Fermi National Accelerator Laboratory

FERMILAB-PUB-88/206-T
December 22, 1988

Probing Vacuum Stability Bounds at the Fermilab Collider

MANFRED LINDNER

Fermi National Accelerator Laboratory,
P.O. Box 500, Batavia, Illinois 60510

MARC SHER AND HELMUT W. ZAGLAUER

Physics Department, Washington University,
St. Louis, MO 63130

ABSTRACT

If the top quark mass is above 86 GeV, then a stringent lower bound on the mass of the Higgs boson arises from the requirement of vacuum stability. Since previous calculations of this bound differ by up to 20 GeV, we calculate the bound as precisely as possible by explicitly solving the renormalization group equations to two-loop order. If the lower bound on the top mass is 100 (110, 120) GeV, the the lower bound to the Higgs mass is found to be 20 (34, 50) GeV. Thus, if the standard model is correct, then a nondiscovery of the top quark at the Fermilab Collider implies that the Higgs boson cannot be discovered at CUSB, SLC or LEP I.

submitted to Physical Review Letters

Operated by Universities Research Association Inc. under contract with the United States Department of Energy

The recent observation of $B - \overline{B}$ mixing at ARGUS[1] and the analysis of UA2 data[2] indicates that the top quark mass must be larger than 50 GeV, a mass region which will be probed in the next few months at Fermilab and CERN. Failure to find the top quark during the current Collider run will imply a lower bound of approximately $100 - 110$ GeV on the top quark mass, m_t. This has led to renewed interest in the theoretical upper bound on m_t. The most stringent of these bounds comes from requiring that the standard model vacuum be stable, and gives a bound which will actually be tested during the current Collider run.

The instability[3] is generated by loop corrections to the Higgs potential. The tree level potential is

$$V_o = -\frac{1}{2}\mu^2\phi^2 + \frac{1}{4}\lambda\phi^4 \tag{1}$$

where $\frac{1}{2}\phi^2 \equiv \Phi^\dagger\Phi$. The one-loop leading logarithm corrections to the potential were first calculated by Coleman and E. Weinberg[4] and give (ignoring scalar loops for the moment)

$$V = V_o + \frac{1}{64\pi^2}B\phi^4 \ln\frac{\phi^2}{M^2} \tag{2}$$

where M is the renormalization scale and

$$B = \frac{6m_W^4 + 3m_Z^4 - 12m_t^4}{\sigma^4} = \frac{9}{16}g^4 + \frac{3}{8}g^2g'^2 + \frac{3}{16}g'^4 - 3g_Y^4. \tag{3}$$

Here, $\sigma \simeq 246\ GeV$ is the minimum of the potential, g, g' and g_Y are the $SU(2)$, $U(1)$ and top quark Yukawa couplings respectively. One can see that if m_t is large enough, then B is negative, and the potential is unbounded for large ϕ, i.e. the vacuum is unstable.

Although this demonstrates the existence of a possible instability for large top quark masses, the above expression *cannot* be used to reliably determine the bound. The reason[5] is that the above loop expansion is an expansion in powers of $\alpha \ln\frac{\phi}{M}$, where α is the largest coupling in the theory, and not an expansion in powers of α. To determine whether the vacuum is stable, large regions of field space must be considered, and thus the logarithm can be quite large. Since the Yukawa coupling is also relatively large, the above loop expansion is unreliable. As shown in Ref. 5, using running couplings in the potential can change the bounds that would be obtained from eq. (2) by as much as a factor of 3.

A simple procedure does exist for including the large logarithms – the renormalization group approach. In Refs. 6 and 7, the renormalization group equation for the effective potential was explicitly solved, and one-loop beta functions and the one-loop anomalous dimension were input; the resulting potential is an expansion in powers of α, without large logarithms. The bound obtained on m_t in Refs. 6 and 7 increases with the Higgs mass m_H, intercepting the $m_H = 0$ axis at $80 - 90$ GeV. Both references quoted an uncertainty in the bound of approximately 10 GeV (and the two differed by that amount).

It is easy to see how this uncertainty arises. Suppose, for example, that eq. (2) were used to determine the bound. One can see that the bound on m_t is roughly linearly dependent on m_W and m_Z. Given that the latter are experimentally uncertain by several percent (or were when Ref. 6 appeared), that radiative corrections to their masses (which would appear in next order) are several percent, and that threshold effects, Higgs mass corrections, etc. were not included, it is not surprising that previous bounds are uncertain by 10 GeV.

This Letter is motivated by the fact that the experimental lower bound on m_t could reach 100 GeV or even more in the next few months. A experimental lower bound on m_t in excess of $80 - 90$ GeV will give a very stringent lower bound to the Higgs mass. Furthermore the lower bound to the Higgs mass is very sensitive to the precise bound on m_t. In fact, a 10 GeV uncertainty in the upper bound of m_t converts into an uncertainty as large as 20 GeV in the derived lower bound to the Higgs mass. The objective of this Letter is to calculate the top quark mass bound as precisely as possible. Our final result can then be used to convert an experimental lower bound or a measurement of m_t to a theoretical lower bound (caused by requiring vacuum stability) on the Higgs mass. We emphasize that we are working in the context of the minimal standard model. If there are additional Higgs fields or supersymmetric particles, then this work is irrelevant.[8]

We will explicitly solve the renormalization group equation for the effective potential, use two-loop beta functions and anomalous dimension, and use one-loop boundary conditions to the differential equations. The renormalization group equation for the effective potential is nothing more than the statement that the potential cannot depend on the choice of renormalization scale:

$$\left(M\frac{\partial}{\partial M} + \beta_\lambda \frac{\partial}{\partial \lambda} + \sum_i \beta_{g_i}\frac{\partial}{\partial g_i} + \beta_\mu \mu^2 \frac{\partial}{\partial \mu^2} + \gamma\phi\frac{\partial}{\partial \phi}\right) V(\phi) = 0 \qquad (4)$$

where the sum is over the gauge and Yukawa couplings, $\beta_\lambda \equiv M\frac{\partial\lambda}{\partial M}$, $\beta_{g_i} \equiv M\frac{\partial g_i}{\partial M}$ and $\beta_\mu \mu^2 \equiv M\frac{\partial \mu^2}{\partial M}$. γ is the anomalous dimension of the scalar field. It should be noted that we are working in the Landau gauge, thus the scale-dependence of the gauge parameter is zero.

The exact solution of this equation is given by

$$V(\phi) = \frac{1}{2}\mu^2(t)G^2(t)\phi^2 + \frac{1}{4}\lambda(t)G^4(t)\phi^4, \qquad (5)$$

where

$$\frac{dg_x}{dt} = \frac{\beta_{g_x}}{1-\gamma} \text{ ; for } g_x = \lambda,\ g\ ,g'\ ,g_s\ ,g_Y$$

$$\frac{d\mu^2}{dt} = \mu^2 \frac{\beta_\mu}{1-\gamma} \qquad (6)$$

$$G(t) \equiv \exp\left(- \int_0^t dt' \ \frac{\gamma}{1-\gamma}\right) .$$

Each of the beta functions and the anomalous dimension is a function of all of the other couplings, which are functions of $t \equiv \ln \frac{\phi}{M}$.

This solution is *exact*. Perturbation theory enters when one inputs the perturbative expansions for the beta functions and anomalous dimension. It is instructive to derive eq. (2) from this solution. Suppose one assumes (without justification) that $\gamma = \beta_\mu = 0$ and that β_λ is a constant. Then the first equation of eq. (6) for λ can be trivially integrated, and plugged into eq. (5). The result is eq. (2).

We will choose our renormalization scale to be the Z mass and we will input the two-loop expressions for the beta functions and gamma. To solve for the potential, boundary conditions for the six first order differential equations are needed. These six conditions will be the initial values of $g, g', \lambda, g_Y, \mu^2$ and g_s (the strong coupling constant). As our input parameters, we choose the Fermi constant, G_F, the Z mass, m_Z, the strong coupling $\alpha_s(m_Z)$, the fine-structure constant $\alpha_{em}(0)$ and the physical top-quark and Higgs masses. Given these six input parameters, the potential can be examined for stability. For given top quark and Higgs masses, the only significant uncertainties are in the values chosen for m_Z and $\alpha_s(m_Z)$.

The two-loop anomalous dimension and the two-loop beta functions can all be found in the works of Machacek and Vaughn.[9−12] We have used their results (in Landau gauge).

The initial values for g and g' can be extracted from the value of $G_F m_Z^2$ and $\alpha_{em}(0)$. The expressions for g and g', evaluated at our renormalization scale m_Z, can be found explicitly in the works of Marciano and Sirlin.[13,14] The value of m_Z will be taken to be 91.6 GeV; the uncertainty caused by this choice will be discussed shortly.[15]

For the initial value of the strong coupling, we take $\alpha_s(m_Z) = 0.115 \pm 0.015$, as suggested by a combination of measurements and theoretical calculations.[16] This corresponds roughly to a range of Λ_{QCD}, in the five-flavor $\overline{MS}$ scheme to two-loops, of $50 - 300$ MeV. Results will be presented for the central value (which corresponds to $\Lambda_{QCD} \simeq 150$ MeV); the uncertainties will be discussed later.

Since μ^2 is the only scale in the theory, its initial value is irrelevant; one adjusts the value to get the correct minimum of the potential. Rather than attempt to use an expression for the Higgs mass in terms of the initial value of λ, we adopt the following procedure. An initial value of λ is chosen, and the potential is numerically calculated. The curvature of the potential at the minimum can then be found. However, the curvature is *not* necessarily the square of the Higgs mass. The inverse scalar propagator can be written as

$$-iG^{-1}(p) = p^2 - m^2 - \Sigma(p) \ ,$$

where $\Sigma(p)$ is the scalar self-energy. If we expand $\Sigma(p)$ about $p^2 = m^2$, and then recognize that the curvature of the potential at the minimum is the inverse propagator at *zero* external momentum, we find that the pole of the propagator occurs at

$$m_H^2 = \left(1 + \frac{d\Sigma}{dp^2}\bigg|_{p^2=m^2}\right) V''(\phi = \sigma). \tag{7}$$

The p-dependent part of Σ can easily be found, and the Higgs mass extracted. The correction factor is generally less than one percent.

Finally, we need the initial value of the Yukawa coupling, given the top quark mass. There are many different possible definitions of the top quark mass; the mass given by half the energy to pair-produce a top quark will differ from the mass given by the energy needed (in virtual W decay) to make a single top quark. Since the only precise bound for the top quark mass will come from pair-production, we will *define* the top quark mass to be

$$m_t \equiv g_Y(q^2 = 4m_t^2)\frac{\sigma}{\sqrt{2}}, \tag{8}$$

where $\sigma = 246.225$ GeV. We also require that the beta function for the Yukawa coupling vanish for $q^2 < 4m_t^2$.

We now have all of the ingredients. The potential is evaluated numerically up to a cutoff scale of $\Lambda = 10^{15}$ GeV (other cutoffs will be discussed shortly). The results are shown in Fig. 1. We have included a line giving the lower bound to the Higgs mass for lighter top quarks (which is $\sqrt{2}$ times the "Linde-Weinberg" bound; see Ref. 8 for an extensive review). We find for $\Lambda = 10^{15}$, for a given lower bound on the top quark mass of 90 (100, 110, 120) GeV, that there is a lower limit to the Higgs mass of 7.4 (19.8, 33.8, 49.6) GeV.

The sensitivity of the results to the input parameters can be easily determined. For light Higgs masses, the bound is roughly proportional to the choice of m_Z; a 1% increase (decrease) in m_Z corresponds to a 1% increase (decrease) in the bound on m_t. As the Higgs mass increases, this correction becomes smaller as scalar loops dominate. Thus, the current 1% uncertainty in m_Z will affect the value of the bound on m_t by at most 1 GeV. The uncertainty due to the choice of α_s is somewhat more severe. For a top quark mass of 100 GeV, the lower bound on the Higgs mass is 17.6, 19.8, 22.0 GeV for $\alpha_s = .100,\ .115,\ .130$. For a top quark mass of 120 GeV, the bound is 45.1, 49.6, 53.8 GeV for $\alpha_s = .100,\ .115,\ .130$. This uncertainty is much larger than the uncertainty caused by neglecting three loop beta functions and two-loop boundary conditions and gauge or scheme dependences and thus this calculation is as precise as possible given current data on α_s.

To test the sensitivity of the calculation to our assumptions about thresholds, we performed a second independent analysis where we replaced the one loop boundary conditions by massless one or two loop conditions with θ-functions for thresholds. This enabled us in the one loop case to compare full thresholds with the massless approximation, where there is no gauge or scheme dependence. Next, we compared massless one loop boundary conditions with the corresponding two loop conditions. With the second method we could also test the contribution of individual terms in the β-functions to the final result in a consistent way. Our tests show us that we can determine the bounds with an systematic uncertainty less than 1 GeV for a given set of input data.

We had cutoff the potential at $\Lambda = 10^{15}$ GeV. From Fig. 1 we can see that modifications in the bounds for different Λ start at a threshold value of m_t which decreases for smaller Λ. The effect of lower cutoffs is to reduce the vacuum stability bounds above this threshold somewhat. For a top quark mass of 120 GeV, the lower bound on the Higgs mass was 49.6 GeV. If the cutoff is lowered to $10^5, 10^4, 10^3$ GeV, then the bound decreases to 48.7, 42.8, 29.5 GeV. Thus, the choice of the value of the cutoff is irrelevant, unless it is below a given threshold. It is interesting to observe that the bounds for small Higgs mass do not depend on the cutoff at all. This is related to the fact, that the cutoff represents the scale where the effective λ gets negative. For small Higgs masses this happens either at low scales or never. Since we are restricting ourselves to the standard model, we must assume that no new physics appears until at least 1-10 TeV (or else the model is not the standard model); our results are thus quite insensitive to the choice of the cutoff.

Finally, in the case of N generations m_t^4 must be replaced approximately by the sum of the fourth power of all heavy quarks. For N = 4 nondiscovery of any new quark would imply $m_{t'} \geq m_t \geq 110$ GeV. If we assume $m_{b'} \geq m_t$ we get $m_H \geq 96$ GeV.

We conclude that the vacuum is unstable unless the Higgs mass is above the curves shown in Fig. 1.[17] In the current Fermilab Collider run, a limit as high as 110 GeV on the top quark mass should be reached, which would then imply a lower bound of 33.8 GeV on the Higgs mass. In the second run, scheduled for the fall of 1990, this bound will be increased beyond 50 GeV. Since it will be several years before LEP I can experimentally search for Higgs bosons with masses above 30 GeV, we conclude that if the top quark is not discovered at the Fermilab Collider, then, within the context of the minimal standard model, the Higgs boson cannot be discovered at CUSB, SLC or LEP I. This result, as can be seen from Fig. 1, is independent of the choice of the scale at which new physics might enter, as long as this scale exceeds 1 TeV, which it must if the minimal standard model is valid.

REFERENCES

1. H. Albrecht et al., *Phys. Lett.* **192B** (1987), 245
2. C. Albajar et al, *Z. Phys. C* **37** (1988), 505
3. I.V. Krive and A.D. Linde, *Nucl. Phys.* **B117** (1976), 265; P.Q. Hung, *Phys. Rev. Lett.* **42** (1979), 873; H.D. Politzer and S. Wolfram, *Phys. Lett.* **82B** (1979), 242; N.V. Krasnikov, *Yad. Fiz.* **28** (1978), 549; A.A. Anselm, *JETP Lett.* **29** (1979), 590
4. S. Coleman and E. Weinberg, *Phys. Rev.* **D7** (1973), 1888
5. N. Cabibbo, L. Maiani, G. Parisi and R. Petronzio, *Nucl. Phys.* **B158** (1979), 295; R.A. Flores and M. Sher, *Phys. Rev.* **D27** (1983), 1679
6. M.J. Duncan, R. Philippe and M. Sher, *Phys. Lett.* **153B** (1985), 165
7. M. Sher and H.W. Zaglauer, *Phys. Lett.* **206B** (1988), 537
8. All of the above references, as well as generalizations to multiple Higgs models, are discussed in detail in M. Sher, *Phys. Reports* **xxx** (1989), xxx, to be published.
9. M.E. Machacek and M.T. Vaughn, *Nucl. Phys.* **B222** (1983), 83
10. M.E. Machacek and M.T. Vaughn, *Nucl. Phys.* **B236** (1984), 221
11. M.E. Machacek and M.T. Vaughn, *Nucl. Phys.* **B249** (1985), 70
12. The reader is warned that one cannot extract the two-loop anomalous dimension by finding the λ-dependent part of the scalar beta function, as was done by H. Alhendi, *Phys. Rev.* **D37** (1988), 3749, (his results are not appreciably changed by using the correct anomalous dimension), as can be seen from Ref. 11.
13. A. Sirlin, *Phys. Rev.* **D22** (1980), 971
14. W.J. Marciano and A. Sirlin, *Phys. Rev.* **D22** (1980), 2695
15. P. Langacker, W.J. Marciano and A. Sirlin, *Phys. Rev.* **D36** (1987), 2191

16. K. Ellis, Fermilab-Conf-88/162-T, Summary talk given at the Conference on Higher Twists and High p_T Physics, Paris, Sept. 1988

17. One might accept the possibility that the vacuum is unstable as long as it was sufficiently long-lived. However, as discussed in detail in Ref. 7, cosmic rays have much greater energy than the size of the barrier and would have induced the decay of an unstable vacuum (except in a very narrow sliver of parameter space).

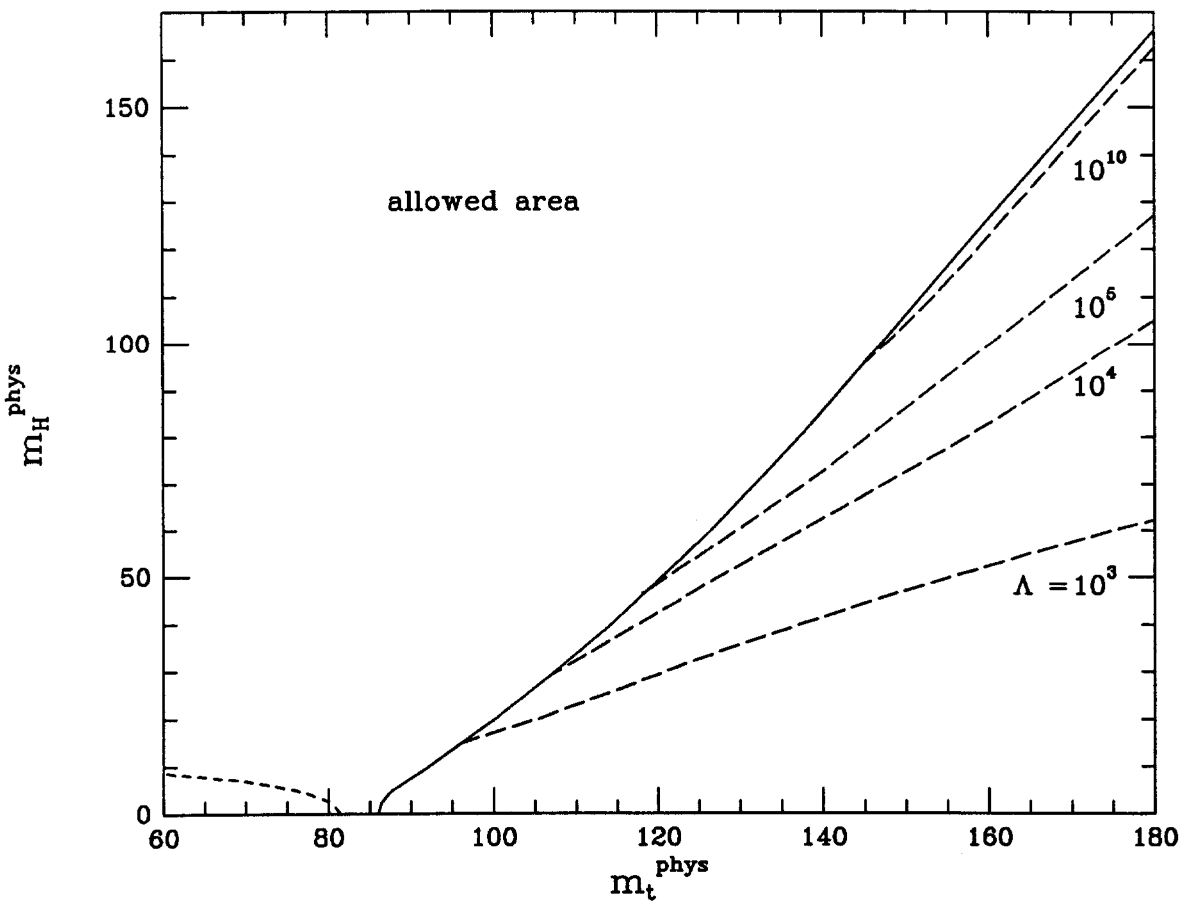

Fig. 1: The vacuum stability bounds at the two loop level with one loop boundary conditions for different cutoffs. The solid line is for $\Lambda = 10^{15}$ GeV and the dashed lines represent lower cutoffs. Systematic uncertainties are smaller than 1 GeV. The dominant uncertainty is the current knowledge of α_s (see text). The lower bound for smaller top quark masses (the "Linde-Weinberg" bound; see ref. 8) is also shown as a dotted line.