Fermi National Accelerator Laboratory

FERMILAB-Pub-89/139-A
June 1989

ERRATUM

Kim Griest

Cross Sections, Relic Abundance, and Detection Rates for Neutralino Dark Matter, *Phys. Rev.* **D38**, 2357 (1988).

At least three typographical errors appear in eq. (4), the annihilation cross section of neutralinos into fermions. In the last line $-\frac{2}{3}$, should read $\frac{2}{3}$, and $v'c_R$ should read $-v'c_R$. The definition of r should read $r = m_{\tilde{X}}^2/(M_{\tilde{q}}^2 + m_{\tilde{X}}^2\beta')$, not as given.

More importantly, it was pointed out by Lars Bergstrom that in our paper we (inadvertently) left out the $-p^\mu p^\nu/m_Z^2$ part of the Z^0 propagator. Including this gives additional terms in the matrix element and the annihilation cross section. For much of the supersymmetric parameter space the resulting corrections are very small, but for pure Higgsinos a maximum correction of around 9% can occur. There are no corrections to the elastic scattering cross sections, the pure photino limit of the annihilation cross section or to the production ($e^+e^- \rightarrow \tilde{\chi}\tilde{\chi}$) cross sections. The changes to the annihilation cross section can result in up to a 9% change in the dark matter detection rates. However, as Bergstrom and Snellman[1] point out, when using this annihilation cross section for present day annihilation of dark matter in the halo (or in the body of the Earth or Sun), the additional terms can be quite important. In particular, at the Z^0 pole ($m_{\tilde{X}} = m_Z/2$), the term proportional to $(m_q/m_{\tilde{X}})^2$ cancels. Since the remaining terms are proportional to $v^2 \approx 10^{-6}$ there is a very large dip in the cross section here rather than a large enhancement which the published cross section would predict. This dip does not occur for annihilation in the early universe where $v^2 \approx 1/3$. The following corrections should be made.

The squared matrix element (eq. 3) should include the additional terms

$$|\mathcal{M}'|^2 = 16g^4 \Bigg\{ \frac{(Z_{13}^2 - Z_{14}^2)^2 (c_L - c_R)^2}{\cos^4\theta_w (m_Z^2 - s)^2} \frac{m_X^2 {m_q}^2}{m_Z^4} \left(\frac{s^2}{4} - \frac{s m_Z^2}{2} \right)$$
$$+ \frac{(Z_{13}^2 - Z_{14}^2)(c_L - c_R)}{4\cos^2\theta_w (m_Z^2 - s)} \frac{m_X m_q}{m_Z^2} \Bigg[\left(\frac{1}{M_{\tilde{q}}^2 - t} + \frac{1}{M_{\tilde{q}}^2 - u} \right) [w' s^2 + 2(u' + v') s m_X m_q]$$
$$+ \left(\frac{1}{M_{\tilde{q}}^2 - t} - \frac{1}{M_{\tilde{q}}^2 - u} \right) (4w')[(p_1 k_1)^2 - (p_1 k_2)^2] \Bigg] \Bigg\}, \tag{1}$$

where all symbols were defined in the paper.

The non-relativistic expansion of the neutralino annihilation cross section (eq. 4) should include the additional terms

$$\sigma'_{ann} v = \sum_q \frac{4}{\pi} G_F^2 c_q m_X^2 \beta' \Bigg\{ (Z_{13}^2 - Z_{14}^2)^2 x'^4 \frac{(c_L - c_R)^2 z^2}{4} \frac{m_X^2}{m_Z^2}$$
$$\times \left[\frac{16 m_X^2}{m_Z^2} - 8 + 2v^2 \left(\frac{2m_X^2}{m_Z^2}(1 + x^2) - x^2 \right) \right]$$
$$+ (Z_{13}^2 - Z_{14}^2)(c_L - c_R) x'^2 y'^2 z \frac{m_X^2}{m_Z^2}$$
$$\times \Bigg[\left(2 + \frac{1}{2} v^2 (x^2 - 2r + \frac{4}{3} \beta'^2 r^2) \right) (4w' + 2z(u' + v'))$$
$$+ 2w'v^2 - \frac{4}{3} w' v^2 \beta'^2 r \Bigg] \Bigg\}. \tag{2}$$

In the limit $v^2 = 0$, relevant for present day annihilation, the total cross section is simply

$$\sigma_{ann}^{tot}(v^2=0) = \sum_q \frac{4}{\pi} G_F^2 c_q m_X^2 \beta' \Bigg[y'^2 \left(2w' + z(u'+v')\right) + \frac{x'^2}{2}(Z_{13}^2 - Z_{14}^2)(c_R - c_L) z \left(1 - 4\frac{m_X^2}{m_Z^2}\right)\Bigg]^2 . \tag{3}$$

Finally, for completeness eq. A1 should include

$$\mathcal{M}'_z = \frac{-2m_X m_q g^2 (c_R - c_L)(Z_{13}^2 - Z_{14}^2)}{2\cos^2\theta_w (m_Z^2 - s) m_Z^2} \bar{v}(p_2)\gamma_5 u(p_1) \bar{u}(k_1)\gamma_5 v(k_2), \qquad (A1)'$$

and eq. A2 should contain the following terms

$$\begin{aligned}
|\mathcal{M}'_z|^2 =& \frac{16 g^4 (Z_{13}^2 - Z_{14}^2)^2}{\cos^4\theta_w (m_Z^2 - s)^2} (c_R - c_L)^2 \frac{m_X^2 m_q^2}{m_Z^4} \left(\frac{s^2}{4}\right) \\
2\mathrm{Re}\mathcal{M}_z \mathcal{M}'^*_z =& -\frac{8 g^4 (Z_{13}^2 - Z_{14}^2)^2}{\cos^4\theta_w (m_Z^2 - s)^2} (c_R - c_L)^2 \frac{m_X^2 m_q^2 s}{m_Z^2}, \\
2\mathrm{Re}\mathcal{M}_a \mathcal{M}'^*_z =& \frac{-8 g^4 (Z_{13}^2 - Z_{14}^2)(c_R - c_L) m_X m_q}{\cos^2\theta_w m_Z^2 (m_Z^2 - s)(M_{\tilde{q}L}^2 - t)} \Bigg(2\epsilon ab[\frac{s^2}{4} + (p_1 k_1)^2 - (p_1 k_2)^2] \\
&+ s m_q m_X (a^2 + b^2)\Bigg), \\
2\mathrm{Re}\mathcal{M}_b \mathcal{M}'^*_z =& \frac{-8 g^4 (Z_{13}^2 - Z_{14}^2)(c_R - c_L) m_X m_q}{\cos^2\theta_w m_Z^2 (m_Z^2 - s)(M_{\tilde{q}R}^2 - t)} \Bigg(-2\epsilon ac[\frac{s^2}{4} + (p_1 k_1)^2 - (p_1 k_2)^2] \\
&+ s m_q m_X (a^2 + c^2)\Bigg), \qquad (A2)' \\
2\mathrm{Re}\mathcal{M}_c \mathcal{M}'^*_z =& \frac{-8 g^4 (Z_{13}^2 - Z_{14}^2)(c_R - c_L) m_X m_q}{\cos^2\theta_w m_Z^2 (m_Z^2 - s)(M_{\tilde{q}L}^2 - u)} \Bigg(2\epsilon ab[\frac{s^2}{4} - (p_1 k_1)^2 + (p_1 k_2)^2] \\
&+ s m_q m_X (a^2 + b^2)\Bigg), \\
2\mathrm{Re}\mathcal{M}_d \mathcal{M}'^*_z =& \frac{-8 g^4 (Z_{13}^2 - Z_{14}^2)(c_R - c_L) m_X m_q}{\cos^2\theta_w m_Z^2 (m_Z^2 - s)(M_{\tilde{q}R}^2 - u)} \Bigg(-2\epsilon ac[\frac{s^2}{4} - (p_1 k_1)^2 + (p_1 k_2)^2] \\
&+ s m_q m_X (a^2 + c^2)\Bigg).
\end{aligned}$$

Note that these should be added with signs appropriate for $\mathcal{M} = \mathcal{M}_z + \mathcal{M}'_z - \mathcal{M}_a - \mathcal{M}_b + \mathcal{M}_c + \mathcal{M}_d$.

We would like to thank Marc Kamionkowski for help with this erratum. This work was supported in part by the DoE (at Chicago) and by the DoE and NASA (grant NAGW-1340) (at Fermilab).

REFERENCES

1. L. Bergstrom, preprint USITP-88-12 (1988); L. Bergstrom and H. Snellman, *Phys. Rev.* **D37**, 3737 (1988).