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A Non-perturbative Approach to the Direct Photon Spectrum Near $x = 1$ in the Inclusive Radiative Decay of J/ψ *

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Abstract

A Born approximation calculation of the direct photon spectrum for $x \lesssim 1$ in the inclusive decay $J/\psi \to \gamma + hadrons$ is given in the framework of QCD multipole expansion in which bound state contributions to the intermediate states are taken into account. The obtained photon spectrum in the $x \lesssim 1$ region is quite close to the experimental data.

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Conventional theoretical studies of inclusive radiative decays of ground state heavy quarkonia $^3S_1(Q\overline{Q})$ are based on perturbation calculations in QCD.[1,2] In the Born approximation, the inclusive decay $^3S_1(Q\overline{Q}) \to \gamma + hadrons$ is calculated through $^3S_1(Q\overline{Q}) \to \gamma + g + g$, where the two gluons are treated as free on-shell particles. Let Γ_{tot} be the total width of $^3S_1(Q\overline{Q})$, $x = 2\omega_\gamma/M$, where ω_γ is the energy of the directly emitted photon and M is the mass of the quarkonium $^3S_1(Q\overline{Q})$. The direct photon spectrum in $^3S_1(Q\overline{Q}) \to \gamma + hadrons$ is expressed as

$$\frac{1}{\Gamma_{tot}}\frac{d\Gamma(^3S_1 \to \gamma + hadrons)}{dx}. \tag{1}$$

For the $b\overline{b}$ system, the perturbative QCD approach is successful. The direct photon spectrum in $\Upsilon \to \gamma + g + g$ calculated from leading order perturbative QCD[2] is already quite close to the experimental data.[3] One-loop QCD correction makes a further small improvement.[4] With a gluon shower correction to the Born approximation,[5] the theoretical curve of the direct photon spectrum is in very good agreement with the recent ARGUS data.[3] However, for the $c\overline{c}$ system, the situation is different. The direct photon spectrum in $J/\psi \to \gamma + g + g$ calculated from leading order perturbative QCD is considerably harder than the experimental data[6] (cf. the dashed curve in Fig. 3). The disagreement between the theory and the experiment in the range $x < 0.8$ is expected since recent experiment shows that there are a lot of resonances in the hadron channel in this region[7] and these have not been taken into account in the Born approximation calculation. In the range $x > 0.8$, the disagreement does imply the shortcoming of the simple perturbation approach. The reason why the $b\overline{b}$ and $c\overline{c}$ cases are so different can be seen as follows. When $x \simeq 1$, the typical momentum of the gluon in $\Upsilon \to \gamma + g + g$ is

$$k(\Upsilon \to \gamma gg) \sim M_\Upsilon/4 \sim 2.4GeV,$$

which is large enough for implementing perturbative QCD. While in $J/\psi \to \gamma + g + g$, the typical momentum of the gluon, when $x \simeq 1$, is only

$$k(J/\psi \to \gamma gg) \sim M_\psi/4 \sim 770MeV.$$

At such a low momentum scale, perturbative QCD is not a good approximation. Therfore, for studying the photon spectum near $x \simeq 1$ in $J/\psi \to \gamma + g + g$, a different theoretical approach suitable for soft gluons is needed.

From the experience of studying hadronic transitions in heavy $Q\overline{Q}$ systems,[8,9] we know that QCD multipole expansion[10] works well for calculating emission of gluons with momenta in the few hundred MeV range. In this paper we will give a Born approximation calculation of the direct photon spectrum in $J/\psi \to \gamma + g + g$ in the region near $x = 1$ based on the general formula for QCD multipole expansion given in Ref. 11). In this approach, bound state contributions to the intermediate states are taken into account, and in this sense the approach is non-perturbative. We shall see that these bound state contributions make the spectrum quite different from that in the perturbation theory and the photon distribution in the region $x > 0.8$ in our approach is indeed very close to the experimental data. Of course, Born approximation may not be good at this low momentum scale. Inspired by Ref. 5), we expect that corrections to Born approximation in $J/\psi \to \gamma + hadrons$ will further soften the photon spectrum. Our calculation just shows that bound state contribution to the intermediate states is an important effect for explaining the smallness of the photon distribution near $x = 1$. We do not expect our calculation can apply to the $x < 0.8$ region because resonance contributions are neglected in the Born approximation and in the small x region the gluons are harder so that multipole expansion is poorer.

According to the general formula given in Ref. 11), the S-matrix element for the inclusive decay

$J/\psi \rightarrow \gamma + g_1 + g_2$ is

$$\begin{aligned}
\langle \gamma g_1 g_2 | S | J/\psi \rangle \; = \; & -i2\pi\,\delta(\omega_\gamma + \omega_1 + \omega_2 - M_\psi) \\
& \{ \langle \gamma g_1 g_2 | He.m. \frac{1}{M_\psi - H_0 + i\partial_0 - H_1} H_2 \frac{1}{M_\psi - H_0 + i\partial_0 - H_1} H_2 | J/\psi \rangle + \\
& + \langle \gamma g_1 g_2 | H_2 \frac{1}{M_\psi - H_0 + i\partial_0 - H_1} H_2 \frac{1}{M_\psi - H_0 + i\partial_0 - H_1} He.m. | J/\psi \rangle + \\
& + \langle \gamma g_1 g_2 | H_2 \frac{1}{M_\psi - H_0 + i\partial_0 - H_1} He.m. \frac{1}{M_\psi - H_0 + i\partial_0 - H_1} H_2 | J/\psi \rangle \},
\end{aligned} \tag{2}$$

where ω_1 and ω_2 are the energies of the gluons g_1 and g_2, respectively,

$$H_0 \int d^3x_1\, d^3x_2\, \Psi^\dagger(\vec{x}_1, t)\Psi(\vec{x}_1, t)\hat{H}\Psi^\dagger(\vec{x}_2, t)\Psi(\vec{x}_2, t), \tag{3}$$

$$H_1 = g_s \int d^3x \Psi^\dagger(\vec{x}, t)\frac{\lambda a}{2}\Psi(\vec{x}, t)A_0^a(\vec{X}, t), \tag{4}$$

$$\begin{aligned}
H_2 = & -g_s \int d^3x(\vec{x} - \vec{X})\Psi^\dagger(\vec{x}, t)\frac{\lambda a}{2}\Psi(\vec{x}, t) \cdot \vec{E}^a(\vec{X}, t) - \\
& -\frac{1}{2}g_s \int d^3x(\vec{x} - \vec{X})\text{x}\Psi^\dagger(\vec{x}, t)\vec{\gamma}\frac{\lambda a}{2}\Psi(\vec{x}, t) \cdot \vec{B}^a(\vec{X}, t) -
\end{aligned} \tag{5}$$

$$He.m. = \frac{2}{3}e \int d^3x \overline{\Psi}(\vec{x}, t)\gamma^\mu\Psi(\vec{x}, t)\mathcal{A}_\mu(x). \tag{6}$$

In (3)-(6), $\Psi(\vec{x}, t)$ is the constituent quark field,[10,11] $\hat{H}$ is the quantum mechanical Hamiltonian of the $c\bar{c}$ system,[10,11] $\lambda a (a = 1, 2, ...8)$ is the SU(3) Gell-Mann matrix, $\vec{X}$ is the coordinate of the center of mass of c and $\bar{c}$, A_μ^a is the gluon field, $\vec{E}^a$ and $\vec{B}^a$ are the color electric and magnetic field, respectively, e is the electromagnetic coupling constant and $\mathcal{A}_\mu$ is the photon field. The operator

$i\partial_0$ in (2) is understood that it operates only on the gauge fields. The constituent quark field $\Psi(\vec{x}, t)$ is quantized as[11)]

$$\{\Psi(\vec{x}, t), \Psi^\dagger(\vec{x}', t)\} = \delta^3(\vec{x} - \vec{x}')$$

$$\text{others anticommute} \tag{7}$$

The quantum mechanical Hamiltonian $\hat{H}$ is specified by taking a certain potential model for the $c\bar{c}$ system. In this paper, we take the Cornell Coulomb plus linear potential[12)] to do the calculation. To evaluate the matrix elements in (2), we insert complete sets of intermediate states into the matrix elements. Take the first matrix element in (2) as an example. By inserting intermediate states, the process can be described as: J/ψ first emits a gluon via H_2 and transits into an intermediate state in which c and $\bar{c}$ are in color octet, then it emits the second gluon via H_2 and transits into an intermediate state in which c and $\bar{c}$ are in color singlet, and finally these c and $\bar{c}$ annihilate into a photon via $He.m.$. The meanings of the other two matrix elements in (2) can be seen in a similar way. The intermediate state containing a gluon and a $c\bar{c}$ pair in color octet is difficult to study from the first principle of QCD since there should be strong interactions between the three bodies. As we used to do in Ref. 8), we take the vibrational state of the quark-confining-string model[13)] to represent this state. The Hamiltonian $H_0 - i\partial_0 + H_1$ for this system will then be taken to be the energy eigenvalue of the string vibrational state. When we do this, the matrix element will depend on the model of the intermediate states. To compensate this model dependence, we take the multipole gluon emission coupling constants to be effective phenomenological coupling constants normalized by taking certain known hadronic transition rates as inputs. We then write H_2 (cf.(5)) as

$$H_2 = -g_E \int d^3x(\vec{x} - \vec{X})\Psi^\dagger(\vec{x},t)\frac{\lambda a}{2}\Psi(\vec{x},t)\cdot\vec{E}^a(\vec{X},t) -$$
$$-\frac{1}{2}g_M \int d^3x(\vec{x} - \vec{X})\text{x}\Psi^\dagger(\vec{x},t)\vec{\gamma}\frac{\lambda a}{2}\Psi(\vec{x},t)\cdot\vec{B}^a(\vec{X},t) - \ldots, \tag{8}$$

where g_E and g_M are the effective electric dipole emission and magnetic dipole emission coupling constants, respectively. $\alpha_E = g_E^2/4\pi$ can be determined by taking the datum of $\Gamma(\psi' \to J/\psi\pi\pi)$ as input. For the Cornell model it is[8)]

$$\alpha_E = 0.54. \tag{9}$$

There is not yet an ideal datum for determining $\alpha_M = g_M^2/4\pi$. A reasonable range for α_M is[8)]

$$\alpha_E \le \alpha_M \le (2-3)\alpha_E. \tag{10}$$

After this normalization, different models for the intermediate states give very similar results.[14)]

The three terms in (2) correspond to three kinds of diagrams shown in Fig. 1. In the perturbation theory where the internal quark lines are free propagators, the three kinds of diagrams are not essentially different. While in our present approach, the intermediate states are considered as certain bound states, the three kinds of diagrams are physically different. In the following calculation, we take into account only the leading $E1$ gluon emissions (the first term in (8)). Fig. 1(a) contains two $E1$ gluon transitions and one electromagnetic annihilation vertex, so that its amplitude is proportional to $eg_E^2 f_{n0}(\vec{0})$, where $f_{n0}(\vec{0})$ is the wave function at the origin of the intermediate state $\psi(n^3S_1)$. Fig. 1(b) and (c) are different. They contain one $E1$ gluon transition, one electromagnetic transition and a vertex at which the color octet $c\bar{c}$ pair annihilates into a gluon. In the non-relativistic approach, $c\bar{c}$ annihilation in the present theory is exactly the same as that in perturbative

QCD[11]), so that the coupling constant g_s at this annihilation vertex should be taken to be the conventional QCD coupling constant at the scale M_ψ instead of being g_E. Then the amplitudes of Fig. 1(b) and (c) are proportional to $eg_Eg_s f'_{n0}(\vec{0})$, where $f'_{n0}(\vec{0})$ is the wave function at the origin of the n^3S_1 string vibrational state. We know that $g_s < g_E$ and $|f'_{n0}(\vec{0})| < |f_{n0}(\vec{0})|$. Therefore the amplitudes of Fig. 1(b) and (c) are suppressed relative to that of Fig. 1(a) in our approach. This suppression makes the photon spectrum so different from that in the perturbation theory.

The calculation is in the Coulomb gauge. The evaluation of (2) and the calculation of the direct photon spectrum (1) are straight forward but lengthy. The result is

$$\begin{aligned}\frac{1}{\Gamma_{tot}}\frac{d\Gamma(J/\psi\rightarrow\gamma gg)}{dx} &= \frac{2\alpha\alpha_E^2}{15\Gamma_{tot}}\left(\frac{M_\psi}{3}\right)^6\{h_1^2x[x^4+30(1-x)^2]+\\ &+\frac{5}{2}\sqrt{\frac{\alpha_s}{\alpha_E}}h_1h_2[x^2(2-x)(x^2-2x+2)-24(1-x)^3(\frac{x^2-2x+2}{x(1-x)}ln(1-x)+\frac{(2-x)^3}{4(1-x)^2})]+\\ &+\frac{5}{4}\frac{\alpha_s}{\alpha_E}h_2^2[5x^3(x^2-2x+2)-24(1-x)^2(\frac{x^2-2x+2}{2-x}ln(1-x)+\frac{x(2-x)^2}{4(1-x)})]\},\end{aligned} \tag{11}$$

where

$$h_1 \equiv \sum_{n,K}\frac{\langle R_{n0}|r|R'_{K1}|\langle R'_{K1}|r|R_{10}\rangle}{(M_\psi-E'_{K1})(M_\psi-E_{n0}-\omega_1-\omega_2)}f_{n0}(\vec{0}). \tag{12a}$$

$$\begin{aligned}h_2 \equiv &\sum_{K'n}\frac{\langle R'_{K'0}|r|R_{n1}\rangle\langle R_{n1}|r|R_{10}\rangle}{(M_\psi-E_{n1}-\omega_\gamma)(M_\psi-E'_{K'0}-\omega_\gamma}f'_{K'0}(\vec{0})+\\ &\sum_{K',K}\frac{\langle R'_{K'0}|r|R'_{K1}\rangle\langle R'_{K1}|r|R_{10}\rangle}{(M_\psi-E'_{K1})(M_\psi-E'_{K'0}-\omega_\gamma)}f'_{K'0}(\vec{0}).\end{aligned} \tag{12b}$$

In (12), R_{nl} and E_{nl} are the radial wave function and the energy eigenvalue of the charmonium state with principle quantum number n and angular momentum quantum number l, R'_{KL} and

E'_{KL} are the radial wave function and the energy eigenvalue of the $n = 1$ mode string vibrational state with principle quantum number K and angular momentum quantum number L, $f_{n0}(\vec{0})$ and $f'_{K'0}(\vec{0})$ are the wave function at the origin of the charmonium state and string vibrational state, respectively.

In (11), we take Γ_{tot} to be its experimental value.[15] The wave functions at the origin $f_{n0}(\vec{0})$, $f'_{K'0}(\vec{0})$ in (12) are to be normalized by the experimental values of related decay widths to include effectively QCD corrections. $f_{n0}(\vec{0})$ is related to the leptonic width of the n^3S_1 state charmonium by

$$\Gamma(n^3S_1 \to l^+l^-) = \frac{64\pi\alpha^2}{9M_n^2}|f_{n0}(\vec{0})|^2, \tag{13}$$

where M_n is the mass of the n^3S_1 state charmonium. We expect that QCD corrections will not vary seriously with n as is inspired by the cases of $n = 1$ and $n = 2$.[11] So we take the experimental value of $\Gamma(J/\psi \to l^+l^-)$ (or $\Gamma(\psi' \to l^+l^-)$) to normalize the $f_{n0}(\vec{0})$'s as what is done in Ref. 11). The wave function at the origin of the string vibrational state is related to the hadronic width $\Gamma(J/\psi \to ggg)$ in the present approach, and it is

$$\Gamma(J/\psi \to ggg) = 160\alpha_s\alpha_E^2(\pi^2 - \frac{29}{3})(\frac{M_\psi}{6})^6 h_3^2, \tag{14a}$$

where

$$h_3 \equiv \sum_{K',K} \frac{\langle R''_{K'0}|r|R'_{K1}\rangle\langle R'_{K1}|r|R_{10}\rangle}{(M_\psi - E''_{K'0})(M_\psi - E'_{k1})} f''_{K'0}(\vec{0}). \tag{14b}$$

In (14b), $R''_{K'0}$, $E''_{K'0}$ and $f''_{K'0}$ are the radial wave function, energy eigenvalue and the wave function at the origin of the $n = 2$ mode string vibrational state, respectively. We also expect that QCD corrections to most of the low lying string vibrational states are approximately the same, so that we normalize the wave functions at the origin of the string vibrational states by the experimental value of $\Gamma(J/\psi \to ggg)$ which can be extracted from the experimental values of Γ_{tot} and $BR(J/\psi \to e^+e^-)$ [15] by

$$\Gamma(J/\psi \to ggg) \simeq \Gamma_{tot}[1 - (2 + R)BR(J/\psi \to e^+e^-)], \tag{15}$$

where $R \equiv \sigma(e^+e^- \to hadrons)/\sigma(e^+e^- \to \mu^+\mu^-)$. The electromagnetic transition matrix elements $\langle R_{n1}|r|R_{10}\rangle$ and $\langle R'_{K'0}|r|R'_{K1}\rangle$ in (12b) should also be normalized by experimental data. We know that the electromagnetic $E1$ transition rates in the $c\bar{c}$ system calculated from the Cornell mode are greater than the data by roughly a factor or 2. So that normalized by the data, the transition elements should be smaller by roughly a factor of $1/\sqrt{2}$.

The terms in (11) can be calculated numerically in the Cornell potential model. We take into account the contributions of the first five low lying states in each summation of the intermediate states. The contributions of Fig. 1(a) (first term in (11)) and Fig. 1(b) and (c) (last term in (11)) are plotted in Fig. 2 together with the corresponding contributions obtained from leading order perturbative QCD[2] with the same normalization for comparison. We see that the contribution of Fig. 1(a) is not much different from that in perturbative QCD, while the contributions of Fig. 1(b) and (c) are significantly suppressed relative to those in perturbative QCD, and this improves the large x behavior of the spectrum.

To compare with the experimental data, we should smear the theoretical curve given by (11) by taking convolution according to the resolution of the photon detector $\delta E/E = 0.12E^{-1/2}$. [6] The result is shown in the solid curve in Fig. 3. The leading order perturbative QCD result is also shown in Fig. 3 in dashed curve for comparison. We see that our result is much better than the perturbative QCD result and is very close to the data in the range $x > 0.8$. In the $x \leq 0.8$ range, our curve is below the data, so that there is room for the contributions of the resonances[7] which are not included in the present Born approximation calculation (of course, for small x the present approach is less reliable). There can be other non-perturbative effects, say gluon condensate and final state hadronization effects, that can make the final state gluons different from massless on-shell particles and thus will also make the photon spectrum softer. Our calculation implies that the bound state contribution to the intermediate states is an important one among the various non-perturbative effects. In our calculation, we take into account only discrete state contributions to the intermediate states. In the framework of QCD multipole expansion, the contribution of continuous spectrum to the intermediate states is that of the virtual charmed meson pair bubbles which is just the coupled channel correction to the present calculation. Relativistic corrections and non-leading multipole gluon emissions are also neglected in this calculation. These effects will be considered in future investigations.

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Figure 1: Examples of the three different kinds of diagrams corresponding to the three terms in (2). The solid line, wavy line and spiral line denote c-quark, photon and gluon, respectively. Every gluon or photon can be emitted by both c and $\bar{c}$.

Figure 2: Comparison of the contributions of Fig. 1(a) and Fig. 1(b) and (c) between the present theory (solid line) and leading order perturbative QCD (dashed line). (a) Contribution of Fig. 1(a). (b) Contributions of Fig. 1(b) and (c).

Figure 3: The direct photon spectrum in $J/\psi \to \gamma + g + g$. The solid curve is the result of the present theory. The dashed curve is the result of leading order perturbative QCD.[6)] The experimental data is from Ref. 6).

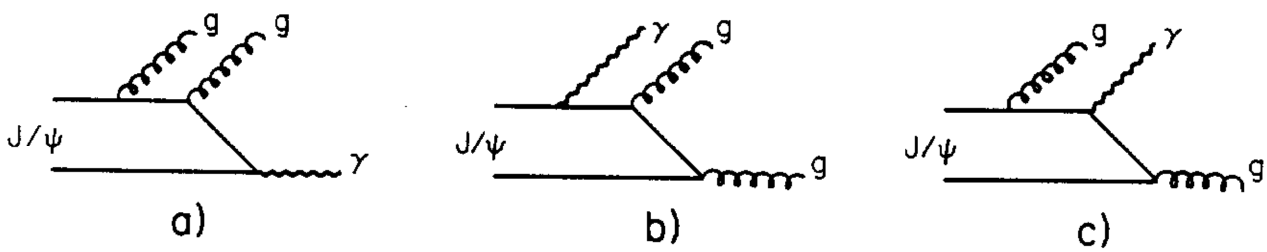

Fig.1

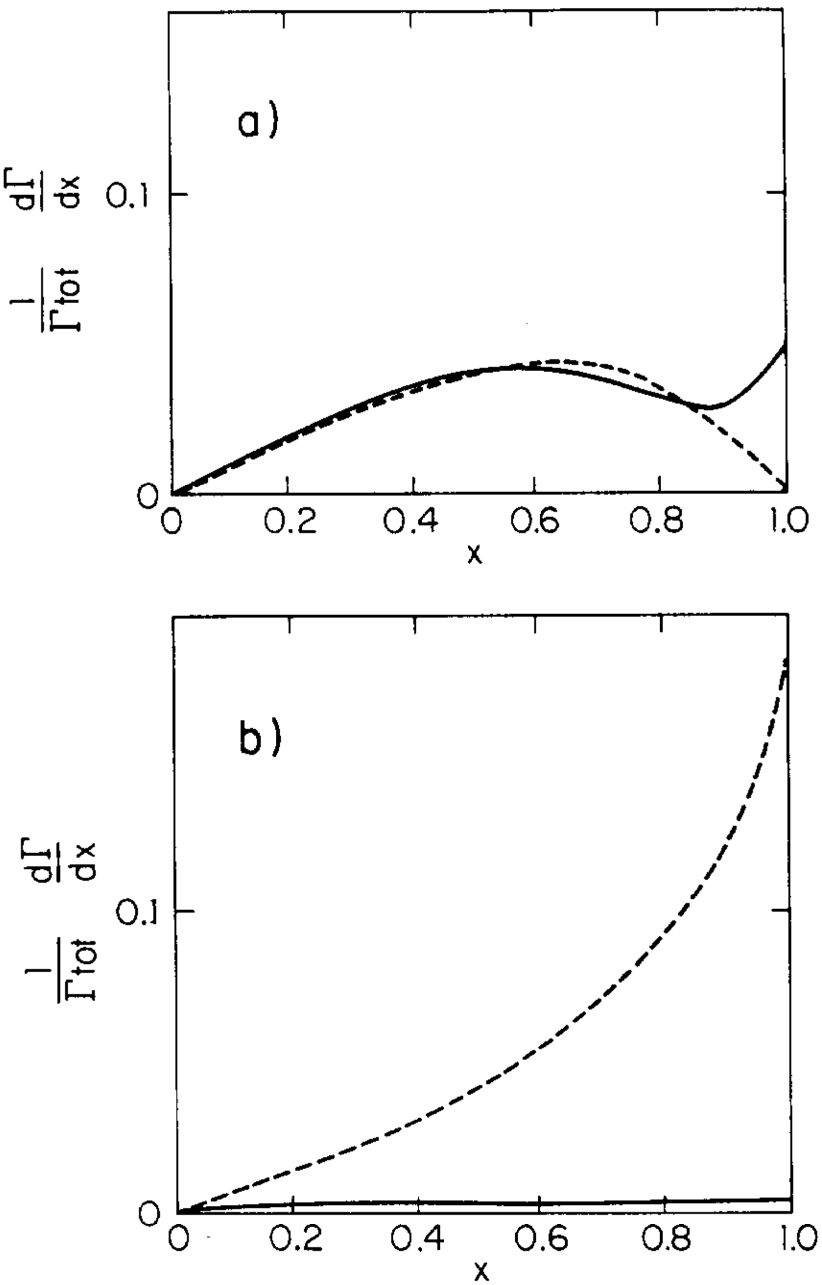
a)
$\frac{1}{\Gamma_{tot}} \frac{d\Gamma}{dx}$
0.1
0
0
0.2
0.4
0.6
0.8
1.0
x
b)
$\frac{1}{\Gamma_{tot}} \frac{d\Gamma}{dx}$
0.1
0
0
0.2
0.4
0.6
0.8
1.0
x

Fig.2

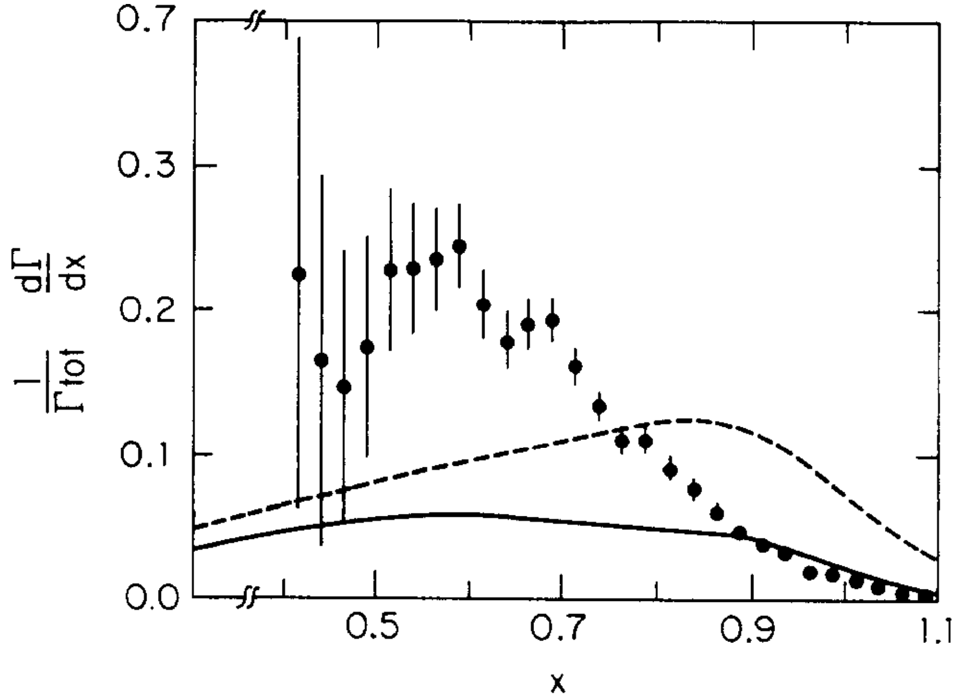

Fig.3