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Analytical Solutions to SSC Coil End Design *

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ABSTRACT

As part of the SSC magnet effort, Fermilab will build and test a series of one meter model SSC magnets. The coils in these magnets will be constructed with several different end configurations. These end designs must satisfy both mechanical and magnetic criteria. Only the mechanical problem will be addressed. Solutions will attempt to minimize stresses and provide internal support for the cable. Different end designs will be compared in an attempt to determine which is most appropriate for the SSC dipole. The mathematics required to create each end configuration will be described. The computer aided design, programming and machine technology needed to make the parts will be reviewed.

ANALYTICAL SOLUTIONS TO SSC COIL END DESIGN

When winding a superconducting cosΘ magnet coil, special attention must be paid to the end configuration. Parts must be made to confine the conductors to a consistent shape. This shape must be defined and described to both the parts manufacturers and those analyzing the magnetic field.

Past Experience With Superconducting Magnet Ends

Over one thousand Tevatron dipoles have been produced at Fermilab. The path which the conductors take as they are wound around the ends on these magnets is defined as the intersection of a sphere and a cylinder (see Figure 1). It can be generated by simply drawing a circle with a compass on the surface of a cylinder. At the center of each turn, the cable is held vertical, that is, perpendicular to the beam path in the yz plane (see Figure 1). This shape has the advantage of being easy to define and inspect. The disadvantage of the "vertical" end is that it results in high internal stresses in the cable. The amount of stress induced in the cable is inversely proportional to the bore diameter. These stresses, although high, were within acceptable limits on the Tevatron magnets. The Tevatron bore diameter is three inches.

The SSC magnets have a much smaller bore diameter than the Tevatron (4 cm). As a result, stresses become unacceptably high. Many problems result from these high stresses. They are listed in Table 1.

Table 1. Problems in Winding COSΘ Magnet Ends

A. Turn-to-Turn Shorts.

Cables which are forced into an unnatural path will cause breakdown of the kapton insulation between turns.

B. Degradation of Strands.

High stresses cause the strands to stretch, and in the worst cases, break. Strands will also come "out of lay" causing the cable to take a shape other than its intended keystone shape.

C. Difficult to wind.

D. Difficult to maintain magnet-to-magnet consistency in conductor placement.

E. Tendency of cables to move when magnet is powered, causing quenches.

F. Tendency of turns to move into the bore after curing.

The cable tries to take a position which is less stressful than that into which it was wound. This usually results in the inner coil turns moving into the bore area (see Figure 2).

Solutions

It is necessary to create an end configuration which produces less internal stress in the cable. Internal stress is proportional to the amount of strain in the cable. "Strain" is defined as the amount of deformation per unit length to which the cable is subject as it is wound around the end. The symbol for strain is $\Delta L/L$, where ΔL is the deformation and L is the total perimeter of the end. There are two different approaches to reducing strain.

1. Decrease the difference between the inside and outside perimeters of the respective turns (decreasing ΔL).
2. Increase the total perimeter of the end by making the end flared (increasing L).

The following geometries will attempt to decrease strain by the first approach.

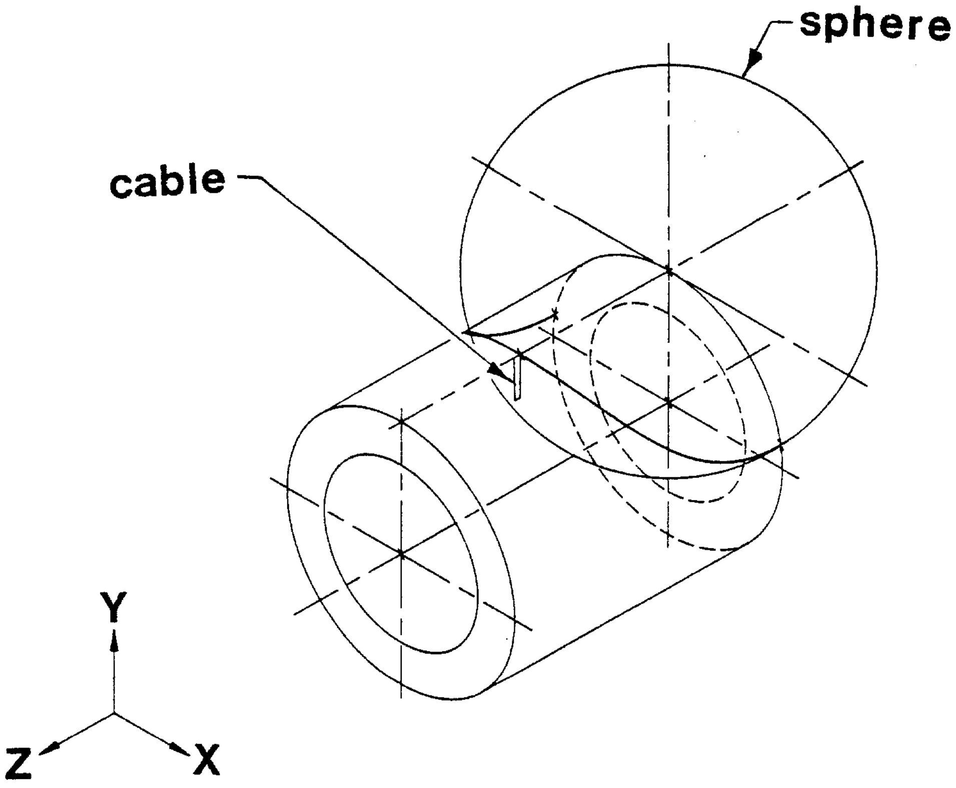


Fig. 1. Intersection of sphere and cylinder.

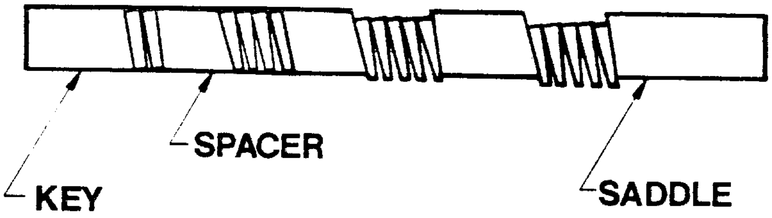


Fig. 2. Turns moving into bore.

Creating Geometries

A. Method of Defining Base Curve.

The base curve is defined as the path which an individual conductor takes as it is wound around the end of a coil. It is represented by a one dimensional line in three dimensional space.

1. Ellipse on Cylinder Method.

We begin by laying out an ellipse in the flat conditon (see Figure 3). b1 is equal to the arc length of the outer surface of the coil cross section at that point. a1 is chosen for magnetic reasons. The ellipse is then wrapped on the cylindrical surface.

The shape of the ellipse can be adjusted to conform to empirical evidence by changing the "order" or the exponent in the ellipse equation. A "superellipse" with an exponent of three,

$$\frac{x^3}{a^3} + \frac{y^3}{b^3} = 1$$

for example, has a "squarer" shape than an ellipse with an exponent of two

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

(see Figure 4). It is important, due to the incomplete nature of our models, to be able to adjust the shape of the base curve to conform to empirical evidence. Some of the shapes, such as the ellipses on cylinders, are generated purely for geometric reasons. Even the best models are seriously flawed. All current models, for example, assume the cable is an infinitely thin, homogeneous strip, and not the composite of shapes and materials which really make up a cable.

The ellipse on cylinder method of defining a base curve has several advantages. It is mathematically simple and easy to define. It readily lends itself to manipulation by changing the order. Its disadvantages are that it does not take into consideration any actual stresses in the cable.

2. Elastica.

A planar elastica is defined as a plane curve into which the central line of a thin elastic rod can be bent by means of forces and couples applied to its ends only. The curve then satisfies the condition that it have the minimum possible strain energy, subject to certain end constraints. In our case we have a cylindrical surface instead of a flat plane, complicating the situation slightly. The curvature of the elastica has a new component perpendicular to the constraining surface. The strain energy corresponding to this component, which was zero when the constraining surface was a flat plane, must now be added to the classical strain energy corresponding to the component

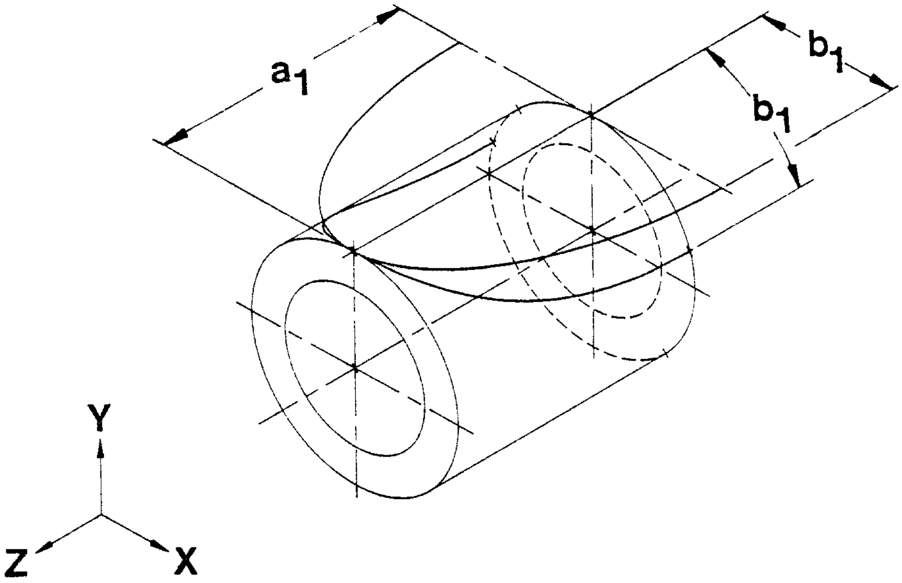


Fig. 3. Ellipse to be wrapped on cylinder.

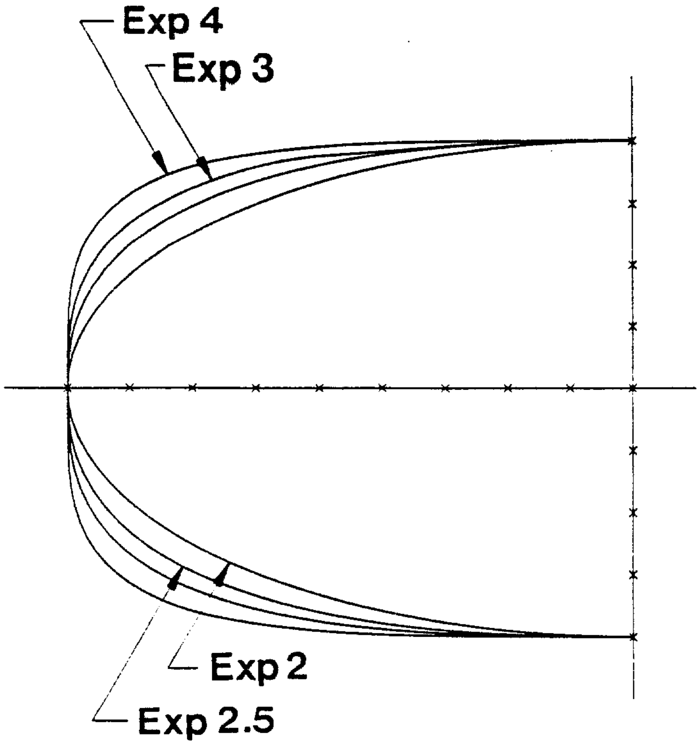


Fig. 4. Superellipses.

of curvature parallel to the constraining surface. The base curve is therefore defined as a smooth line segment with minimum total strain energy. It is confined to a cylindrical surface and must satisfy certain initial and final conditions. These conditions are:

a. The line must begin at a point on the surface of the cylinder and be pointing in the direction of the positive z axis (see Figure 5).

b. The line must end at a point on the top center of the cylinder (at a value of x=0) and be pointing in the direction of the negative x axis (see Figure 5).

The elastica has the advantage of being based on elasticity theory rather than pure geometry. It may create less stress in the cable than the ellipse on cylinder method. It has the disadvantage of being less amenable to mathematical manipulation. Manipulation of this type, based on evidence obtained from inspecting previously wound coils, may be necessary given the incomplete model.

B. Base Curve Position.

The base curve may be placed on either the inside or the outside radius of the layer (see Figure 6). In either case a "gap" appears on the other radius. This happens because the cables are laying at an angle and no longer require as much radial clearance as they would if they were stacked vertically. Fermilab's SSC coils use the outside radius for the base curve. This allows us to use the gap on the inside radius to provide internal support for the conductor. A "shelf" is attached to the spacer, filling the empty space and keeping the turns from moving into the bore.

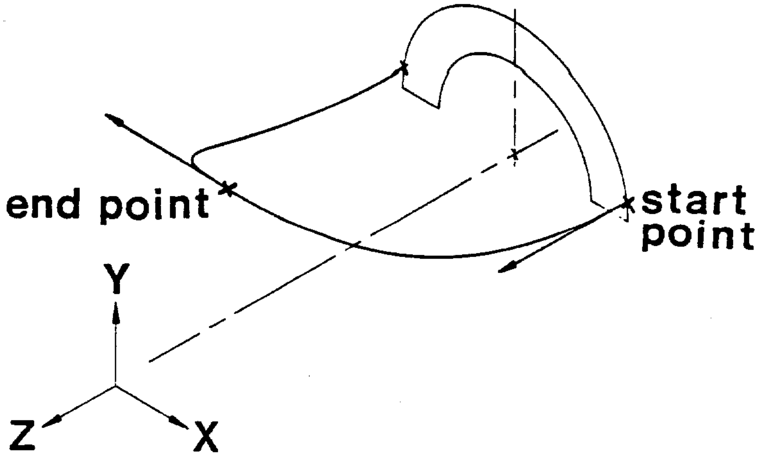


Fig. 5. Elastica

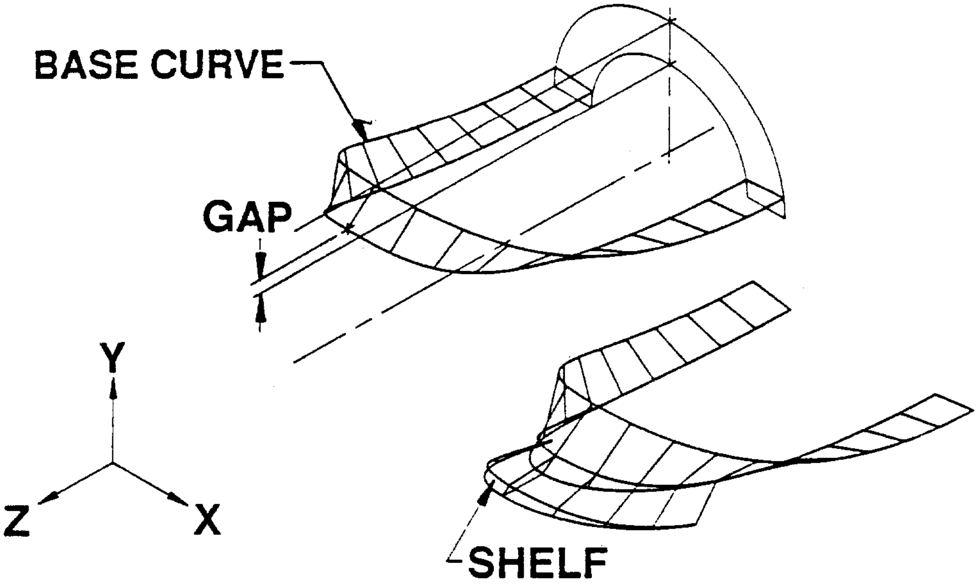


Fig. 6. Base curve positioned on outside of layer.

C. Method of Defining Surfaces.

1. Connecting Lines Between Equally Spaced Points.

a. If Base Curve Is Ellipse On Cylinder.

We begin with a base curve (see Figure 7). a_1 is the longitudinal radius of the ellipse. b_1 is the azimuthal radius of the ellipse. The perimeter of this ellipse is calculated (L_1. We can now solve for another ellipse. We know L_2 because it must be equal in length to L_1. b_2 of our second ellipse is equal to the arc length of the inner surface of the coil cross section. We solve for a_2. The two ellipses are then wrapped around their respective cylinders (see Figure 8). Eleven equally spaced points are drawn on each ellipse half. The points are then connected making ten "facets". The eleven lines are then cut off at a distance of one cable width from the top surface. A curve is drawn connecting the new points. This gives us a new perimeter, L_3. L_3 is shorter than L_1 or L_2 and lies between L_1 and L_2. Then by an iterative process we repeat this procedure increasing a_2 in small steps until $L_3 = L_1$ giving us an average $\Delta L/L$ of zero.

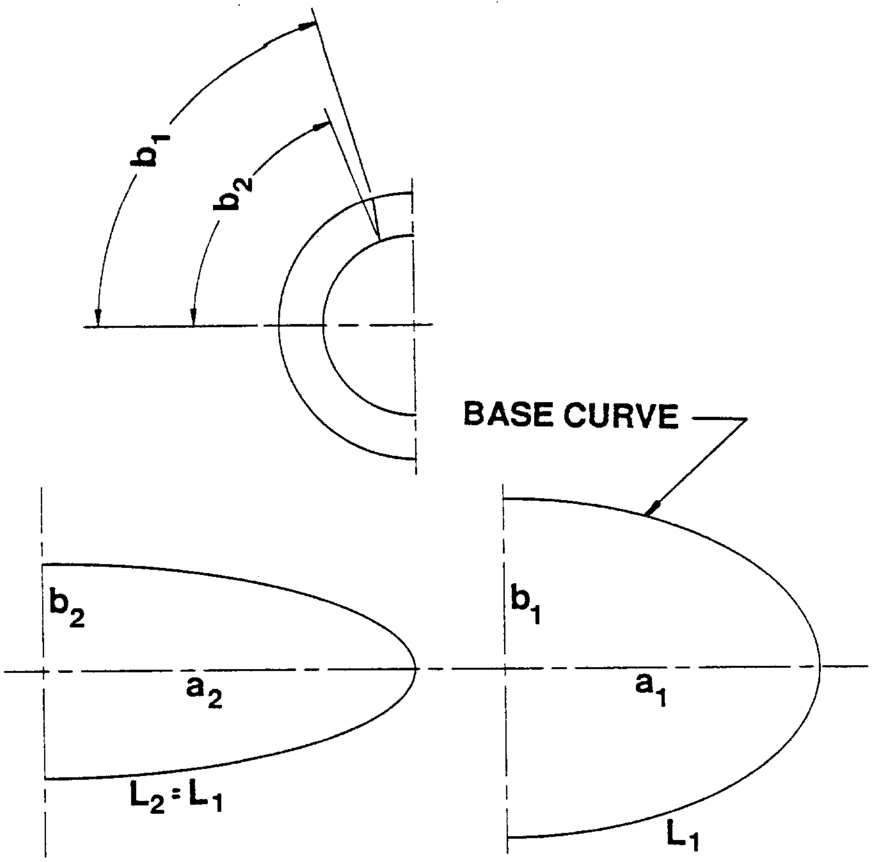


Fig. 7. Ellipse on cylinder as base curve.

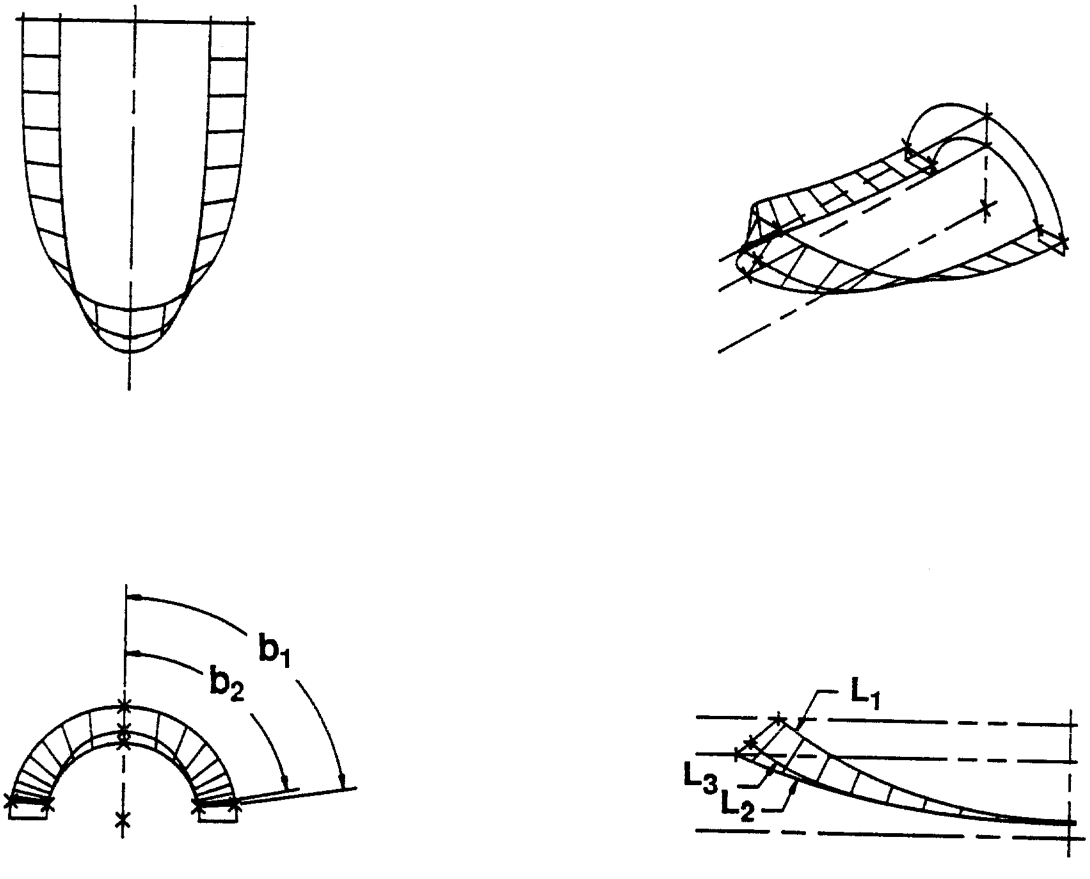


Fig. 8. Ellipses wrapped around their respected cylinders.

b. If Base Curve Is Elastica.

We create two elasticas, one representing the inside edge of the turn and one representing the outside edge of the turn. An equal number of equally spaced points are placed on each curve. Straight lines are then drawn between the corresponding points. The resulting surface is then "trimmed" to the thickness of the cable.

This method is geometrically simple. It does, however, have undesirable qualities. The surface which is created is not "developable", that is, one which can be laid onto a flat surface without distortion. We are making an attempt to create an end configuration which results in a developable surface.

2. Creating a Developable Surface.

A base curve is wrapped on a cylinder (curve L_1 on Figure 9). A set of equally spaced points is placed on the curve (P_1, P_2, ...Pn). Vectors are drawn from the points (V_1, V_2, ...Vn) in such a way that they sweep out a certain surface called by differential geometers the rectifying developable of the curve. It is by definition perpendicular to the direction of curvature of the curve at every point. The cable is modeled by a strip in this surface along the curve.

Two simplifications in our mathematical model allow us to approximate in this way a two dimensional elastic strip by a one-dimensional elastica. First and most drastic, the torsional rigidity of the strip (its resistance to torques with axes parallel to the curve) is taken to be zero. Second its flexural rigidity about an axis perpendicular to the strip is taken to be infinite. The flexural rigidity of the strip about its other axis is then equal to the rigidity of the elastica. In the Fortran program implementing this model the first two assumptions are relaxed slightly: the torsional rigidity is small but non-zero and the first flexural rigidity is large but finite.

After the surface is created in this manner, it is trimmed at the appropriate cable width (L_3).

It is not certain that the developable surface creates a less stressful condition for the cable than connecting the straight lines. Analysis is still needed to determine whether the developable surface is truly more desirable.

D. Method of Stacking Cables.

Independent of the method of defining the shape of the individual surface for a turn, there are two methods of stacking a cable within an end. These are the "individually determined" and "grouped" methods (see Figure 10).

1. Individually Determined.

In this method each turn has its surface calculated individually. This results in the turns not laying directly upon each other. As turns get farther from the pole, the geometry requires that they be stacked at an increasingly larger angle (see Figure 10). This creates small spaces between each turn. These spaces typically become filled with epoxy.

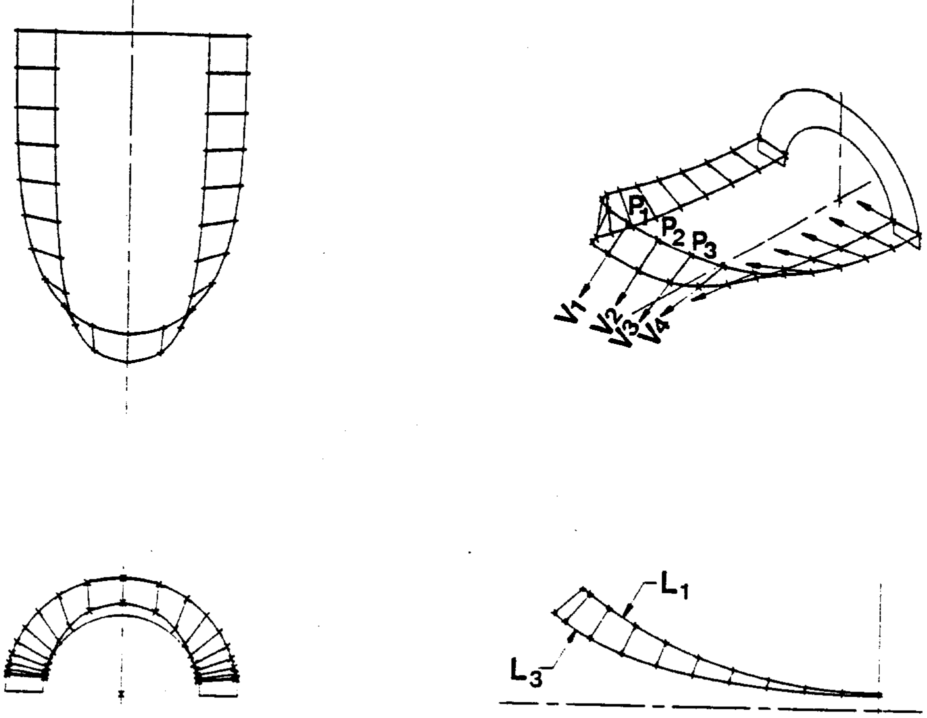
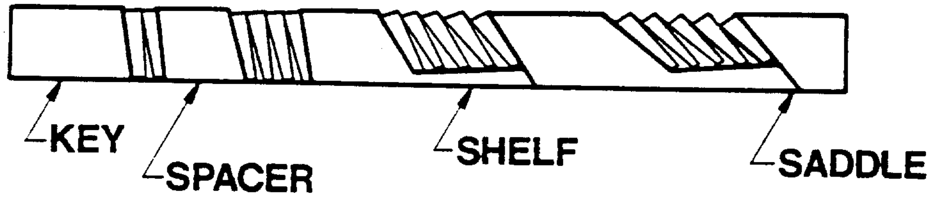
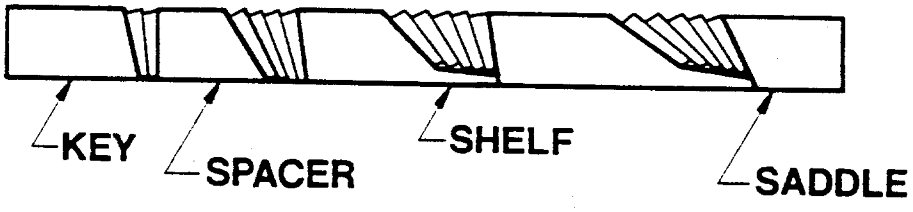


Fig. 9. Developable Surface.



INDIVIDUALLY DETERMINED



GROUPED

Fig. 10. Cable stacking methods.

2. Grouped.

In the grouped method only the center of each current block is a calculated surface. The turns within each block are layed directly on each other with no spaces between them. As the turns get farther away from the center of the current block, their internal stresses become progressively higher.

The advantage to the individually determined method is that the stress in each turn is minimized. The advantage to the grouped method is that the spaces betwen turns are eliminated, further restricting cable movement.

Creating the Drawings

Creating these end geometries would be impossible without modern computer aided drafting and design. Software exists for such functions as "laying an ellipse on a cylinder", "creating a spline in three dimensions", and "placing equally spaced points on a curve". Solid modeling programs allow the designer to create these various parts, visually inspect and analyse them, then send the data to a numerically controlled machine.

Much effort has been put into automating the design of these parts. Having a clear mathematical description of the curves allows us to write programs which create the curves based on an initial input. The designer must answer several prompts concerning the two dimensional coil cross section and the longitudinal current block positions on the ends. Curves are created by the program. The output consists of a data file of coordinate points from which three dimensional surfaces are defined and tool paths automatically created for the numerically controlled machine.

Such a program has been completed for an end design based on straight lines connecting ellipses. An upcoming program will deal with elasticas and developable surfaces.

Manufacturing

All parts are presently being produced by numerically controlled machines. Raw material is G-10 tubing. Both Fermilab's machine shops and outside vendors are making parts.

The most efficient way of machining these parts is by using a five axis machine with a cylindrically shaped cutting tool. All surfaces could be machined in a single pass by this method. Due to the unavailability of five-axis machines, we have been presently producing them with three-axis machines. We must use a spherically shaped tool and cut the surfaces in several passes. This process requires more complicated programming and longer machining time than would be required by the five-axis machine.

It is clear that machining parts is unacceptable for quantities as large as would be required for the SSC. Other fabrication techniques must be pursued for the production magnets.

Fermilab is attempting to make molded parts. Several materials are under consideration. Some success has been achieved using phenolic with 50% glass fiber. The material is hydraulically transferred into a mold cavity and cured at 350 degrees farenheit for 40 seconds. Molded parts have not yet been used in a magnet.

Conclusion

There are many possible solutions to the problem of winding the end of an SSC coil. The Fermilab short model program is choosing several and attempting to compare them in the most experimental way possible. One meter coils will be built with various combinations of "ellipse vs. elastica", "straight line vs. developable" and "grouped vs. individually determined" configurations. These magnets will be tested cold. The coils will also be potted and sectioned to closely examine the placement of conductors. Analysis of the results will determine which type of end will be used in the Fermilab SSC long models.

REFERENCES

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