$$\frac{\delta\rho}{\rho} = \frac{H^2}{\dot{\sigma}}. \tag{3.11}$$

Using the slow-roll equations of motion, $3H\dot{\sigma} = -V'$ and $H^2 = 8\pi G_N V/3$, we can rewrite this as

$$\frac{\delta\rho}{\rho} = 3\left(\frac{8\pi}{3M_P^2}\right)^{3/2} \frac{(V)^{3/2}}{V'}. \tag{3.12}$$

Using the fact that the extended inflation potential is $V = \rho_V \exp(4\sigma/f)$, and using the expressions for $a(t)$ and σ from Eqs. (3.4), we find that

$$\frac{\delta\rho}{\rho} = \frac{3}{4}\left(\frac{8\pi}{3}\right)^{3/2} \hat{f} \left(\frac{\mu}{M_P}\right)^2 \frac{\exp(2\sigma_0/f)}{(1+Bt)}. \tag{3.13}$$

We would like to relate the time that a scale crosses outside the horizon with its co-moving size λ. The distance to the horizon is about H^{-1}. In addition, the physical size of a scale increases linearly with the scale factor of the universe. Since the physical size of a scale grows much faster than H^{-1}, a scale λ leaves the horizon when $\lambda\, a(t)/a(0) \simeq H^{-1}$. Using the expression for $a(t)$, the horizon-crossing for a scale λ occurs when $(1+Bt) \simeq (B\pi \hat{f}^2 \lambda)^{-1/(\pi\hat{f}^2-1)}$. Putting this into our expression for $\delta\rho/\rho$, we thus find the expression for the density perturbation as a function of the co-moving scale λ:

$$\left(\frac{\delta\rho}{\rho}\right)_\lambda \propto \lambda^{1/(\pi\hat{f}^2-1)}, \tag{3.14}$$

where the proportionality constant depends on $\hat{f}$. We will use this result in the next section.

IV. EVOLUTION OF THE DILATON IN THE POST-EXTENDED-INFLATION ERA

After extended inflation is over, the dilaton field will in general be displaced from its ground state at $\langle\sigma\rangle/f = -1/4 - \rho_T/\Delta(\langle\bar{\phi}\rangle)$. In fact, we can compute the value of the dilaton at the end of the extended inflationary period. Recall that during extended inflation σ has the following time dependence:

$$\sigma(t) = \sigma_0 - \frac{f}{2}\ln(1+Bt). \tag{4.1}$$

Thus, we can use our result for $1 + Bt_{\rm end}$ derived in the previous section to compute $\sigma(t_{\rm end})$. Doing this yields

$$\sigma(t_{\rm end}) = \frac{f}{2}\left[\ln\left(\frac{3-(\pi\hat{f}^2)^{-1}}{8\pi}\right) + \ln\left(\frac{\lambda_0^{1/2} M_P^2}{\rho_V}\right)\right], \tag{4.2}$$

where, again, λ_0 is the nucleation rate when $\sigma = 0$. Note that $\sigma(t_{\rm end})$ is independent of the initial value of σ prior to the onset of extended inflation. It is very interesting to estimate a value for $\sigma(t_{\rm end})$. Clearly the argument of the logarithm of the first term in Eq. (4.2) is of order unity and can probably be ignored. If we choose a mass scale μ for the inflaton, then $\rho_V \sim \mu^4$, $\lambda_0 \sim \mu^4 \exp(-S_B)$, and $\sigma(t_{\rm end})/f \sim -1.09 + \ln(M_P/\mu) - S_B/4$. Now if we make the traditional assumption that $\mu \sim 10^{14}$GeV, then $\sigma(t_{\rm end})/f \sim 10.4 - S_B/4$. Picking a value for S_B is a much riskier enterprise than guessing a value for μ. Since S_B is positive, clearly $\sigma(t_{\rm end})/f$ cannot be too large a positive number. While in principle it could be a very large negative number, it would be just as reasonable to pick $S_B = \mathcal{O}(20)$ which would result in $\sigma(t_{\rm end}/f) = \mathcal{O}(+5)$.

Now the natural question that arises at this point is how quickly does the dilaton field reach its minimum value? If it does so quickly enough and radiates its potential energy efficiently enough, then nothing of any cosmological significance takes place due to dilaton evolution. However, there is also the possibility that the dilaton *slowly rolls* to its minimum, allowing for a *second* inflationary era. It is this possibility that we wish to explore here.

What conditions must be satisfied for the dilaton to induce this second epoch of inflation? First, the dilaton potential energy density must exceed that of the surrounding radiation field generated by bubble collisions at the end of extended inflation. Since bubble collisions may be rather efficient at reheating the Universe, we expect that $\rho_R \simeq \exp[4\sigma(t_{\rm end})/f]\rho_V$. Thus, our first condition for slow roll to begin is that $V_{\rm eff}(\sigma(t)) \gg \exp(4\sigma(t_{\rm end})/f)\rho_V(a(t_{\rm end})/a(t))^4$ where $V_{\rm eff}(\sigma)$ was defined in Eq. (2.23). Numerical integrations of the equations of motion show that if radiation has this form, it only dominates for a small amount of time.

Our second constraint is that the slow-roll conditions[20] must also obtain. In general, these read:

$$\left|\frac{V''_{\rm eff}(\sigma)}{V_{\rm eff}}\right| \ll \frac{24\pi}{M_P^2}$$
$$\left|\frac{V'_{\rm eff}(\sigma)}{V_{\rm eff}}\right| \ll \frac{\sqrt{48\pi}}{M_P}. \tag{4.3}$$

Applied to our potential as given in Eq. (2.23), we arrive at the following conditions:

$$\left|\frac{1+2\bar{\sigma}/f}{\exp(-1-4\bar{\sigma}/f)+4\bar{\sigma}/f}\right| \ll \frac{3\pi}{4}\hat{f}^2$$
$$\left|\frac{1+4\bar{\sigma}/f}{\exp(-1-4\bar{\sigma}/f)+4\bar{\sigma}/f}\right| \ll (3\pi)^{1/2}\hat{f} \tag{4.4}$$

where we have shifted the dilaton field to $\bar{\sigma}/f = \sigma/f + \rho_T/\Delta(\langle\tilde{\phi}\rangle)$. When one of the these conditions is not satisfied, slow-rollover inflation ends. A graph of $V_{\rm eff}(\sigma)$ is shown in Fig. 2, and the regions where the slow-roll approximations are not valid are indicated. Note that if σ slowly rolls on the flat side of the potential (i.e., $\sigma < \langle\sigma\rangle$), an excessive number of e-folds of slow-rollover inflation results, erasing any memory of the extended inflation era.

Given all of the above information, we can compute all of the relevant quantities for the slow-rollover inflationary era, such as the number of e-folds, the density fluctuation spectrum, and the reheating temperature.

The number of e-folds is given by

$$N_{\rm slow-roll} = \int_{t_i}^{t_f} dt H(t) = -\frac{8\pi}{M_P^2}\int_{\bar{\sigma}_i}^{\bar{\sigma}_f} d\bar{\sigma}\,\frac{V_{\rm eff}(\bar{\sigma})}{V'_{\rm eff}(\bar{\sigma})}$$
$$= -2\pi\hat{f}^2\int_{x_i}^{x_f} dx\frac{4x+\exp(-1-4x)}{1+4x}, \tag{4.5}$$

where $x = \bar{\sigma}/f$. Here $\bar{\sigma}_i$ and $\bar{\sigma}_f$ are the values of the dilaton field at the beginning and the end of the slow-roll phase. In order to actually calculate $N_{\rm slow-roll}$, we need some more information. The problem is that $\bar{\sigma}_i$ depends on quantities such as the bounce action, ρ_V, $\rho_T/\Delta(\langle\tilde{\phi}\rangle)$ and $\hat{f}$. Let us assume some values for some of these quantities. We take $\hat{f} = 1$ and $\rho_T = 0$ as an example. Then $\bar{\sigma} = \sigma$. We can find numerically the value of

σ/f for which the slow-rollover conditions break down. This yields $x_f \equiv \sigma/f \simeq -0.38$ for $\sigma/f < 0$, and $x_f \simeq +0.025$ for $\sigma/f > 0$. The value for $x_i \equiv \sigma_i/f$ may also be found: $x_i \simeq -1.09 + \ln(M_P/\mu) - S_B/4$. Taking $\mu \simeq 10^{14}$ GeV, say, we find that $x_i \simeq 10.4 - S_B/4$. As an example, if we take $S_B = 20$, we find that $N_{\rm slow-roll} = 30$. Thus, by varying the bounce action we can obtain differing amounts of e-folds during this phase. A graph of $N_{\rm slow-roll}$ versus S_B for the above choices of μ and ρ_T, and for $\hat{f} = 1$ and 2 appears as solid lines in Fig. 3. Also shown as solid dots are the number of e-folds of inflation found by the result of numerical integration of the equations of motion. We should note that if this slow-roll phase occurs, the constraint that we have at least 65 e-folds during *extended inflation* is obviated.

Next, we compute the density fluctuation spectrum for this phase. The density contrast at horizon crossing is given in Eq. (3.11). Using the slow-roll equation of motion for σ and the relation between H^2 and $V_{\rm eff}$ as in Eq. (3.12), we may write this as

$$\begin{aligned} \left(\frac{\delta\rho}{\rho}\right)_\lambda &= 3\left(\frac{8\pi}{3M_P^2}\right)^{3/2} \frac{(V_{\rm eff})^{3/2}}{V'_{\rm eff}} \\ &= \hat{f}\sqrt{\frac{\pi}{192}}\left(\frac{m_{\tilde{\phi}}}{M_P}\right)^2 \exp(-1/2 - 2\rho_T/\Delta(\langle\tilde{\phi}\rangle)) \exp[2(\sigma_\lambda - \langle\sigma\rangle)/f] \\ &\quad \times \frac{[4(\sigma_\lambda - \langle\sigma\rangle)/f - 1 + \exp[-4(\sigma_\lambda - \langle\sigma\rangle)/f]]^{3/2}}{(\sigma_\lambda - \langle\sigma\rangle)/f}, \end{aligned} \tag{4.6}$$

where we have used the expression for $V_{\rm eff}$ in Eq. (2.23), and the fact that $\langle\sigma\rangle/f = -1/4 - \rho_T/\Delta(\langle\tilde{\phi}\rangle)$. Here, σ_λ refers to the value of σ at the time the physical scale λ crossed outside the horizon, as discussed in Section III. We see that the scale for the density contrast is set by the quantity

$$\left(\frac{\delta\rho}{\rho}\right)_\lambda \propto \exp[-1/2 - 2\rho_T/\Delta(\langle\tilde{\phi}\rangle)](\pi\hat{f}^2)^{1/2}\left(\frac{m_{\tilde{\phi}}}{M_P}\right)^2. \tag{4.7}$$

We see then that for $\rho_T \gtrsim 0$, and taking $m_{\tilde{\phi}} \simeq 10^{14}$ GeV, these fluctuations are rather small. However, their magnitude can be enhanced in theories where $\rho_T < 0$. In fact this can be used to place a constraint on how large (and negative) ρ_T can be.

In the previous section we found that $(\delta\rho/\rho)_\lambda \propto \lambda^{1/(\pi\widehat{f}^2-1)}$ or that $\log(\delta\rho/\rho)_\lambda = \text{const} + 1/(\pi\widehat{f}^2 - 1)\log\lambda$ in extended inflation [Eq. (3.14)]. It will also be true in slow-roll inflation that $\log(\delta\rho/\rho)_\lambda = \text{const} + 1/(\pi\widehat{f}^2 - 1)\log\lambda$, with a different additive constant but with the same slope, $1/(\pi\widehat{f}^2 - 1)$. The reason for this is the following: the slow-roll potential, $V(\sigma)$, is proportional to $[\sigma\exp(4\sigma/f)]$, rather than to $[\exp(4\sigma/f)]$ as in extended inflation — the new feature here being the multiplicative factor σ. From Eq. (4.6), $V^{3/2}/V' \propto [\sigma^{1/2}\exp(2\sigma/f)]$ during slow-roll inflation. But $[\exp(2\sigma/f)]$ is an exponentially steeper function than $[\sigma^{1/2}]$. Thus we are justified in ignoring the time-dependence of $[\sigma^{1/2}]$ relative to $[\exp(2\sigma/f)]$ in calculating the slope for $\log(\delta\rho/\rho)_\lambda$ versus $\log(\lambda)$ in slow-roll inflation. Thus the power of the scale λ will be very nearly the same in extended inflation and slow-roll inflation. Numerical integrations confirm this, as can be seen in Fig. 4.

We must now calculate the relative magnitude of the ratio of adiabatic density fluctuations formed during extended inflation and those formed during slow-roll inflation. First we define λ_t to be the co-moving scale which defines the end of extended inflation and the beginning of slow-roll inflation. Assuming that the slow-roll conditions hold continuously during extended and slow-roll inflation, we can calculate $\delta\rho/\rho$ evaluated at λ_t for both of these inflationary periods.

Since we have asssumed that σ is constant during tunneling and reheating, we can calculate this quantity as in Ref. (18). Taking as an example $\rho_T = 0$ and $\sigma \gg f/4$, we then have

$$
\begin{aligned}
(\delta\rho/\rho)_{\lambda_t \text{ slow-roll}} &\simeq f\frac{3}{4}\left(\frac{8\pi}{3M_P^2}\right)^{3/2}\left(\Delta(\langle\bar{\phi}\rangle)\exp(4\sigma_{\lambda_t})\frac{\sigma_{\lambda_t}}{f}\right)^{1/2} \\
(\delta\rho/\rho)_{\lambda_t \text{ exten-infl}} &\simeq f\frac{3}{4}\left(\frac{8\pi}{3M_P^2}\right)^{3/2}(\exp(4\sigma_{\lambda_t}/f)\rho_V)^{1/2}.
\end{aligned}
\tag{4.8}
$$

Using Eq. (4.2) for the expression for $\sigma(t_{\text{end}})$, and substituting in $\lambda_0 = \rho_V\exp(-S_B) = \mu^4\exp(-S_B)$, σ_{λ_t} becomes:

$$\sigma_{\lambda_t} = -\frac{1}{2}\ln\left(\frac{8\pi^2\widehat{f}^2}{3\pi\widehat{f}^2-1}\right) - \ln\left(\frac{\mu}{M_P}\right) - \frac{S_B}{4}. \tag{4.9}$$

Taking $\widehat{f} \simeq 1$ in the weak dependence of the log term in the above expression for σ_{λ_t}, and using the fact that $\Delta(\langle\widetilde{\phi}\rangle) = 2m_{\widetilde{\phi}}^4/(8\pi)^2$, we finally end up with

$$\frac{(\delta\rho/\rho)_{\lambda_t\ \text{exten-infl}}}{(\delta\rho/\rho)_{\lambda_t\ \text{slow-roll}}} = \frac{8\sqrt{2}\pi(\mu/m_{\widetilde{\phi}})^2}{|-2.19-4\ln(\mu/M_P)-S_B|^{1/2}}. \tag{4.10}$$

Thus we see that we can have more power on large scales by setting the above ratio greater than 1. Using only the constraint that $S_B > 0$, this is easy to satisfy.

In Fig. 4, we plot the density perturbation spectrum for the case of $\mu \simeq 10^{14}$GeV, $m_{\widetilde{\phi}}/\mu = 2.86$, $\widehat{f} = 1.5$ and $S_B = 25$; the above ratio equals 1 in this special case. We can see from the plot that the beginning of the slow-roll period of inflation does not satisfy the slow-roll conditions exactly—the curve attains a constant slope only after a few e-folds. But the density perturbations formed during the initial onslaught of slow-roll inflation are expected to be heavily dependent on the effects of reheating from bubble collisions, which we have ignored anyway. We thus cannot be too sure at present about the dependence of $\delta\rho/\rho$ on λ at the beginning of slow-roll inflation.

Extrapolating the constant slope part of the slow-roll density fluctuations back to $\lambda = \lambda_t$ gives us $(\delta\rho/\rho)_{\lambda_t\ \text{exten-infl}}$. In addition, $\delta\rho(\lambda_t)/\rho$ is defined as the value of $\delta\rho/\rho$ at the beginning of slow-roll inflation, which depends on the effects of reheating as we have already discussed in the previous paragraph. The squiggly line is drawn in to represent the presently uncalculated density perturbations generated from bubble collisions on the scale λ_t. In the most general case of S_B and $m_{\widetilde{\phi}}/\mu$, the two curves will be displaced from each other at λ_t (neglecting the initial slow-roll behavior), even though they will have the same slope of $1/(\pi\widehat{f}^2-1)$.

Thus we have seen that the larger scales leave the horizon during extended inflation, while the smaller scales leave during slow-roll inflation. Futhermore, even though the perturbation spectrum for each inflationary period is roughly scale invariant, the magnitude of the fluctuations can be very different for each. This leads to the natural idea that we can have more power on large scales. We must be careful, however, not to put too much power on these scales so as not to come into conflict with measurements of the temperature fluctuations in the microwave backround at large angular separations. We see then that the combination of the two inflationary phases lead to natural "double inflationary" spectra studied earlier.[21]

Finally, we compute the reheating temperature of the Universe due to the decay of the dilaton. First, we note that the dilaton can couple to a massive Higgs scalar (such as the colored Higgs triplet of GUT models) or to fermions with *bare* mass terms. The reason the fermions must have a bare mass is that Yukawa couplings between fermion and bosons are automatically scale invariant. Thus the dilaton would not couple to such fermions. We will consider the bosonic case here for concreteness. If χ denotes the Higgs boson under consideration, and m_χ its mass, then the dilaton will couple to χ as follows:

$$-\mathcal{L}_{\rm int} = m_\chi^2 \chi^2 \exp(2\sigma/f) = \frac{2m_\chi^2}{f}\chi^2\sigma + \dots. \tag{4.11}$$

Thus, the dilaton can decay into two χ's as long as the this decay is kinematically allowed, i.e., $m_\sigma > 2m_\chi$. For $m_{\tilde{\phi}}$ in the GUT range, this requires that m_χ be less than about 10^{11} to 10^{12} GeV. This is not a very stringent requirement, even if χ represents a color triplet that can mediate proton decay (and hence baryogenesis, depending on CP violation parameters).[22]

What must be checked now is whether the reheating process is fast or slow. The decay rate for the dilaton is given by

$$\Gamma_\sigma = \frac{1}{8\pi}\frac{m_\chi^4}{f^2 m_\sigma}\sqrt{1-\frac{4m_\chi^2}{m_\sigma^2}}. \tag{4.12}$$

Using $m_\sigma \simeq 10^{12}$GeV, $m_\chi \simeq 10^{11}$GeV, we find that $\Gamma_\sigma/H \simeq 10^{-5} \ll 1$ so that reheating proceeds slowly here and the reheating temperature is approximately equal to[23]

$$T_{\rm RH} \simeq \sqrt{\Gamma_\sigma M_P} \simeq 10^6 {\rm GeV}. \tag{4.13}$$

Although this temperature seems low, one can still generate a baryon asymmetry via the "way out" of equilibrium decays of the dilaton into Higgs colored triplets.[24]

V. CONCLUSIONS

From our calculations above, we see that theories with scale invariance realized in the non-linear mode can be used to construct a large class of theories which can undergo "safe" extended inflation. Little if any fine tuning required to solve the standard cosmological problems or the problems associated with extended inflationary models. Furthermore, as we saw in the previous section, extended inflation via hidden scale invariance can be used to set up initial conditions for a second phase of slow-rollover inflation. The combination of the two inflationary phases might be able to give rise to "designer" density fluctuation spectra, but in a far more natural way than has been done previously.[25]

The only "fly in the ointment" that we see with this class of models is that we do not know how to construct the larger theory (with the spontaneously broken scale invariance restored) of which our models are effective parametrizations. It would be nice to know if there is some class of scale-invariant theories that can give rise to a non-scale-invariant gravitational Lagrangian such as the one we appear to need in order to make our scenarios work. However, we expect that this will only be accomplished once we have a more detailed understanding of gravity at short distance scales. What we *can* say however, is that if we want to construct inflationary models that perform the tasks they were constructed to perform without any extreme fine-tuning, then we should take models with hidden scale invariance very seriously.

To conclude then, we have constructed a large class of models that realize extended inflation in an extremely natural fashion. As long as there is a field in the theory with a suitable potential, i.e., with a metastable false vacuum state, extended inflation will occur with none of the problems usually associated with inflationary models. These models are proof positive that the inflationary paradigm can actually be realized in practice.

ACKNOWLEDGEMENTS

R.H and Y.W. were supported in part by the Department of Energy grant # DE-AC02-76ER03066 while that of E.W.K. and S.V. was supported in part by the Department of Energy and NASA (grant #NAGW-1340). R.H. and E.W.K would like to thank the Aspen Center for Physics for hospitality while this work was in progress. We would also like to acknowledge useful discussions with Roberto Peccei, Michael Turner, and Frank Wilczek.

APPENDIX A

In this appendix, we will show that the extended inflationary equations of motion derived from either inflaton Lagrangian in Eq. (2.13) are the same. This is an important result since, a priori, there is no reason to choose either one.

The equations of motion derived from the lagrangian $\mathcal{L}_{\tilde{\phi}}$ are given in Section III. We will now derive the equations of motion obtained from $\mathcal{L}_{\phi}$, where

$$\mathcal{L}_{\phi} = \sqrt{-g}\left\{g^{\mu\nu}\frac{1}{2}\partial_\mu\phi\partial_\nu\phi - \exp(4\sigma/f)V_0[\exp(-\sigma/f)\phi)]\right\}. \tag{A.1}$$

In Section III, we found extended inflationary solutions to the equations of motions when we assumed that the vacuum energy density ρ_V was the dominant form of energy, and nglected the anomaly and cosmological constant terms. We *also* set $\tilde{\phi} = 0$. This we did by setting the metastable minimum for the potential at $\tilde{\phi} = 0$ and seeing that high temperature effects could allow for this. Thus it is important here to determine the potential (with 1-loop corrections) so that we may find its value of $\tilde{\phi}$ in its metastable minimum. Fortunately, the potentials are equivalent, and thus both minima occur at $\tilde{\phi} = \exp(-\sigma/f)\phi = 0$! We can see this by evaluating the operator from Eq. (2.16) which determines the 1-loop corrections. Since the dilaton σ, is being taken as a constant background field, we end up with the same operator and thus the same 1-loop corrections. Thus, the metastable minimum for our potential, $\hat{V}(\exp(-\sigma/f)\phi,\sigma) = \exp(4\sigma/f)[V(\exp(-\sigma/f)\phi) + \Delta(\exp(-\sigma/f)\phi)\sigma/f] + \Lambda$, occurs for $\exp(-\sigma/f)\phi = 0$.

Let us write down the entire action, again letting scale invariance be a symmetry only of the inflaton sector as we did in sections III and IV. It is

$$\begin{aligned} S[g,\phi,\sigma] \ = \ \int d^4x\,\sqrt{-g}\,\Bigg\{&-\frac{R}{16\pi G_N} + \frac{1}{2}g^{\mu\nu}\partial_\mu\sigma\partial_\nu\sigma + \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi \\ &- \exp(4\sigma/f)\left[V(\exp(-\sigma/f)\phi) + \Delta(\exp(-\sigma/f)\phi)\frac{\sigma}{f}\right] \end{aligned}$$

$$-\frac{(8\pi)^2\Delta(\langle\exp(-\sigma/f)\phi\rangle)}{2f^2}\exp\left(-1-\frac{4\rho_T}{\Delta(\langle\exp(-\sigma/f)\phi\rangle)}\right)\Bigg\}. \qquad (A.2)$$

We thus find the equations of motion to be

$$\begin{aligned}
\left(\frac{\dot a}{a}\right)^2+\frac{k}{a^2} &= \frac{8\pi G_N}{3}\left[\frac{1}{2}\dot\sigma^2+\frac{1}{2}\dot\phi^2+\widehat{V}\right]\\
\ddot\phi+3\frac{\dot a}{a}\dot\phi &= -\exp(4\sigma/f)\left[\frac{\partial V}{\partial\phi}+\frac{\partial\Delta}{\partial\phi}\frac{\sigma}{f}\right]\\
\ddot\sigma+3\frac{\dot a}{a}\dot\sigma &= \frac{1}{f}\Big[-4\widehat{V}-\exp(4\sigma/f)\Delta\\
&\quad -\exp(4\sigma/f)\left(\frac{\partial V}{\partial\sigma}+\frac{\partial\Delta}{\partial\sigma}\frac{\sigma}{f}\right)\Big], \qquad (A.3)
\end{aligned}$$

where we have left out the functional dependences: $\widehat{V} = \widehat{V}(\exp(-\sigma/f)\phi,\sigma)$, $V = V(\exp(-\sigma/f)\phi)$ and $\Delta = \Delta(\exp(-\sigma/f)\phi)$. Using the fact that $\dot\phi = \exp(\sigma/f)(\dot{\bar\phi}+\bar\phi\dot\sigma/f)$ and $\ddot\phi = \exp(\sigma/f)(\ddot{\bar\phi}+2\dot{\bar\phi}\dot\sigma/f+\bar\phi(\dot\sigma/f)^2+\bar\phi\ddot\sigma/f)$, we change variables from ϕ to $\bar\phi$ in our above equations of motion. Then since we are trapped in the false vacuum state, we set $\exp(-\sigma/f)\phi = \bar\phi = 0$. We end up with

$$\begin{aligned}
\left(\frac{\dot a}{a}\right)^2+\frac{k}{a^2} &= \frac{8\pi G_N}{3}\left[\frac{1}{2}\dot\sigma^2+\frac{1}{2}\exp(2\sigma/f)\dot{\bar\phi}^2+\widehat{V}(0,\sigma)\right]\\
\ddot{\bar\phi}+3\frac{\dot a}{a}\dot{\bar\phi} &= -2\frac{\dot\sigma}{f}\dot{\bar\phi}-\exp(-2\sigma/f)\frac{\partial\widehat{V}(0,\sigma)}{\partial\bar\phi}\\
\ddot\sigma+3\frac{\dot a}{a}\dot\sigma &= \frac{1}{f}\left[-4\widehat{V}(0,\sigma)-\exp(4\sigma/f)\Delta(0)\right], \qquad (A.4)
\end{aligned}$$

where we have used the fact that $\partial V/\partial\sigma = -\bar\phi V'/f$, the prime indicating the derivative with respect to $\bar\phi$. It is clear that since $\dot{\bar\phi} = 0$ and $\ddot{\bar\phi} = 0$ are solutions to our equations of motion (as they should be in extended inflation!), we end up with the same equations of motion as we had previously using $\mathcal{L}_{\bar\phi}$ (see Eqs. (3.3)). Thus, our cosmological extended-inflation solutions will be the same if we use either the Weyl or non-Weyl-invariant ϕ or $\bar\phi$ kinetic terms.

APPENDIX B

In this appendix we would like to show that if all sectors of the theory are scale invariant (except for the anomaly term from the 1-loop corrections to the potential which explicitly breaks the scale invariance of the theory), we do not have extended inflation. The action, which has been considered previously in different contexts, is

$$\begin{aligned} S[g,\tilde{\phi},\sigma] &= \int d^4x\,\sqrt{-g}\,\Bigg\{-\frac{\exp(2\sigma/f)\,R}{16\pi G_N}+\frac{f^2}{2}g^{\mu\nu}\partial_\mu\exp(\sigma/f)\,\partial_\nu\exp(\sigma/f) \\ &\quad +\frac{1}{2}g^{\mu\nu}\exp(2\sigma/f)\partial_\mu\tilde{\phi}\partial_\nu\tilde{\phi}-\exp(4\sigma/f)\left[V(\tilde{\phi})+\Delta(\tilde{\phi})\frac{\sigma}{f}\right] \\ &\quad -\exp\left(-1-4\rho_T/\Delta(\langle\tilde{\phi}\rangle)\right)\frac{m_\phi^4}{f^2}\Bigg\}. \end{aligned} \tag{B.1}$$

We refer the reader to Section II for the derivation of the above action. We can see that this action is *almost* that of the Brans-Dicke theory by making the following redefinition of the dilaton field σ in terms of the Jordan-Brans-Dicke (JBD) field Φ. Setting

$$\Phi=\frac{1}{16\pi G_N}\exp(2\sigma/f), \tag{B.2}$$

we can rewrite the action as

$$\begin{aligned} S[g,\Phi,\chi] &= \int d^4x\,\sqrt{-g}\,\Bigg\{-\Phi R+\omega g^{\mu\nu}\frac{\partial_\mu\Phi\partial_\nu\Phi}{\Phi}+\frac{1}{2}(16\pi G_N\Phi)g^{\mu\nu}\partial_\mu\tilde{\phi}\partial_\nu\tilde{\phi} \\ &\quad -(16\pi G_N\Phi)^2\left[V(\tilde{\phi})+\Delta(\tilde{\phi})\ln(16\pi G_N\Phi)^{1/2}\right] \\ &\quad -\exp\left(-1-4\rho_T/\Delta(\langle\tilde{\phi}\rangle)\right)\frac{m_\phi^4}{f^2}\Bigg\}. \end{aligned} \tag{B.3}$$

where now $\omega=2\pi(f/M_P)^2$. Except for the fact that the inflaton potential is now coupled to the JBD field, we see that we end up with a Brans–Dicke action. However it is precisely this coupling that prevents extended inflation.

Upon working out the equations of motion in this frame (the so-called *Jordan conformal frame*), we find that they do not lead to sensible analytical solutions. Thus in search of simple analytical solutions, we perform a conformal transformation $\Omega^2\overline{g}_{\mu\nu}=g_{\mu\nu}$,

$R = \Omega^{-2}\overline{R} - 6\Omega^{-3}\overline{\Box}\Omega$, into the Einstein frame. We then find that $\Omega^2 = (16\pi G_N \Phi)^{-1} = \exp(-2\sigma/f)$, so that the action becomes

$$\begin{aligned}
\overline{S}[\overline{g},\psi,\tilde{\phi}] &= \int d^4x\sqrt{-\overline{g}}\,\Big[-\frac{1}{16\pi G_N}\overline{R} + \frac{1}{2}\overline{g}^{\mu\nu}\,\partial_\mu\psi\partial_\nu\psi \\
&\quad + \frac{1}{2}\overline{g}^{\mu\nu}\partial_\mu\tilde{\phi}\partial_\nu\tilde{\phi} - \Big[V(\tilde{\phi}) + \frac{1}{2}\Delta(\tilde{\phi})\psi/\psi_0\Big] \\
&\quad - \exp(-2\psi/\psi_0)\exp\left(-1 - 4\rho_V/\Delta(\langle\tilde{\phi}\rangle)\right)\frac{m_\phi^4}{f^2}\Big], \qquad \text{(B.4)}
\end{aligned}$$

where

$$\begin{aligned}
\psi &= \psi_0 \ln(16\pi G_N\Phi) = [(3+2\omega)/16\pi G_N]^{1/2}\ln(16\pi G_N\Phi) \\
&= (1 + (3M_P^2)/(4\pi f^2))^{-1/2}\sigma. \qquad \text{(B.5)}
\end{aligned}$$

Since we are assuming that the false vacuum energy ρ_V dominates the energy density in the metastable vacuum at $\tilde{\phi} = 0$, we will take $V(\tilde{\phi}) = \rho_V$, and $\Delta(0) = \Lambda = 0$. We then find that $\dot{\tilde{\phi}} = \ddot{\tilde{\phi}} = 0$ is a solution to the equation of motion for $\tilde{\phi}$; we are indeed "stuck" in the false vacuum as required. We are left with the equations governing the evolution of the scale factor and the dilaton. They are

$$\begin{aligned}
\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} &= \frac{8\pi G_N}{3}\left[\frac{1}{2}\dot{\psi}^2 + \rho_V\right] \\
\ddot{\psi} + 3\frac{\dot{a}}{a}\dot{\psi} &= 0. \qquad \text{(B.6)}
\end{aligned}$$

Clearly $\psi =$ const is not a viable solution, as the JBD field is then constant and we are led to Guth's "old" exponential inflation and the "graceful exit problem."

Since $\dot{\psi} = \dot{\psi}(0)(a(t)/a(0))^{-3}$, the solution for $k = 0$ is

$$\frac{a(t)}{a(0)} = \cosh\left(\sqrt{24\pi G_N\rho_V}\; t\right) + \sqrt{1 + \frac{\dot{\psi}^2(0)}{2\rho_V}}\sinh\left(\sqrt{24\pi G_N\rho_V}\; t\right) \qquad \text{(B.7)}$$

while the Hubble parameter is

$$\begin{aligned}
H(t) &= \frac{24\pi G_N\rho_V}{3} \\
&\quad \times \left[\frac{\sinh(\sqrt{24\pi G_N\rho_V}\;\, t) + \sqrt{1+\dot{\psi}^2(0)/(2\rho_V)}\cosh(\sqrt{24\pi G_N\rho_V}\;\, t)}{\cosh(\sqrt{24\pi G_N\rho_V}\;\, t) + \sqrt{1+\dot{\psi}^2(0)/(2\rho_V)}\sinh(\sqrt{24\pi G_N\rho_V}\;\, t)}\right]. \qquad \text{(B.8)}
\end{aligned}$$

Thus, exponential growth is achieved for large t. It is found numerically that even letting the arbitrary parameters vary, exponential inflation with ψ = constant (or σ = constant) is achieved after only a few e-folds. However, since constant ψ leads to a constant Hubble parameter, we are driven to an old inflation type of scenario suffering as usual from the graceful exit problem. This result also holds when a more careful accounting of the time dependence of the bubble nucleation rate is done.[26]

REFERENCES

1. A. D. Linde, *Phys. Lett.* **108B**, 389 (1982); A. Albrecht and P. J. Steinhardt, *Phys. Rev. Lett.* **48**, 1220 (1982).
2. A. D. Linde, *Phys. Lett.* **129B**, 177 (1983).
3. D. La and P. J. Steinhardt, *Phys. Rev. Lett.* **62**, 376 (1989); *Phys. Lett.* **220B**, 375 (1989).
4. A. H. Guth, *Phys. Rev. D* **23**, 347 (1981).
5. A. H. Guth and E. J. Weinberg, *Nucl. Phys.* **B212**, 321 (1983).
6. P. Jordan, *Zeit. Phys.* **157**, 112 (1959); C. Brans and C. H. Dicke, *Phys. Rev.* **24**, 925 (1961).
7. D. La, P. J. Steinhardt, and E. Bertschinger, *Phys. Lett.* **231B**, 231 (1989); E. J. Weinberg, *Phys. Rev. D* **40**, 3950 (1989).
8. R. D. Reasenberg, et al., *Astrophys. J.* **234**, L219 (1979).

9. F. S. Accetta and J. J. Trester, *Phys. Rev. D* **39**, 2854 (1989); P. J. Steinhardt and F. S. Accetta, *Phys. Rev. Lett.* **64**, 2740 (1990); R. Holman, E. W. Kolb, and Y. Wang, *Phys. Rev. Lett.* **65**, 17 (1990).

10. W. Buchmüller and N. Dragon, *Phys. Lett.* **195B**, 417 (1987); R. D. Peccei, J. Solà, and C. Wetterich, *Phys. Lett.* **195B**, 183 (1987).

11. B. Gradwohl and G. Kälbermann, *Phys. Lett.* **208B**, 198 (1988).

12. S. Coleman, *Aspects of Symmetry* (Cambridge University Press, Cambridge, 1985).

13. J. H. Lowenstein, *Phys. Rev.* **D4**, 2281 (1971); R. Crewther, *Phys. Rev. Lett.* **28**, 1421 (1972); M. S. Chanowitz and J. Ellis, *Phys. Lett.* **40B**, 397 (1972).

14. P. Ramond, *Field Theory, a Modern Primer* (Benjamin/Cummings, New York, 1981).

15. N. D. Birrell and P. C. W. Davies, *Quantum Fields in Curved Space* (Cambridge University Press, Cambridge, 1982).

16. W. Buchmüller and D. Wyler, *Phys. Lett.* **B249**, 281 (1990).

17. R. Holman, E. W. Kolb, S. L. Vadas, Y. Wang, and E. J. Weinberg, *Phys. Lett.* **237B**, 37 (1990).

18. E. W. Kolb, D. S. Salopek, and M. S. Turner, Fermilab preprint FERMILAB-PUB-90-116-A, *Phys. Rev. D* (to be published).

19. A. Guth and S.-Y. Pi, *Phys. Rev. Lett.* **49**, 1110 (1982); S. W. Hawking *Phys. Lett.* **115B**, 295 (1982); J. Bardeen, P. J. Steinhardt, and M. S. Turner, *Phys. Rev. D* **28**, 679 (1983); A. A. Starobinski, *Phys. Lett.* **117B**, 175 (1982).

20. P. Steinhardt and M. S. Turner, *Phys. Rev. D* **29**, 2162 (1984).

21. J. Silk and M. S. Turner, *Phys. Rev. D* **35**, 419 (1987).

22. G. D. Coughlan, et al., *Phys. Lett.* **158B**, 401(1985); G. D. Coughlan, et al., *Phys. Lett.* **160B**, 249(1985).

23. E. W. Kolb and M. S. Turner, *The Early Universe*, (Addison-Wesley, Redwood City, Ca., 1990).

24. A. Albrecht, P. J. Steinhardt, M. S. Turner, and F. Wilczek, *Phys. Rev. Lett.* **48**, 1437 (1982); L. Abbott, E. Farhi, and M. Wise, *Phys. Lett.* **117B**, 29 (1982); A. Dolgov and A. D. Linde, *Phys. Lett.* **116B**, 329 (1982).

25. D. S. Salopek, J. R. Bond, and J. M. Bardeen, *Phys. Rev. D* **40**, 1753 (1989); L. A. Kofman and A. D. Linde, *Nucl. Phys.* **B282**, 555 (1987); H. M. Hodges, G. R. Blumenthal, L. A. Kofman, and J. R. Primack, *Nucl. Phys.* **B335** 197 (1990).

26. R. Holman, E.W. Kolb, S.L. Vadas, and Y. Wang, *Phys. Lett.* **250B**, 24 (1990).

FIGURE CAPTIONS

Fig. 1: The number of e-folds in *extended* inflation versus $S_B + 4\sigma_0/f$ for three values of f. Here, S_B is the bounce action and f is the dilaton decay constant. The energy scale of extended inflation is choosen to be that typical of GUT scales. Note that a wide range of parameters give a number of e-folds in the range 30 to 65.

Fig. 2: The dilaton potential during slow-roll inflation for $\rho_T = 0$. Bars indicate regions where the slow-roll approximations break down for the indicated values of f.

Fig. 3: The number of e-folds in *slow-roll* inflation versus the bounce action S_B for different values of f. The curves are calculations using the slow-roll approximations, while the dots are the results from numerical integrations of the full equations of motion. There is no inflation for $S_B \simeq 42$ because this corresponds to the dilaton starting from the minimum of its potential. If $S_B > 42$, the dilaton started rollover inflation on the flat side of the potential, and many e-folds of inflation result.

Fig. 4: An example of the spectrum of density fluctuations for $f = 1.5M_P$ and $S_B = 25$. The fluctuations have a slope $\delta\rho/\rho \propto \lambda^{1/(\pi\hat{f}^2-1)}$ for both the extended and slow-roll inflationary periods. The squiggly line indicates the length scale of the transition between the two epochs. Fluctuations formed from bubble collisions and reheating should appear around this scale. The density fluctuations from both epochs are shown with the same amplitude if extrapolated to the same scale. The relative amplitude actually depends upon $m_{\tilde{\phi}}/\rho_V^{1/4}$. The scale λ_t depends upon the number of e-folds of slow-roll inflation: $\lambda_t \sim \exp(N_{\rm slow-roll} - 45)$ Mpc. In this example $N_{\rm slow-roll} = 48$, and $\lambda_t \sim 20$ Mpc.

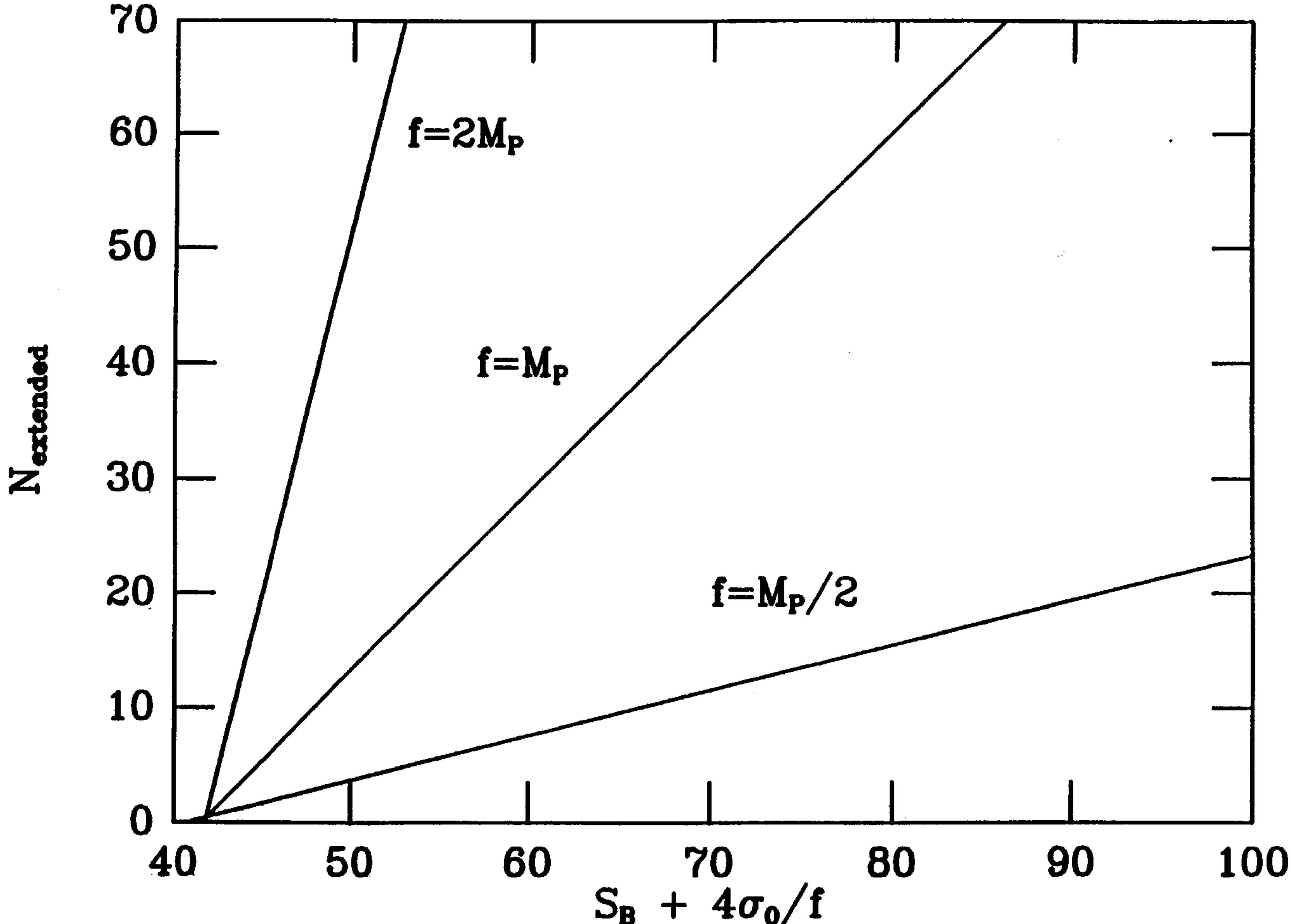

70
60
50
40
30
20
10
0
40
50
60
70
80
90
100
N extended
S B + 4σ 0 /f
f=2M P
f=M P
f=M P /2

FIGURE 1.

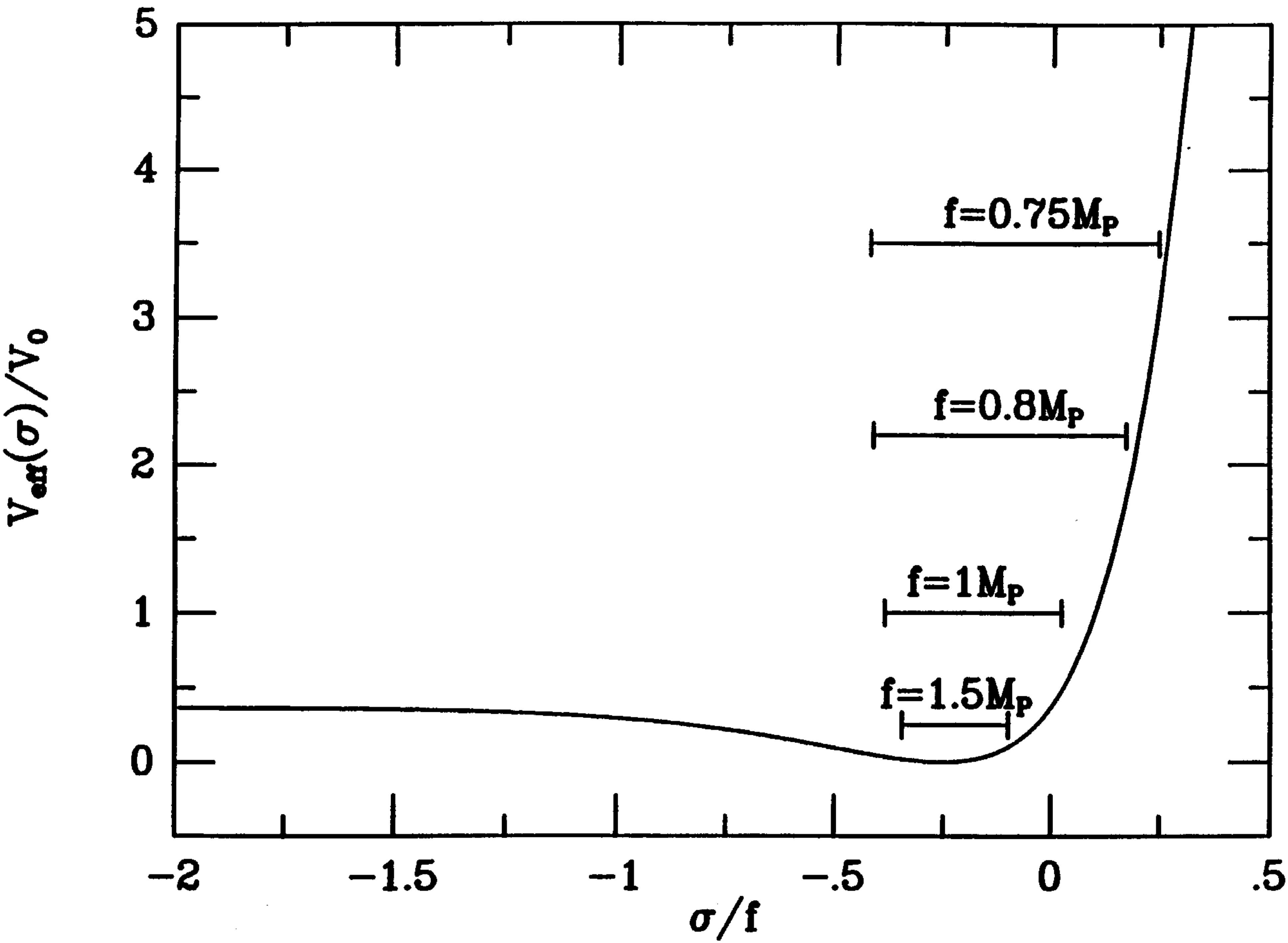
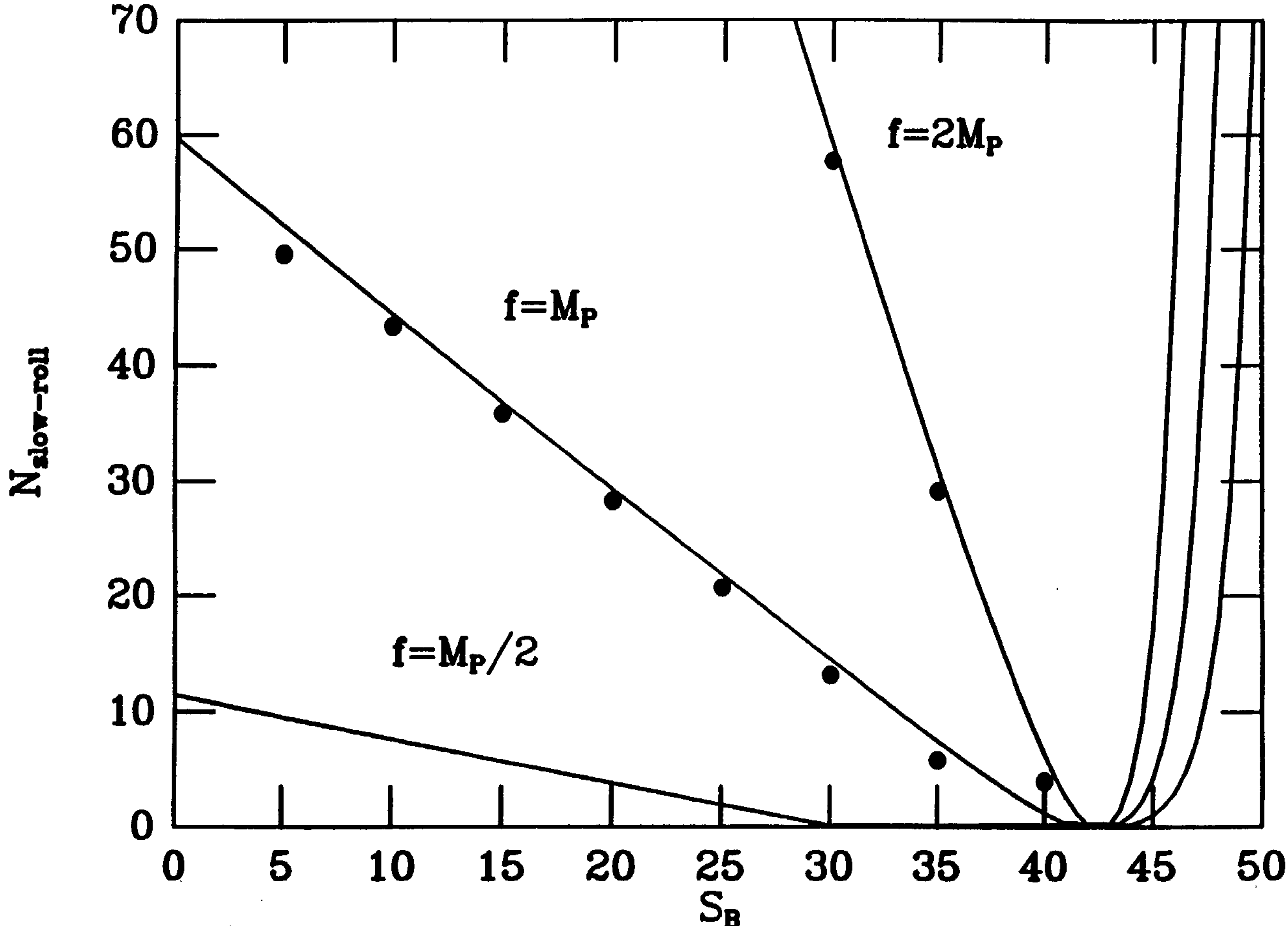
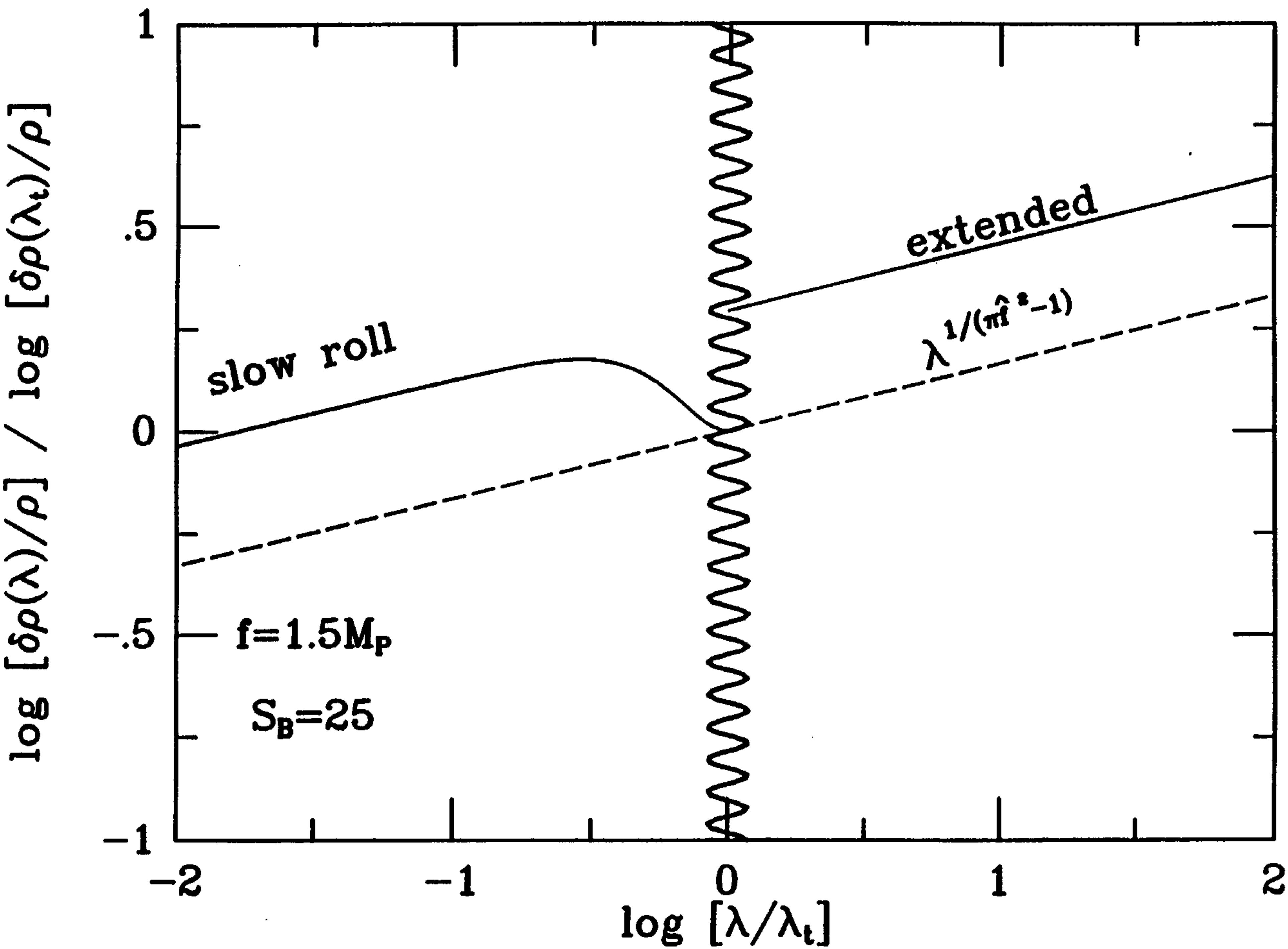


FIGURE 1.



70
60
50
40
30
20
10
0
0
5
10
15
20
25
30
35
40
45
50
S_B
$N_{slow-roll}$
$f=2M_P$
$f=M_P$
$f=M_P/2$

FIGURE 3.



extended
slow roll
$\lambda^{1/(\pi\hat{f}^2-1)}$
$f=1.5M_P$
$S_B=25$
1
.5
0
-.5
-1
-2
-1
0
1
2
$\log [\lambda/\lambda_t]$
$\log [\delta\rho(\lambda)/\rho]$ / $\log [\delta\rho(\lambda_t)/\rho]$

FIGURE 4.