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Ultrahigh-Energy Particle Flux from Cosmic Strings

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Abstract

We estimate the expected flux of ultrahigh-energy ($> 10^{18} \text{eV}$) protons in the present epoch due to a process which involves collapse or multiple self-intersections of a special class of closed cosmic string loops in the universe. We compare this flux with the observed flux of ultrahigh-energy cosmic rays, and discuss the implications.



1. Introduction

The purpose of this paper is to show that, under certain circumstances, Cosmic Strings(CS)[1]^{#1} could be the sources of extremely energetic particles in the universe. Production of ultrahigh-energy(UHE)(energy $> 10^{18} eV$) particles from CS's was considered in two recent papers[3,4] which dealt with a particular process, namely, the so-called "cusp evaporation" process[5]. In this paper we shall consider another process of UHE particle production which is due to collapse and/or multiple self-intersections of a special class of "primary" closed CS loops in the universe. (The "primary" loops are formed due to intersections of long, i.e., greater-than-horizon-length CS's; see below). We shall estimate the flux of UHE protons in the present epoch due to this process (further discussed below) and compare this flux with the observed UHE cosmic-ray flux. This will give us, for reasonable values of other parameters in the problem, an upper limit, $f < 4.3 \times 10^{-4}$, on the average fraction f of the total energy contained in all primary loops formed at any time, that may be released in the form of particles due to collapse and/or multiple self-intersections of some of these loops. We also discuss the implications of our results for the flux of UHE cosmic rays above the so-called "Greisen-Zatsepin-Kuz'min(GZK) cutoff"[6] energy $\sim 6 \times 10^{19} eV$ (see below).

2. Formation, collapse, and multiple self-intersections of closed CS loops

Analytical as well as numerical studies of the energetic consistency of the evolution of CS's show[7-10] that a certain average number β of subhorizon-sized "primary" closed loops of CS's must form continually by the process of intersections of long strings (or self-intersections of superhorizon-sized closed loops) per horizon-sized volume of the universe per Hubble time at any time t . The loops are thought to form as the intersecting string segments "exchange partners"[1] and reconnect the other way. The rate of formation of closed CS loops is thus given by^{#2}

$$\frac{dn_f}{dt_f} = \beta \frac{1}{t_f^4}, \quad (1)$$

where n_f is the number density at the time of formation t_f . The typical length L_f of the

string loop at any time of formation t_f is given by

$$L_f = \alpha t_f = M_f / \mu, \quad (2)$$

where $\alpha \lesssim O(1)$, M_f is the total energy of the loop at formation, and μ is the energy per unit length of the string, which is determined by the energy-scale at which the string-forming phase transition takes place[2].

Immediately after its formation a closed loop will execute complicated motion due to the tension(= μ) in the string. For subhorizon-sized loops the expansion of the universe can be neglected[8] and the motion of the loop can be described(with a suitable 'gauge' choice[11]) by the "wave equation" $\ddot{\mathbf{X}}(\sigma, t) = \mathbf{X}''(\sigma, t)$, together with the constraint equations $\dot{\mathbf{X}} \cdot \mathbf{X}' = 0$, $\dot{\mathbf{X}}^2 + \mathbf{X}'^2 = 1$. Here $\mathbf{X}(\sigma, t)$ denotes the spatial coordinates of points of the string at time t , the points being parametrized by the values of the parameter $\sigma \in [0, L]$, L being the total length of the string, and the primes and dots indicate derivatives *w.r.t.* σ and t , respectively. In absence of any energy loss a closed loop will execute periodic motion with a fundamental period[11] $L/2$. Kibble and Turok[11] showed that any initially static[i.e., $\dot{\mathbf{X}}(\sigma, 0) = 0$] loop(static in its c.m.frame) completely *collapses* after half a period of oscillation, i.e., $\mathbf{X}(\sigma, \frac{L}{4}) = \mathbf{X}(\sigma + \frac{L}{2}, \frac{L}{4})$, $\forall \sigma$. (Here $t = 0$ is the time of formation of the loop). Complete collapse is also the fate of any initially non-static loop which has only single-frequency waves on it[11]^{#3}. It is difficult, if not impossible, to have a precise estimate of the fraction of all loops that could be formed at any time in these collapsing configurations—all one can say is that this fraction is likely to be very small because these loops correspond to rather special configurations. However, since expansion of the universe irons out[8] higher-frequency waves on long(i.e., length > horizon scale) strings (the intersections of which give rise to the loops we are considering), it is probably not inconceivable that a small but nevertheless finite fraction of the loops formed may have only the lowest-frequency waves on them and so will collapse as described above. If a closed CS loop completely collapses, the energy contained in the entire loop would be released[12] mostly in the form of massive gauge- and higgs bosons(of the underlying spon-

taneously broken gauge theory) as well as possible heavy fermions that could be coupled to the string-forming higgs field[13]. The decay products of these massive particles(these primary massive particles are hereafter referred to as X-particles)would be present in today's universe in the form of UHE particles, to estimate the flux of which is the aim of this paper.

As mentioned above, the completely collapsing loops are rather special. As a somewhat more general situation, one may consider closed CS loops which do not completely collapse but rather self-intersect, not just at one point, but at multiple(in principle, very large number of)points, i.e., there may be a large number of pairs(σ, σ') which satisfy the condition $X(\sigma, t) = X(\sigma', t)$ at some value of $t \in [0, \frac{L}{2}]$, $t = 0$ being the time of formation of the loop. In such a case, one large loop will break up into a large number of small loops at once. Intersections of CS's would be accompanied by particle production; because of the finite width of the string the two intersecting segments overlap at any point of intersection, and the underlying microphysical interactions of the fields "constituting" the string would cause particle production at these overlapping regions near the points of intersection—the more the number of intersection points, the more will be the amount of energy released. Numerical simulations[14] of CS intersections do indeed "see" such energy release from intersecting CS's. Again, it is hard to make a precise estimate of the amount of energy released. In the following, we shall simply assume that a certain small average fraction f of the total energy of all primary loops formed at any time is released in the form of particles within one period of oscillation(specifically, at the end of half-period[11]) either due to a small fraction of loops completely collapsing and/or due to some loops breaking up into large number of small loops (due to multiple self-intersections) at once. We shall see that the observed UHE cosmic-ray flux gives an upper limit to this fraction f .

3. Proton injection rate and flux in the present epoch

The loops which collapse or self-intersect themselves at time t are the ones that were formed at time $t_f = t(1 + \alpha/4)^{-1}$. The X-particles are assumed to be produced instan-

taneously at the time of collapse or (multiple)self-intersections of the loops. So the rate of production of X-particles at time t , $dn_X(t)/dt$, where $n_X(t)$ is the number density, is given by the rate of loop formation at time t_f , eq.(1):

$$\frac{dn_X}{dt}(t) = f \cdot \left(\frac{dn_f}{dt_f} \right)_{t_f} \left(\frac{dt_f}{dt} \right) \left(\frac{a(t_f)}{a(t)} \right)^3 \frac{M_f}{m_X}. \quad (3)$$

In eq.(3), $a(t)$ is the scale factor of the universe, and m_X is the average energy of a single X-particle(we ignore here a possible spectrum of the emitted X-particles). We have also neglected the energy loss of the loops through gravitational radiation because the loops we are considering do not survive more than one oscillation period. We need to consider times t and t_f which are in the matter-dominated epoch only, because the particles produced at redshifts $z \gg O(1)$ essentially lose all energy during propagation through the cosmic medium and so they do not survive as UHE particles in the present epoch(see below). So, $a(t_f)/a(t) = (t_f/t)^{2/3}$. Eqs.(1)–(3) then give

$$\frac{dn_X(t)}{dt} = f \alpha \beta \mu m_X^{-1} t^{-3}. \quad (4)$$

Now, each X-particle will decay into quarks and leptons. The quarks will hadronize producing jets of hadrons. The latter will be mostly pions, but a small fraction will be nucleons. We will not consider here the possibly important effects of gamma rays and neutrinos that will result from the decays of pions, but concentrate only on the nucleons. The neutrons of even the highest energies considered($\sim 10^{11} GeV$) will beta-decay to protons during the propagation times of our interest. We shall estimate the proton production(i.e., injection)spectrum $\Phi(E_i, t_i)$ (=the number density of injected protons per unit energy interval at the injection energy E_i per unit time at any injection time t_i) by using the following QCD-motivated hadronic jet fragmentation formula[15], namely

$$\frac{dN_h}{dx} \simeq 0.08 \exp \left(2.6 \sqrt{\ln(1/x)} \right) (1-x)^2 / \left(x \sqrt{\ln(1/x)} \right), \quad (5)$$

which fits the GeV–TeV collider well; we shall simply assume this formula to be reasonably valid at the energies of our interest. In eq.(5), $x \equiv E_i/E_{jet}$, where $E_{jet} \simeq m_X/2$ is the total

energy in the jet (assuming that each X-particle decays into a quark and a lepton, each of which shares energy roughly equally, and each quark produces one hadronic jet), and N_h is the number of hadrons carrying a fraction x of the total energy in a jet. Assuming a $\sim 3\%$ nucleon content of the jet [15], eqs. (4) and (5) give

$$\Phi(E_i, t_i) \simeq 0.06(f\alpha\beta\eta) \left(\frac{M_{Pl}}{m_X}\right)^2 \left(\frac{dN_h}{dx}\right) t_i^{-3}, \quad (6)$$

where $\eta \equiv G\mu$ is the dimensionless CS parameter.

Now, let $j(E_0)$ denote the expected flux, i.e., the number of protons per unit energy interval at an observed energy E_0 , crossing per unit area per unit solid angle per unit time in the present epoch (t_0), due to the process described above. Then assuming an isotropic CS loop distribution in an Einstein-de Sitter “flat” ($\Omega_0 = 1$) universe, we get [3]

$$j(E_0) = \frac{3}{8\pi} ct_0 \int_0^{z_{i,max}} dz_i (1+z_i)^{-5.5} \Phi(E_i, z_i) \left(\frac{dE_i(E_0, z_i)}{dE_0}\right)_{E_0}, \quad (7)$$

where z_i is the redshift corresponding to the injection time t_i . The protons suffer energy loss as they traverse the cosmic medium. Therefore, in order for a proton injected at a redshift z_i to appear today at any given energy E_0 , it must have a definite injection energy $E_i \equiv E_i(E_0, z_i)$ such that $E_0 < E_i < m_X/2$. The upper limit $z_{i,max}(E_0)$ on the integral in eq. (7) is defined such that

$$E_i(E_0, z_{i,max}(E_0)) = m_X/2. \quad (8)$$

The dominant energy loss of UHE protons propagating through the cosmic medium at an epoch of redshift z is due to the following processes^{#4}: (i) Cosmological redshift due to expansion of the universe, (ii) e^+e^- -pair production off the background photons ($p + \gamma \rightarrow p + e^+ + e^-$), and (iii) photopion production off the background photons ($p + \gamma \rightarrow \pi + N$). Thus one can write [17]

$$\frac{1}{E} \frac{dE}{dz} = (1+z)^{-1} + H_0^{-1} (1+z)^{\frac{1}{2}} [\beta_{0,pair}((1+z)E) + \beta_{0,pion}((1+z)E)], \quad (9)$$

where the function $\beta_0(E) \equiv -\frac{1}{E} \left(\frac{dE}{dt}\right)_{z=0}$ denotes the energy-loss rate (divided by the energy) in the present epoch ($z = 0$) of a proton of energy E ; the subscripts “pair” and “pion”

refer respectively to the processes (ii) and (iii) mentioned above. Note[17,3] that the arguments of the functions β_0 appearing in eq. (9) are $(1+z)E$, not E . In eq. (9), H_0 is the Hubble constant in the present epoch (we take $H_0 = 75 \text{ Km.s}^{-1}.\text{Mpc}^{-1}$).

The energy-loss function $\beta_0(E)$ has been calculated by several authors. For a nice summary see Figure 1 of ref. [17] whose results we shall use below. For $E \lesssim 6 \times 10^{19} \text{ eV}$, $\beta_{0,pair}$ dominates over $\beta_{0,pion}$; $\beta_{0,pair}^{-1}(E)$ decreases from $\sim 10^{11} \text{ yr}$ at $E \simeq 10^{18} \text{ eV}$ to $\sim 7.8 \times 10^9 \text{ yr}$ at $E \simeq 4.6 \times 10^{18} \text{ eV}$. For $5 \times 10^{18} \text{ eV} \lesssim E \lesssim 6 \times 10^{19} \text{ eV}$, $\beta_{0,pair}^{-1}(E)$ has a weak energy dependence remaining roughly constant at $\sim 5 \times 10^9 \text{ yr}$. At $E \gtrsim 6 \times 10^{19} \text{ eV}$, $\beta_{0,pion}$ becomes dominant and it rises very steeply with increasing E ; $\beta_{0,pion}^{-1}$ decreases from $\sim 4.7 \times 10^9 \text{ yr}$ at $E \simeq 6 \times 10^{19} \text{ eV}$ to $\sim 7.9 \times 10^7 \text{ yr}$ at $E \simeq 2 \times 10^{20} \text{ eV}$. Above $\sim 10^{21} \text{ eV}$, $\beta_{0,pion}^{-1}$ reaches a roughly constant value at $\sim 3.9 \times 10^7 \text{ yr}$. The sharp fall-off of the time scale of energy loss through photopion production for $E \gtrsim 6 \times 10^{19} \text{ eV}$ is the basis of the well-known GZK[6] prediction of an expected onset of a cutoff of the UHE cosmic ray proton spectrum at $E \gtrsim 6 \times 10^{19} \text{ eV}$, provided the sources are extragalactic.

Knowing $\beta_{0,pair}(E)$ and $\beta_{0,pion}(E)$, we solve eq. (9) numerically to find the value of E_i for given values of z_i and E_0 . The derivatives (dE_i/dE_0) are also evaluated numerically. The injection spectrum is then calculated from eqs. (5) and (6), and the numerical evaluation of the z_i -integral in eq. (7) gives the required flux. The value of $z_{i,max}$ defined by eq. (8) increases as E_0 decreases. But for all values of E_0 of our interest, $z_{i,max}$ remains below ~ 1.5 . Actually the contribution to the flux from those values of z_i for which $E_i(E_0, z_i) \gg 6 \times 10^{19} \text{ eV}$ are small because of the sharp fall-off of the injection spectrum [see eqs. (5) and (6)] above the onset of photopion energy-loss dominance at $\sim 6 \times 10^{19} \text{ eV}$ [3].

4. Results and Discussions

Our results are shown in Figure 1 for a fiducial value of $m_\chi = 10^{15} \text{ GeV}$ (GUT scale). The observational data on the UHE cosmic-ray flux differ among the various experimental groups. For definiteness we consider here the best-fit power-law result for the UHE cosmic-

ray flux given by the “Fly’s Eye” group[18], which is represented by the dashed line. Requiring that the flux due to CS’s remain below the observed flux at all energies, we find an upper limit on the product $f\alpha\beta\eta \leq 1.7 \times 10^{-9}$; the solid curve in Figure 1 represents our results normalized at $f\alpha\beta\eta = 1.7 \times 10^{-9}$. The values of the parameters α and β are uncertain. To have some idea of the kind of numbers likely to be involved, it is perhaps reasonable to take[10] the *product* $\alpha\beta \approx 1$ (although the individual values of α and β are more uncertain). Then taking $\eta < 4 \times 10^{-6}$ (given recently by Bennett and Bouchet[10] from the consideration of the null results for the expected pulsar timing variation due to the stochastic gravitational wave background that would be created by oscillating CS loops), we get $f < 4.25 \times 10^{-4}$. This limit weakens by an order of magnitude if we instead use the upper limit on η given by Albrecht and Turok[10]^{#5}.

Note also from Fig. 1 that even for the upper-limit value of $f\alpha\beta\eta = 1.7 \times 10^{-9}$, the CS’s contribute a very small amount to the UHE particle flux at $E_0 \lesssim 3 \times 10^{19} eV$. However, above this energy, the CS’s *can* contribute significantly to the observed flux. But perhaps the most interesting feature of the calculated spectrum is that there is really no complete GZK[6]-cutoff. There is, as expected, a sharp fall-off of the flux at $E_0 \sim 6 \times 10^{19} eV$; but the fall-off is a power law behaving approximately as $j(E_0) \propto E_0^{-4.4}$ which *flattens* for $E_0 \gtrsim 1 \times 10^{20} eV$ to approximately $j(E_0) \propto E_0^{-3.2}$ with a further flattening to approximately $j(E_0) \propto E_0^{-1.94}$ for $E_0 \gtrsim 2 \times 10^{20} eV$. The flattening of the spectrum at $E_0 \gtrsim 1 \times 10^{20} eV$ and again at $E_0 \gtrsim 2 \times 10^{20} eV$ are due to corresponding decrease of slope of the curve for $\beta_{0,pion}$ at these energies. In the language of refs. [16,17], we have in Figure 1 a “photopion pile up” at $E_0 \sim 5 \times 10^{19} eV$ and a “photopion dip” at $E_0 \sim 2 \times 10^{20} eV$. Observationally, the Fly’s Eye group[18] essentially sees no events above the energy $7 \times 10^{19} eV$ (the “cutoff”), whereas the Haverah Park experiment[19], for example, reports events at $10^{20} eV$ and higher. We are in no position to shed any light on this conflicting observational situation. However, we would like to point out here that exotic processes, such as the one involving CS’s described above, can indeed give

rise to energetic events, albeit at a small rate, beyond the GZK cutoff in a natural way—the photopion energy-loss cannot completely overwhelm the particle production rate of the source(CS loops in the present case). Moreover, no complicated mechanism for accelerating the particles to ultrahigh energies is needed—for example, for CS's formed at a GUT scale energy $\sim 10^{15} GeV$, the produced particles can automatically have a maximum energy of this order at least at the time of production. From our calculations above, we can make a crude estimate of the expected *integral flux* of UHE protons above any given energy due to the particular CS scenario described above. We predict, for $f\alpha\beta\eta = 1.7 \times 10^{-9}$ (see above), an integral flux $J(E_0 > 6 \times 10^{19} eV) \simeq 9.1 \times 10^{-16} m^{-2}.s^{-1}.sr^{-1}$, which gives an event rate $N(E_0 > 6 \times 10^{19} eV) \simeq 9 \times 10^{-2} yr^{-1}.Km^{-2}$. These numbers are certainly small, but perhaps not entirely beyond the possible future UHE cosmic-ray experiments.

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Footnotes

#¹ See ref. [2] for a review.

#² Wherever appropriate, we use natural units, $c = \hbar = 1 = M_{Pl}\sqrt{G}$, where M_{Pl} is the Planck mass, and G is Newton's constant.

#³ A special case of this is an exactly circular loop which completely collapses to a point. Of course, in reality, the string has a finite width ($\sim \mu^{-\frac{1}{2}}$), and when the radius of the loop becomes on the order of the width, the loop will probably turn into a massive particle which will then decay and produce energetic particles.

#⁴ See, e.g., refs. [16,17].

#⁵ It is interesting to note that if one takes $f = 1$, then with $\alpha\beta \approx 1$, one gets $\eta < 1.7 \times 10^{-9}$. In other words, if *all* CS loops were to collapse or release all their energy in the form of particles within one period of oscillation from the time of their birth, then the existence of GUT-scale CS's ($\eta \sim 10^{-6}$) would be incompatible with the observational results on UHE cosmic rays. One may thus view this as another argument for requiring $f \ll 1$, at least for "GUT scale" CS's, i. e. , that a large fraction of loops must be in non-selfintersecting configurations.

Figure Captions

Fig.1. The estimated UHE proton flux $j(E_0)$ (multiplied by E_0^3) for the upper limit value of the multiplicative factor $f\alpha\beta\eta = 1.7 \times 10^{-9}$ (solid line). The dashed line represents the best fit power-law result for the observed UHE cosmic-ray spectrum given by the “Fly’s Eye” group[18].

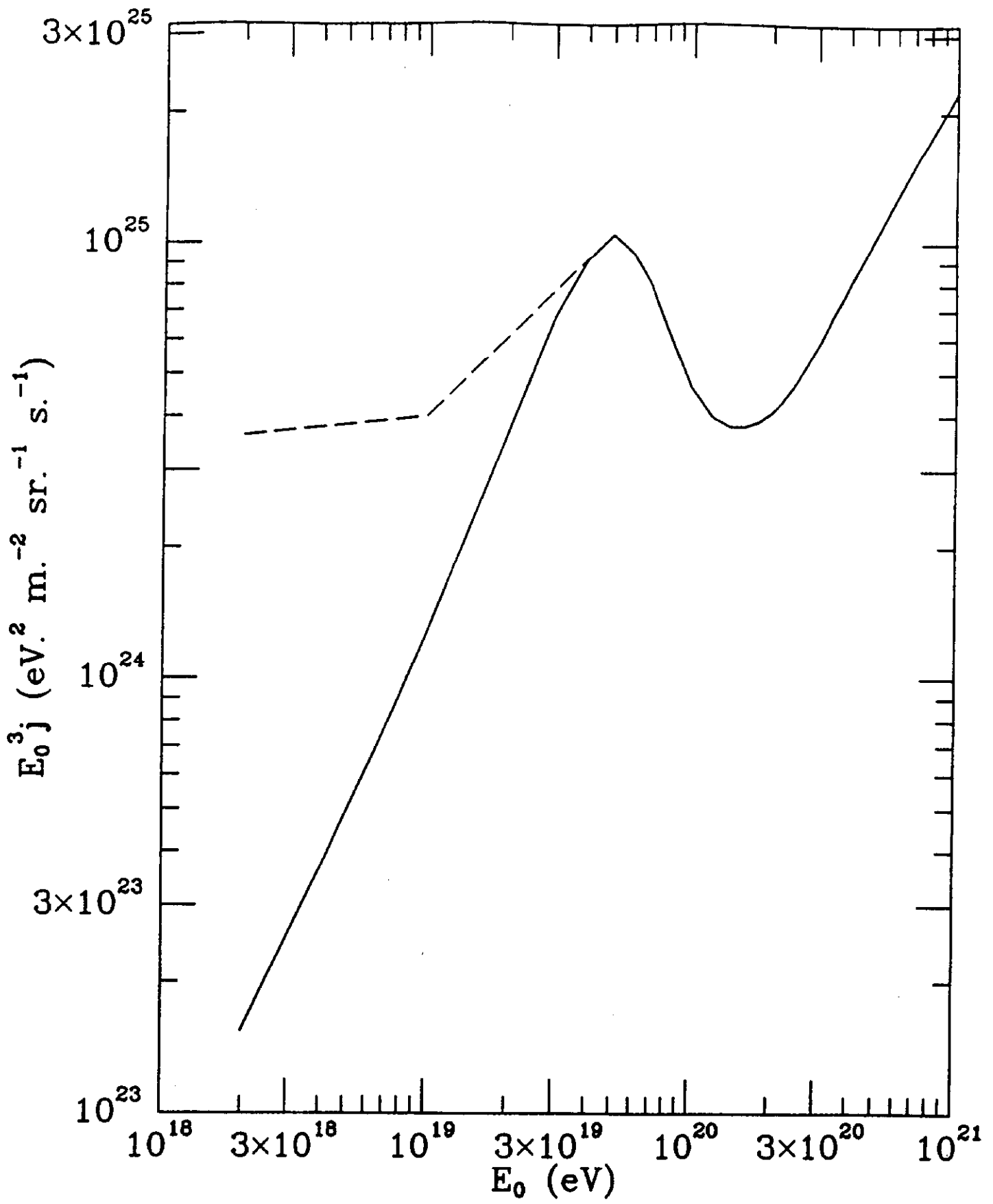


Fig. 1