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COSMOLOGY OF SPONTANEOUSLY BROKEN GAUGE FAMILY SYMMETRY

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Abstract

The $SU(3)_H \otimes U(1)_H$ model with $SU(3)$ being spontaneously broken local family symmetry and $U(1)_H$ being the associated global symmetry is considered as a simplest version of realistic quantum flavourdynamics, giving a reasonable reproduction of the mass hierarchy and mixing pattern of quarks and leptons. The model predicts: existence of the neutrino Majorana masses with regular hierarchy, existence of familon being simultaneously invisible axion (or arion) and Majoron, relationship between neutrino lifetimes relative to familon decays. Thereby, the model provides the unified physical ground for all the main types of dark matter, considered in the theory of large scale structure of the Universe.

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1. The problem of fermion families remains one of the central problems of particle physics. The standard $SU(3) \otimes SU(2) \otimes U(1)$ model, as well as its possible "vertical" extensions in the one family framework like $SU(5)$, $SO(10)$ etc., does not contain any deep physical grounds for the explanation of fermion mass hierarchy and their weak mixing pattern due to arbitrariness of Yukawa couplings. The identity of quark and lepton families:

$$(u, d, e, \nu_e), \quad (c, s, \mu, \nu_\mu), \quad (t, b, \tau, \nu_\tau) \tag{1}$$

relative to strong and electroweak interactions strongly suggests the existence of "horizontal" symmetry between them. The concept of local horizontal symmetry $SU(3)_H$ with left-handed quark and lepton components transforming as $SU(3)_H$ triplets and the right-handed ones - as antitriplets, first proposed in [1], is attractive to be considered (generalization on the case of n families, $SU(n)_H$, is trivial). In this approach the hypothesis of horizontal hierarchy (HHH) [2-4] is reasonable, according to which the structure of fermion mass matrices is determined by the pattern of horizontal symmetry breaking (i.e., by the structure of vacuum expectation values (VEV) of "horizontal" scalars, maintaining $SU(3)_H$ breaking) and the mass hierarchy between families is related to a definite hierarchy in this breaking. Indeed, the mass terms transform as $3 \times 3 = \overline{3} + 6$ and hence may arise as a result of $SU(3)_H$ breaking only.

The simplest realization of HHH was suggested in [3,4] with the so called "see-saw" approach providing that the quark and lepton masses are induced due to their mixing with some additional superheavy fermions. As it was shown in [5,6], in this approach along with the local $SU(3)_H$ the global $U(1)_H$ symmetry could be included naturally. Its breaking results in the existence of Goldstone boson α, which is simultaneously axion (or arion), singlet Majoron and familon. Depending on the heavy fermion content there are two possibilities, with the quark and lepton mass hierarchy being in direct or inverted relation with the hierarchy of the $SU(3)_H \otimes U(1)_H$ symmetry breaking, which are called the direct and inverse hierarchy models, respectively. It may be shown [6], that in these approaches it is possible to reproduce all the popular ansatz's for quark and lepton mass matrices.

In the present paper the whole pattern of physical and cosmological implications of the inverse hierarchy model is analysed and confronted with the possibilities of their experimental and astronomical search. Flavour nondiagonal transitions with α emission induce the decays $\mu \to e\alpha$, $\tau \to \mu\alpha$, $K \to \pi\alpha$, $\mathrm{B} \to K\alpha$ etc., being available for experimental searches. On the other hand, accounting for the prediction of the neutrino Majorana mass spectrum with ordinary hierarchy $m_{\nu e} \ll m_{\nu\mu} \ll m_{\nu\tau}$ and the presence of familon and axion, the model provides the unified description of all the main types of dark matter, considered in the theory of the cosmological large scale structure: i) hot dark matter (HDM) in the form of massive ν_τ with mass about 20 eV, ii) cold dark matter (CDM) in the form of oscillating primordial axion field and iii) unstable dark matter (UDM) in the form of neutrinos with mass about $50 \div 100$ eV and lifetime $10^{15} \div 10^{16} s$, decaying into lighter neutrino and familon with the dominance of relativistic or nonrelativistic decay products in the modern Universe.

2. Let us consider the standard $SU(3) \otimes SU(2) \otimes U(1)$ model with local chiral horizontal symmetry $SU(3)_H$ [1] between the families (1). Quarks and leptons are in the following representations of $SU(2) \otimes U(1) \otimes SU(3)_H$:

$$f_{L\alpha}: \quad \begin{pmatrix} u \\ d \end{pmatrix}_{L\alpha} (2, 1/3, 3), \quad \begin{pmatrix} \nu \\ e \end{pmatrix}_{L\alpha} (2, -1, 3)$$

$$f_R^\alpha: \quad u_R^\alpha(1, 4/3, \overline{3}), \quad d_R^\alpha(1, -2/3, \overline{3}), \quad e_R^\alpha(1, -2, \overline{3}) \tag{2}$$

where we retain family ($SU(3)_H$) index: $\alpha = 1, 2, 3$. We can choose scalars breaking the horizontal symmetry as $SU(3)_H$ sextets and triplets. All of them should be $SU(2) \otimes U(1)$ singlets in order to prevent electroweak symmetry breaking at $SU(3)_H$

scale. To generate realistic quark and lepton mass matrices no less than three such horizontal scalars are needed. At least one of them with the greatest VEV should be a sextet: $\xi^{(0)}_{\{\alpha\beta\}}$, $\alpha, \beta = 1,2,3$. Otherwise, triplet fields only generate unrealistic mass matrices. The other two scalars $\xi^{(1)}$ and $\xi^{(2)}$ may be either sextets $\xi^{(n)}_{\{\alpha\beta\}}$ or (anti)triplets $\xi^{(n)}_{[\alpha\beta]} = \epsilon_{\alpha\beta\gamma}\xi^{(n)\gamma}$ ($n = 1,2$). We shall not concretize further their $SU(3)_H$ content, mentioning only those cases, when sextet and triplet representation result in different consequences. Let us introduce additional fermions in the form [3]:

$$F^{\alpha}_{L}: \quad U^{\alpha}_{L}(1,4/3,\overline{3}), \quad D^{\alpha}_{L}(1,-2/3,\overline{3}), \quad E^{\alpha}_{L}(1,-2,\overline{3}), \quad N^{\alpha}_{L}(1,0,\overline{3})$$

$$F_{R\alpha}: \quad U_{R\alpha}(1,4/3,3), \quad D_{R\alpha}(1,-2/3,3), \quad E_{R\alpha}(1,-2,3) \tag{3}$$

Note, that these fermions cancel the $SU(3)_H$ anomalies of quarks and leptons (2). The most general Yukawa couplings allowed by the gauge symmetry are:

$$g_f \overline{f}_{L\alpha} F_{R\alpha}\phi^o + G_{nF}\overline{F}_{R\alpha}F^{\beta}_{L}\xi^{(n)}_{\alpha\beta} + G_f \overline{F}^{\alpha}_{L} f^{\alpha}_{R}\eta + h.c.; \quad n = 0,1,2 \tag{4}$$

for quarks and charged leptons ($f = u, d, e,\ F = U, D, E$) [3,6] and

$$g_\nu \overline{\nu}_{L\alpha} N_{R\alpha}\phi^o + G_{nN}\overline{N}_{R\alpha}N^{\beta}_{L}\xi^{(h)}_{\{\alpha\beta\}} + h.c. \tag{5}$$

for neutrinos ($N_R \equiv C\overline{N}_L,\ \nu_R \equiv C\overline{\nu}_L$) [7]. Here ϕ^o is the neutral component of the standard $SU(2)\otimes U(1)$ Higgs doublet ϕ $(2,-1,1)$ ($<\phi^o>\equiv v = (\sqrt{8}G_F)^{-1/2} = 174\ GeV$) and η is the real singlet scalar ($<\eta>\equiv \mu$).

Yukawa couplings (4), (5) are invariant relative to global axial $U(1)_H$ transformations:

$$f_L \to e^{i\omega} f_L, f_R \to e^{-i\omega} f_R, F_L \to e^{-i\omega} F_L, F_R \to e^{i\omega} F_R,$$

$$\phi \to \phi, \quad \eta \to \eta, \quad \xi^{(n)} \to e^{2i\omega}\xi^{(n)}; \quad n = 0,1,2 \tag{6}$$

This $U(1)_H$ symmetry will be maintained also by the Higgs potential provided that there are no trilinear couplings between the horizontal scalars $\xi^{(n)}$. Such couplings are not induced by any other (gauge or Yukawa) interactions, so their absence in the Lagrangian seems to be natural [5,6].

If the constants of Hermitian quartic interactions of the scalars ξ are larger than the constants of the non-Hermitian ones, then the horizontal $VEVs$ matrix could have the pure Fritzch structure [8].

$$\hat{V}_H = < \xi^{(0)} + \xi^{(1)} + \xi^{(2)} > = \begin{pmatrix} r_1 & p_1 & 0 \\ \pm p_1 & 0 & p_2 \\ 0 & \pm p_2 & 0 \end{pmatrix} \tag{7}$$

(where $+$ and $-$ signs correspond to the cases of sextet and triplet $\xi^{(1)}$, $\xi^{(2)}$, respectively) with the natural (about 10-fold) hierarchy $r_1 > p_1 > p_2$. The account for non-Hermitian couplings results in the general structure (see for details [2,5,6]):

$$\hat{V}_H = < \xi^{(0)} + \xi^{(1)} + \xi^{(2)} > = \begin{pmatrix} r_1 & p_1 & p_3 \\ \pm p_1 & r_2 & p_2 \\ \pm p_3 & \pm p_2 & r_3 \end{pmatrix} \tag{8}$$

where $r_2 \leq 0(p_1^2/r_1)$, $p_3 \leq 0(p_2)$ and $r_3 \leq 0(p_3^2/r_1)$.

Inserting the scalar $VEVs$ into Yukawa couplings (4), (5) one obtains full 6×6 fermion mass matrices:

$$\begin{array}{c} \\ \overline{f}_L \\ \overline{F}_L \end{array} \!\!\begin{array}{c} \begin{array}{cc} f_R & F_R \end{array} \\ \begin{pmatrix} 0 & | & g_f v \\ G_f \mu & | & \hat{M}_F \end{pmatrix} \end{array}, \quad \begin{array}{c} \\ \overline{\nu}_L \\ \overline{N}_L \end{array} \!\!\begin{array}{c} \begin{array}{cc} \nu_R & N_R \end{array} \\ \begin{pmatrix} 0 & | & g_\nu v \\ g_\nu v & | & \hat{M}_N \end{pmatrix} \end{array} \tag{9}$$

Where $\hat{M}_F = \sum G_{nF} < \xi^{(n)} >$, $F = U, D, E, N$ (note, that only sextet $\xi^{(n)}$ scalars contribute into Majorana mass matrix $\hat{M}_N$). So, one has Dirac "see-saw" mechanism of the quark and lepton mass generation and ordinary Majorana "see-saw" for neutrino masses, where N_R play the role of right-handed neutrinos. The mass matrices obtained from the block-diagonalization of (9) have the form:

$$\hat{m}_f = g_f G_f v \mu \hat{M}_F^{-1} \quad (f = u, d, e), \quad \hat{m}_\nu = (g_\nu v)^2 \hat{M}_N^{-1} \tag{10}$$

Therefore, the mass hierarchy between the families appears to be inverted with respect to hierarchy of $SU(3)_H \otimes U(1)_H$ symmetry breaking

$$SU(3)_H \otimes U(1)_H \xrightarrow{v_3} SU(2)_H \otimes U(1)'_H \xrightarrow{v_2} U(1)''_H \xrightarrow{v_1} I \quad (11)$$

where $v_3 \cong r_1$, $v_2 \cong p_1$ and $v_1 \cong (p_2^2 + p_3^2)^{1/2}$. Here the intermediate $SU(2)_H \otimes U(1)'_H$ horizontal symmetry is maintained between the second and third families and the remaining $U(1)''_H$ is appropriate to the third family only. Therefore, according to (10), the hierarchy of quark and lepton masses appears to be inverted with respect to the hierarchy of horizontal symmetry breaking (11). The latter hierarchy could be estimated, factor the difference in Yukawa coupling constants G_{nF}, which are supposed to be of the same order of magnitude, if one accepts e.g. the Fritzch pattern (7) for the VEV matrix $\hat{V}_H$ (and, consequently, the inverted Fritzsch structure [3,6] for the mass matrices $\hat{m}_{u,d,e}$):

$$v_1 : v_2 : v_3 \sim \begin{cases} 1 : (m_\tau/m_e)^{1/2} : (m_\tau/m_e)^{1/2}(m_\mu/m_e)^{1/2} = 1 : 60 : 240 \\ 1 : (m_b/m_d)^{1/2} : (m_b/m_d)^{1/2}(m_b/m_s)^{1/2} = 1 : 30 : 150 \\ 1 : (m_t/m_u)^{1/2} : (m_t/m_u)^{1/2}(m_t/m_c)^{1/2} = 1 : 100 : 650 (m_t = 60 \text{ GeV}) \end{cases} \quad (1$$

3. The breaking of global $U(1)_H(U(1)''_H)$ symmetry results in the existence of Nambu-Goldstone boson α, having both flavour diagonal and flavour non-diagonal couplings with quarks and leptons and thus being the "singlet" familon of the type [9,10], different from the octet familons arising in the context of spontaneously broken global $SU(3)_H$ symmetry [11]. The non-diagonal couplings can be pure scalar, pseudoscalar, or their mixture, depending on the structure of fermion mass matrices. The typical values for the Yukawa coupling constants of α with quarks and leptons can be estimated if assume the Fritzsch pattern (7) for the $VEVs$ matrix $\hat{V}_H$. E.g. for the charged leptons we have:

$$g_{\tau\tau} \simeq 2m_\tau/v_1, \quad g_{\mu\mu} \simeq 2m_\mu(m_\mu/m_\tau)/v_1, \quad g_{ee} \simeq 2m_e(m_e m_\mu/m_\tau^2)/v_1,$$

$$g_{\tau\mu} \simeq \sqrt{m_\mu m_\tau}/v_1, g_{\tau e} \simeq \sqrt{m_e m_\tau}(m_e/m_\mu)/v_1, g_{\mu e} \simeq (m_e/m_\tau)\sqrt{m_e m_\mu}/v_1 \quad (13)$$

The couplings of α with quarks are the similar.

In our minimal $SU(2) \otimes U(1) \otimes SU(3)_H$ version with the fermion sets (2), (3) QCD and electromagnetic anomalies of global $U(1)_H$ current are cancelled in parallel with $SU(3)_H$ anomaly. So α is the arion type particle [12] having no couplings αgg and $\alpha\gamma\gamma$ induced by fermion triangles. Its interactions with the ordinary matter (first family quarks and leptons) are highly suppressed removing the strong astrophysical restrictions on the scale v_1. The strongest restriction follows from the analysis of ν-signal from supernova SN 1987A, giving the lower bound on this scale $v_1 \geq 10^5$ GeV.

Somewhat stronger restrictions follow from the analysis of familon decays $\mu \to e\alpha$, $K \to \pi\alpha$, $\tau \to \mu\alpha$, $B \to K(K^*)\alpha$. For the typical values of familon coupling constants given in (13) the branching ratios of these decays can be estimated as

$$Br(\mu \to e\alpha) \simeq 3 \cdot 10^{-8}(10^6 \mathrm{GeV}/v_1)^2, \quad Br(\tau \to \mu\alpha) \simeq 3 \cdot 10^{-3}(10^6 \mathrm{GeV}/v_1)^2,$$

$$Br(K \to \pi\alpha) \simeq 3 \cdot 10^{-6}(10^6 \mathrm{GeV}/v_1)^2, \quad Br(B \to K\alpha) \simeq 3 \quad 10^{-2}(10^6 \mathrm{GeV}/v_1)^2 \tag{14}$$

Then the experimental upper limits $Br(\mu \to e\alpha) < 2.6 \cdot 10^{-6}$ [14] $Br(K \to \pi\alpha) < 3.8 \cdot 10^{-8}$ [15], $Br(\tau \to \mu\alpha) < 2.7 \cdot 10^{-2}$ [16], $Br(B \to K\alpha) < 0.35$ [17] turn into lower bounds on the scale v_1:

$$v_1 > 1.1 \cdot 10^5 \mathrm{GeV}(\mu \to e\alpha) \quad v_1 > 3.3 \cdot 10^5 \mathrm{GeV} \quad (\tau \to \mu\alpha)$$

$$v_1 > 7 \cdot 10^6 \mathrm{GeV}(K \to \pi\alpha) \quad v_1 > 3.1 \cdot 10^5 \mathrm{GeV}(B \to K\alpha) \tag{15}$$

It should be noted that in the case of pseudoscalar nondiagonal couplings of α the strongest restriction from $K \to \pi\alpha$ decay is removed due to vanishing matrix element $< K|\bar{s}\gamma_5 d|\pi >$ [9,10].

The only possible upper limit on v_1 could be obtained from the analysis of primordial black hole (PBH) formation by the lightest of heavy metastable neutral leptons N at the stage of their dominancy in the very early Universe: $v_1 \leq 10^{14}$ GeV [5,6].

In the case of α being arion $U(1)_H$ is not related to Peccei-Quinn symmetry [18], so the question of strong CP violation in QCD remains opened. However, there are two ways to solve this problem in the given model. The first solution is related to the singlet scalar field η. For η complex Yukawa couplings (4) acquire the additional chiral global symmetry $U(1)_{PQ}$:

$$f_L \to e^{i\sigma} f_L; \quad f_R \to e^{-i\sigma} f_R; \quad F_L \to e^{i\sigma} F_L; \quad F_R \to e^{i\sigma} F_R;$$
$$\phi \to \phi, \quad \eta \to e^{2i\sigma}\eta, \quad \xi^{(n)} \to \xi^{(n)}, \quad n = 0,1,2. \tag{16}$$

This symmetry, playing the role of Peccei-Quinn symmetry, is broken at the scale $v_{PQ} = \mu$, which leads to the appearance of invisible axion α' of the Zhitnitsky-Dine-Fishler-Srednicki (ZDFS) type [19] with $x = 1$, having only diagonal pseudoscalar couplings with quarks and leptons - $g_{dd} = m_d/\mu$, $g_{uu} = m_u/\mu$ etc.

Astrophysical lower bounds from red giants $v_{PQ} > 10^9$ GeV [20] and from SN 1987A $v_{PQ} > 10^{10}$ GeV [21] and cosmological upper bound $v_{PQ} < 10^{12}$ GeV [22] leave rather narrow window between 10^{10} GeV and 10^{12} GeV for v_{PQ} variation.

Another possibility is to leave the scalar η real, but to introduce some additional set of heavy fermions, e.g.

$$\begin{pmatrix}\mathcal{U}\\ \mathcal{D}\end{pmatrix}^{\alpha}_{L}(2,1/3,\bar{3}), \begin{pmatrix}\mathcal{N}\\ \mathcal{E}\end{pmatrix}^{\alpha}_{L}(2,-1,\bar{3}), \begin{pmatrix}\mathcal{U}\\ \mathcal{D}\end{pmatrix}_{R\alpha}(2,1/3,3), \begin{pmatrix}\mathcal{N}\\ \mathcal{E}\end{pmatrix}_{R\alpha}(2,-1,3) \tag{17}$$

Some kind of such additional set arises naturally at the extension of considered scheme to GUTs. For example, in $SU(5) \times SU(3)_H$ with quarks and leptons (2) arranged in left handed multiplets $(\bar{5} + 10, 3)_L$, [1,2] the heavy fermions (17) form together with (3) the representations $(5 + \bar{5}, \bar{3})_L$ and $(10 + \overline{10}, \bar{3})_L$ [3] (the cancellation of $SU(3)_H$ anomalies requires also introduction of pure $SU(3)_H$ fermions, e.g. $\Psi^{\gamma}_{L\{\alpha\beta\}}$, $(1, \overline{15})_L$ and $\Psi_{L\alpha}$ $(1,3)_L$).

In this case owing to the presence of the extra heavy fermions (17) triangle diagrams induce αgg and $\alpha\gamma\gamma$ vertices. $U(1)''_H$ turns out to be the Peccei-Quinn symmetry with scale $v_{PQ} = v_1$ and α becomes the invisible axion of the nearly hadronic type

[23] having strongly suppressed tree-level couplings with the first family fermions (see (13)). But diagonal couplings with quarks are enhanced due to loop effects - $g_{qq} \sim m_q/v_{PQ}$, $(q = u, d)$ and α acquires the mass $m_\alpha \sim \Lambda_{QCD}^2/v_1$. The scale $v_1 = v_{PQ}$ is then restricted from below by astrophysical evaluations of stellar energy losses due to axion emission: $v_1 > 3 \cdot 10^6$ GeV (Sun and red giants [24] and the data from SN 1987A excludes the range 10^7 GeV $< v_{PQ} < 10^{10}$ GeV [21], so there is an allowed window $3 \cdot 10^6$ GeV $< v_{PQ} < 7 \cdot 10^6$ GeV, for which axion decays (14) could be still observable. Certainly, the window 10^{10} GeV $< v_{PQ} < 10^{12}$GeV is also possible.

So, in the framework of our model both popular types of axion (ZDFS and hadronic) can be included. The difference of our hadronic axion from the standard one [23] removes the cosmological problem of the latter, connected with the overabundance of superheavy stable quarks in the Universe, since such quarks (3) and (17) are unstable owing to their mixing with light quarks.

4. According to (10) the hierarchy of neutrino Majorana masses is similar to the ordinary quark and lepton mass hierarchy:

$$m_{\nu_e} : m_{\nu_\mu} : m_{\nu_\tau} \sim m_e : m_\mu : m_\tau \tag{18}$$

If the mass of heaviest neutrino is larger than 1 eV, then, accounting for the existing upper limits on neutrino mixings from the searches for neutrino oscillations, it is natural to expect that the relationship (18) is almost exact. Note, that MSW solution for solar neutrino problem [25] requires in view of the hierarchy (18), $m_{\nu_\tau} \le 1$ eV. According to (10) the mass of heaviest neutrino is determined by the mass of the lightest of heavy neutral leptons N: $m_{\nu_\tau} = (g_\nu v)^2 M_{N_3}^{-1}$, so that accounting for (18) we have

$$m_{\nu_\tau} \simeq \left(\frac{g_\nu^2}{G_N}\right) \frac{v^2}{v_1} \sqrt{\frac{m_\tau}{m_\mu}} \simeq \frac{g_\nu^2}{G_N} \left(\frac{10^8 \mathrm{GeV}}{v_1}\right) \mathrm{MeV} \tag{19}$$

Familon α appears to be the singlet Majoron type particle [26] connected with the lepton number violation due to the appearance of the Majorana masses of right-handed neutrinos $N_R = C\overline{N}_L$. For the sextet $\xi^{(1)}$, $\xi^{(2)}$ the neutrino mass matrix $\hat{m}_\nu$ (11) is non-diagonal and the familon decays $\nu_H \to \nu_L\alpha$ are possible with the lifetime

$$\tau(\nu_H \to \nu_L\alpha) = 16\pi/g^2_{HL}m_H \tag{20}$$

where $g_{\nu_\mu\nu_\tau} \simeq \sqrt{m_{\nu_\mu}m_{\nu_\tau}}/v_1$ etc. analogously to (13). Note, that the lightest of neutral leptons $N(N_3)$ is metastable, and its lifetime is determined by the mass of the heaviest neutrino $(\nu_\tau) : \tau_{N_3} \propto m^{-1}_{\nu_\tau}$ [5,6]. The analysis [5,6] of PBH formation in the very early Universe due to dominance of these particles leads to an upper limit on the scale $v_1 : v_1 \lesssim 10^{14}$ GeV and, correspondingly, to the lower bound on the ν_τ mass: $m_{\nu_\tau} \gtrsim 0.1$ eV [30].

5. The present model exhibits the simplest variant of unified physical framework for analysis of practically all the main types of dark matter considered in the cosmological theory of large scale structure formation. The model predicts the hierarchy of neutrino masses and lifetimes relative to familon decays and the existence of axion. Relative contribution of neutrinos and axions into the cosmological density is determined by the parameters of the model and, first of all, by the scale v_1 (when α is an axion), or v_1 and μ (for the case of coexisting axion α and ZDFS axion α').

Not accounting for the second case, with the total ρ_{tot} and baryon ρ_B modern densities being fixed, the relationship:

$$\rho_\alpha(v_1) + \rho_\nu(v_1) + \rho_{d\nu}(v_1) + \rho_{d\alpha}(v_1) + \rho_B = \rho_{tot} \tag{21}$$

turns to be the equation for the value of v_1 fixing the discrete set of cosmological models with different types of dark matter, forming the structure of the Universe. Since baryonic forms of dark matter are rather improbable, in the framework of considered model one has six realistic possibilities:

1) The primordial axion density ρ_α dominance (CDM): primordial axion field oscillations contribute the modern cosmological density as $\rho_\alpha \simeq (v_{PQ}/4 \cdot 10^{11}\text{GeV})\rho_{cr}$

[22]. One can easily estimate that the density of primordial thermal axions is always small: $\rho_\alpha^T = m_\alpha n_\alpha^T$, where $m_\alpha \sim f_\pi m_\pi / v_{PQ}$ and $n_\alpha^T \leq 0.1 n_\nu \leq 0.03 n_\gamma$. According to [27] the intensive axion emission by decaying axion cosmic string structure may increase the cosmological axion density, so that its modern value is equal to $\rho_\alpha = (v_{PQ}/2\ 10^{10}\mathrm{GeV})\rho_{cr}$.

The sensitivity of ρ_α to the existence of the cosmic string network makes it possible to probe the conditions of $U(1)''_H$ phase transition in their relationship with the inflational stage. Indeed, typical scale of $\theta = \alpha/v_{PQ}$ variation on 2π, needed for axion string formation, is of the order of horizon size for the period of $U(1)_{PQ}$ phase transition. Since this scale is usually very small, averaging over it gives the estimation $\theta \simeq 1$ for the mean amplitude of axion coherent oscillations, and the contribution of axion emission of strings is essential. If such transition takes place on inflational stage (or earlier), this scale extends exponentially, so that the string network is too rarefied to give any significant contribution into the axion density. In this case the axion density is determined by axion condensate with the oscillation amplitude θ, fixed by its arbitrary value at the beginning of phase transition, so that very small values of this amplitude $\theta \ll 1$ are possible [28].

2) massive stable neutrino density ρ_ν dominance (HDM): ν_τ with mass $m_{\nu_\tau} \simeq 20$ eV when its lifetime $\tau(\nu_\tau \to \nu_\mu \alpha)$ exceeds the age of the Universe $t_U \cdot \rho_\nu = m_\nu n_\nu$, where $n_\nu = (3/11) n_\gamma$ is the standard big bang neutrino number density [29]. Accounting for (12) and (19) and varying g_ν^2/G_N from 10^{-4} to 1 the corresponding scale v_1 varies in the range $10^8 \div 10^{12}$ GeV.

Note, that for the definite range of the parameter (g_ν^2/G_N) it is possible to realize the combined scenario CDM + HDM with the dominance in the Universe of comparable amounts of CDM and HDM.

3) the dominance of density of axions $\rho_{d\alpha}$ and neutrinos $\rho_{d\nu}$: relativistic products

of decay of massive unstable ν_τ with mass $m_{\nu\tau} = m_o = 50 \div 100$ eV and lifetime $t_U > \tau(\nu_\tau \to \nu_\mu \alpha) = \tau_o = 10^{15} \div 10^{16}$ sec, $\rho_{d\alpha} \sim \rho_{d\nu} \propto v_1^{3/2}$. From (20) with typical $g_{\nu_\mu \nu_\tau} \sim \sqrt{m_{\nu_\mu} m_{\nu_\tau}}/v_1$ we estimate that the scale $v_1 \sim$ a few 10^8 GeV. This fixes the value $\frac{g_\nu^2}{G_N} \sim 10^{-4}$.

4) the dominance of nonrelativistic ν_μ from ν_τ decay and primordial ν_μ for $m_{\nu_\mu} \geq 1/10\ m_{\nu_\tau}$.

5) the dominance of relativistic axions $\rho_{d\alpha}$ and neutrinos $\rho_{d\nu}$ from the decay of massive unstable ν_μ with $m_{\nu_\mu} = m_o$ and $\tau(\nu_\mu \to \nu_e \alpha) = \tau_o$. For typical $g_{\nu_\mu \nu_e} \sim \sqrt{m_{\nu_e} m_{\nu_\mu}}/v_1 (m_{\nu_e}/m_{\nu_\tau})$ this corresponds to the scale $v_1 \sim$ a few 10^6 GeV and $g_\nu^2/G_N \sim 10^{-4}$. Then the condition of fast ν_τ decays with $m_{\mu_\tau} \sim (1 \div 10)$ keV and $\tau(\nu_\tau \to \nu_\mu \alpha) \leq 10^8 \div 10^{10}$ s is satisfied [5].

6) the dominance of non-relativistic (or semirelativistic) axions with mass of few eV $\rho_{d\alpha}$, being the products of early ν_τ decays and of ν_μ decays with $\tau(\nu_\mu \to \nu_e \alpha) = \tau_o$ and $\sqrt{\tau/t_U} > 2m_\alpha/m_{\nu_\mu}$, $m_\alpha \sim m_\pi f_\pi/v_1$.

The main contribution into the inhomogeneously distributed dark matter is provided from nonrelativistic axions from early ν_τ decays with the axion concentration n_α^i, being equal to the concentration of primordial ν_τ, $n_{\nu_\tau}^{prim}$. Since the concentration of nonrelativistic axions from $\nu_\mu \to \nu_e \alpha$ decay, being uniformly distributed in the Universe, is $n_\alpha^u = n_{\nu_\mu}$ and n_{ν_μ} is determined by the concentration of primordial ν_μ, $n_{\nu_\mu}^{prim}$, and by the concentration of ν_μ from early $\nu_\tau \to \nu_\mu \alpha$ decays, being equal to $n_{\nu_\tau}^{prim}$, one obtains for $n_{\nu_\mu}^{prim} = n_{\nu_\tau}^{prim}$ $\Omega^u \geq 2\Omega^i$, so that the total density being equal to the critical one the density of dark matter in the inhomogeneities should be no more, than $\Omega_{dm}^i < 0.3$.

So, the considered model leaves the special room for the scenario of unstable dark matter in the form of unstable neutrinos (ν_τ in the cases 3) and 4), or ν_μ in the cases 5) and 6)) with mass about 50-100 eV and lifetime $\tau(\nu_H \to \nu_L \alpha) \sim 10^8$yr [31].

For $\rho_{tot} = \rho_{cr}$ and $\rho'_B \simeq 0.1\rho_{cr}$ one has the cosmological dark matter density dependence on the scale v_1 shown on Fig. 1 with the solutions of equation (21) considered above.

Note, that the cases 5) and 6) correspond to a nontrivial scenario of the cosmological evolution of inhomogeneities, combining the attractive features of both the models of early neutrino decays [32] and of unstable dark matter scenarios [31] and deserving special consideration. In this scenario short period of ν_τ dominancy in the Universe at $t \sim 10^6 \div 10^8$ s provides the survival and further development of short-wave ν_μ and axion density perturbations at the scales $\sim$ 100 kpc and long (from 10^{11}s to 10^{16}s) period of ν_μ dominancy gives birth to the formation of the cosmological large scale structure (at the scales $\sim 10 \div 100$ Mpc). The condition of the growth of initial density fluctuations into the observed structure for the observed isotropy of the microwave thermal background is satisfied for the neutrino mass and lifetime hierarchy, predicted by the presented model, if either $m_{\nu_\tau} \leq 100$ eV, or $(v_1/10^8 \text{ GeV})^2(100 \text{ eV}/m_{\nu_\mu}) < 10^{-9}$ (see [5,6]). The account for $\alpha \to \gamma\gamma$ electromagnetic decays with $\tau_\alpha \sim \tau^\circ_\pi(m_\pi/m_\alpha)^3(v_1/f_\pi)^2$ results in the prediction of nonthermal electromagnetic background, ionizing the matter at $z > 50$ and reducing thus the predicted anisotropy of the thermal background (see cf [31]). It should be emphasized, that the cases 3)-4), requiring $v_1 \sim 10^6$ GeV, are excluded in view of SN 1987A data, if α is axion. However, for α being arion these cases become allowed, but the cases 1) and 6) are absent, and the case 1) may be realized by the additional ZDFS α' axion only. Since the scale related to α' is restricted from below $\mu > 10^{10}$ GeV, significant contribution of CDM into the cosmological density is guaranteed in this situation. Thus, for appropriate choice of the scale v_1 interesting possibilities of combined HDM + CDM or UDM + CDM cosmological models. The case 6) cannot be realized, since arion is practically massless particle.

6. We have considered the simplest model of quantum flavourdynamics based on local $SU(2) \otimes U(1) \otimes SU(3)_H$ symmetry with associated global$U(1)_H$ satisfying the requirement of HHH: after the horizontal symmetry breaking at the scale $v_H > 10^6$ GeV it reduces to standard $SU(2) \otimes U(1)$ scheme. The Yukawa couplings of the only standard Higgs doublet ϕ are then determined by the VEV matrix $\hat{V}_H$ of horizontal scalars. As a consequence, the flavour-changing neutral scalar currents are naturally suppressed [33] at the electroweak scale.

The model contains a rather wide set of parameters. But: i) the number of these parameters is smaller than in the standard model without horizontal symmetry and it will be reduced with the extention to GUTs; ii) the bulk of these parameters is fixed by the experimental data on quark and lepton properties; and, finally, iii) the set of new nontrivial phenomena, predicted by the model, provides in principle the complete check of the model and determination of all the parameters. These phenomena arise at a high energy scale of horizontal symmetry breaking $v_H > 10^6$ GeV which cannot be achieved even in the far future at accelerators. However, combination of experimental searches of their indirect effects in the processes with known particles (rare familon decays) $\mu \to e\alpha$, $\tau \to \mu\alpha$ etc., ν-oscillations, $2\beta_{\sigma\nu}$-decay and so on) together with the search of their cosmological and astrophysical effects (detection of solar axions with axion helioscopes or CDM primordial axions with axion haloscopes [34]) makes it possible to study physics, predicted at this scale. Determination in astronomical observations of the dominant form of dark matter, formed the structure of the Universe, is the way of precise measurement of the magnitude v_H.

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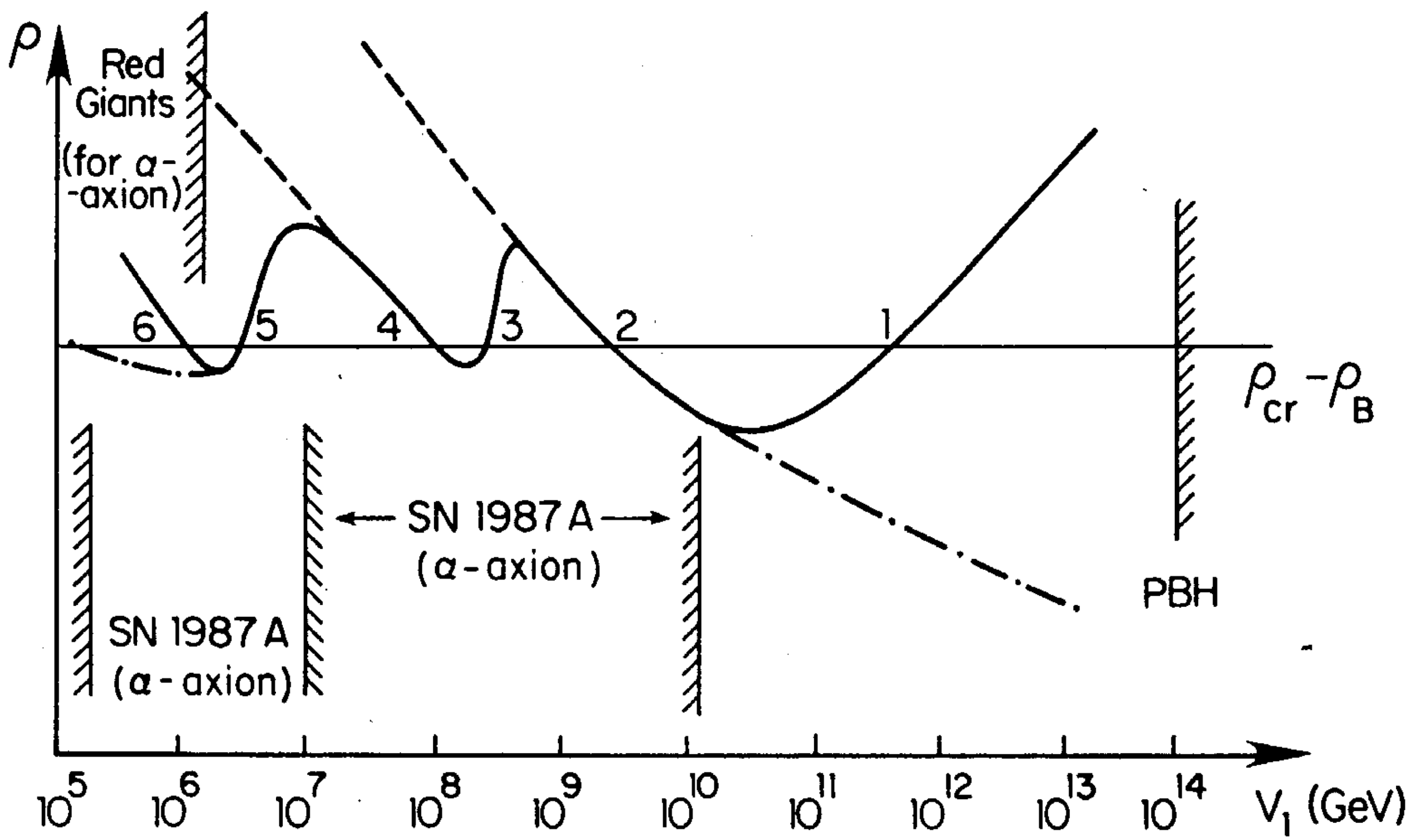
Figure Caption

Fig. 1. Cosmological density dependence on the scale v_1. Here the solid line corresponds to cosmological density in the case, when $\xi^{(1)}$, $\xi^{(2)}$ are sextets and α is axion. (1) - (6) correspond to the cases 1) - 6) in the text. The dashed line corresponds to triplet $\xi^{(1)}$, $\xi^{(2)}$, for which the corresponding neutrino decays absent and, finally, the dash-dotted line corresponds to the case α is arion (in the latter case CDM may be provided by ZDFS α' axion with $M > 10^{10}$ GeV). The astrophysical and cosmological restrictions on v_1 are also shown.


ρ
Red
Giants
(for α-
-axion)
6
5
4
3
2
1
ρcr - ρB
SN 1987 A
(α - axion)
SN 1987 A
(α - axion)
PBH
10^5
10^6
10^7
10^8
10^9
10^10
10^11
10^12
10^13
10^14
V1 (GeV)