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Neutrino Magnetic Moment and Nonabelian Discrete Symmetry

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Abstract

We propose a mechanism which will give rise to naturally a small mass but a large magnetic moment for the neutrino such that the solar neutrino deficit problem can be explained. The idea is a discrete version of Voloshin's $SU(2)$ mechanism. An example of such mechanism using the quaternion group is illustrated.

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A. Introduction. Recently there has been a lot of attention on the mechanism which may generate a large magnetic moment for the neutrino while maintaining its small mass. It was motivated by the proposal[1] that if the neutrino has a large magnetic moment, $\mu_\nu \geq 10^{-11}\mu_B$ where μ_B is the Bohr magneton, then it can provide a solution for the solar neutrino deficit problem[2]. In this mechanism, the electron neutrino oscillates in the solar magnetic field into other neutrinos such as the sterile right handed neutrino or the muon neutrino. This idea is further encouraged by the apparent anti-correlation between the sun spot activity and the observed neutrino flux. Later, it has been observed[3] that the oscillation into the right handed neutrino may provide a mechanism for the energy loss of the supernova explosion in contradiction with the recent observation of SN 1987A data. Therefore the muon neutrino seems to be a more reasonable choice.

Typically any model that may provide a large magntic moment naturally gives rise to a large neutrino mass which is strongly constrained by experiment. Voloshin[7] suggested a possible mechanism which may suppress the neutrino mass while allow the magnetic moment. He observed that, if one considers the neutrinos as Weyl fermions, then in the magnetic moment term the two neutrinos are necessarily in the antisymmetric combination while in the case of the neutrino mass they are symmetric. Voloshin adopted a global or local $SU(2)$ symmetry. If the left-handed electron neutrino belongs to a doublet representation of this symmetry, then the magnetic moment can be a singlet of the symmetry while the mass will be a triplet and therefore forbidden. We shall call such symmetry which allows μ_ν while forbids the neutrino mass custodial symmetry. Such custodial symmetry is very difficult to implement in practice however. Voloshin himself provided no example. Recently some progresses in this direction has been made. Barbieri and Mohapatra[4] used a local $SU(3)$ as custodial symmetry which contains the usual $SU(2)_L$. Babu and Mohapatra[5] used instead a local $SU(2)_H$ horizontal symmetry. However due to the phenomenological constraints, it is necessary to break these custodial symmetries at scales much higher than the weak scale. Therefore, in general, these custodial symmetries fail to protect the models from the neutrino mass constraints unless extra finetunings are also enforced. The natrualness problem of these models have been pointed out in the literature[6,8]. In particular, Leurer and Golden[6] provide a model using $U(1)$ symmetry. However since the $U(1)$ is insufficient as a custodial symmetry, some unnatural finetunings are still needed in their model.

Roughly speaking, the custodial symmetry has to be broken at high scale if it is a local symmetry because it gives rise to flavor changing neutral currents just like any local horizontal symmetry. If the symmetry is global then the astrophysical constraint from the stellar energy loss due to the Goldstone boson emmission requires the symmetry scale to be very high also. In this paper we like to provide an alternative mechanism which allows us to suppress neutrino mass even at the energy scale lower than the weak scale. The alternative is to use a special class of nonabelian discrete symmetry[9] as the custodial symmetry.

Following Voloshin, we shall assume that the electron neutrino belongs to the doublet representation of the custodial symmetry, G. This already implies that G cannot be abelian. Voloshin's $SU(2)$ symmetry only serves to provides a mechanism such that when two doublets are coupled together only the antisymmetric combination contains a trivial identity representation. Since the $SU(2)$ symmetry is in general too large to implement naturally, one should look for a smaller symmetry to reach the same goal. There are indeed many nonabelian discrete symmetries which may provide the mechanism. Two simplest groups for this purpose are the quaternion group Q of order 8 or dicyclic group Q_6 of order 12. Both of them can be easily embedded into $SU(2)$. Note that this mechanism does not give rise to Goldstone boson or flavor changing neutral current, therefore we do not have to break this symmetry until very low energy. We shall see later that, quite independent of the group we use, as long as we put the left-handed electron and muon doublets into a G-doublet, the electron mass provides a natural lower bound of the G symmetry breaking scale.

Here we shall use the quaternion group Q to illustrate the implementation of this mechanism. By the end of our discussions it should be clear that the same mechanism can be implemented for any discrete group with the antisymmetrization property. The group has 5 irreducible representations. One of them is two dimensional, denoted by D; and the other four one dimensional, denoted as R_1, R_2, R_3, R_4. R_1 is the trivial identity representation. The antisymmetric combination two doublets D couples only to R_1, while the symmetric combinations couple to $R_2 + R_3 + R_4$.

To suppress neutrino mass, we shall put ν_e and ν_μ into a D representation.[5] Here we assume that τ's lepton number is conserved and therefore τ does not interfere with the physics of the first two generations. Without inventing any more neutrinos, the transitional magnetic moment between ν_e and ν_μ is allowed by the Q symmetry while the neutrino masses are forbidden. Therefore, generally speaking, to make neutrino mass small enough we only have to make sure that Q symmetry is protected to low enough energy.

B. The Model. To be as conservative as possible, we choose the standard model gauge group augmented by the discrete group Q. The leptonic sector consists of the usual three family, with the transformation properties for the electron and the muon as

$$L_D = \begin{pmatrix} \nu_e & \nu_\mu \\ e & \mu \end{pmatrix}_L \quad : \quad (-\tfrac{1}{2}, 2, 1_c; D); \tag{1}$$

together with $e_R(-1, 1, 1_c; R_2)$ and $\mu_R(-1, 1, 1_c; R_3)$. The numbers inside parentheses denote the representation content under $U(1)_Y \times SU(2)_L \times SU(3)_c \times Q$. The usual up and down quarks can be taken as Q singlet R_1. Also we introduce an extra vectorial pair of $SU(2)_L$ singlets g_L and g_R, which transform as $(-\frac{1}{3}, 1, 3_c; R_4)$. The g quarks are not the only choice; their existence is natural from the popular grand unified theories based on E_6. The Higgs sector of the theory must also be enriched. Besides the conventional $SU(2)$–doublet ϕ_s which belongs to R_1 of Q, we need an extra Q–doublet ϕ_D and a pair of Q–doublet lepto–quark bosons H_1 and H_2.

$$\begin{aligned} \phi_D \ &: (\tfrac{1}{2}, 2, 1_c; D); \\ H_1 \ &: (-\tfrac{1}{6}, 2, \bar{3}_c; D); \\ H_2 \ &: (\tfrac{5}{6}, 2, \bar{3}_c; D) \ . \end{aligned} \tag{2}$$

The Yukawa couplings and the mass terms of the theory are given by

$$\begin{aligned} L_Y = &\quad f_e(\bar{L}_D\phi_D)_{R_2}e_R + f_\mu(\bar{L}_D\phi_D)_{R_3}\mu_R + m_g\bar{g}_L g_R \\ &+ \ h_1(\bar{L}_D H_1)_{R_4}g_R + h_2(L_D H_2)_{R_4}g_L + H.c. \end{aligned} \tag{3}$$

where the parentheses and the subscripts indicate which representations the fields are coupled into. Some relevant terms in the Higgs potential are

$$\begin{aligned} V \ &= \ \cdots + \lambda_s(H_1H_2^*)_{R_1}\phi_s^2 + \textstyle\sum_{i=1}^4 \lambda_D^i(H_1H_2^*)_{R_i}(\phi_D\phi_D)_{R_i} \\ &+ \ \textstyle\sum_{i=1}^4 \delta_1^i(H_1H_1)_{R_i}(H_1\phi_D)_{R_i} + \sum_{i=1}^4 \delta_2^i(H_1H_2)_{R_i}(H_1\phi_D^*)_{R_i} \quad . \end{aligned} \tag{4}$$

Notice that the λ_i terms violate the lepton number conservation, and the δ^i terms break the continuous g number symmetry (more about it below).

We consider the symmetry breaking scenario $\langle\phi_s\rangle \sim G_F^{\frac{1}{2}} \gg \langle\phi_D\rangle = \Lambda_Q$. The muon gets its mass from $\langle\phi_D\rangle$ and $m_\mu \simeq f_\mu\Lambda_Q$. In order to utilize the custodial Q symmetry to its limit, we choose $f_\mu \simeq 0.1$ and $\Lambda_Q \simeq 1$ GeV.

The leading order contribution to μ_ν is given by the one-loop diagram in Fig. 1 and the diagram with the internal g-lines interchanged. The photon line can also be attached to H_1 and H_2. Of course, this is a transitional ν_e-ν_μ magnetic moment. It is easy to see that the similar set of diagrams for m_ν (with the photon line removed) add up to zero. since the neutrino mass is not a singlet under Q. The leading diagram for m_ν given in Fig. 2 invloves two $\langle\phi_D\rangle$ insertions and so $m_\nu/\mu_\nu \propto \langle\phi_D\rangle^2 / \langle\phi_s\rangle^2$. This is the origin of the smallness of m_ν.

An order of magnitudes estimate in the non-diagonal particle basis gives

$$\mu_\nu \simeq 2em_g\frac{h_1h_2}{16\pi^2}\frac{\lambda_s\langle\phi_s\rangle^2}{m_{H_1}^2 m_{H_2}^2} \ , \tag{5}$$

$$m_\nu \ \simeq C\frac{\mu_\nu\Lambda_Q^2}{2e}\frac{\lambda_D}{\lambda_s}\frac{m_{H_1}m_{H_2}}{\langle\phi_s\rangle^2} \ . \tag{6}$$

Here λ_D stands for the generic λ_D^i couplings. The coefficient C which is of order about one is given by $C = x(x^2-1)^{-1}\ln x^2$ with $x = m_{H_1}/m_{H_2}$. Compare this with the value for μ_ν in the standard model with the Dirac neutrino through the W–exchange [10], *i.e.* $\mu_\nu^{SM} \simeq 3\times10^{-19}\mu_B(m_\nu/1\text{ eV})$. Assuming typical values $\langle\phi_s\rangle^2 \simeq m_{H_1}m_{H_2} \sim G_F^{-1}$, in Eq.(5), we estimate in the present model, $\mu_\nu \simeq 10^{-8}h_1h_2\lambda_s(m_g/100\text{ GeV})\mu_B$. Thus for $m_g \simeq 100$ GeV, it can easily accomodate $\mu_\nu \simeq 10^{-11}\mu_B$ for $h_1h_2\lambda_s \sim 10^{-3}$. The neutrino magnetic moment is large in our model. This is not surprising as it has been achieved by previous attempts[11] on this issue. What is new is the smallness of m_ν, namely, for $\mu_\nu \simeq 10^{-11}\mu_B$. We estimate from Eq.(6) that

$$m_\nu \simeq C(\lambda_D/\lambda_s)(\Lambda_Q/1\text{ GeV})^2 \times 5\text{eV} \ . \tag{7}$$

In order for μ_ν to flip $\nu_e\to\nu_\mu$ in the magnetic field of the sun, the mass difference between ν_e and ν_μ should be rather small[12]: $\Delta m^2 \lesssim 10^{-4}$ $(\text{eV})^2$; otherwise the energy gap would be too large. We demand then the typical mass matrix element to be $(m_\nu) \lesssim 10^{-2}$eV. Assuming $\Lambda_Q \simeq m_\mu \simeq 1$GeV and $C \simeq 0.2$, we can achieve the constraint for λ_D/λ_s of order 10^{-2}.

The model as it is has a problem related to its two neutral Higgs bosons in ϕ_D. As $\langle\phi_D\rangle \simeq 1$ GeV, their masses are naturally of that order after the spontaneous breaking of the Q symmetry driven by a negative mass term for ϕ_D^2. Hence, the Z^0 gauge boson will decay into these neutral Higgs bosons with a partial width equivalent to that of one light neutrino generation. Experimentally, this has been ruled out[13]. This technical difficulty, however, can be circumvented as follows. We can introduce another Q doublet, s_D, which is a $SU(2)_L\times U(1)_Y$ singlet, with a negative mass term to drive the spontaneous breaking of Q symmetry. In that case the mass term for ϕ_D^2 can be chosen to be positive in the Lagrangian. A nonzero VEV of ϕ_D will be induced by the trilinear Higgs boson coupling $m\phi_s^*(s_D\phi_D)_{R_1}$. Therefore s_D picks up a nonzero VEV, $\langle s_D\rangle = \Lambda_Q$ that defines the Q symmetry breaking scale. And ϕ_D picks up a nonzero VEV, $\langle\phi_D\rangle = m\langle\phi_s\rangle\langle s_D\rangle/m_D^2$, that gives rise to the tree level charged lepton masses although its bare mass is positive. Thus, we can arrange the mass m_D of ϕ_D to be at the scale $\simeq M_W$ and a small $\langle\phi_D\rangle \lesssim \langle s_D\rangle = \Lambda_Q$, with the choice of m slightly below the weak scale. There are mixings, $\theta = \langle\phi_D\rangle/\langle s_D\rangle$, between ϕ_D^0 and s_D.

Therefore the contribution of s_D to Z^0 width only corresponds to a small fraction θ^2 of that of a neutrino generation. This scenario is confirmed by a detail analysis of the Higgs potential as long as $\langle\phi_D\rangle \lesssim \langle s_D\rangle$. Note however that we really do not have much freedom in choosing our mass scales even in this modified model. The lower bound for the smallest VEV, $\langle\phi_D\rangle$, is controlled by the muon mass while the larger VEV, $\langle s_D\rangle$, is restricted by its contribution to the one loop neutrino masses. For $\langle\phi_D\rangle \simeq 0.1$GeV and $\langle s_D\rangle \simeq 1$GeV one can barely satisfy the requirements of $\mu_\nu \simeq 10^{-11}\mu_B$ and $(m_\nu) \lesssim 10^{-2}$eV if one allows some of the quartic couplings involving s_D to be of the order 10^{-2}.

C. Discussion.

(a) The reader may wonder why we introduce a new g quark but not make use of the τ lepton in the loop of Fig. 1. The trouble is that in this case we would end up with ν_e-ν_τ (ν_μ-ν_τ) mass terms $\propto \langle\phi_D\rangle$, instead of $\langle\phi_D\rangle^2$. This would obviously render m_ν too large.

(b) The δ^i in Eq.(4) terms break the continuous g-number symmetry, but still keep a discrete symmetry, $g_{L,R} \to e^{+2\pi i/3} g_{L,R}$ and $H_{1,2} \to e^{-2\pi i/3} H_{1,2}$, which prevents right-handed down quarks from coupling to $\bar{L}_D H_1$ (Q symmetry cannot do this job by itself). The symmetry is needed, unfortunately in order to forbid rare decays such as $K_L \to \mu\bar{e}$.

(c) The new charged scalar bosons from ϕ_D modify the $V - A$ nature of $\mu \to e\bar{\nu}_e\nu_\mu$ decay and/or imply the limit $f_e f_\mu < 10^{-2}$. This is in accord with the electron receiving mass term from ϕ_D; since $m_e m_\mu \simeq f_e f_\mu \Lambda_Q^2 \simeq f_e f_\mu m_\mu^2$ and so $f_e f_\mu < 10^{-2}$.

(d) By producing a large transitional ν_e-ν_μ magnetic moment, we must address dangerous $\mu \to e\gamma$ and $\mu \to ee\bar{e}$ decays. It turns out that both are safe. First, $\mu \to e\gamma$ is a helicity flipping process and the simplest such operators are $(\bar{L}_D\phi_D)\sigma_{\mu\nu}e_R F^{\mu\nu}$ and $(\bar{L}_D\phi_D)\sigma_{\mu\nu}\mu_R F^{\mu\nu}$. But these operators have the same flavor structure as the Yukawa couplings and so are simultaneously diagonalized. They cannot change lepton flavor. The next order operators will be suppressed by another factor of $\Lambda_G^2/\langle\phi_s\rangle^2$. A rough estimate gives

$$\Gamma(\mu \to e\gamma)/\Gamma(\mu \to e\nu\bar{\nu}) \simeq m_\mu^4/M_W^4 \sim 10^{-12} \quad ,$$

which is certainly within the experimental limit. Next, let us analyze $\mu \to ee\bar{e}$ decay. Here the situation is a bit more tricky, since we have two $SU(2)$ doublet in ϕ_D and so we expect flavor violation in the neutral Higgs sector. Let the VEV's of ϕ_1^0 and ϕ_2^0 in ϕ_D be v_1 and v_2 respectively. It is easy to show that the vacuum structure can be arranged consistently with the choice $v_2 = 0$. Since ϕ_1^0 has flavor diagonal coupling and ϕ_2^0 has only $\bar{\mu}e$ type couplings, in order to induce $\mu \to ee\bar{e}$ decay, we need the mixing between ϕ_1^0 and ϕ_2^0. However, this mixing is proportional to $v_1 v_2$ and so it vanishes. Also, it can be checked easily that, to one loop order, the $\mu \to ee\bar{e}$ amplitude is proportional to the neutrino mass m_ν and thus it is much smaller than the experimental bound.

(e) The choice, $v_2 = 0$, has another important consequence. The quaternion structure in the one loop neutrino mass diagrams implies that the two ϕ_D's must couple flavor diagonally so that the product of the VEV's is nonzero. As a result, the two neutrino doublets in the external lines must also couple flavor diagonally. That means to one loop order the neutrino mass matrix is diagonal and the mixing is suppressed. Therefore in our model, the Mikheyev–Smirnov–Wolfenstein[14] mechanism of resonant neutrino oscillation is suppressed and the magnetic moment mechanism is the dominant one for explaining solar neutrino flux depletion. For the neutrino mass about 10^{-2}eV which we took, the oscillation most likely happens in the radiation zone[12] and therefore the solar neutrino depletion may not be correlated with the sun spot activity. To explain the correlation, a small neutrino mass is required and it can be obtained by assuming a smaller quartic coupling, λ_D, in Eq.(7).

(f) It is also possible to implement the neutrino mass to be of Dirac type in our mechanism by using a large enough discrete group, which contains a discrete subgroup of the lepton number symmetry. An example based on dicyclic group is being investigated by the authors.

(g) We do not consider the domain wall problem at the phenomenological level. However, to live with it, one would have to either disregard the standard big–bang model at the temperature $T \gtrsim 100\text{MeV}$, or complicate further the Higgs sector in order to achieve symmetry non-restoration[15] for $T > \Lambda_Q$. Yet another possibility is to break the Q symmetry softly with $d = 2$ or 3 terms. This could also be used to get rid of the s_D scalars.

To conclude, our model indicates that the mechanism seemingly requires a proliferation of both fermions and scalars. Also the parameters of the model must clearly be stretched in order to make a small neutrino mass. Nevertheless, there is no need of fine tuning which is more unnatural than those in the standard model to account for the fermion masses. The weaknesses in our model should be interpreted as saying that our model is not the last word for the mechanism. A better group and a more appealing model should be sought for.

During the writing of this manuscript, we learned that two other groups, Babu and Mohapatra (Maryland Preprint MdDP–PP–90–077) and Ecker, Grimus and Neufeld (CERN preprint CERN–TH.5485/89), were also studying the simliar problem. We thank J. Liu for useful discussions and D. Seckel for bringing up the problem of light scalars. This research was supported in part by the U.S. Department of Energy.

FIGURE CAPTIONS

1. A diagram for the neutrino transitional magnetic moment.

2. A diagram for the neutrino mass.

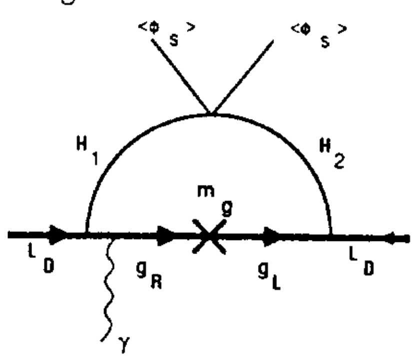

Figure 1

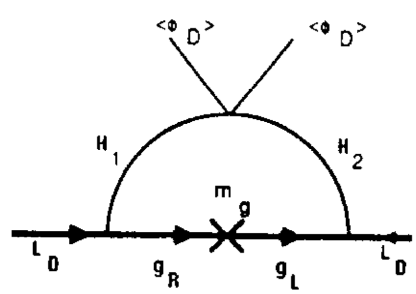

Figure 2

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