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Is the Sub-millisecond Pulsar Strange?

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The interpretation of 1968.629 Hz optical pulsations[1] from SN 1987A as a rapidly rotating neutron star poses a challenge to models of nuclear matter. The simultaneous requirement that a star with angular velocity $\Omega_{SN} = 1.237 \times 10^4\ s^{-1}$ be stable against nonaxisymmetric perturbations and that the maximum non-rotating neutron star mass exceed $1.44 M_\odot$ (the mass of the binary pulsar), appears to rule out all standard equations of state.[2] QCD calculations[3] suggest that three-flavor quark matter, or "strange matter," may be absolutely stable,[4] the ground state of the hadrons. Neutron stars could convert to strange stars,[5–7] which have a very different equation of state and mass-radius relation from conventional neutron star models. For a range of hadronic parameters, the maximum rotation rate of secularly stable strange stars may exceed that of the half-millisecond pulsar, while the non-rotating maximum mass $M_{max} > 1.52 M_\odot$. The low-mass companion(s) to SN 1987A, inferred from the 8-hour modulation of the optical signal, could be stable strange matter lump(s) ejected around the time of collapse.

Nascent neutron stars can convert to strange matter by a variety of routes,[5,6] *e.g.*, via formation of two-flavor quark matter, clustering of Λ particles, Kaon condensation,[8] burning of neutron matter, or sparking by cosmic ray neutrinos. Once a seed of strange matter of baryon number $A \gtrsim 10 - 100$ is formed at high density, it grows without bound and converts the star to strange matter.[6] Strange stars are thus fundamentally distinguished from neutron stars with quark cores by the assumption that quark matter is stable down to *zero* pressure. In the context of a bag model calculation, this assumption has been shown to be plausible[3] for a range of values of the strange quark mass m_s, the bag constant B (the difference in energy density between the perturbative vacuum and the true vacuum), and the strong coupling constant α_c. (In Fig. 1, we show contours of fixed energy per baryon number $E/A = 899$ and 939 MeV in the limit $\alpha_c = 0$.) Although these three parameters have been estimated by fitting the bag model to the mass spectrum of light hadrons, these fits introduce additional variables, making it difficult to extract the values appropriate for bulk quark matter.[3] In addition, the parameters m_s and α_c drop with increasing density, so fits made at low density may not be adequate for assessing strange matter stability in neutron stars. (Furthermore, the bag model is probably a crude approximation to the behavior of bulk quark matter; more appropriate might be a model in which B is a dynamical variable, as in an $SU(3)$ sigma model.[9]) In exploring stellar models based on strange matter, we shall therefore only impose the criterion of self-consistency, choosing parameters which lie in the estimated window of strange matter stability, $E/A < 939$ MeV. (Presumably, as one moves away from the stable strange matter parameter region, neutron stars have shrinking quark cores.)

The equation of state for strange matter can be written

$$p = \frac{1}{3}(\rho - 4B) - \frac{1}{3}\varepsilon(\rho - B) \quad . \tag{1}$$

The function $\varepsilon(m_s(4B)^{-1/4}, \rho/4B) < 0.14$ is a measure of the correction to the equation of state due to non-zero m_s and vanishes in the limits $m_s = 0$ (where s, u and d quarks have equal abundances) and $m_s \simeq 1.6(4B)^{1/4}$ (since the s quark abundance is suppressed as m_s approaches μ_s, the chemical potential). In calculating ε, we shall neglect $\mathcal{O}(\alpha_c)$ corrections due to quark interactions within the bag. These effects are small for the EOS, but they shift the window of strange matter stability to lower values of B.[3] We have constructed spherical general relativistic stellar models using this equation of state. Happily, for an EOS of the form (1), the QCD energy scale $B^{1/4}$ can be scaled out of the Oppenheimer-Volkoff equation. Fig. 2, which shows the resulting mass-radius relation in dimensionless units, is converted to physical units by a choice of bag constant. Mass sequences are characterized by the single dimensionless ratio $m_s/(4B)^{1/4}$. A number of consequences follow from this scale invariance. For example, for fixed $m_s/(4B)^{1/4}$, the ratio of the stellar radius to the Schwarzschild radius for maximum mass models, $R_{max}/2GM_{max}$, is independent of B.

In terms of a fiducial bag constant $B_o^{1/4} = 145$ MeV ($B_o = 57$ MeV/fm^3), parameters of the maximum mass models are given in Table 1. Also, in Fig. 1 we show contours of fixed radius $R = 8.7, 8.8, 9$ km for these models. Since the mass scales as $M \sim B^{-1/2}$, the maximum mass reaches a lower bound at the edge of the strange matter stability window, $E/A = 939$ MeV; there, $M_{max} \geq 1.52 M_\odot$ (with $R_{max} > 8.52$ km). In contrast to some soft equations of state, this is large enough to account for the observed mass[10] of PSR 1913+16 and is consistent with other inferred binary masses; neutron star mass observations to date place no constraints on the allowed parameter space.

The maximum rotation rates of neutron stars depend sensitively on the mass distribution and thereby on the equation of state. Generally, soft models are more centrally condensed, have smaller radii and maximum masses, and can rotate faster than stiff models. In this sense, strange matter is an oddball, embodying elements of both soft and hard EOS. For example, at the low mass end, where gravity plays essentially no role in their structure, strange stars are approximately uniform density spheres with $\rho \simeq 4B$ and $M \sim R^3$. Low mass strange stars, and the outer regions of more massive strange stars, are much stiffer than all neutron star models. In the central regions of strange stars near the maximum mass, however, $\rho_c \simeq 5(4B)$ and the equation of state gets very soft (relativistic fermi gas).

Massive strange stars are thus hard on the outside, soft on the inside. The stiffness is reflected in the relative flatness of the density profile: for maximum mass models, $\rho_c/\bar{\rho} = 2.1 - 2.7$, lower than or comparable to the stiffest neutron star models. On the other hand, for large values of B, the maximum mass strange stars have global features comparable to soft nuclear models. For example, consider models with constant radius $R = 8.7$ km in Fig. 1. At $m_s = 0$, we find $M_{max} = 1.59 M_\odot$, with average density $\bar{\rho} = 1.15 \times 10^{15}$ gm cm^{-3} and moment of inertia $I = 1.15 \times 10^{45}$ gm cm^2. As one moves up to $m_s/(4B)^{1/4} = 0.75$, these values drop by less than 3%, while the central density decreases from 3.09 to 2.75×10^{15} (the density profile flattens). By comparison, for the Reid soft core[11] (EOS A in the Arnett-Bowers[12] collection), $M_{max} = 1.655 M_\odot$, $R_{max} = 8.426$ km, with $\rho_c = 3.98 \times 10^{15}$, $\bar{\rho} = 1.31 \times 10^{15}$ gm cm^{-3}, and $I = 1.05 \times 10^{45}$ gm cm^2. In addition, at $m_s = 0$, for the maximum mass strange star, the pressure-averaged adiabatic index $\bar{\Gamma} = \bar{\rho}/(\bar{\rho} - 4B) = 2.3$, which is moderately soft.

How fast can strange stars spin? A lower bound on the maximum rotation rate is given by the low mass models with $m_s = 0$. For $M < 0.5(B/B_o)^{-1/2} M_\odot$, the ratio of central to average density $\rho_c/\bar{\rho} < 1.06$, the pressure $p_c < 0.06\rho_c$, the average polytropic index $\bar{n} = (\bar{\Gamma} - 1)^{-1} < 0.1$, and the post-Newtonian corrections to gravity are negligible. When they rotate, these ultra-stiff low mass stars are the nearest incarnations of Newtonian Maclaurin spheroids in Nature (the Maclaurin approximation improves as the mass decreases). Since the average density is bounded from below by $\bar{\rho} > 4B$, the maximum rotation rate,† set by the dynamical instability,[13] is $\Omega_{dyn} = 0.67(\pi G \bar{\rho})^{1/2} > 6.22 \times 10^3 (B/B_o)^{1/2}$ s^{-1}. The nonaxisymmetric secular $m = 2$ instability, driven by gravitational radiation reaction (GRR)[14], sets in at $\Omega_{sec} = 0.91\Omega_{dyn}$. Although there are also modes with $m > 2$ which become unstable at lower angular velocity, strange stars can rotate fast enough to account for the 1.6 ms pulsar.

More massive strange stars can rotate faster since they have higher densities. For strange stars near the maximum mass, we estimate the maximum rotation speed using the results of Friedman, Ipser, Parker[2] and Butterworth[15]. An absolute upper limit is set by the onset of equatorial (Keplerian) mass shedding. Due to relativistic effects,[15] at fixed density $\bar{\rho}$ the Keplerian limit Ω_K can be larger than Ω_{dyn}. In addition, we expect that the relatively high density strange matter surface may rotate faster than lower surface density neutron star models of the same radius. By comparing with models of comparable M, $\bar{\rho}$, and R (e.g., EOS A),[2] we infer a maximum equatorial velocity in the range $v_{eq}/c =$

†Although low mass strange stars are bound (against evaporation into free neutrons) principally by the strong interactions, not gravity, their stability against rotationally induced fission into smaller fragments *is* determined by gravity. (For very small strange matter lumps, the surface tension also helps prevent rotational fission.)

0.45 − .48. This is compatible with the results of ref. 15 on rapidly rotating uniform density stars: for a spherical star with $p_c/\rho_c = 0.27$ (comparable to the central pressure-density ratio in maximum mass strange stars), they find $v_{eq}/c = 0.45$ for the corresponding rotating star. In addition, for neutron stars with comparable stiffness to strange matter, the equatorial radius of the maximum mass rotating model is in the range[2] $a = 1.26 - 1.3R$, where R is the radius of the spherical maximum mass model. Thus, for maximum mass strange stars, we expect the Keplerian limit to be $\Omega_K = (1.03 - 1.14) \times 10^4 (10\ \mathrm{km}/R)\ \mathrm{s}^{-1}$. To account for the apparent angular velocity of the pulsar in SN 1987A, the radius of the spherical model must satisfy the constraint $R < 8.4 - 9.2$ km. Clearly, a more accurate determination awaits the construction of relativistic rapidly rotating strange stars. Here we are suggesting that, given the broad range of QCD parameter space (Fig. 1) for which $R_{max} < 8.7 - 9$ km, such an investigation would be of utmost interest.‡

Although some conventional neutron star models, such as those based on EOS A or F,[2] also satisfy the constraint $\Omega_K > \Omega_{SN}$, they are secularly unstable to perturbations in the $m = 4$ or 5 mode.[2,17–20] Since these GRR-driven instabilities are damped by viscosity, the origin of the difficulty is the relatively low viscosity of hot young neutron matter. In this regard, strange matter offers two advantages. First, at equal temperature and density, the shear viscosity of strange matter[21] $\eta_s \simeq 9.7 \times 10^{16} (\alpha_c/0.13)^{-3/2} (\rho/10^{15})^{5/3} (T/10^{10}K)^{-2}$ g cm^{-1} s^{-1}, is roughly an order of magnitude larger than that of nuclear matter,[22] $\eta_{nuc} \simeq 9.6 \times 10^{15} (\rho/10^{15})^{9/4} (T/10^{10}K)^{-2}$. Here, the strange matter viscosity due to quark-quark scattering has been calculated[21] to lowest order in α_c, assuming the magnetic field $B \lesssim 10^{10}(T/10^9 K)^2\ G$. (This is consistent with the observational upper bound on the field of SN 1987A due to the continued decline in bolometric luminosity.) Second, neutrino cooling in strange stars is significantly more efficient than the modified URCA process.[23] As a result, the core temperature of a two year-old strange star is estimated to be $\simeq 10^8$ K, as opposed to $\simeq 10^9$ K for neutron stars. Since the shear viscosity scales as T^{-2},

‡This is in contrast to a recent claim that strange stars cannot account for the pulsar in SN 1987A.[16]

the present strange star viscosity, $\bar{\nu} \simeq 6 \times 10^5$ cm^2 s^{-1}, is a factor $\sim 10^3$ larger than that for young neutron stars. Consequently, GRR-driven secular instabilities are more effectively damped in strange stars. Using previous estimates of the viscous damping timescale,[17,18,20] we find that the $m \geq 4$ modes are stable. For the $m = 3$ mode, we infer[18] $\Omega_{m=3} \geq 0.96\Omega_K$ for this range of viscosity. This reduces the maximum non-rotating radius estimated above to the range $R < 8.1 - 8.8$ km.

The reported 8 hour modulation of the optical pulsar period has been interpreted as evidence for a Jupiter-mass companion to SNR 1987A.[1] Since strange stars are bound by the strong force rather than gravity, they have no minimum stable mass. Thus, planetary mass strange matter debris ejected in an asymmetric supernova collapse, or shed in a rapidly rotating (super Keplerian) early post-collapse phase, would not disintegrate. Furthermore, strange matter lumps have comparable density to strange stars, so they can be ejected without being disrupted by tidal forces. This would circumvent the difficulties of more contrived planet formation scenarios, *e.g,* those which must invoke rapid disk formation and instability.

The 1987A pulsar data also appears to have a second amplitude and phase modulation, with a period of $\simeq 2$ hours. If this modulation is attributed to a second companion, it implies a Neptune-mass planet on a highly eccentric orbit with semi-major axis around 4×10^5 miles.[24] In order to survive tidal disruption, such a satellite must have a mean density $\bar{\rho} \gtrsim 25$ g cm^{-3}. This is much higher than the density of any planet-like object, while the inferred mass is well below the lower mass limit for white dwarfs. The only object we know of that can fit this scenario is a strange matter lump. An alternative explanation is modulation due to precession of the pulsar; this hypothesis can be tested with additional data. If precession is ruled out, the case for strange matter would be impelling.

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Figure Caption

Fig. 1.- The strange matter parameter space. Solid curves marked 899 and 939 are contours of constant energy per baryon number (in MeV) for bulk strange matter (from Ref. 3). Dashed curves are contours of constant radius $R = 9, 8.8, 8.7$ km for non-rotating maximum mass models. Also shown are lines of constant $m_s/(4B)^{1/4} = 1.2, 1.0$ and 0.5.

Fig. 2.- Mass $M(4BG^3)^{1/2}$ versus radius $R(4BG)^{1/2}$ relation for strange stars, for strange quark masses $m_s/(4B)^{1/4} = 0, .5, 1.0, 1.2$. Crosses mark maximum mass models. Also shown is the minimum radius for stable uniform density stars, $R = (9/8)2GM$.

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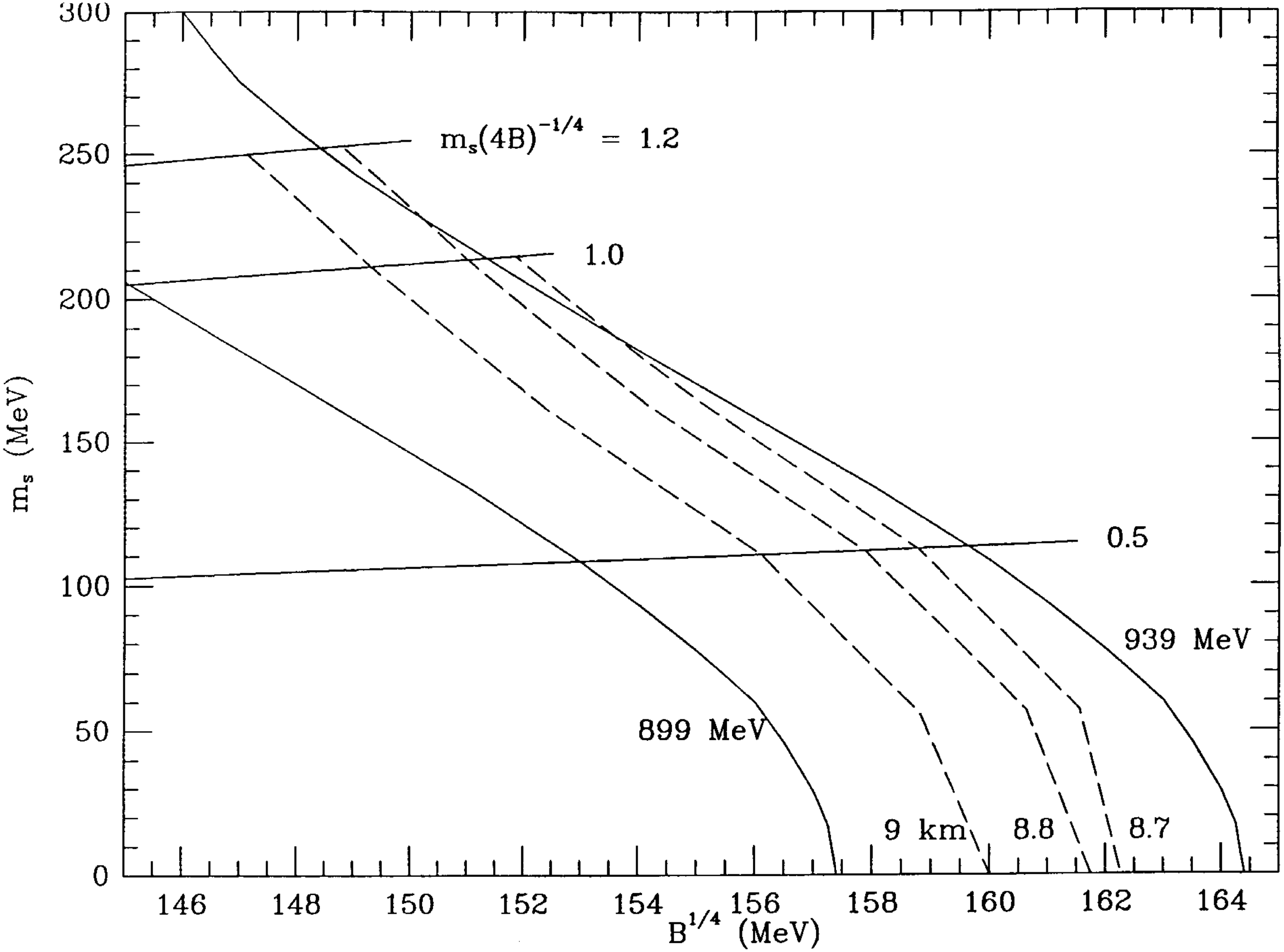
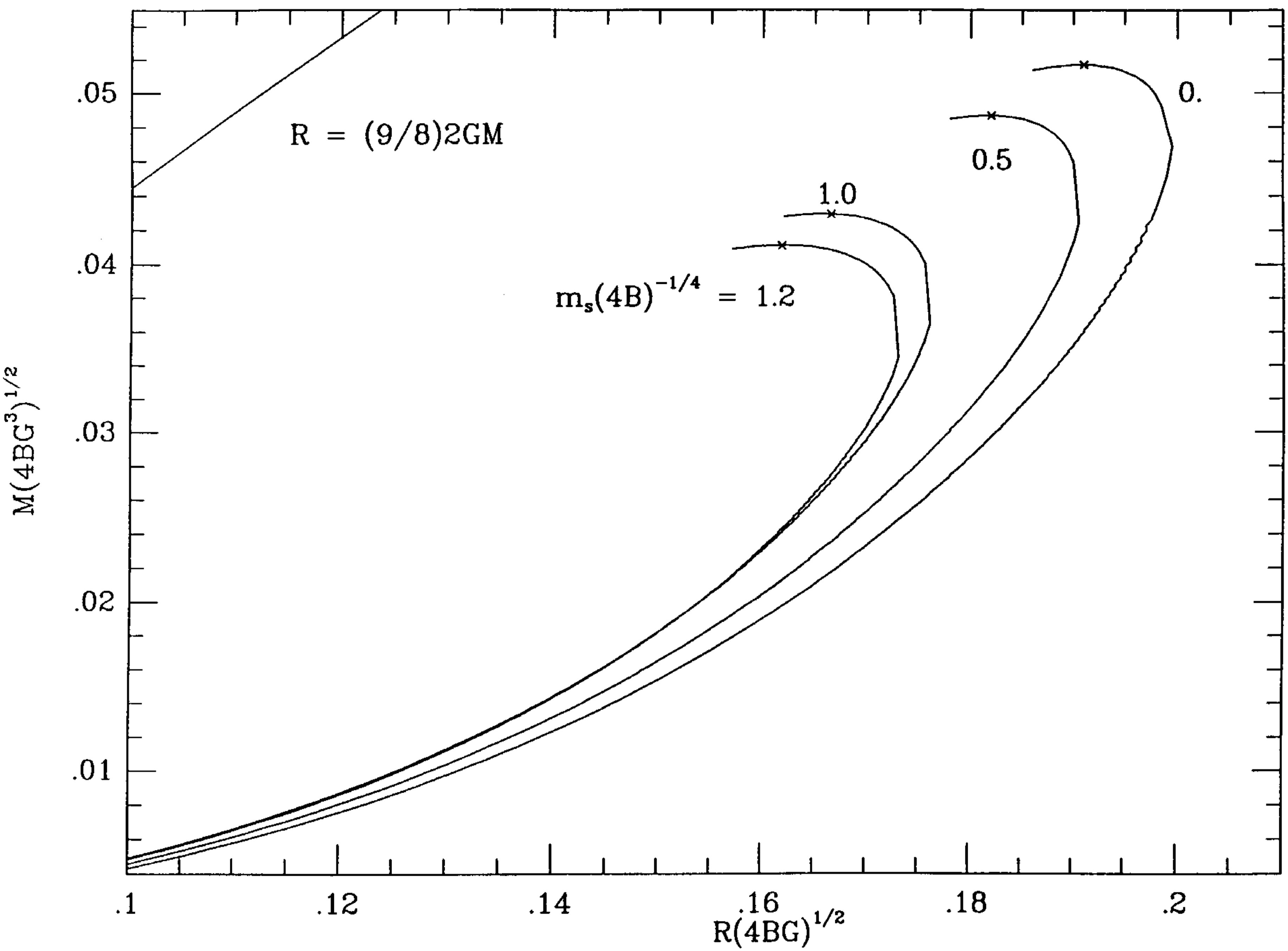
Table 1 - Maximum Mass Models*

$m_s(4B)^{-1/4}$	$M_{max}(B/B_o)^{1/2}$	$R_{max}(B/B_o)^{1/2}$	$\rho_c(B/B_o)^{-1}$ g cm^{-3}	$\bar{\rho}(B/B_o)^{-1}$	$R_m/2GM_m$
0	2.006 $M_\odot$	10.95 km	1.95×10^{15}	7.27×10^{14}	1.847
0.25	1.973	10.80	1.97	7.44	1.854
0.5	1.887	10.43	2.02	7.90	1.871
0.75	1.775	9.94	2.11	8.58	1.897
1.0	1.665	9.54	2.09	9.10	1.940
1.2	1.594	9.27	2.05	9.52	1.968

*Fiducial bag constant $B_o^{1/4} = 145$ MeV.



m_s (MeV)
$B^{1/4}$ (MeV)
$m_s(4B)^{-1/4}$ = 1.2
1.0
0.5
899 MeV
939 MeV
9 km
8.8
8.7
0
50
100
150
200
250
300
146
148
150
152
154
156
158
160
162
164



M(4BG^3)^1/2
R(4BG)^1/2
R = (9/8)2GM
m_s(4B)^-1/4 = 1.2
1.0
0.5
0.
.01
.02
.03
.04
.05
.1
.12
.14
.16
.18
.2