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Critical Reanalysis of CP Asymmetries in B^0 Decays to CP Eigenstates.

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Abstract

The time integrated CP asymmetry in the decay of a neutral B-meson to a CP eigenstate has been claimed to be free of uncertainties arising from hadronic matrix elements (modulo the mixing parameter $\Delta M/\Gamma$). That is, it is a direct measure of KM angles. We scrutinize this claim, and question its generality. To this end we compute the effective hamiltonian for $\Delta B = 1$, charm and up conserving processes, in the leading logarithmic approximation. Enhancement of hadronic matrix elements of 'penguin' operators could easily invalidate the claim.

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1. Introduction A nonvanishing time–integrated asymmetry

$$a = \frac{\Gamma(B^0 \to f) - \Gamma(\bar{B}^0 \to \bar{f})}{\Gamma(B^0 \to f) + \Gamma(\bar{B}^0 \to \bar{f})} \tag{1}$$

indicates a violation of CP invariance. If the final state f is a CP eigenstate, $f^{CP} = \pm f$, the expression for the predicted asymmetry takes on a very simple form[1]:

$$a = \frac{\Delta M/\Gamma}{1+(\Delta M/\Gamma)^2} \operatorname{Im}\left(\frac{q}{p}\rho_f\right) \tag{2}$$

Here q and p are standard notation[2] for the $\bar{B}^0$ and B^0 components, respectively, of the mostly CP even physical state B_L $(q/p = (1-\epsilon)/(1+\epsilon))$. The mass difference per width $\Delta M/\Gamma$ indicates B^0–$\bar{B}^0$ mixing, and has been measured[3] in two different experiments to be $\sim 65\%$. More interesting is the remaining factor

$$\rho_f = \frac{\langle f|\mathcal{H}|\bar{B}^0\rangle}{\langle f|\mathcal{H}|B^0\rangle}. \tag{3}$$

It has been pointed out[1] that ρ_f may be independent of hadronic matrix elements. If this is the case then ρ_f is given simply in terms of fundamental mixing angles! It is the purpose of this letter to study just under what conditions this conclusion is valid.

2. CPT. The implications from the CPT theorem are straightforward. For the sake of generality we consider momentarily the case where f is not necessarily a CP eigenstate, $f \neq f^{CP}$. It is convenient to define

$$\bar{\rho}_f = \frac{\langle \bar{f}|\mathcal{H}|B^0\rangle}{\langle \bar{f}|\mathcal{H}|\bar{B}^0\rangle}. \tag{4}$$

Then, *if* $|f\rangle$ *is an eigenstate of the S matrix*, one has

$$\langle f|\mathcal{H}|\bar{B}^0\rangle = \exp(-2i\delta_f)\left\langle f^{CPT}\right|\mathcal{H}\left|\bar{B}^{0\,CPT}\right\rangle^*, \tag{5}$$

where δ_f is the f–phase shift. It follows that

$$\rho_f = \bar{\rho}_f^*. \tag{6}$$

For self-conjugate final states one has in addition $\rho_f = 1/\bar{\rho}_f$, and therefore $|\rho_f| = 1$. These results do not hold for a final state which is not a strong interactions eigenstate, as can be seen by expanding such state in terms of strong eigenstates[1].

In deriving eq (2) it is generally assumed that both $|p/q| = 1$ and $|\rho_f| = 1$. More generally, for self-conjugate final states

$$a = \frac{(1+\frac{1}{2}(\Delta M/\Gamma)^2)(1-|\rho|^2) + \frac{1}{2}(\Delta M/\Gamma)^2(|\rho|^2\left|\frac{q}{p}\right|^2 - \left|\frac{p}{q}\right|^2) + \Delta M/\Gamma\,\mathrm{Im}(\frac{q}{p}\rho - \frac{p}{q}\rho^*)}{(1+\frac{1}{2}(\Delta M/\Gamma)^2)(1+|\rho|^2) + \frac{1}{2}(\Delta M/\Gamma)^2(|\rho|^2\left|\frac{q}{p}\right|^2 + \left|\frac{p}{q}\right|^2) + \Delta M/\Gamma\,\mathrm{Im}(\frac{q}{p}\rho + \frac{p}{q}\rho^*)}. \tag{7}$$

The real part of ϵ is of the order[4] of $\Delta\Gamma/\Gamma \sim 10^{-2}$ while the imaginary part could be much larger. Therefore it is appropriate to use $|p/q| \approx 1$. Then (7) reduces to

$$a = \frac{(1-|\rho|^2) + 2\Delta M/\Gamma\,\mathrm{Im}\left(\frac{q}{p}\rho\right)}{(1+|\rho|^2)(1+(\Delta M/\Gamma)^2)} \tag{8}$$

We see that if $|\rho| \neq 1$ one may be in large error if using eq (2).

3. CP. An alternative approach is to exploit the CP invariance of strong and electromagnetic interactions. If the $\Delta B = \pm 1$ hamiltonian

$$\mathcal{H} = \mathcal{H}^{\Delta B=1} + \mathcal{H}^{\Delta B=1\dagger} \tag{9}$$

satisfies

$$(CP)\mathcal{H}^{\Delta B=1}(CP)^\dagger = \exp(-i\alpha)\mathcal{H}^{\Delta B=1\dagger} \tag{10}$$

then

$$\rho_f = \mp\exp(i\alpha), \tag{11}$$

for $f^{CP} = \pm f$. Note that f need not be a strong interaction eigenstate. If condition (10) holds, then ρ_f is computable in terms of parameters of the weak hamiltonian only.

4. Standard Model Predictions. In this section we consider condition (10) in the context of the standard model of electroweak interactions. The interaction lagrangean is

$$\mathcal{L}_{int} = \frac{g_2}{\sqrt{2}} W^\mu\,(\bar{u},\bar{c},\bar{t})_L\,\gamma_\mu(1-\gamma_5)V\begin{pmatrix} d \\ s \\ b \end{pmatrix}_L \tag{12}$$

[1] Except, of course, if all the phase shifts for these states happen to coincide.

where V is a 3×3 unitary matrix that arises because the quark fields must be redefined to diagonalize their mass matrices. V can be written in terms of four angles $\theta_1, \theta_2, \theta_3$ and δ

$$V = \begin{pmatrix} c_1 & -s_1 c_3 & -s_1 s_3 \\ s_1 c_2 & c_1 c_2 c_3 - s_2 s_3 e^{i\delta} & c_1 c_2 s_3 + s_2 c_3 e^{i\delta} \\ s_1 s_2 & c_1 s_2 c_3 + c_2 s_3 e^{i\delta} & c_1 s_2 s_3 - c_2 c_3 e^{i\delta} \end{pmatrix} \tag{13}$$

Here $s_i \equiv \sin\theta_i$ and $c_i \equiv \cos\theta_i$. Experimental information on nuclear β decay, semileptonic hyperon decay and B-meson decay implies that the angles θ_1, θ_2 and θ_3 are small.

Obviously there are many different $\Delta B = 1$ terms with different complex coefficients. Condition (10) is not satisfied, unless only terms with no relative phase in their coefficient contribute to the matrix element for $B \to f$. In practice we need only consider this matrix element to leading order in Fermi's constant, G_F, and to all orders in strong interactions. We should really be asking whether (10) holds for $\mathcal{H}_{\text{eff}}$, the low energy effective weak hamiltonian to order G_F. Moreover we are only concerned here with CP self-conjugate final states, so we should focus on the $\Delta B = 1$, $\Delta C = \Delta U = 0$ part of the hamiltonian.

The calculation of $\mathcal{H}_{\text{eff}}$ is straightforward. We account for anomalous dependence on heavy masses in the leading logarithmic approximation using standard methods[5]. We will integrate out the W boson and top quark simultaneously. To the extent that the masses of these particles differ by a factor of order 1, this is the only consistent way to compute in leading-logs. Tree level matching at the scale $\mu \approx M_W$ gives

$$\mathcal{H}_{\text{eff}} = \frac{4G_F}{\sqrt{2}} \sum_{q=u,c} \sum_{q'=d,s} V_{qb}^* V_{qq'} (\bar{b}_{L\alpha} \gamma^\mu q_{L\alpha})(\bar{q}_{L\beta} \gamma_\mu q'_{L\beta}) \tag{14}$$

Here α and β are color indices, and L(R) stands for left-(right-) handed fields. The cases $q' = d$ or s can be treated separately. We will do explicitly the former, and the result for the latter can be read off from this. For scales $\mu < M_W$ the hamiltonian is of the form

$$\mathcal{H}_{\text{eff}} = \frac{4G_F}{\sqrt{2}} \sum_i c_i(\mu) O_i \tag{15}$$

where

$$O_1 = (\bar{b}_{L\alpha}\gamma^{\mu}d_{L\alpha})(\bar{c}_{L\beta}\gamma_{\mu}c_{L\beta}) \tag{16}$$

$$O_2 = (\bar{b}_{L\alpha}\gamma^{\mu}d_{L\beta})(\bar{c}_{L\beta}\gamma_{\mu}c_{L\alpha}) \tag{17}$$

$$O_3 = (\bar{b}_{L\alpha}\gamma^{\mu}d_{L\alpha})[(\bar{u}_{L\beta}\gamma_{\mu}u_{L\beta}) + \cdots + (\bar{b}_{L\beta}\gamma_{\mu}b_{L\beta})] \tag{18}$$

$$O_4 = (\bar{b}_{L\alpha}\gamma^{\mu}d_{L\beta})[(\bar{u}_{L\beta}\gamma_{\mu}u_{L\alpha}) + \cdots + (\bar{b}_{L\beta}\gamma_{\mu}b_{L\alpha})] \tag{19}$$

$$O_5 = (\bar{b}_{L\alpha}\gamma^{\mu}d_{L\alpha})[(\bar{u}_{R\beta}\gamma_{\mu}u_{R\beta}) + \cdots + (\bar{b}_{R\beta}\gamma_{\mu}b_{R\beta})] \tag{20}$$

$$O_6 = (\bar{b}_{L\alpha}\gamma^{\mu}d_{L\beta})[(\bar{u}_{R\beta}\gamma_{\mu}u_{R\alpha}) + \cdots + (\bar{b}_{R\beta}\gamma_{\mu}b_{R\alpha})] \tag{21}$$

$$O_7 = (\bar{b}_{L\alpha}\gamma^{\mu}d_{L\alpha})(\bar{u}_{L\beta}\gamma_{\mu}u_{L\beta}) \tag{22}$$

$$O_8 = (\bar{b}_{L\alpha}\gamma^{\mu}d_{L\beta})(\bar{u}_{L\beta}\gamma_{\mu}u_{L\alpha}) \tag{23}$$

$$O_9 = em_b(\bar{b}_{R\alpha}\sigma^{\mu\nu}d_{L\alpha})F_{\mu\nu} \tag{24}$$

$$O_{10} = gm_b(\bar{b}_{R\alpha}\sigma^{\mu\nu}T^a_{\alpha\beta}d_{L\beta})G^a_{\mu\nu}\ . \tag{25}$$

Here $F_{\mu\nu}$ and $G^a_{\mu\nu}$ are the electromagnetic and strong interaction field strength tensors, respectively, and e and g are the corresponding coupling constants. We will neglect the effect of O_9 and O_{10}, the former because its effects are suppressed by e^2/g^2, and the latter because its anomalous dimension[6] makes its coefficient small for $\mu \ll M_W$. The functional dependence of the coefficients c_i is determined from

$$\mu\frac{d}{d\mu}c_i(\mu) = -\sum_j \gamma^T_{ij}c_j(\mu) \tag{26}$$

and the boundary condition eq (14). For the anomalous dimensions matrix γ

with n active quark flavors we obtain, to one loop order,

$$\gamma = \frac{g^2}{8\pi^2}\begin{pmatrix} -1 & 3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 3 & -1 & -1/9 & 1/3 & -1/9 & 1/3 & 0 & 0 \\ 0 & 0 & -11/9 & 11/3 & -2/9 & 2/3 & 0 & 0 \\ 0 & 0 & 3-n/9 & -1+n/3 & -n/9 & n/3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -3 & 0 & 0 \\ 0 & 0 & -n/9 & n/3 & -n/9 & -8+n/3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 3 \\ 0 & 0 & -1/9 & 1/3 & -1/9 & 1/3 & 3 & -1 \end{pmatrix}. \tag{27}$$

With

$$\xi_c = V_{cb}^* V_{cd} \approx s_1 s_3 + s_1 s_2 e^{-i\delta} \tag{28}$$

$$\xi_u = V_{ub}^* V_{ud} \approx -s_1 s_3 \tag{29}$$

$$\xi_t = -(\xi_u + \xi_c) \approx -s_1 s_2 e^{-i\delta} \tag{30}$$

the result at $\mu = m_b$ is ($M_W = 85\text{GeV}$, $m_b = 5\text{GeV}$, $\Lambda^{(5)}_{\text{QCD}} = 0.153\text{GeV}$)

$$\mathcal{H}_{\text{eff}} = \frac{4G_F}{\sqrt{2}}\{\xi_c(-0.26O_1 + 1.11O_2) \\ -\xi_t(0.012O_3 - 0.026O_4 + 0.008O_5 - 0.033O_6) + \xi_u(-0.26O_7 + 1.11O_8)\}. \tag{31}$$

5. Discussion The effective hamiltonian (31) does not satisfy (10). But it is probably an excellent approximation to neglect the effects of O_7 and O_8 when the final state is, say, D^+D^- or $D^0\bar{D}^0$. Then, to the extent that we are willing to neglect the effect of penguins, the phase α in eq (11) is twice the phase of ξ_c. The problem is that we don't know how large the matrix elements of the 'penguin' operators O_3-O_6 might be. In particular, O_6, with the largest coefficient, has a different chiral structure. An enhancement in the matrix element of O_6 by a factor of 3 could lead ρ to differ from one by 10%, and a 20% error in the estimate of the asymmetry would ensue (*cf.*, eq (8)). More precisely, defining (for $f = D\bar{D}$)

$$\omega = 0.030\frac{\langle f|O_6|B^0\rangle}{\langle f|O_2 - 0.24O_1|B^0\rangle}, \tag{32}$$

one obtains

$$\begin{aligned}\rho_f &\approx \frac{\xi_c}{\xi_c^*}\left(1+2i\omega\,\mathrm{Im}\,\frac{\xi_t}{\xi_c}\right)\\ &=\frac{\xi_c}{\xi_c^*}\left(1+2i\omega\frac{s_2s_3s_\delta}{|s_3+s_2e^{i\delta}|^2}\right).\end{aligned} \tag{33}$$

If $|\omega|\sim 10\%$ then $|\rho|$ can differ from 1 by as little as 1% or as much as 20% according to whether ω is mostly real or mostly pure imaginary.

The calculation of the effective hamiltonian in (31) is done in a consistent expansion (*i.e.*, the leading-log approximation), up to our neglect of the operators O_9 and O_{10}. Unfortunately, for a heavy top quark the next to leading-logs can be just as important. There is a 1 loop matching contribution to $c_i(M_W)$, for $i=3,\ldots,6$. It arises from graphs with an internal top quark (*e.g.*, a top penguin graph) and is easily estimated to be $\sim 0.01\xi_t$. This is of the same order of magnitude as the result in eq (31). Therefore, a reliable estimate really requires inclusion of the sub-leading logarithms[2].

While ω could be measured in numerical simulations of lattice QCD, and the sub-leading-logs could be computed, one can alternatively *assume* these corrections are small. The hypothesis could be tested experimentally by comparing asymmetries in different processes (*e.g.*, when the final states are $D^0\bar{D}^0$, D^+D^- and $\eta_c\pi^0$)

6. Conclusions. We have argued that, generally, the CP asymmetry in neutral B-meson decays to CP eigenstates cannot be obtained without knowledge of certain hadronic matrix elements. Nevertheless, if matrix elements of penguin operators are not enhanced, the asymmetry is given to good approximation in terms of KM angles only. This is a "catch 22" situation: we do not need to know the hadronic matrix elements, but only provided that we know that some of them are not much larger than others! The situation could be improved

[2]Simply running down these coefficients obtained by 1 loop matching using the 1 loop anomalous dimension (27) is inconsistent. The procedure would yield a scheme dependent result. The situation is analogous to that described in ref. [7]. There, the electromagnetic penguin contribution to $b\to se^+e^-$ from a heavy top in the loop is sub-leading, but still numerically important. The scheme dependence of the incomplete sub-leading-log approximation was discussed in some detail in ref. [7].

(worsen) if the next to leading-log corrections decrease (increase) the coefficients of the penguin operators in the effective hamiltonian.

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