Fermi National Accelerator Laboratory

FERMILAB-Pub-89/252-A
December 1989

Big Bang Nucleosynthesis and the Quark-Hadron Transition

Hannu Kurki-Suonio
Department of Physics
Drexel University
Philadelphia, PA 19104

Richard A. Matzner
Center for Relativity
and
Department of Physics
University of Texas
Austin, TX 78712-1081

Keith A. Olive
School of Physics and Astronomy
University of Minnesota
Minneapolis, MN 55455

and

David N. Schramm
University of Chicago
5640 S. Ellis Avenue
Chicago, IL 60637
and
NASA/Fermilab Astrophysics
Fermi National Accelerator Laboratory
Box 500
Batavia, IL 60510

In press *Astrophysical Journal*, 1989

Abstract

An examination and brief review is made of the effects of quark-hadron transition induced fluctuations on big bang nucleosynthesis. It is shown that cosmologically critical densities in baryons are difficult to reconcile with observation, but the traditional baryon density constraints from homogeneous calculations might be loosened by as much as 50%, to 0.3 of critical density, and the limit on the number of neutrino flavors remains about $N_\nu \lesssim 4$. To achieve baryon densities $\gtrsim 0.3$ of critical density would require initial density contrasts $R \gg 10^3$, whereas the simplest models for the transition seem to restrict R to $\lesssim 10^2$.

The possibility that effects due to the confinement of quarks in the early Universe could create significant changes[1,2] to the standard homogeneous big bang nucleosynthesis results[3,4] has received a great deal of recent attention. In the standard homogeneous-isotropic big bang nucleosynthesis calculation, a weak and nuclear reaction network is numerically followed for a uniform fluid, cosmologically expanding and cooling in the early universe to predict light element abundances. The success of such calculations is one of the central ingredients to the current overwhelming support found for the big bang model itself. While the basic weak and nuclear reactions are measured in the lab to reasonable accuracy and are thus not seriously questioned, the assumption of a homogeneous-isotopic fluid has been questioned many times (cf. ref. 3). Recent work on the quark-hadron transition has given a physically derived motivation to such questioning. In particular the transition from the early "quark-soup" to normal hadronic nuclear matter should take place at $T \geq 100$ MeV at just prior to the nucleosynthesis epoch, at $T \leq 1$ MeV. Witten and others[5)] had noted that if the quark-hadron transition is a first order phase transition then density fluctuations would naturally result. The possible effects of these fluctuations on big bang nucleosynthesis calculations is the reason for the current excitement.

The purpose of this paper is to briefly review the previous quark-hadron inspired results and compare them with the traditional homogeneous results and then to present a new set of calculations which explicitly show the sensitivity of the resultant light element abundances to the parameters of the quark-hadron transition. We will show that even if the transition is first order, the result is unlikely to significnatly alter the key predictions from homogeneous nucleosynthesis as long as one continues to require agreement with the observed light element abundances, particularly ^7Li and ^4He. The persistence of the nucleosynthesis conclusions despite the addition of new initial conditions with several additional parameters shows the robustness of big bang nucleosynthesis.

Traditional big bang nucleosynthesis had become one of the cornerstones of big bang cosmology because of its remarkable agreement with light element abundance observations, spanning a dynamical range of over 9 orders of magnitude in its predictive powers. This success, coupled with its prediction of the number of neutrino families[6,7,3] is an important vindication of the "particle physics connection" in the study of the early universe. Furthermore, standard big bang nucleosynthesis arguments using deuterium[8)] and later helium-3[3)] and lithium[3,9)] constrain the density, Ω_b, of normal matter, baryons[11)], in units of the critical density to $\Omega_b \sim 0.1$. More precisely, the ratio of baryons to photons, $n_b/n_\gamma \equiv \eta$ is constrained[3,9)] to

$$3 \times 10^{-10} \lesssim \eta \lesssim 4 \times 10^{-10} \qquad (1)$$

for current population II stellar lithium abundances and current limits on D and ^3He*. The fact that $\Omega_b \sim 1$ is excluded is one of the prime driving forces behind the current searches for non-baryonic dark matter[11)].

*The upper limit of $\eta < 6 \times 10^{-10}$ in ref. 3 from ^7Li was reduced to $\eta < 5 \times 10^{-10}$ in ref. 9 using newer lithium rates. The latest rates[10)] yield the upper limit in eq. (1).

With so much at stake, the initial claims[1,2] that a quark-hadron transition inspired model could yield an $\Omega_b = 1$ Universe compatible with light element abundances created tremendous interest. Some preliminary lattice gauge calculations implied that the quark-hadron transition may indeed be a first order phase transition. Applegate et al[1] noted that due to the proton's electric charge there is preferential diffusion of neutrons versus protons out of the high density fluctuations produced by such a quark-hadron transition. This could lead to big bang nucleosynthesis occurring under conditions with both inhomogeneities and variable neutron/proton, n/p, ratios. The result is that the nucleosynthesis in the high density regions occurs with a low n/p ratio while the low density region has a high n/p. Regions with n/p > 1 have qualitatively different nucleosynthesis then standard homogeneous nucleosynthesis (where n/p ~ 1/7). If n/p > 1, the number of protons rather then neutrons becomes the constraining parameter on the reaction network flow towards ^{4}He.

In the first round of calculations[1,2] these groups claimed that such mixed conditions might allow $\Omega_b \sim 1$ while fitting the observed primordial abundances of ^{4}He, D, ^{3}He but with an overproduction of ^{7}Li. Since ^{7}Li is the most recent of the cosmological abundance constraints and has a different observed abundance in population I stars versus the traditionally more primitive population II stars[12] some argued that perhaps some special depletion process might have occurred to reduce the excess ^{7}Li. Reeves and Audouze et al[13] each argued against such processes and tried to turn the argument around and use the lithium abundances to constrain properties of the quark-hadron transition.

On this basis, Reeves concluded that the ^{7}Li abundances required that the ratio R, of baryon densities in the high to low density regions satisfy $R < 2$-4. These limits in principle imply constraints on the transition temperature $T_c \geq 150$ MeV. The limit on T_c is however based on naive assumptions made in estimating the density contrast as a function of the transition temperature T_c[5,2]. The main ingredient neglected was the interactions in the hadron phase (indeed without these one would conclude the existence of a high temperature hadron phase). When the effects of the finite size of hadrons due to repulsive interactions are included[14] one finds that for a first order transition $R \gtrsim 7$ for all values of T_c. This means that possible constraints from nucleosynthesis must be on the more detailed aspects of the phase transition. One should also note that the baryon density contrast across the phase boundary during the transition does not necessarily translate directly into the density contrast remaining after the transition[15].

At first it appeared that if the lithium constraint could be surmounted then the constraints of standard big bang nucleosynthesis might disintegrate. Although the number of parameters needed to fit the light elements was somewhat larger for the non-standard models, nonetheless a non-trivial loophole appeared to be forming. To further stimulate the flow through the loophole, Malaney and Fowler[16] showed that in addition to looking at the diffusion of neutrons out of high density regions one must also look at the subsequent effect of neutrons diffusing back into the high density regions as free neutrons are depleted at a much slower rate in the low density regions in nucleosynthesis. (The initial calculations treated the two regions separately.) Malaney and Fowler argued that for certain phase transition parameter values, (eg. nucleation site separations ~ 10m at the time of the transition) this back diffusion could destroy much of the excess lithium produced as ^7Be via $^7Be(np)^7Li(p\alpha)^4He$ in the high density regions. However, it has been recently argued[17,18,19] that in detailed diffusion models, the back diffusion not only affects ^7Li but also the other light nuclei as well. Those calculations found that for $\Omega_b \sim 1$, ^4He is also overproduced (although it does go to a minimum for similar parameter values as does the lithium).

One can understand why these models tend to overproduce ^4He and ^7Li by remembering that in standard homogeneous big bang nucleosynthesis, high baryon densities lead to excesses in these nuclei. As back diffusion evens out the effects of the initial fluctuation the averaged result should approach the homogeneous value. Furthermore, any narrow range of parameters, such as those which yield relatively low lithium and helium, are unrealistic since in any realistic phase transition there is a distribution of parameter values (distribution of nucleation sites, separations, density fluctuations etc.). Therefore narrow minima are washed out[20] which would bring the ^7Li and ^4He values back up to excessive levels for parameter values with $\Omega_b \sim 1$. We stress this point since diffusive effects are only important in lowering the nuclear abundance in a narrow window of parameter space.

After the above review of the current situation and the new apparent difficulties in making $\Omega_b \sim 1$, we have decided to address the quark-hadron transition with a more traditional approach. Namely, instead of setting $\Omega_b \sim 1$ and seeing what excesses may or may not occur, let us believe the light element abundance observations and see how the traditional big bang nucleosynthesis constraints might vary as quark-hadron transition parameters are explored. (This is similar to the approaches of Reeves and Audouze et al[13] however we are using the more detailed dynamical code of Kurki-Suonio and Matzner[18] which explicitely includes multizone forward and backward diffusion). Indeed, one might worry that because $R \geq 7$ for all values of T_c, the allowed set of parameters in standard big bang nucleosynthesis might be altered (e.g. the range in η). We will therefore test the standard model parameters in the presence of baryon inhomogeneities.

In these calculations we did not explore the exciting possibility[21] that quark-hadron fluctuations might enable big bang nucleosynthesis to make elements heavier than ^{7}Li which are blocked in the conventional model. If such synthesis is possible for the allowed parameter space that fits the light element abundances this would be very exciting and might explain some abundance patterns in metal-poor stars and provide an independent test of whether or not the transition was indeed first order.

We have also not explored the remaining fundamental physics questions about the transition itself. Is it a first order phase transition? What is the relationship between nucleation sites, density fluctuations, etc., and the fundamental QCD parameter Λ_{QCD}? We have also followed the previous calculations and assumed basically isothermal fluctuations, however differential temperature diffusion should be explored.

We follow the parameterization of Kurki-Suonio and Matzner[18] which treats the transition in a very phenomenological manner. (For the relationship of these parameters to certain bag models see Alcock et al[2].) Thus our aim in this paper is not to make specific statements about the physics of the quark-hadron transition (although some inferences might be made) but instead to see what effects the transition might have on the traditional big bang nucleosynthesis constrained quantities, in particular on η. To this end, we will use the results of Kurki-Suonio and Matzner[18] for the calculated abundances of D, ^{3}He, ^{4}He and ^{7}Li in a nucleosynthesis model in the presence of baryon inhomogeneities with diffusion taking place before and during nucleosynthesis. Because the details of the quark-hadron transition are largely unknown, we explore a parameter space to find the largest possible set of primordial abundances.

The phenomenological parameters we explored which can affect nucleosynthesis are the following:

(1) The average baryon to photon ratio, η;

(2) The average density contrast R

(3) The average distance, scale of the inhomogeneities, ℓ and

(4) The average volume fraction of the high density regions, f_v.

(Note that only η is a parameter in the homogeneous case.) Furthermore the geometry of the high density regions can also have an effect. We consider planar, spherical and cylindrical geometries. In this paper we did not consider fractal-like boundaries which might also result in such transitions and could further enhance surface diffusion effects. Our results are displayed for a baryon density contrast between the high and low density phases, R = 100. Increasing (decreasing) R, brings the resultant abundances further from (closer to) the homogeneous results[18]. For example, reducing the contrast to R = 10 depending on the volume fraction involved, reduces the deviation from the homogeneous results to about one-half and to about one-quarter for R = 6 (when $f_v = \frac{1}{4}$). We also consider a range f_v = 1/4 - 1/64 for the volume fraction of the high density region. Specifically, our

results use data from the following choices of parameters $f_v = 1/4$, $f_v = 1/8$ and $f_v = 1/16$ for planar geometries and $f_v = 1/8$ and $f_v = 1/64$ for spherical geometries. These were chosen so as to minimize and maximize the elemental abundances. As f_v goes to 1 or 0, the results approach the homogeneous results. (For larger R, smaller f_v would have to be considered.) The distance scale ℓ is given in meters at 100 MeV after the phase transition between the centers of high and low density regions.
For a given value of η and ℓ, we have varied f_v and the geometry so as to find a maximal range for the calculated abundances. We will find that only for a limited range in ℓ and η are the derived abundances in agreement with observational determinations.

In the figure, we show the allowed region in the ℓ - η plane from the constraints given by the abundances of D, ^3He, ^4He and ^7Li. The observational constraints we use are the following[4]: $D/H \geq 10^{-5}$ by number, $(D + {}^3He)/H \leq 10^{-4}$ by number, $0.224 \leq Y_{^4He} \leq 0.254$, where $Y_{^4He}$ is the ^4He abundance by mass and $^7Li/H \leq 2 \times 10^{-10}$ by number for population II and $^7Li/H \leq 2 \times 10^{-9}$ by number for population I. For standard big bang nucleosynthesis ($\ell = 0$) the bounds on η may be read from the bottom of the figure; they are the results giving rise to eq. (1).

The calculated abundances are for a neutron half-life of $\tau_n = 10.35$ min. The weak n <--> p rates are obtained by numerical integration, and multiplied with a Coulomb correction factor[22] 0.98. Additional small corrections calculated by Dicus et al.[23] are represented by subtracting 0.001 from all ^4He mass fractions. The strong reaction rates used are from the recent compilation by Caughlan and Fowler[10]. The new rates for $^2H(d,n)^3He$, $^2H(d,p)^3H$, $^3He(d,p)^4He$, $^4He(t,\gamma)^7Li$, and $^7Be(n,p)^7Li$ lead to a higher estimate for produced ^{7}Li further narrowing the range of η allowed by population II ^{7}Li. Rates for (n,γ) reactions and $^{7}Be(n,\alpha)^{4}He$ not included in this compilation are those used by Schramm and Wagoner[24], except the newer estimate for $^{7}Li(n,\gamma)^{8}Li$ by Malaney and Fowler[25] is used. Abundances for $A > 7$ isotopes are not calculated but their maximum effect on $A \leq 7$ isotopes was controlled by including the reactions leading to $A > 7$ as sinks. For the density range discussed here the effect of these sinks on final ^{7}Li was at most a few per cent. (Except that in the $f_v = 1/64$, $\eta = 7 \times 10^{-9}$ case, where the high density region had the highest density, ^{7}Li came 15-45% lower with sinks than without sinks. A full network would give a result in between. Since these yields were an order of magnitude above the population I upper limit, this inaccuracy does not affect the results reported here.) Because reactions occur in thin layers near the original high/low density boundary[26], a fairly fine zoning was necessary for accurate results (in most cases 64 zones was found to be sufficient, compared with only 8 zones for the Livermore group[20] and 2 for the Tokyo group[19]).

When we compare these to the observational constraints, we find the contours shown in the figure. Consider for example the contour found from $Y_{^4He} < 0.254$. As the inhomogeneity is turned on, the ^{4}He abundance increases. The rise in ^{4}He for small ℓ has a straightforward explanation. Because of the small distance ℓ, all the neutrons can find their way to a high density to react prior to their decay. ^{4}He is raised in the high density regions. Since the computations are for fixed averaged baryon density, the result is that nucleosynthesis occurs in overdense regions giving enhanced ^{4}He. For <u>very</u> small ℓ, the protons as well as the neutrons diffuse, giving the uniform baryon density result when nucleosynthesis begins. For an optimal value of ℓ in the range 10-100, one sees the original effect claimed in refs. 1 and 2. However, as claimed there, the back diffusion does not allow the drop in $Y_{^4He}$ to be as pronounced and only a modest increase in the limit on η based on $Y_{^4He}$ is seen. For larger values of ℓ, diffusion becomes irrelevant and one has strictly an inhomogeneous nucleosynthesis model and one finds a larger ^4He abundance[3,27,6] (and hence a tighter constraint on η).

For the cases of D and D + ^3He, aside from a slight decrease in D (for relatively low η) both D and D + ^3He increase with ℓ. This shifts allowed the values of η to a higher range. For $\ell = 0$, standard nucleosynthesis, D and D + ^3He require $3 \times 10^{-10} \leq \eta \leq 10 \times 10^{-10}$ whereas, for $\ell \geq 100$ this range moves up to $4.5 \times 10^{-10} \leq \eta \leq 30 \times 10^{-10}$. The dip in D for $\ell \sim 10$ allows a drop in the bound in η, $\eta \geq 2.2 \times 10^{-10}$.

The ^7Li abundances, as has been known all along in this type of investigation, rise with increasing ℓ; the effect of which is to decrease the allowed range for η. In the case of the population II ^7Li abundances, we see rather dramatically the constraint $\ell \lesssim 150$, for any value of η. When $\ell \sim 10$, we find (using the D + ^3He abundances for the lower limit) $2.2 \times 10^{-10} \leq \eta \lesssim 3 \times 10^{-10}$. There is a gap which excludes values of ℓ from 30-100. For ℓ between 100 and 150, we have $4 \times 10^{-10} \leq \eta \leq 7 \times 10^{-10}$, limits which are comparable or tighter than the standard nucleosynthesis bounds. The bound from $^7Li_{II}$, $\eta \lesssim 7 \times 10^{-10}$ for $\ell \sim 100$ is evidence of the Malaney-Fowler[16] effect, a maximization of back diffusion destruction of ^7Li (actually ^7Be, which produces ^7Li by e^--capture). The ^7Li abundance is determined late in nucleosynthesis, when neutron abundance is very low. For

$\eta \leq 10^{-9}$ the neutron fraction in the low-density region is diminishing slowly enough that sufficient neutrons are available to diffuse into the high-density regions and destroy most of the ^7Be there. This effect is very sensitive to the distance scale. If distances are too short, diffusion depletes the neutron resevoir too early. If distances are too long, diffusion will not be efficient. For higher densities, e.g. $\eta = 7 \times 10^{-9}$ with R = 100, even the low-density region is too dense for a sufficient number of neutrons to survive long enough to have a dramatic effect. For the population I abundances, though we do not find a limit on ℓ, the bounds on η are again comparable to the standard results. In either case, $\eta \lesssim 20 \times 10^{-10}$ or $\Omega_b \lesssim 0.3$ remain upper bounds for all values of the parameters considered. We conclude once more that the Universe can not be closed by baryons. (The lower bound of Ω_b drops by only ~ 25%; thus still being greater than Ω in visible matter.)

Diffusion effects on nucleosynthesis could be stronger if the density contrast were much higher than R = 100. Because the details of the confinement transition are poorly understood, it is difficult to make a convincing calculation of R from first principles. One approach[5)] has been widely used; namely calculate R assuming chemical equilibrium during the phase transition. With this assumption (and only with this assumption) can one calculate unambiguously the density contrast. In this case, it was shown[14)] that for $T_c \geq 100$ MeV, $R \leq 100$.

Kurki-Suonio[15)] considered possibilities for the evolution of baryon number fluctuations assuming that the equilibrium ratio is maintained at the phase boundary but only extends a diffusion length from the boundary. Depending on the distance scales of nucleation, coalescence, and diffusion, he obtained various possibilities, with the most probable being to form final density contrast $R = (w_H/w_Q)R_{eq}$, where (w_H/w_Q) is the ratio of enthalpy densities of the two phases, which is less than 1, and R_{eq} is the equilibrium baryon density ratio. To obtain significant inhomogeneities with R much larger that R_{eq} would seem to require extremely efficient baryon transport in the quark phase, the more likely outcome being that the final inhomogeneity involves only an insignificant fraction of the total baryon number.

In the models of Ref. 18, the dependence on R is rather weak and results for R = 1000 do not appear very different. In. Ref. 19, $R = 10^3$ 10^4, was claimed to allow $\Omega_B = 1$ if $h_o < 0.5$. However this possibility is achieved only for an extremely narrow range in the parameter f_v. (We remind the reader of our previous comment with regard to results which are valid only in narrow windows.) This conclusion is based on a two-zone calculation (in contrast to the 64-zone calculation in ref. 18) and uses constraints $Y_{^4He} < 0.26$ and $^7Li/H < 10^{-9}$ (we assume that $Y_{^4He} < 0.254$ and $^7Li/H < 2 \times 10^{-10}$ for population I and $^7Li/H < 2 \times 10^{-9}$ for population II). The homogeneous value of $Y_{^4He}$ in ref 19 also falls short by about 0.005 of the homogeneous calculations used here and in ref. 3. On this basis, we do not feel that there is any real disagreement between those results and the ones quoted here. Mathews et. al.[28] have studied the effect of extreme density contrasts $R = 10^5$ and report that with suitable parameter values ^2H, ^3He, ^4He can be brought to simultaneous agreement with observations. From comparisons with the work of other groups, it would seem that deviations from our conclusions only begin to occur for $R >> 10^3$ which we consider unrealistic.

It is also interesting to note that in addition to altering the bounds on η, baryon inhomogeneities and neutron diffusion could in general alter the limits on the number of neutrino flavors from nucleosynthesis. The current limit of $N_\nu < 4.2$ based on $Y_{^4He} < 0.254$, $\tau_n > 10.2$ min and $\eta > 3 \times 10^{-10}$. Depending on the value of η and ℓ, the limit could increase or decrease. For example, at $\eta = 4 \times 10^{-10}$ and $\ell \sim 100$, $N_\nu < 4.6$ and $\ell \sim 10$ for the same value of η, $N_\nu < 3.9$, while for $\eta = 2.2 \times 10^{-10}$ with $\ell \sim 10$, $N_\nu < 4.3$. Thus again, we find only minor fluctuations from the traditional conclusion.

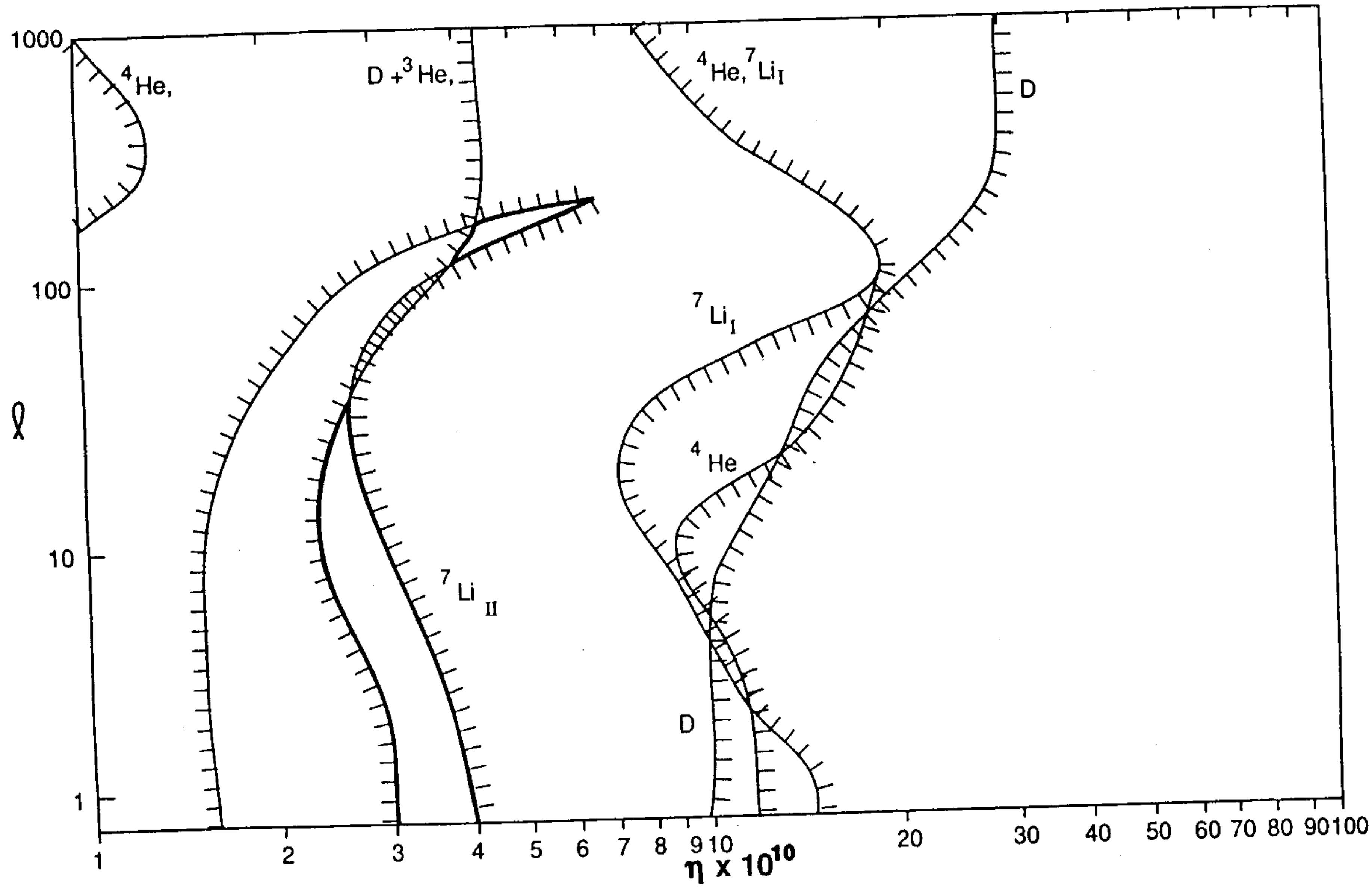
Although from nucleosynthesis abundances we can not calculate a limit to R or the possibly related parameter T_c, the limit on ℓ is an interesting constraint (though not a terribly strong one) on the quark hadron transition. The distance scale ℓ has been estimated in terms of transition parameters such as the transition temperature T_c, the surface tension associated with the fluctuations, σ, and the latent heat of the transition, L. Assuming $L \approx 15\ T_c^4$, Fuller et al[29] find $\ell \sim (4 \times 10^4)\ (\sigma/\mathrm{MeV}^3)^{3/2}\ (T_c/\mathrm{MeV})^{-13/2}$. (We have here corrected for the error in the numerical factor in the approximate solution for the supercooling parameters in Fuller et. al, which was too large by a factor ~ 4, making their distance scale estimates 50 times too large[30]. The surface tension σ has been estimated[31] $\sigma^{1/3} \lesssim 70$ MeV so that for $T_c > 100$ MeV we expect that $\ell \lesssim 1$, (and note the strong temperature dependence) well below our nucleosynthesis bound of $\ell \lesssim 150$.

In conclusion, we find that for reasonable values of the baryon density contrast $R \lesssim 100$, it remains possible to be consistent with observational determinations of the light element abundance (including population II ^7Li)

if the mean separation of the fluctuations is $\ell \lesssim 150$. In addition the standard nucleosynthesis constraints on η and N_ν remain largely intact. For $\ell \approx 10$, η may be as low as 2.2×10^{-10} (but less than 3×10^{-10}). The upper bound on η is $\eta < 7 \times 10^{-10}$ for all values of ℓ for population II ^7Li abundances. This upper limit is increased to $\eta \lesssim 20 \times 10^{-10}$ for the population I ^7Li abundances. In all cases we find $\Omega_b \sim 1$ still excluded by big bang nucleosynthesis.

Acknowledgements

This work was supported in part by National Science Foundation and Department of Energy grants at Drexel, Texas and Minnesota; by NASA Grant #NAGW-1321 and NSF Grant #AST 88-22595 at Chicago; and by the DoE and NASA Grant # NAGW 1340 at the NASA/Fermilab Astrophysics Center.



1000
100
10
1
ℓ
^{4}He,
D + ^{3}He,
^{4}He, ^{7}Li$_I$
D
^{7}Li$_I$
^{4}He
^{7}Li $_{II}$
D
1
2
3
4
5
6
7
8
9 10
20
30
40
50
60 70 80 90 100
η x 10^{10}

Figure Caption

Allowed regions in the ℓ-η plane from the observational constraints on D, ^3He, ^4He and ^7Li (from both population I and II stars). The area outlined by bold lines are the only regions consistent with all observations.

References

1. J.H. Applegate, C.J. Hogan and R.J. Scherrer, Phys. Rev. D35, 1151-1160(1987).

2. C. Alcock, G.M. Fuller, and G.J. Mathews, Ap. J. 320, 439-447(1987).

3. J. Yang, M.S. Turner, G. Steigman, D.N. Schramm and K.A. Olive, Ap. J. 281, 493-511(1984).

4. A. Boesgaard and G. Steigman, Ann. Rev. Astron. Astrophys. 23, 319-378 (1985).

5. E. Witten, Phys. Rev. D30, 272-285(1984); M. Crawford and D.N. Schramm, Nature, 298(1982)538.

6. G. Steigman, D.N. Schramm and J. Gunn, Phys. Lett. 66B, 202-204(1977).

7. G. Steigman, K.A. Olive, D.N. Schramm and M.S. Turner, Phys. Lett. 176B, 33-38(1986).

8. H. Reeves, J. Audouze, W.A. Fowler and D.N. Schramm, Ap. J. 179, 909-930(1973); R. Gott, J. Gunn, D.N. Schramm and B. Tinsley, Ap. J. 194, 543-553(1974); R. Epstein, J. Lattimer and D.N. Schramm, Nature 263, 198-202(1976).

9. L. Kawano, D.N. Schramm and G. Steigman, Ap. J. 327, 750-754(1988).

10. G.R. Caughlan and William A. Fowler, Atomic Data and Nuclear Data Tables, 40, 283-334(1988).

11. B. Sadoulet, D. Seckel and J. Primack, Ann. Rev. Nuc. & Part. Sci. 38, 751-807(1988).

12. H. Spite and F. Spite, Astron. & Astrophys. 115, 357-366(1982); R. Rebolo, P.M. Caro, J. Beckman, Astron. & Astrophys. 192, 192-205(1988); L. Hobbs and D. Duncan, Ap. J. 317, 796-809(1987).

13. H. Reeves, Dark Matter, Proc. of the XXIIIrd Recontre de Moriond, eds. J. Audouze and J. Tran Thanh Van (Editions Frontiers, 1989) pp. 287-302. J. Audouze, P. Delbourgo-Salvador, and P. Salati, ibid, pp.227-286(1988).

14. J. Kapusta and K.A. Olive, Phys. Lett. B209, 295-299(1988).

15. H. Kurki-Suonio, Phys. Rev. D35, 2104-2110(1988); G.M. Fuller, G.J. Mathews, C.R. Alcock, Dark Matter, Proc. of the XXIIIrd Recontre de Moriond, eds. J. Audouze and J. Tran Thanh Van (Editions Frontiers, 1989) pp. 303-318.

16. R.A. Malaney and W.A. Fowler, Ap. J. 333, 14-20(1988).

17. H. Kurki-Suonio, R.A. Matzner, J.M. Centrella, T. Rothman, and J.R. Wilson, Phys. Rev. D38, 1091-1099(1988).

18. H. Kurki-Suonio and R.A. Matzner, Phys. Rev. D39, 1046-1053(1989).

19. N. Terasawa and K. Sato, University of Tokyo preprints UTAP77/1988, UTAP79/1988 and UTAP83/1988.

20. B. Meyer, G. Mathews, C. Alcock, G. Fuller, private communication 1988.

21. J.H. Applegate and C.J. Hogan and R.J. Scherrer, Ap. J. 329, 572-579(1988).

22. R.V. Wagoner, Ap. J. 179, 343-360(1973).

23. D.A. Dicus, E.W. Kolb, A.M. Gleeson, E.C. G. Sudarshan, Vigdor L. Teplitz, and M.S. Turner, Phys. Rev. D26, 2694-2706(1982).

24. D.N. Schramm and R.V. Wagoner, Ann. Rev. Nucl. Sci. 27, 37-74(1977).

25. R.A. Malaney and W.A. Fowler, in Origin and Distribution of the Elements, edited by G.J. Mathews (World Scientific, Singapore, 1988).

26. J.H. Applegate, private communication (1988).

27. G.R. Gisler, E.R. Harrison and M.J. Rees, MNRAS 166, 663-671(1974); R.I. Epstein and V. Petrosian, Ap. J. 197, 281-284(1975). J.D. Barrow and J. Morgan, MNRAS 175, 359-370(1976).

28. G.J. Mathews, G.M. Fuller, C.R. Alcock, and T. Kajino, Dark Matter, Proc. of the XXIIIrd Recontre de Moriond, eds. J. Audouze and J. Tran Thanh Van (Editions Frontiers, 1989) pp. 319-328, C. Alcock, G.M. Fuller, G.J. Mathews, and B. Meyer, submitted to Proceedings of the Seventh International Conference on Ultrarelativistic Nucleus-Nucleus Collisions (Quark Matter '88), Lenox, MA, September 26-30, 1988.

29. G.M. Fuller, G.J. Mathews and C.R. Alcock, Phys. Rev. D37, 1380-1400(1988).

30. K. Kajantie and C. Alcock, private communication.

31. C. Alcock and E. Fahri, Phys. Rev. D32, 1273-1279(1985).