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NATURAL INFLATION WITH PSEUDO-NAMBU-GOLDSTONE BOSONS

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ABSTRACT

We show that a pseudo-Nambu-Goldstone boson, with a potential of the form $V(\phi) = \Lambda^4[1 \pm \cos(\phi/f)]$, can naturally give rise to an epoch of inflation in the early universe. Successful inflation can be achieved if $f \sim m_{pl}$ and $\Lambda \sim m_{GUT}$. Such mass scales arise in particle physics models with a large gauge group that becomes strongly interacting at a scale $\sim \Lambda$, *e.g.*, as can happen in the hidden sector of superstring theories. The density fluctuation spectrum is non-scale-invariant, with extra power on large length scales.

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The inflationary universe model was proposed [1] to solve several cosmological puzzles, notably the horizon, flatness, and monopole problems. During the inflationary epoch, the energy density of the universe is dominated by a (nearly constant) false vacuum energy term $\rho \simeq \rho_{vac}$, and the scale factor $R(t)$ of the universe expands exponentially: $R(t) = R(t_1)e^{H(t-t_1)}$, where $H = \dot{R}/R$ is the Hubble parameter, $H^2 = 8\pi G\rho/3 - k/R^2$ ($\simeq 8\pi G\rho_{vac}/3$ during inflation), and t_1 is the time at the beginning of inflation. If the interval of exponential expansion satisfies $t - t_1 \gtrsim 60H^{-1}$, a small causally connected region of the universe grows to a sufficiently large size to explain the observed homogeneity and isotropy of the universe, to dilute any overdensity of magnetic monopoles, and to flatten the spatial hypersurfaces, $\Omega \equiv 8\pi G\rho/3H^2 \to 1$.

To satisfy a combination of constraints on subsequent inflationary models [2], in particular, sufficient inflation (≥ 60 e-foldings of the scale factor) and microwave background anisotropy limits [3] on the generation of density fluctuations, the potential of the field responsible for inflation (the *inflaton*) must be very flat. For a general class of inflation models involving a single slowly-rolling field (including new [4], chaotic [5], and double field inflation [6]), the ratio of the height to the (width)^4 of the potential must satisfy [7]

$$\chi \equiv \Delta V/(\Delta\phi)^4 \leq \mathcal{O}(10^{-6} - 10^{-8}), \tag{1}$$

where ΔV is the change in the potential $V(\phi)$ and $\Delta\phi$ is the change in the field ϕ during the slowly rolling portion of the inflationary epoch. (For extended inflation, $\chi \leq \mathcal{O}(10^{-15})$ [8].) Consequently, the inflaton must be extremely weakly self-coupled, with effective quartic self-coupling constant $\lambda_\phi < \mathcal{O}(\chi)$ [7] (in realistic models, $\lambda_\phi < 10^{-12}$).

While a number of workable inflation models (satisfying Eqn.(1)) have been proposed [9], none of them is compelling from a particle physics standpoint. In some cases, the tiny coupling λ_ϕ is simply postulated *ad hoc* at tree level, and then must be fine-tuned to keep it small in the presence of radiative corrections. But this merely replaces a cosmological naturalness problem with unnatural particle physics. The situation is improved in models where the smallness of λ_ϕ is protected by a symmetry, *e.g.*, supersymmetry. In these cases [10], λ_ϕ may arise from a small ratio of mass scales; however, the required mass hierarchy, while stable, is itself unexplained, and is postulated solely in order to generate successful inflation. It would be preferable if such a hierarchy, and thus inflation itself, arose dynamically in particle physics models, instead of being imposed upon them.

Nambu-Goldstone bosons are ubiquitous in particle physics models: they arise whenever a global symmetry is spontaneously broken. If there is additional explicit symmetry breaking, these particles become pseudo-Nambu-Goldstone bosons (PNGBs). In models with a large global symmetry breaking scale f, PNGBs are very weakly interacting, since their couplings are suppressed by inverse powers of f. For example, in 'invisible' axion models [11,12] with Peccei-Quinn scale $f_{PQ} \sim 10^{15}$ GeV, the axion self-coupling is $\lambda_a \sim (\Lambda_{QCD}/f_{PQ})^4 \sim 10^{-64}$. (This simply reflects the hierarchy between the QCD and GUT scales, which arises from the slow logarithmic running of α_{QCD}.) Due to the nonlinearly realized global symmetry, the potential for PNGBs is exactly flat at tree level. The symmetry may be explicitly broken by loop corrections, as in schizon [13] and axion [11] models. In the case of axions, for example, the PNGB mass arises from non-perturbative gauge-field configurations (instantons) through the chiral anomaly. When the associated gauge group becomes strong at a mass scale Λ, instanton effects give rise to a periodic potential of height $\sim \Lambda^4$ for the PNGB field [14]. Since the nonlinearly realized symmetry is restored as $\Lambda \to 0$, the flatness of the PNGB potential is natural in the sense of 't Hooft [15].

The resulting PNGB potential is generally of the form

$$V(\phi) = \Lambda^4[1 \pm \cos(N\phi/f)]\,. \tag{2}$$

We will take the positive sign in Eqn.(2) (this choice has no effect on our results) and, unless otherwise noted, assume $N = 1$, so the potential, of height $2\Lambda^4$, has a unique minimum at $\phi = \pi f$ (we assume the periodicity of ϕ is $2\pi f$). We show below that, for appropriately chosen values of the mass scales, namely $f \sim m_{pl}$ and $\Lambda \sim m_{GUT} \sim 10^{15}$ GeV, the PNGB field ϕ can drive inflation. (Note that this is consistent with Eqn.(1), since $\chi \sim (\Lambda/f)^4 \sim 10^{-13}$.) These mass scales can arise naturally in particle physics models. For example, in the hidden sector of superstring theories, if a large non-Abelian group remains unbroken, the running gauge coupling can become strong at the GUT scale [16]; then the role of the PNGB inflaton might be played, *e.g.*, by the model-independent axion [17].

For temperatures $T \lesssim f$, the global symmetry is spontaneously broken, and the field ϕ describes the phase degree of freedom around the bottom of a Mexican hat. Since ϕ thermally decouples at a temperature $T \sim f^2/m_{pl} \sim f$, we assume it is initially laid down at random between 0 and $2\pi f$ in different causally connected regions. Within each Hubble volume, the evolution of the field is described by

$$\ddot{\phi} + 3H\dot{\phi} + \Gamma\dot{\phi} + V'(\phi) = 0\ , \tag{3}$$

where Γ is the decay width of the inflaton. In the temperature range $\Lambda \lesssim T \lesssim f$, the potential $V(\phi)$ is dynamically irrelevant, because the forcing term $V'(\phi)$ is negligible compared to the Hubble-damping term. (In addition, for axion models, $\Lambda \to 0$ as $T/\Lambda \to \infty$ due to the high-temperature suppression of instantons [14].) Thus, in this temperature range, aside from the smoothing of spatial gradients in ϕ, the field does not evolve. Finally, for $T \lesssim \Lambda$, in regions of the universe with ϕ initially near the top of the potential, the field starts to roll slowly down the hill toward the minimum. In those regions, the energy density of the universe is quickly dominated by the vacuum contribution ($V(\phi) \simeq 2\Lambda^4 \gtrsim \rho_{rad} \sim T^4$), and the universe expands exponentially. Since the initial conditions for ϕ are random, our model is closest in spirit to the chaotic inflationary scenario [5].

To successfully solve the cosmological puzzles of the standard cosmology, an inflationary model must satisfy a variety of constraints.

1) *Slow-Rolling Regime.* The field is said to be slowly rolling (SR) when its motion is overdamped, *i.e.*, $\ddot{\phi} << 3H\dot{\phi}$ (*n.b.*, we assume $\Gamma \ll H$). Two necessary conditions must be satisfied for the motion to be overdamped [2]:

$$|V''(\phi)| \lesssim 9H^2\ ,\ \text{i.e.,}\quad \sqrt{\frac{2\,|\cos(\phi/f)|}{1+\cos(\phi/f)}} \lesssim \frac{\sqrt{48\pi}f}{m_{pl}} \tag{4}$$

and

$$\left|\frac{V'(\phi)m_{pl}}{V(\phi)}\right| \lesssim \sqrt{48\pi}\ , \text{ i.e.,} \quad \frac{\sin(\phi/f)}{1+\cos(\phi/f)} \lesssim \frac{\sqrt{48\pi}f}{m_{pl}}\ . \tag{5}$$

From Eqns. (4) and (5) the existence of a broad SR regime requires $f \geq m_{pl}/\sqrt{48\pi}$ (required below for other reasons). The SR regime ends when ϕ reaches a value ϕ_2, at which one of the inequalities (4) or (5) is violated. For example, for $f = m_{pl}$, the end of the SR epoch is at $\phi_2/f = 2.98$ (nearly at the minimum of the potential), while for $f = m_{pl}/\sqrt{24\pi}$, $\phi_2/f = 1.9$. Clearly, as f grows, ϕ_2/f approaches π. (Here and below, we assume inflation begins at a field value $0 < \phi_1/f < \pi$; since the potential is symmetric about its minimum, we can just as easily consider the case $\pi < \phi_1/f < 2\pi$.)

2) *Sufficient inflation.* We demand that the scale factor of the universe inflates by at least 60 e-foldings during the SR regime (in principle, there could be some additional inflation later),

$$\begin{aligned} N_e(\phi_1,\phi_2,f) \equiv \ln(R_2/R_1) &= \int_{t_1}^{t_2} H dt = \frac{-8\pi}{{m_{pl}}^2}\int_{\phi_1}^{\phi_2}\frac{V(\phi)}{V'(\phi)}d\phi \\ &= \frac{16\pi f^2}{{m_{pl}}^2}\ln\left[\frac{\sin(\phi_2/2f)}{\sin(\phi_1/2f)}\right] \geq 60\ . \end{aligned} \tag{6}$$

Using Eqns. (4) and (5) to determine ϕ_2 as a function of f, the constraint (6) determines the maximum value (ϕ_1^{max}) of ϕ_1 consistent with sufficient inflation. For example, the initial value of the field must satisfy $\phi_1/f \leq 2.1, 0.6, 10^{-2}$ and 10^{-40} for $f = 3m_{pl}, m_{pl}, m_{pl}/2$, and $m_{pl}/\sqrt{24\pi}$ respectively.

The fraction of the universe with $\phi_1 \in [0, \phi_1^{max}]$ will inflate sufficiently. If we assume that ϕ_1 is randomly distributed between 0 and πf from one horizon volume to another, the probability of being in such a region is ϕ_1^{max}/π. For $f = 3m_{pl}, m_{pl}$, and $m_{pl/2}$, the probability is 0.7, 0.2, and 3×10^{-3}. Since the fraction of the universe that inflates sufficiently drops precipitously with decreasing f, the scenario is only tenable (in the sense that initial conditions do not need to be severely fine-tuned) for f near m_{pl}.

3) *Density Fluctuations.* Inflationary models generate density fluctuations with amplitude [18]

$$\delta\rho/\rho \simeq H^2/\dot{\phi}\ , \tag{7}$$

where the right hand side is evaluated at the time when the fluctuation crossed outside the horizon during inflation, and $\delta\rho/\rho$ is the amplitude of the perturbation when it crosses back inside the horizon after inflation. Fluctuations on observable scales are produced during the time period (60 - 50) e-foldings before the end of inflation. The largest amplitude perturbations are produced at 60 e-foldings before the end of inflation,

$$\frac{\delta\rho}{\rho} \simeq \frac{3\Lambda^2 f}{m_{pl}^3}\left(\frac{8\pi}{3}\right)^{3/2}\frac{[1+\cos(\phi_1^{max}/f)]^{3/2}}{\sin(\phi_1^{max}/f)}\ . \tag{8}$$

Constraints on the anisotropy of the microwave background [3] require $\delta\rho/\rho \leq 5\times 10^{-5}$; applying this to Eqn.(8), we find, *e.g.*,

$$\Lambda \leq 5\times 10^{15}\,\text{GeV for } f = m_{pl} \tag{9a}$$

$$\Lambda \leq 9\times 10^{14}\,\text{GeV for } f = m_{pl}/2\ . \tag{9b}$$

Thus, to generate the fluctuations responsible for large-scale structure, Λ should be comparable to the GUT scale, and the inflaton mass $m_\phi = \Lambda^2/f \sim 10^{11} - 2\times 10^{12}$ GeV.

In this model, the fluctuations deviate from a scale-invariant spectrum. For $f \lesssim 3m_{pl}/4$, the amplitude grows with mass scale M as [2] $\delta\rho/\rho \sim M^{m_{pl}^2/48\pi f^2}$. As a result, the primordial power spectrum (at fixed time) is a power law, $|\delta_k|^2 \sim k^n$, with spectral index $n \simeq 1 - (m_{pl}^2/8\pi f^2)$. Since there is extra power on large scales, this may have important implications for large-scale structure: if $n \simeq 0.6$, corresponding to $f \simeq m_{pl}/3$, the observed large-scale velocity flows can be more easily accounted for [19] (than if $n = 1$).

In new inflation models, satisfying the density perturbation constraint usually implies a very large number of e-foldings of the scale factor. Here, there will be many regions of the universe which inflate less than 60 e-foldings and which generate acceptable density fluctuations. Thus, this model might be easily embedded in double-inflation scenarios that also seek to produce extra power on large scales [20].

4) *Quantum Fluctuations.* For the semi-classical treatment of the scalar field to be valid, the initial value of the field must be larger than the characteristic quantum fluctuations in ϕ, *i.e.*, $\phi_1 \geq \Delta\phi = H/2\pi$. For example, this requires $\phi_1/f > 10^{-7}$ for $f = m_{pl}$, and $\phi_1/f > 6\times 10^{-9}$ for $f = m_{pl}/2$. Since $\phi_1^{max} \gg H/2\pi$ over the entire parameter range of interest, this constraint does not place significant restrictions on the model.

5) *Reheating.* At the end of the SR regime, the field ϕ begins to oscillate about the minimum of the potential, and gives rise to particle and entropy production. The decay of ϕ into fermions and gauge bosons reheats the universe to a temperature [2]

$$T_{RH} = \left(\frac{45}{4\pi^3 g_*}\right)^{1/4} \min\left[\sqrt{H(\phi_2)m_{pl}}\ , \sqrt{\Gamma m_{pl}}\right] , \tag{10}$$

where g_* is the number of relativistic degrees of freedom. On dimensional grounds, the decay rate is

$$\Gamma \simeq g^2 {m_\phi}^3/f^2 = g^2\Lambda^6/f^5 , \tag{11}$$

where g is an effective coupling constant. (For example, in the original axion model [12], $g \propto \alpha_{EM}$ for two-photon decay, and $g^2 \propto (m_\psi/m_\phi)^2$ for decays to light fermions ψ.) For $f = m_{pl}$ and $g_* = 10^3$, we find $T_{RH} = \min[6\times 10^{14}\text{ GeV}\ , 10^8 g\text{ GeV}]$. Since we generally expect $g \lesssim 1$, the reheat temperature will be $T_{RH} \lesssim 10^8$ GeV, too low for conventional GUT baryogenesis, but high enough if baryogenesis takes place at the electroweak scale. Alternatively, the baryon asymmetry can be produced directly during reheating through baryon-violating decays of ϕ or its decay products. The resulting baryon-to-entropy ratio is $n_B/s \simeq \epsilon T_{RH}/m_\phi \sim \epsilon g\Lambda/f \sim 10^{-4}\epsilon g$, where ϵ is the CP-violating parameter [9]; provided $\epsilon g \gtrsim 10^{-6}$, the observed asymmetry can be generated.

6) *Spatial Gradients and Topological Defects.* In the discussion above, we assumed that the Universe is vacuum-dominated ($\rho \simeq V(\phi)$) when ϕ begins to roll down the hill. Otherwise, the onset of the inflationary epoch could be delayed or even prevented altogether. There are several sources of energy density which could be problematic: spatial ϕ gradients, global cosmic strings, and domain walls. For a spatial fluctuation

with amplitude $\delta\phi$ and wavelength L, the gradient energy density is $(\nabla\phi)^2 \simeq (\delta\phi/L)^2$. Requiring this to be less than the potential $V(\phi)$ at the onset of inflation leads to the constraint $LH \gtrsim 3(\delta\phi/f)(f/m_{pl})$; in order not to dominate the energy density of the universe, large amplitude gradients ($\delta\phi \sim f$) must have wavelengths longer than the Hubble length, $L \gtrsim H^{-1}$, at $T \sim \Lambda$ [21]. However, gradients on subhorizon scales ($L \lesssim H^{-1}$) are expected to be smoothed out by the beginning of inflation: since the potential is inoperative for $T \gtrsim \Lambda$, these gradients are damped (redshifted away) by the Hubble expansion [22]. Thus, the gradient energy density at $T \sim \Lambda$ is at most comparable to the potential and quickly becomes sub-dominant [23]; the net effect is to delay only slightly the onset of inflation. This conclusion follows as long as there also exists a long-wavelength mode ($L >> H^{-1}$) with amplitude $\delta\phi \sim f$; in this case, there will be regions with $\phi_1 < \phi_1^{max}$ which inflate [23]. Since ϕ is initially Poisson distributed, we expect roughly equal power on all scales at $T \sim f$, *i.e.*, $\delta\phi_L \sim f$ independent of L (at least for $L \gtrsim m_{pl}^{-1}$); consequently, gradients should be innocuous, and the probability for inflation will be given by the estimates in section 2 above.

To be conservative, however, we can assume that we must be in a region of the universe that is homogeneous over at least $\mathcal{O}(1)$ horizon volume at the onset of inflation. To calculate what fraction of the universe has $0 \le \phi_1 \le \phi_1^{max}$ (or the equivalent at other maxima of the potential) over a horizon volume, we model the universe as a tetrahedral lattice with vertices separated by a Hubble length and assume the field is uncorrelated from one vertex point to another. Requiring each point of a tetrahedron to have $\phi_1 \le \phi_1^{max}$, we find that the fraction of the universe that is homogeneous and inflates is $2N(\phi_1^{max}/2\pi f N)^4$, where the number of distinct minima of the potential is N. For $f = m_{pl}$, $\phi_1^{max}/f = 0.6$, and the probability for such a smooth patch is $2 \times 10^{-4} N^{-3}$. For $f = m_{pl}/2$, $\phi_1^{max} \simeq 10^{-2}$, and the probability is only $10^{-11} N^{-3}$. From this argument, the scale f must be very near m_{pl} to avoid fine tuning the initial conditions. On the other hand, if the previous paragraph is correct, the constraints from gradients are not so severe; ultimately, the issue should be settled by numerical simulations.

Initial gradients in ϕ may also lead to global cosmic strings, which form in the symmetry breaking at $T \sim f$, and to domain walls, which form at $T \sim \Lambda$ if the PNGB potential is multiply degenerate ($N > 1$) [24]. The energy density in strings, which correspond to configurations in which ϕ/f winds around 2π and have dimensionless mass per unit length $G\mu \sim (f/m_{pl})^2 \sim 1$, is comparable to the gradient density estimated above, assuming of order one string per horizon. The initial energy density in domain walls, $\rho_{DW} \simeq \sigma H$, where $\sigma \simeq f\Lambda^2$ is the wall surface energy per unit area, will also be of the same order of magnitude. Since their energy densities redshift away, topological defects do not prevent the universe from inflating, but, like gradients, delay briefly the onset of inflation. Once inflation takes place, our observable universe lies deep inside a single domain with $\phi = \pi f$, so both strings and domain walls are inflated away.

In conclusion, a pseudo-Nambu-Goldstone boson, with a potential [eqn.(2)] that arises naturally from particle physics models, can lead to successful inflation if the global symmetry breaking scale $f \simeq m_{pl}$ and $\Lambda \simeq m_{GUT}$.

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REFERENCES

[1] A. H. Guth, *Phys. Rev.* D **23**, 347 (1981).

[2] P. J. Steinhardt and M. S. Turner, *Phys. Rev.* D **29**, 2162 (1984).

[3] Recent measurements include P. Lubin and P. Meinhold, unpublished; A.C.S. Readhead et al, *Ap.J.* **346**, 566 (1989); J. M. Uson and D.T. Wilkinson, *Nature* **312**, 427 (1985). For discussion of $\delta\rho/\rho$, see D. S. Salopek, J. R. Bond, and J. M. Bardeen, *Phys. Rev. D* **40**, 1753 (1989); S. Matarrese, A. Ortolan, and F. Lucchin, *Phys. Rev.* D **40**, 290 (1989).

[4] A. D. Linde, *Phys. Lett.* **108 B**, 389 (1982); A. Albrecht and P. J. Steinhardt, *Phys. Rev. Lett.* **48**, 1220 (1982).

[5] A. D. Linde, *Phys. Lett.* **129 B**, 177 (1983).

[6] F. C. Adams and K. Freese, submitted to *Phys. Rev.* D (1990).

[7] F. C. Adams, K. Freese, and A. H. Guth, submitted to *Phys. Rev.* D (1990).

[8] D. La and P. J. Steinhardt, *Phys. Rev. Lett.* **376**, 62 (1989); *Phys. Lett.* **220 B**, 375 (1989).

[9] For a review, see E. W. Kolb and M.S. Turner, *The Early Universe*, (New York: Addison-Wesley Publishing Co.) (1990).

[10] See, *e.g.*, R. Holman, P. Ramond, and G. G. Ross, *Phys. Lett.* **137B, 343** (1984).

[11] H. Quinn and R. Peccei, *Phys. Rev. Lett.* **38**, 1440 (1977); S. Weinberg, *Phys. Rev. Lett.* 40, 223 (1978); F. Wilczek, *Phys. Rev. Lett.* 40, 279 (1978); J. E. Kim, *Phys. Rev. Lett.* **43**, 103 (1979); M. Dine, W. Fischler, and M. Srednicki, *Phys. Lett.* **104 B**, 199 (1981); M. Wise, H. Georgi, and S. L. Glashow, *Phys. Rev. Lett.* **47**, 402 (1981).

[12] For a review, see J. E. Kim, *Phys. Rep.* **150**, 1 (1987).

[13] C. T. Hill and G. G. Ross, *Phys. Lett.* **203 B**, 125 (1988); *Nucl. Phys.* **B 311**, 253 (1988).

[14] See, *e.g.*, D. J. Gross, R. D. Pisarski, and L. G. Yaffe, *Rev. Mod. Phys.***53**, 43 (1981).

[15] G. 't Hooft, in *Recent Developments in Gauge Theories*, eds. G. 't Hooft, *et al*, (Plenum Press, New York and London, 1979), p. 135.

[16] J. P. Derendinger, L. E. Ibanez, H. P. Nilles, *Phys. Lett.* **155B**, 65 (1985); M. Dine, R. Rohm, N. Seiberg, and E. Witten, *Phys. Lett.* **156 B**, 55 (1985).

[17] E. Witten, *Phys. Lett.* **149B**, 351 (1984); K. Choi and J. E. Kim, *Phys. Lett.* **154B**, 393 (1985).

[18] A. H. Guth and S.-Y. Pi, *Phys. Rev. Lett.* **49**, 1110 (1982); S. W. Hawking, *Phys. Lett.* **115B**, 295 (1982); A. A. Starobinskii, *Phys. Lett.* **117B**, 175 (1982); J. Bardeen, P. Steinhardt, and M.S. Turner, *Phys. Rev.* D **28**, 679 (1983);

[19] N. Vittorio, S. Mattarese, and F. Lucchin, *Ap. J.* **328**, 69 (1988).

[20] J. Silk and M. S. Turner, *Phys. Rev.* D **35**, 419 (1986); L. A. Kofman, A. D. Linde, and J. Einasto, *Nature* 326, 48 (1987).

[21] D.S. Goldwirth and T. Piran, *Phys. Rev. Lett.* **64**, 2852 (1990).

[22] A. Albrecht, R.H. Brandenberger, and R.A. Matzner, *Phys. Rev. D* **32**, 1280 (1985); *ibid.* **35**, 429 (1987).

[23] J. Kung and R. Brandenberger, preprint (1990).

[24] T. W. B. Kibble, *J. Phys.* **A 9**, 1387 (1976); for a review, see A. Vilenkin, *Phys. Rep.* **121**, 263 (1985).

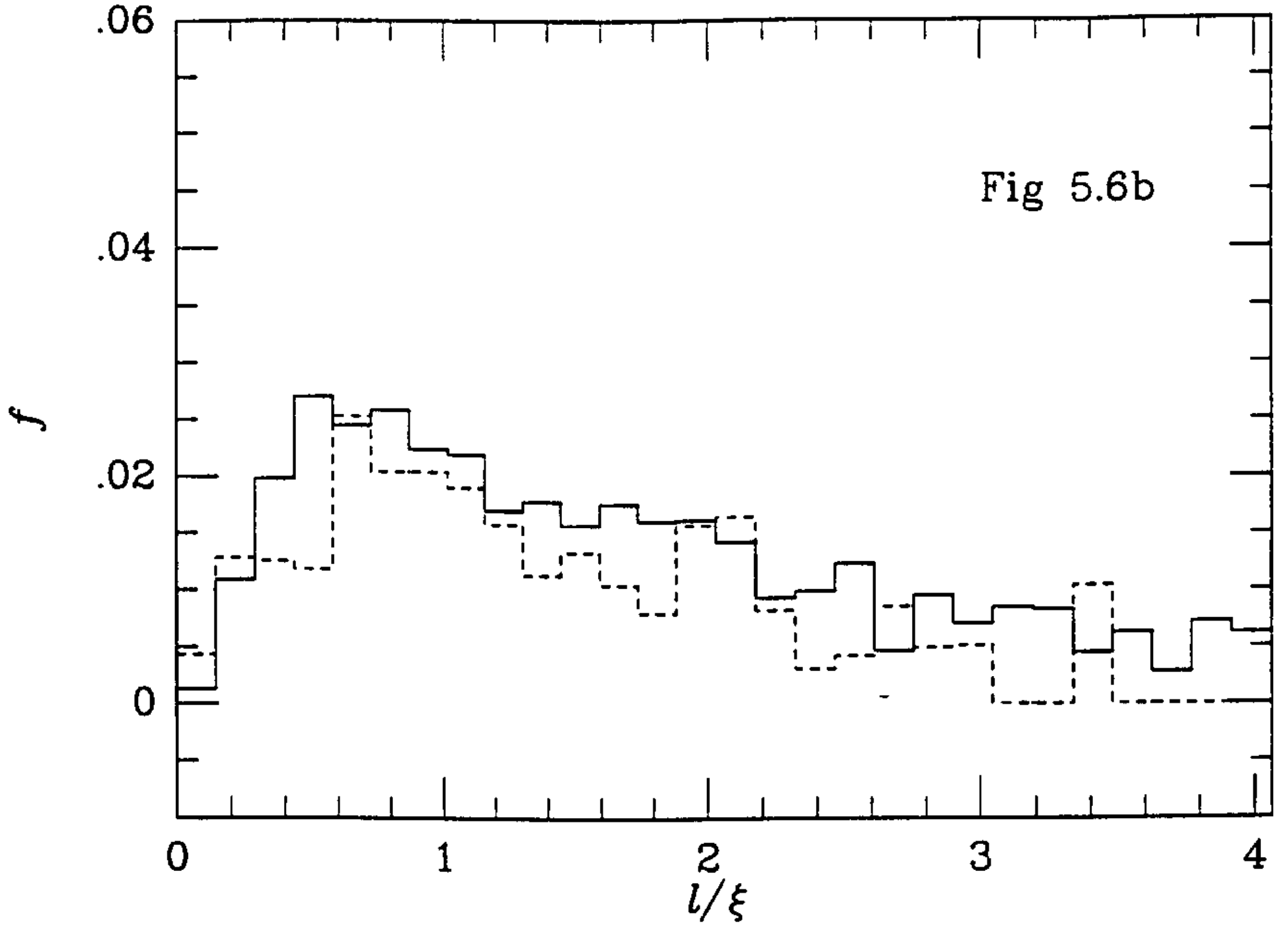

Fig 5.6b
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.04
.02
0
f
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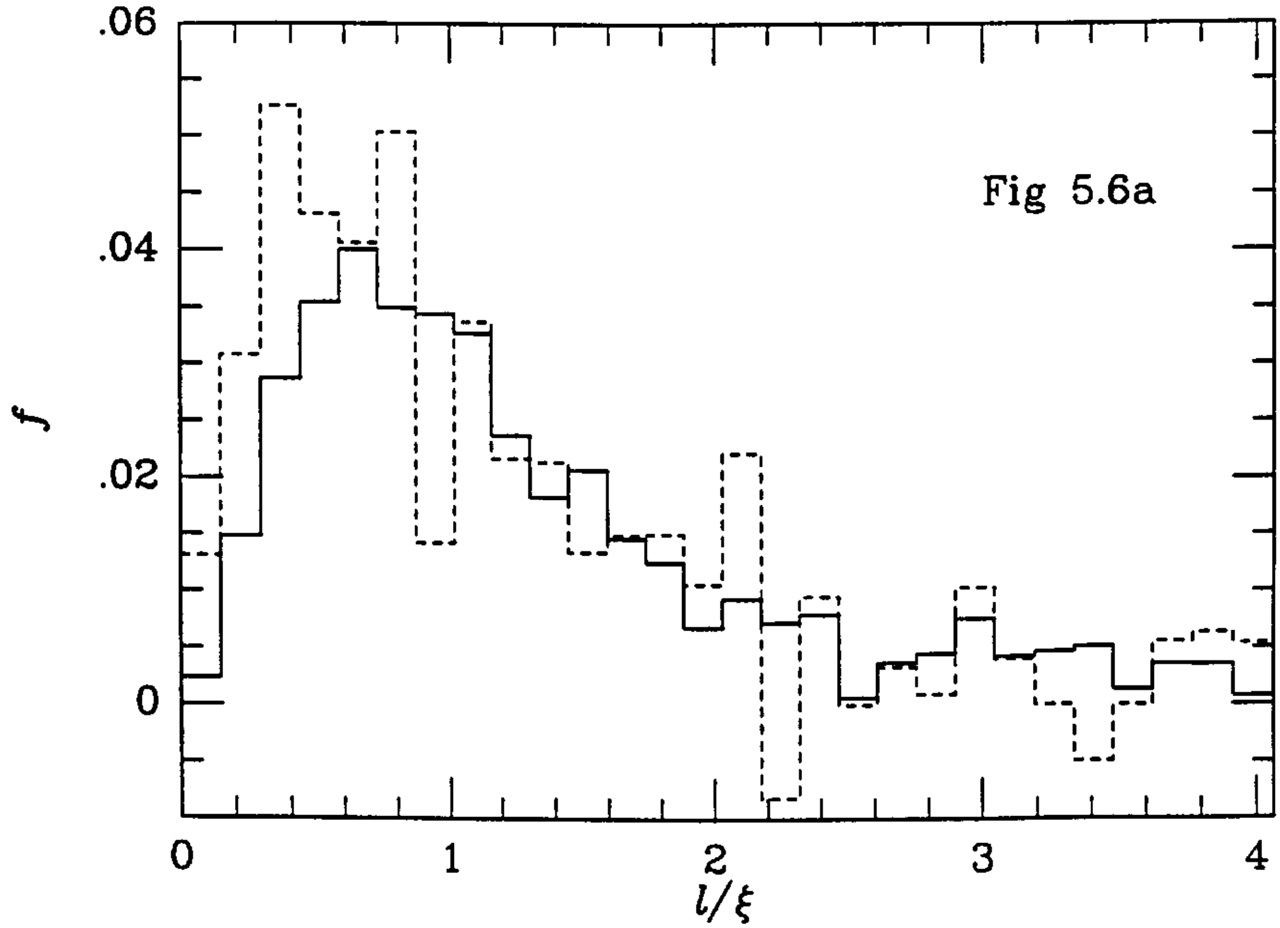
l/ξ



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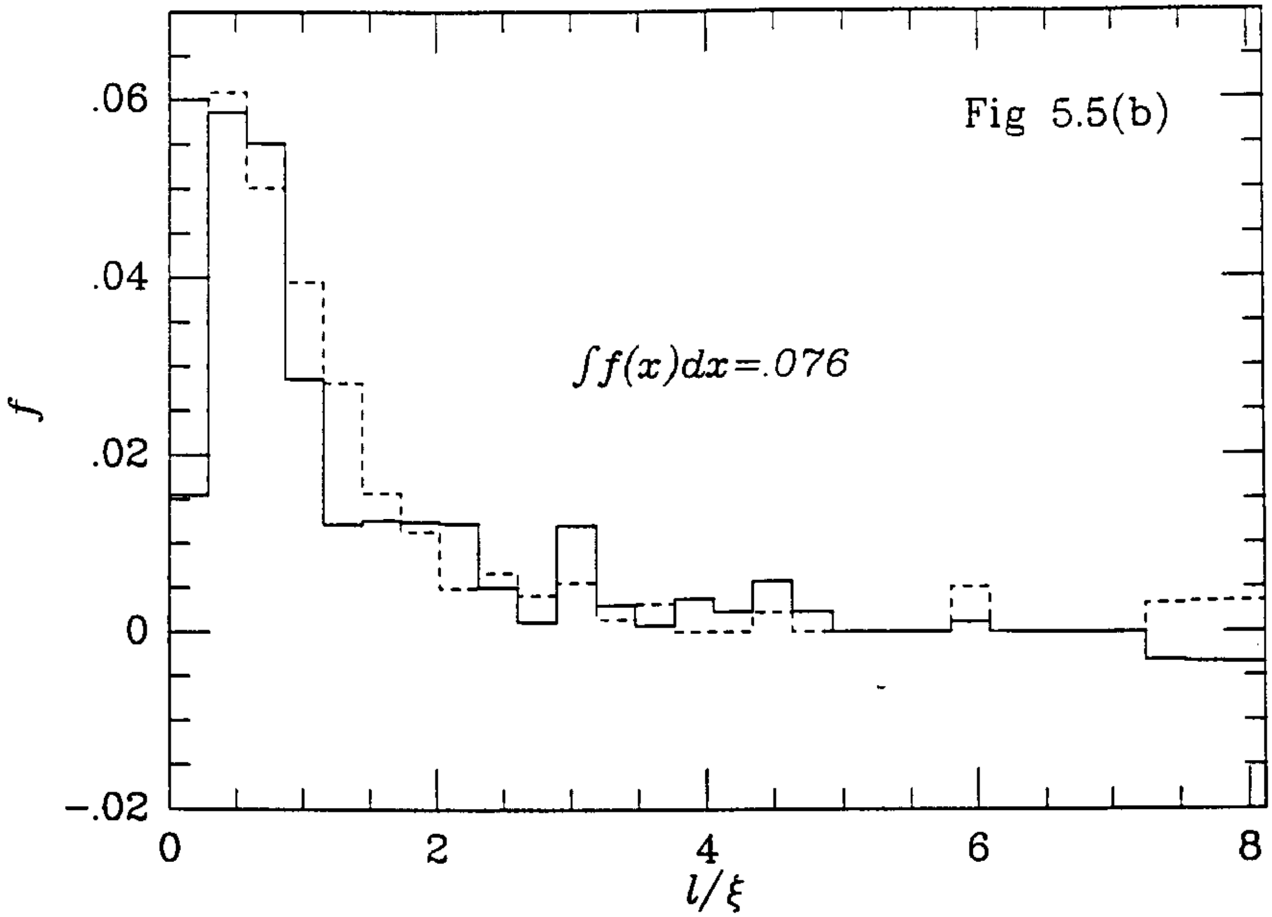
Fig 5.5(b)
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l/ξ
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.02
0
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0
2
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6
8

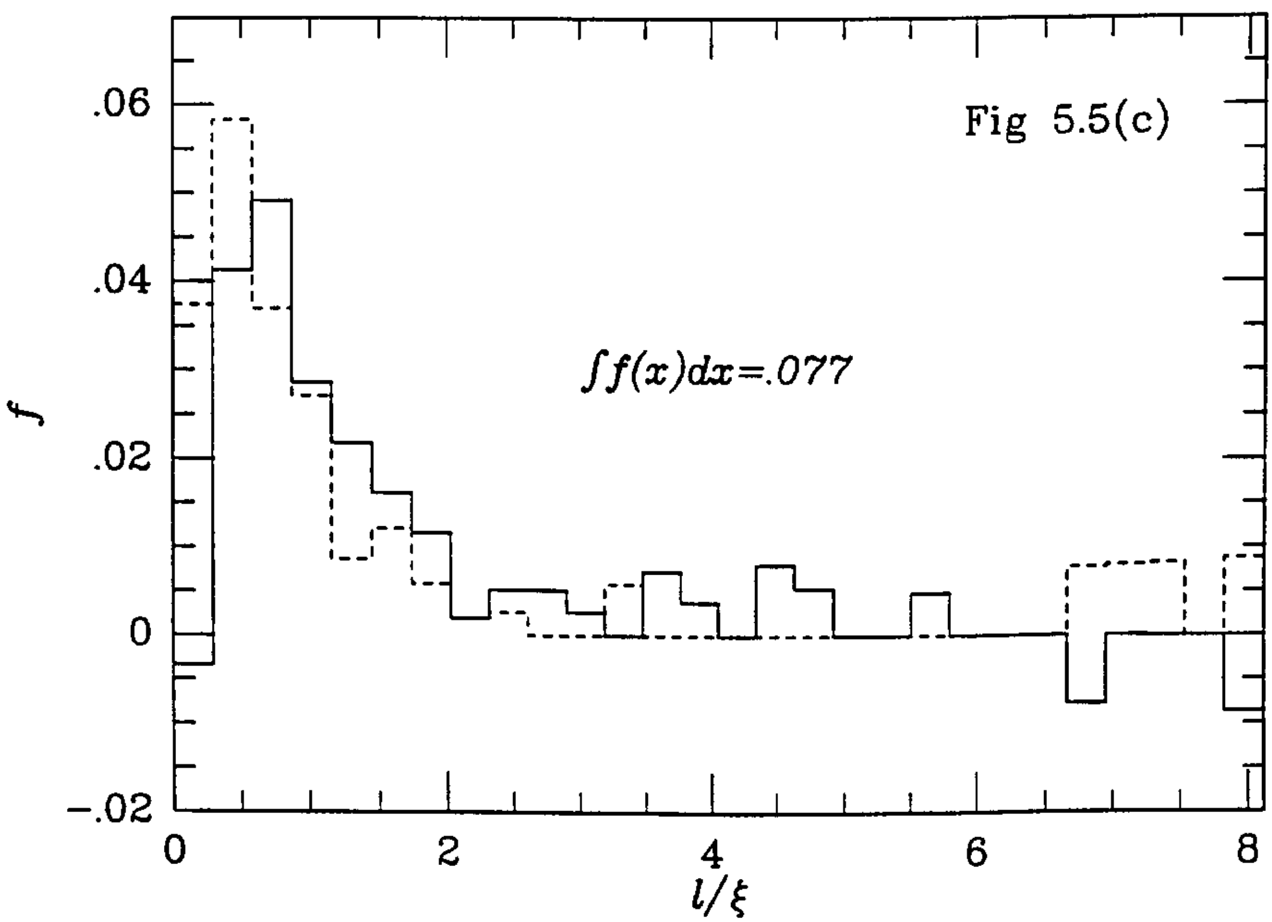
Fig 5.5(c)
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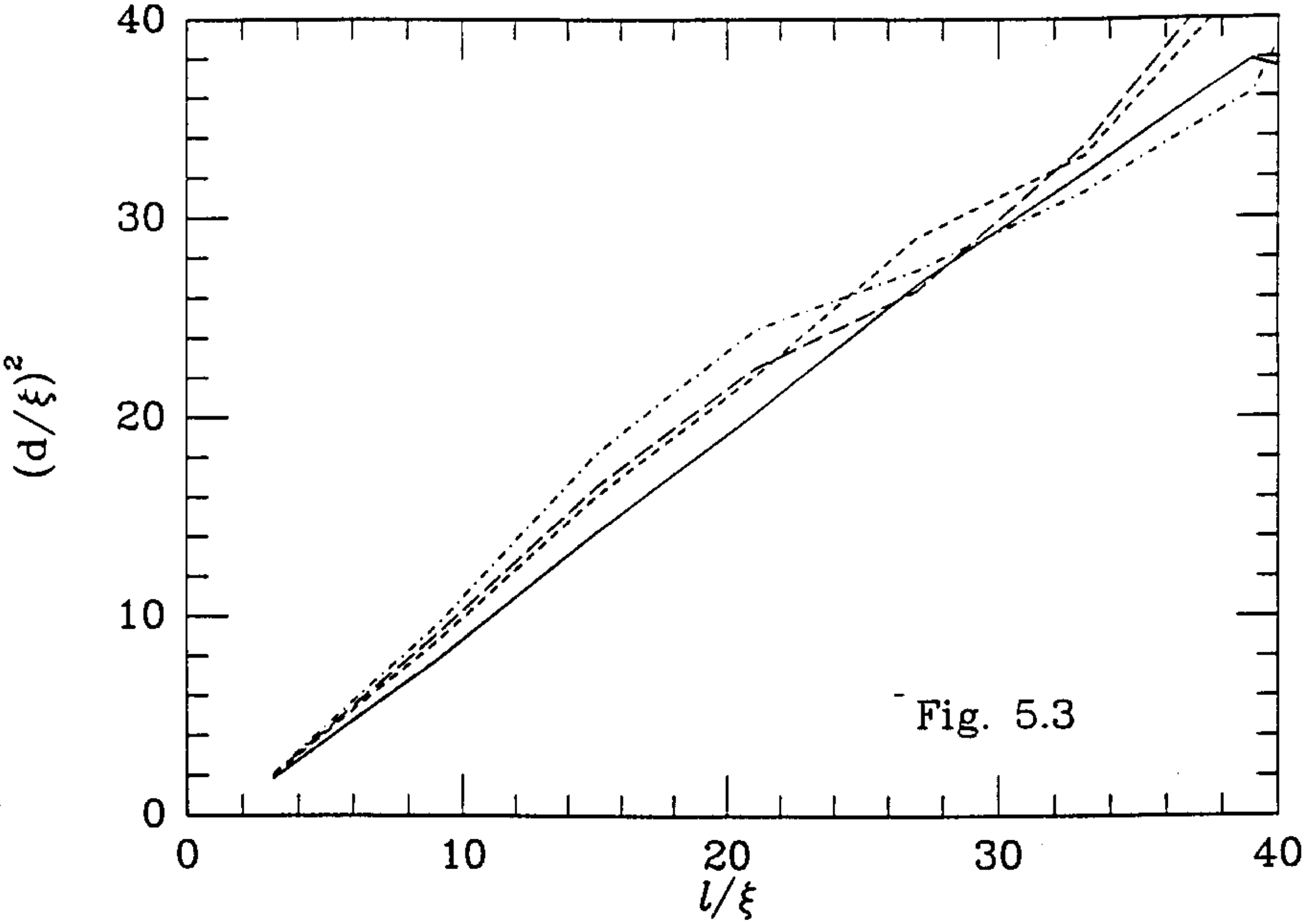

Fig. 5.3

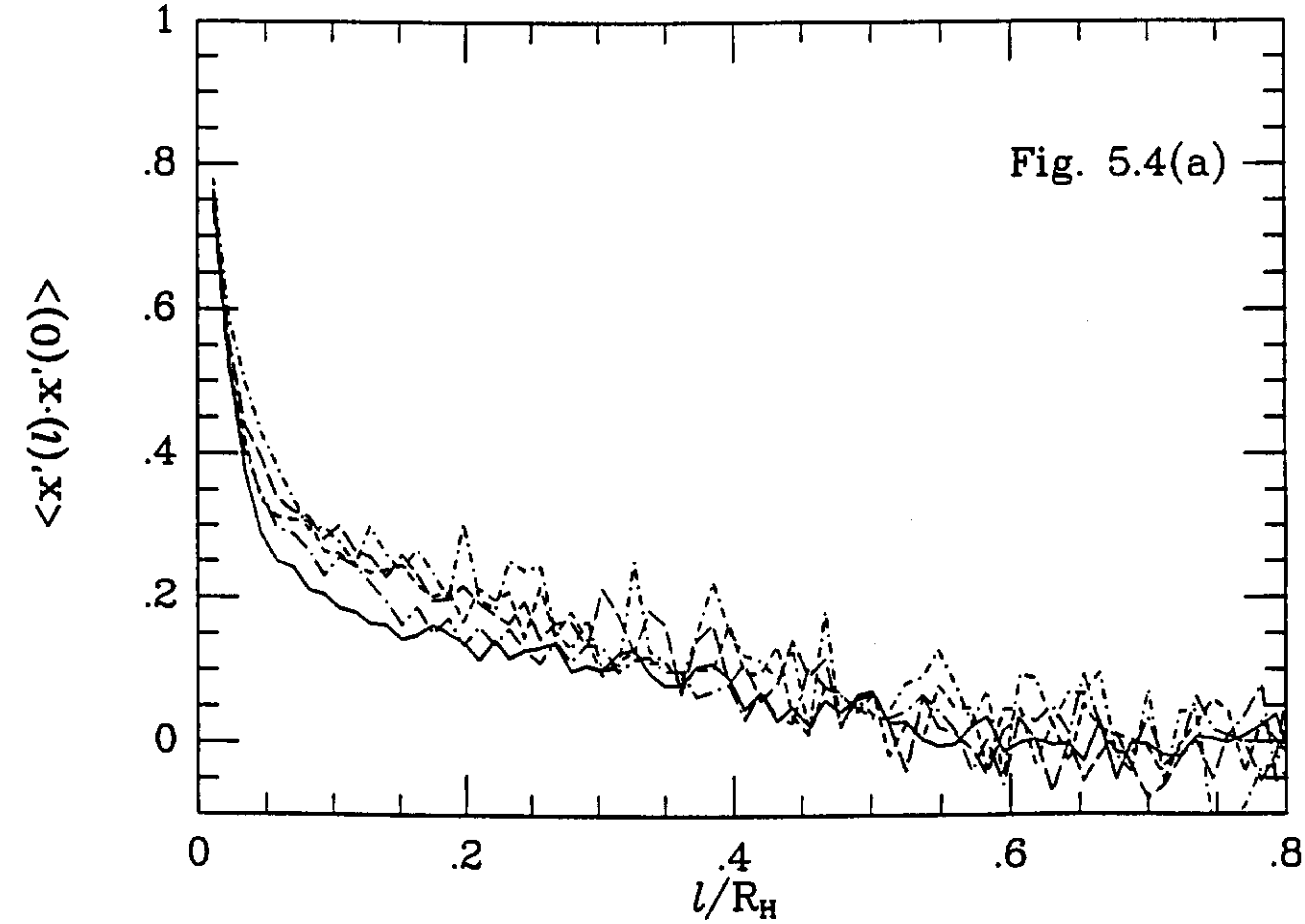

Fig. 5.4(a)

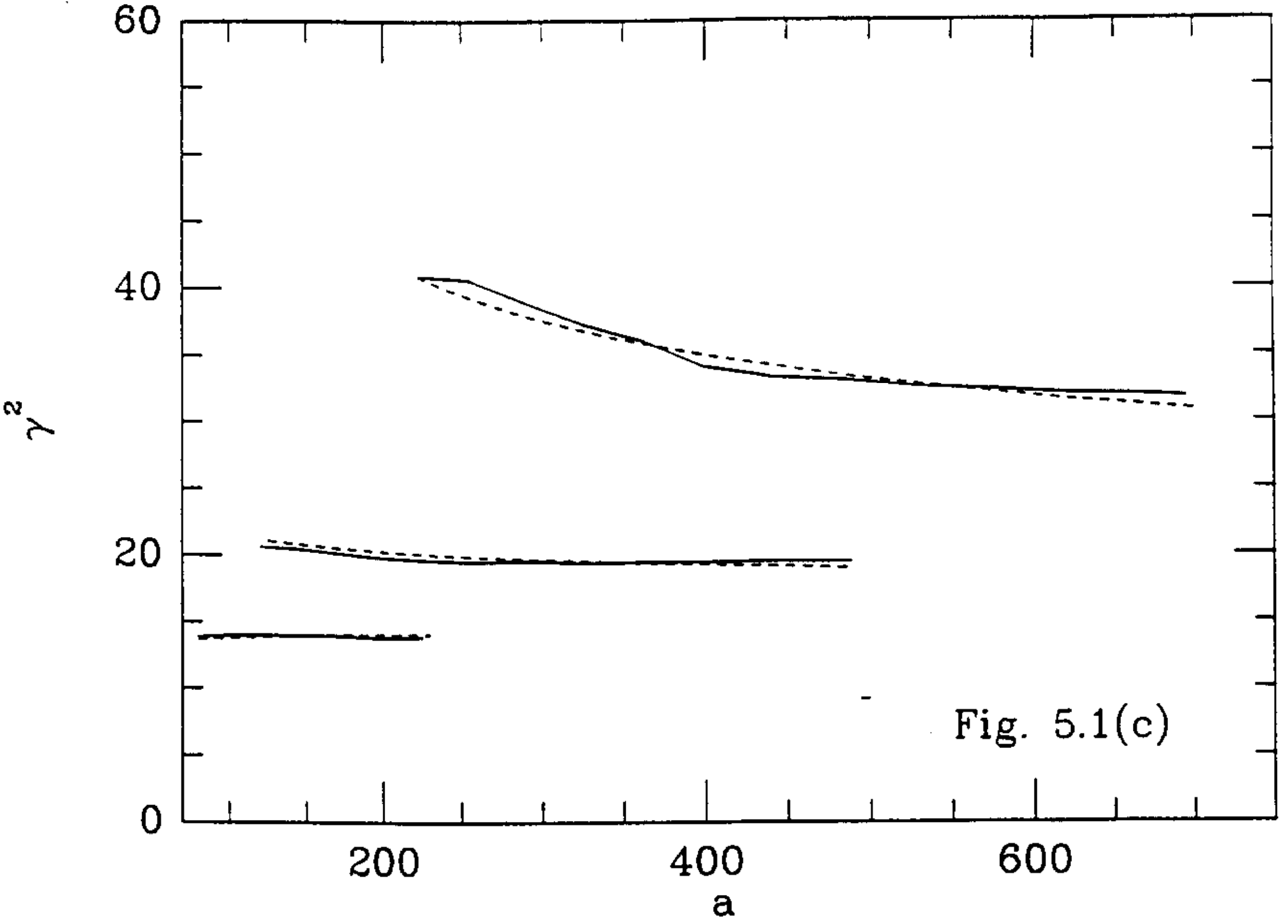

Fig. 5.1(c)

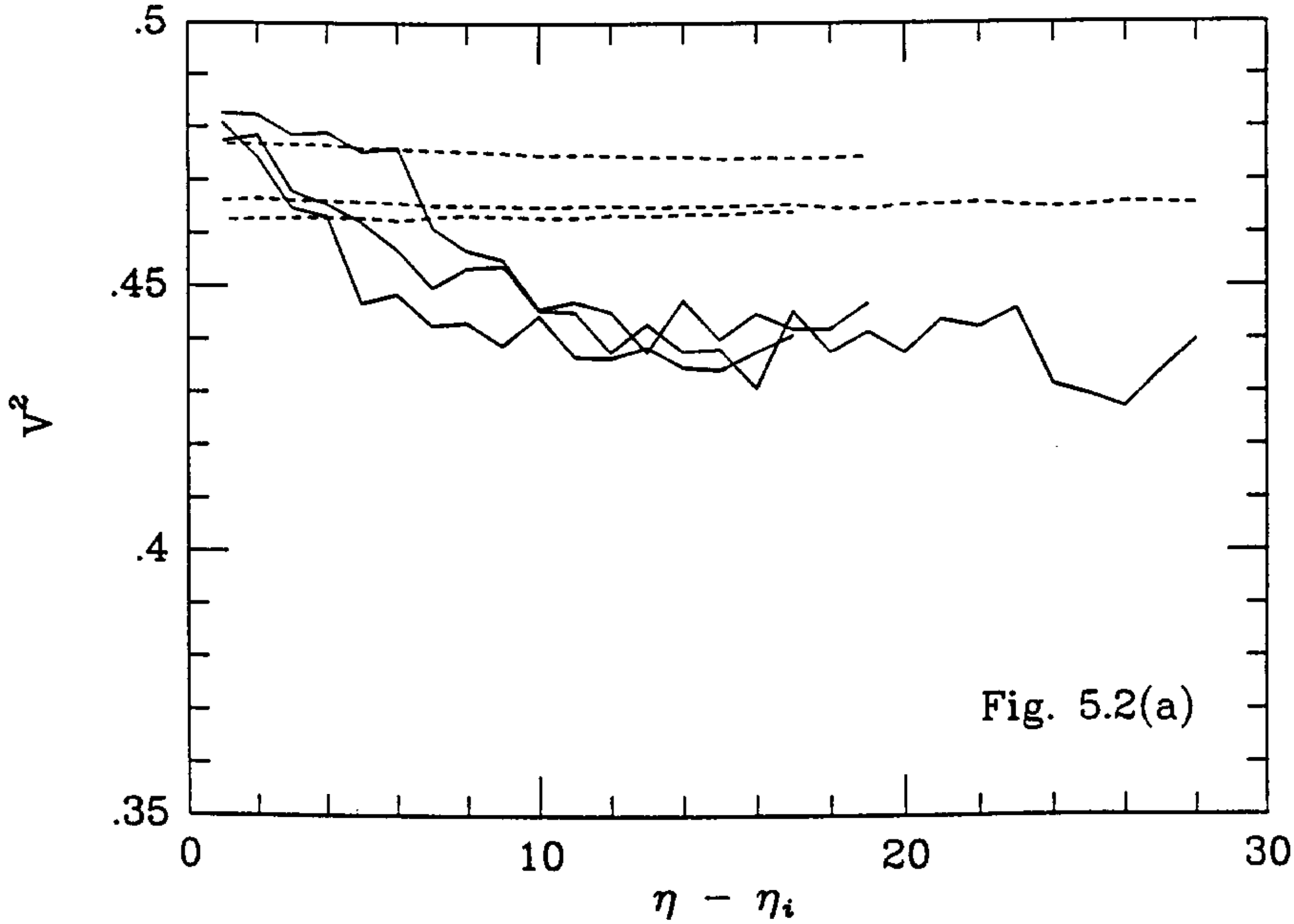

Fig. 5.2(a)

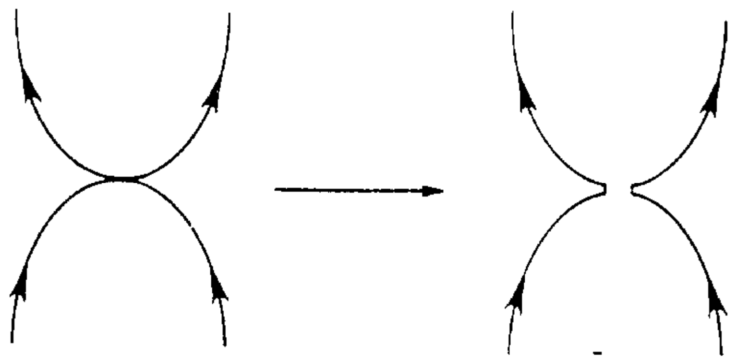

Figure 2.1