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**Exotic Phases of Hadronic Matter
and their Astrophysical Application**

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1. Introduction.

We begin this review by defining what we mean by “exotic phase of hadronic matter.” The meaning of such term is strongly time dependent, since phases of matter that might recently have seemed exotic (e.g. neutron star matter, hot quark-gluon plasma, cold quark matter) are now so widely discussed that they are familiar concepts. In contrast, the notion that quark matter might be absolutely stable is quite new, being raised by Witten¹ in 1984. Absolutely stable quark matter, named “strange matter” by Farhi & Jaffe² because of the important role played by the strange quarks, is by hypothesis the true ground state of the strong interaction, not ⁵⁶Fe. This startling idea has some striking phenomenological consequences that are discussed in this review. “Exotic phase of hadronic matter” means “strange matter” in 1988.

It is curious that the strange matter hypothesis was not raised earlier, given the long history of discussions regarding quark matter. Chin & Kerman³ anticipated some of the ideas, and Chapline & Nauenberg’s calculations indicate stability for some choices of the parameters.^{4,5} There is nothing in Witten’s seminal paper that could not have been done in 1974!

One consequence of the short history of this subject is that it can be reviewed fairly completely. We believe we have read every paper published on the topic, but apologize in advance for any omissions we have made. A brief review has been written by Farhi.⁶

2. The Physics of Strange Matter

2.1 Quark Matter

Nuclear matter is believed to undergo a phase transition to quark matter when subjected to high enough pressure. Quantum Chromodynamics (QCD) suggests that color confinement prevails under normal circumstances; but deconfinement should occur at sufficiently high densities. Theoretical arguments and computer simulations indicate that the phase transition is first order. A discussion of this issue is beyond the scope of this review, but can be found in Ref. 7. The dynamics of this phase transition are not yet well understood because of the difficulties involved in solving nonperturbative QCD. Above some critical density, individual nucleons start to overlap, and the system should be better described by its quark components. In this quark matter phase the uninterrupted presence of a high density of color sources over macroscopic distances allows the quarks to move freely; they are confined to the system as a whole.

Quark matter can be thought of as a Fermi gas of $3A$ quarks that together constitute a single color-singlet baryon with baryon number A . The dynamics of confinement can be approximated by the phenomenological bag model, in which the quarks are separated from the vacuum by a phase boundary and the region in which the quarks live is endowed with a constant universal energy density B .

When formed from protons and neutrons, quark matter is composed mostly of up and down quarks immediately after the phase transition. If the fermi level of the quark gas is higher than the mass of the strange quark, m_s , strange quarks will be created via

weak interactions. The strange quarks introduce more degrees of freedom and lower the energy of the system. Therefore, strange quark matter is more bound than nonstrange quark matter. The possibility that strange quark matter could actually be lower energy than nuclear matter was emphasized by Witten.¹ In this case removing the pressure will not make strange quark matter become nuclear matter; instead strange quark matter is absolutely stable and is called "strange matter".

2.2 Bulk Strange Matter

The above argument on the stability of strange matter depends on the plausibility of the Fermi gas analysis. For large baryon number and m_s smaller than the fermi level of the up and down quark gas, strange quark matter is energetically favored. Strange quarks will be created and the energy of the system may be lowered enough to be below nuclear matter (i.e., 930 MeV for ^{56}Fe). This analysis breaks down for low baryon number when the energy lost in increasing the degrees of freedom does not compensate the the gain in adding a heavier quark to the system. (In the limit of baryon number 1, $m_\Lambda \gg m_n$.)

The stability of bulk strange matter can be studied by treating QCD perturbatively and approximating confinement by the phenomenological bag model. By "bulk" we mean strange matter in aggregates large enough that surface effects can be ignored. One can calculate the thermodynamical potential and the energy per baryon number, E/A , for bulk strange matter as a function of m_s , the bag constant, B , and the strong coupling constant, α_c . The stability can then be tested by requiring that the bulk value of E/A be less than 930 MeV. Farhi & Jaffe² used a renormalization-group-improved perturbation expansion of QCD and found windows in B , m_s , and α_c in which bulk strange matter is stable. For example, they find that if $\alpha_c = 0$ and $m_s = 200\text{MeV}$, B can vary between $(145\text{ MeV})^4$ and $(160\text{ MeV})^4$; while if $\alpha_c = 0.6$ and $m_s = 100\text{MeV}$, B can vary between $(130\text{ MeV})^4$ and $(145\text{ MeV})^4$.

The Farhi & Jaffe analysis is highly suggestive, but certainly not conclusive. The use of perturbation theory in the strong coupling limit is certainly risky. The entire approach was criticized by Bethe, Brown, & Cooperstein,³ who advocated the gas of Λ 's as a model of strange matter. In this model, obviously, strange matter is not stable since $m_\Lambda > 930$ MeV. However, the rigid constraint of absolute local color neutrality in the deconfined phase is arbitrarily imposed, and clearly relaxing (or partially relaxing) a constraint on the system will lower the energy. Hence, their strongly worded conclusion that strange matter cannot be absolutely stable seems overstated. The cautionary remarks regarding the use of perturbation theory are well made. The only conclusion that can be reached with present calculational capabilities is that the stability of strange matter is not known. This uncertainty need not prevent us studying its properties.

If we set $m_s = 0$, the equilibrium configuration has equal numbers of up, down, and strange quarks and is electrically neutral. As m_s grows, strange quarks are depleted and charge neutrality is maintained by a small fraction of electrons.

In quark matter one-gluon exchange is repulsive. However, the repulsive interaction is weaker for massive quarks than for massless quarks. One-gluon exchange therefore shifts the chemical equilibrium in the direction of more strange quarks. Negative hadronic

electric charge in strange matter seems to be allowed by the parameters of the theory (but only for small values of m_s , and large α_c). In this case positrons would maintain charge neutrality, and normal matter would be attracted and converted to strange matter

2.3 Strangelets

Chunks of strange matter containing enough quarks so the Fermi gas approximation is valid, but small enough that the effects of surface tension cannot be neglected are called strangelets. They can have baryon number between 10^2 and 10^7 and typical radius between 5 and 200 fm. Since their radii are less than the electron Compton length, the electrons are outside of the strangelets and Coulomb effects become important.

The charge-to-baryon-number ratio for strangelets is much lower than for nuclei. The Coulomb energy vanishes for $m_s = 0$, and increases with increasing m_s . Even with finite m_s , the charge density is small, and strangelets can support much larger charges than nuclei. For example, a charge of 1000 is reasonable and would correspond to baryon number 10^6 (radius of 112 fm). Since Z grows only slowly with A , one would expect many stable strange isotopes for each value of Z .

Strangelets with positive charge less than 100 will look chemically like ordinary, but unusually heavy atoms. Electrons will surround the core in atomic orbitals and the Bohr radius of the innermost shell will be much larger than the size of the strangelet.

Surface effects can increase the energy per baryon substantially for small A leading to instability. The separation energy dE/dA required to remove a baryon from a strangelet is a good measure of stability. If dE/dA exceeds the mass of a nucleon, m_N , neutrons evaporate from the surface. If it is less than m_N but more than $m_\alpha/4$, then α particles are emitted, though this process is inhibited by a Coulomb barrier.

For some "typical" choice of parameters Farhi & Jaffe² find that for $A < 1900$ strangelets decay by α emission, and for $A < 320$ strangelets emit nucleons. Thus, strangelets with low baryon number might be quasistable, and decay radioactively by chains of α , β , and nucleon emission. For even smaller baryon numbers (less than 100), shell effects become important and stability is even less likely.

2.4 Interactions

Unless m_s is very small and α_c is very large, strange matter is positively charged. It can be inert in contact with ordinary matter, since the Coulomb barrier prevents nuclei from being absorbed. There is no barrier to absorbing neutrons, so strange matter in contact with a neutron-rich environment will grow without bound. The conversion of neutron matter to strange matter will be regulated by how fast strangeness can be equilibrated at the conversion front. Strangeness is necessary for stability and can be provided by diffusion of strange quarks to the interface or by weak reactions $u + d \rightarrow u + s$.

Strangelets with low baryon number can decay by a complicated chain of radioactive decays. Emission of α -particles drives strangelets out of flavor equilibrium and ceases until weak decays reestablish equilibrium. The process resembles the radioactive decay of a heavy nucleus like uranium. The α process is much faster in strangelets because the Coulomb barrier is lower. On the other hand, the β process is slower because strangeness

changing weak interactions are inhibited by a factor of $\sin^2\theta_c = 0.04$. Eventually, as strangelets decay, emission of protons and neutrons become possible. Anomalous patterns of radioactive decay might be a signature for strange matter.¹⁰

If m_s is small and α_c large, strange matter is negatively charged with positrons maintaining charge neutrality. This exceptional case has rather catastrophic phenomenological consequences, since any normal matter encountered by a lump of strange matter will be attracted and converted to strange matter. This happens because the Coulomb barrier is replaced by an attractive well. Any small lump of strange matter placed into a normal environment (such as the Earth!) would grow without bound. We believe that this possibility may be excluded from further discussions.

2.5 Thermodynamics of Bulk Strange Matter

It is useful to contrast the exotic properties of strange matter with the more conventional picture by constructing equilibrium phase diagrams for QCD under the two hypotheses. Phase diagrams are widely used in geophysics and material science, where they are usually constructed from empirical measurements. In contrast, the phase diagram for QCD is almost entirely theoretical.

A theoretical phase diagram is computed by minimizing a thermodynamical potential with respect to all internal degrees of freedom. One such degree of freedom is the phase; for any choice of thermodynamical coordinates the stable phase is the phase with the lower potential, and phase equilibria occur where the potential for the phases are equal. The Gibbs potential $G(T, P)$ and the Landau potential $\Omega(T, \mu)$ are the most useful for this purpose, since the quantities T , P , and μ are continuous across phase boundaries. The free energy $F(T, N_B)$, where N_B is the baryon number density, is less useful because N_B is generally discontinuous at a phase boundary.

A partial representation of the "standard" phase diagram in QCD is shown in Fig. 1. (Not shown, for instance, are the many weak phase transitions in neutron star crusts.¹¹) There is a first-order phase transition separating the free quark phase from the nucleonic or hadronic phase. In the high- T , low- μ region this is generally referred to as quark-hadron phase transition and is the portion of the phase plane that may be studied using relativistic heavy-ion colliders. The universe evolved along a track at very low μ . The high- μ , low- T region may be of importance in the cores of neutron stars, which (in this scenario) have small cores of quark matter.

The phase diagram in the strange matter picture is quite different¹² as shown in Fig. 2. The presumed absolute stability of strange matter ensures that most of the region of the plane that would otherwise be hadronic either comprises bulk strange matter or a gas of strangelets. In the gas of strangelets, most of the volume of the system carries no baryon number, which is concentrated into isolated lumps of strange matter.

The high- T region of the phase plane is occupied by the quark-gluon plasma, as in the standard phase diagram. Of particular interest here is the first-order phase transition to the hadron gas at $T \simeq 100\text{MeV}$, which also occurs in the standard phase diagram. In this region of the phase plane the baryon number of the universe is not carried by strange matter but by ordinary matter. This occurs because strange matter is a low-

entropy configuration, and at high temperatures its low energy is more than offset by its low entropy. The universe has to cool to temperatures $T < 2\text{MeV}$ before strange matter becomes thermodynamically favorable. The balance between a gas of strangelets and a gas of hadrons¹³ is expressed by equations of the form

$$\frac{N(n)N(A)}{N(A+1)} = 2 \left(\frac{M_n T}{2\pi}\right)^{3/2} e^{-I/T} ;$$

where $N(n)$ is the number density of neutrons, $N(A)$ the number density of strangelets of baryon number A , and I the energy needed to liberate one neutron from a large strange lump. (This sort of equation is known as the Saha equation in astrophysics and the Law of Mass Action in chemistry.)

In the low- T portion of the hadronic region of the phase plane, $N(n) \simeq N_B/2$. It is then easy to find the temperature T_R at which $N(A) = N(A+1)$. At $T > T_R$, $N(A) > N(A+1)$ and the baryon number density in strangelets is small. At $T < T_R$, the balance shifts to $N(A) < N(A+1)$ and thermodynamic equilibrium favors the growth of strangelets and the disappearance of neutrons and protons. For $I = 20\text{MeV}$, $T_R = 2\text{MeV}$; T_R is much less than I because N_B is very low. This is also the that cosmic recombination of hydrogen occurs at $T \simeq 1\text{eV}$, much cooler than the recombination energy 13.6eV .

The change from hadron gas to strangelet gas is not a phase transition. It is also clear that the change from a gas of strangelets to bulk strange matter is not a phase transition. However, there is good reason, as stated above, to believe that the change from a hot quark-gluon plasma to a hadron gas at $T \simeq 100\text{MeV}$ is a first-order phase transition. Hence, the quark-hadron coexistence line must terminate in the phase plane; this termination point will be a critical point for the system. Presumably (but not certainly!) this critical point is near $\mu \simeq T$. The consequences of this new source of critical behavior in QCD have not been explored. There is no analogue in the standard model.

3. Laboratory Searches for Strange Matter

3.1 Heavy-Ion Activation

Heavy-ion accelerators provide one means of searching for small abundances of strangelets in terrestrial materials. The search strategy arises in the observation by Farhi & Jaffe¹⁴ that the absorption of a heavy ion by a strangelet has a very unusual and recognizable signature.

Strangelets do not ordinarily react with nuclei because of the $\sim 20\text{MV}$ Coulomb barrier. However, a heavy-ion accelerator may readily accelerate gold or uranium nuclei (to take the two obvious examples) to kinetic energies well in excess of 60MeV per baryon. A uranium nucleus with kinetic energy per baryon of $\sim 100\text{MeV}$ will easily penetrate the Coulomb barrier. A strong interaction with the strangelet will ensue.

What happens next depends only weakly on the baryon number A of the strangelet. For $A \gg 10^3$, the nucleus will be absorbed into the strangelet, and a rapid sequence of β decays will establish weak equilibrium. The excess energy, comprising most of the kinetic energy of the projectile plus the binding energy of the strange matter, will be distributed

among the internal excitations of the strangelet. There are so many of these excitations that we may say that the energy is converted into heat.

This heat will be lost radiatively. Precisely how many photons of which energies will be emitted is unknown, but $\sim 5000\text{MeV}$ will be radiated, most likely in photons of energy $\leq 1\text{MeV}$. This flash of photons would be a unique signature of the presence of a strangelet.

3.2 Mass Spectroscopy

The ratio of charge to baryon number, Z/A , for strangelets is much less than that of ordinary nuclei. Very small strangelets may occur in normal matter, where their chemistry would be determined by their charge Z . The nuclear chemistry of these strangelets would reveal them to be extremely heavy isotopes of the chemical elements selected for study.

It does not appear likely that a successful search strategy using existing mass spectrographs can be devised. Mass spectrographs are precise devices of extreme sensitivity over relatively narrow ranges of Z/A . The masses of these strangelets are so much higher than the masses of the equivalent ions that, most likely, the typical spectrograph magnet would barely deflect them. A special modification would be needed, and the optimal design would be hard to determine given the uncertain Z/A of the object being looked for.

We would be happy to be contradicted with regard to our pessimism on this matter.

3.3 Creation of Strangelets in Heavy-Ion Colliders

The possibility of creating a quark-gluon plasma in the laboratory is one of the principal motivations for the study of relativistic heavy-ion collisions. Should a phase transition to the quark-gluon plasma be observed in the collision of two heavy ions, a powerful new tool for studying QCD will become available. The issues in relativistic heavy-ion collisions have been frequently summarized, for instance by McLerran.⁷

Liu & Shaw¹⁵ suggested that strangelets (or perhaps metastable droplets of quark matter) might be formed in heavy-ion collisions. The probability that a given droplet might form is difficult to compute, and the results turn out to be highly model dependent. Liu & Shaw also suggested an experimental apparatus for detection of strangelets. This involved a spectrometer (to search small Z/A) followed by a target chamber that would search for bursts of Λ 's that follow collisions between strangelets and ions.

The creation of strangelets requires the creation of strange quarks, which occur via the production of $s\bar{s}$ pairs. Recent work suggests strongly that an equilibrium abundance of $s\bar{s}$ pairs at local temperature T is produced in the quark-gluon plasma.¹⁶ The s and \bar{s} quarks must separate before condensation from the quark-gluon plasma in order to prevent their annihilation. This requirement is difficult to satisfy, but may occur as follows.¹⁷ The existence of a larger number of u and d quarks than \bar{u} and \bar{d} means that K^+ and K^0 mesons carry away \bar{s} quarks, leaving behind an excess of s quarks, with which small strangelets might be made.

4. The Astrophysics of Strange Matter

The overwhelming majority of environments in the universe, after 10 *ms* of cosmic time, belong in regions of the phase plane (Fig.2) for which some form of strange matter is thermodynamically favored state. It is therefore important to explore the consequences for astrophysics of strange matter. The contexts in which the astrophysical consequences of strange matter have been discussed are described here.

4.1 Strange Matter in the Early Universe

Strange matter was originally conceived as a dark matter candidate that might have been made in the early universe. Subsequent work has shown that strange matter could not have survived later than one second in the early universe and therefore is not a plausible dark matter candidate.. However, the arguments leading to this conclusion are illuminating and are summarized here.

Witten's model for the formation of strange matter in the early universe requires that the progression from a quark-gluon plasma to a gas of hadrons be via a first-order phase transition, as shown in Fig.2. As the universe cooled through the first-order phase transition, there was a brief epoch of coexistence when part of the universe was in the quark phase and part was in the hadron phase. During this period the temperature of the universe remained fixed at the coexistence temperature $T_c \simeq 100\text{MeV}$. As the universe expanded, the fraction of the universe that was hadronic increased to unity, at which point the universe resumed cooling.

Bulk thermodynamic equilibrium between the two phases required the exchange of entropy and baryon number across the phase boundary. Entropy was exchanged primarily by neutrinos and photons. Baryon number exchange could only occur via the association of three quarks into confined hadrons at the phase boundary, a process that has a characteristic scale of $\sim 1\text{fm}$. If this process of association were inefficient, then the baryon number of the universe would have become trapped into smaller regions of quark phase. One possible outcome of this would have been the creation of lumps of strange matter, also know as quark nuggets.

Quark nuggets were an attractive candidate for the dark matter in the universe. They are a form of cold dark matter, since their velocities with respect to the mean Hubble expansion would have been nonrelativistic. In addition, the quark nuggets would have been only weakly coupled to the photon gas that dominated the universe until $T \simeq 1\text{eV}$, and gravitational perturbations in the ensemble of nuggets would have develop during the radiation epoch. The advantages for theories of galaxy formation of dark matter candidates with these properties are described by Silk.¹⁸

Witten's model for the formation of quark nuggets was criticized by Applegate & Hogan.¹⁹ However, the ultimate viability of this dark matter candidate was most seriously challenged by Alcock & Farhi¹², who showed that even if nuggets of strange matter are formed, they would have evaporated as the universe cooled to $T \simeq 2\text{MeV}$. This occurs as the universe crosses the region of the phase diagram between the quark-hadron phase transition and the strangelet region (see Fig. 2). The computation of the evaporation rate is at first sight straightforward, using detailed balance arguments to relate

the neutron capture rates to the neutron evaporation rates at the surface of the nugget via the equilibrium equation (Section 2.5). The rates turn out to be so large that the evaporation of large lumps was limited by the rate at which neutrino heating could supply the energy needed to emit the neutrons. Alcock & Farhi concluded that all lumps with baryon number $\leq 10^{52}$ would evaporate, and that lumps larger than this could not have formed without violating causality at the epoch of formation.

This calculation was criticized by Madsen, Heiselberg & Riisager²⁰, who pointed out that since only neutrons (and some protons) are emitted at the surface the u and d quarks are depleted but the s quarks are not. Evaporation would have been limited by the rate at which equilibrium could be reestablished among the u, d , and s quarks. This rate of equilibration has since been computed by Heiselberg, Madsen, & Riisager²¹ and is slow enough to suppress surface evaporation significantly. These authors conclude that quark nuggets with baryon number as low as $\sim 10^{46}$ might survive evaporation. This number is smaller than the "causality limit" ($\sim 10^{49}$) but still much larger than the characteristic baryon numbers envisioned by Witten.

What was overlooked in this controversy was that the conversion of quark matter to hadron gas does not occur only at the surface. Strange matter at low pressure and high temperature ($T \geq 10\text{MeV}$) is so far out of thermal equilibrium that bubbles of hadron gas spontaneously nucleate within the quark matter. These bubbles grow at rates that are limited only by the heating rate; the quark matter boils. Since this process occurs throughout the volume of the quark nugget, the weak equilibration limit is no longer significant, and the process is limited by the rate of heating by neutrinos; a baryon number limit similar to the original $\sim 10^{52}$ is obtained when the volume conversion is taken into account.

Our conclusion from this is that strange matter cannot be the dark matter of the universe. Furthermore, since the quark nuggets are so vulnerable at $T \simeq 20\text{MeV}$, it seems unlikely to us that they would have formed in the first place, at $T \simeq 100\text{MeV}$, in agreement with Applegate & Hogan. This conclusion reflects the fact that, as shown in Fig.2, the universe spends a significant amount of time in a region of the phase plane where normal hadrons are the favored constituents.

4.2 Strange stars

If the strange matter hypothesis is correct, neutron stars are metastable with respect to stars made of strange matter. This in turn means that the objects known to astronomers as neutron stars are probably made of strange matter, not of neutron matter, and should be called "strange stars." The properties of strange stars are discussed here.

4.2.1 Global Properties of Strange Stars

The global properties of strange stars have been described by Witten¹, Haensel, Zdunik & Schaeffer²², and Alcock, Farhi & Olinto.²³ These objects have extremely simple structures, because the zero temperature equation of state is, to high accuracy, $P = (\rho - 4B)/3$. This expression is exact in the bag model with massless quarks (either two-flavor or three-flavor). The addition of mass to one of the flavors (the s quark) causes deviations no

greater than 4% from the simple relation because, if the mass is dynamically important, the abundance of the massive quarks becomes small and their contribution to the material insignificant.

This equation of state has the property that as $P \rightarrow 0$, $\rho \rightarrow 4B$. For $B = (145\text{MeV})^4$ this means $\rho = 4 \times 10^{14}\text{g/cm}^3$, slightly greater than nuclear density. Thus, there is a sequence of objects with very low internal pressure and uniform density. Their mass (M) radius (R) relation is $M \propto R^3$.

The pressure at the center of one of these objects is $P_c = 2\pi G\rho^2 R^2/3$, where G is Newton's constant and Newtonian gravity is assumed. For sufficiently large radius R the pressure P_c approaches $4B/3$ and the density increases toward the center of the object. This effect becomes noticeable at $R \simeq 5\text{km}$, $M \simeq 0.1M_\odot$, and the mass radius relation is very different from $M \propto R^3$ for objects with $R \simeq 10\text{km}$ and $M \simeq 1M_\odot$. Relativistic gravity also becomes important in these stars and the Oppenheimer-Volkoff equation for stellar structure must be used to compute the models. The full mass radius relation is shown in Fig. 3. The sequence terminates at the limit of dynamical stability, known as the Chandrasekhar limit. The dynamical stability of relativistic stars is discussed fully by Shapiro & Teukolsky.²⁴

Figure 3 also shows some well-known mass radius relations for neutron stars, which are computed for a variety of different nuclear matter equations of state: MF is a mean field theory calculation; TI is a tensor-interaction model; BJ is a Bethe-Johnson model, which includes hyperons; R is a pure neutron model with a soft core interaction; π is the R model with pion condensate. These models were reviewed by Baym & Pethick.^{25,26} These mass-radius relations are very different from that for strange stars, and the difference arises entirely because, for nuclear matter, $\rho \rightarrow 0$ as $P \rightarrow 0$. One would hope that such a large, qualitative difference could be exploited to discover the truth regarding the strange matter hypothesis.

Nature has not been kind here. All neutron/strange stars for which masses have been determined have masses near $1.4M_\odot$ ²⁷, where the two models of compact stars have very similar radii. Should a very low mass compact star be discovered, the two pictures would be distinguishable.

4.2.2 Surface Properties of Strange Stars

The fact that strange matter is absolutely stable raises the possibility that strange stars are made exclusively of strange matter, and that the surface of the star is exposed quark matter. Early discussions of strange stars^{28,29} presumed that this would be the case, and some interesting consequences for the appearance of these objects are found. However, as we show below, there is also the strong possibility that the surface of a strange star is made of the same material as the surface of a neutron star.

A bare strange surface has very unusual properties. The thickness of the "quark surface" is $\sim 1\text{fm}$; the integrity of this surface is ensured by the strong force. The electrons are held to the quark matter electrostatically, and the thickness of the "electron surface" is several hundred fermis; the electric field in this region is $\sim 5 \times 10^{17}\text{V/cm}$. Since neither component is held in place gravitationally, the traditional "Eddington Limit" to

the luminosity that a static surface may emit does not apply, and these objects may (in principle) have photon luminosities much greater than 10^{38} erg/s .

Alcock, Farhi & Olinto²³ concluded that a strange matter surface would have a low emissivity for X-ray photons. They reached this conclusion by calculating the dispersion relation for photons in strange matter. The result is much like the dispersion relation for photons in an electron plasma, but with characteristic "plasma frequency" $\omega_p = (8\pi\alpha/3)N_u^2/\rho_u$ (where α is the fine structure constant, N_u the number density of up quarks, ρ_u the energy density of up quarks). For typical parameters, $\omega_p \simeq 19 \text{ MeV}$. This means that the surface of a bare strange star is highly reflective in the X-ray region and has a low emissivity. The emissivity has not yet been calculated.

There is a further consequence of the electrical properties of this surface. The very high electric field in the electron surface will exert a strong outward force on an ion. Clearly, a certain amount of normal ionic material can be supported by this electric field. It turns out that a crust of mass up to $\sim 5 \times 10^{28} \text{ g}$ may be supported, with density at the inner edge up to $4 \times 10^{11} \text{ g/cm}^3$.

This upper limit is set by the requirement that nuclear reactions between the crust and the strange matter must be prevented, or else the ions at the base of the crust would be converted to strange matter. This requirement is satisfied if (a) there are no free neutrons in the crust [i.e. there is no "neutron drip"¹¹]; (b) there is a "gap" between the ions at the base of the crust and the quark surface in which a Coulomb barrier prevents direct reactions between the ions and the strange matter.

This thin layer is identical to the "outer crust" of a neutron star. For this reason, a strange star with a crust is not different from a neutron star in its photon emissivity. Furthermore, since the crust is held onto the star by gravitation, this new surface is subject to the Eddington limit.

It seems likely that this latter view of the surface of a strange star is more realistic. The universe is a "dirty" environment, and certainly supernova remnants contain a lot of material that may accrete onto the surface of a newly formed strange star and make a crust. Hence, we are once again driven to conclude that a strange star is very similar to a neutron star in its observable properties.

4.2.3 Pulsar Glitches

Radio pulsars are observed to have periods that steadily increase. This is attributed to the loss of angular momentum by magnetic dipole radiation. In some pulsars small "glitches" in this smooth spin-down are occasionally observed. In a glitch the period abruptly (in less than a day) decreases; over the next 40–80 days most of this decrease is lost as the pulsar appears to "heal" back toward its original spin-down curve.

A model has been developed for this phenomenon involving the behavior of superfluid neutrons in the inner crust of a neutron star: see Pines & Alpar³⁰ for a review. There is no equivalent for this model involving strange stars. It is not clear how seriously the lack of a model for glitches should be taken; this may reflect only lack of imagination on our part. It is certainly disingenuous to claim that the success of the superfluid neutron model provides a model-independent argument against the strange matter hypothesis.³¹

4.2.4 Conversion of Neutron Matter to Strange Matter

A variety of “routes” from neutron matter to strange matter have been suggested.^{23,32} These include conversion via two-flavor quark matter, clustering of lambda’s, kaon condensates, direct “burning”, and seeding from the outside. The uncertainties in each of these are so large that estimates of conversion rates cannot be made with confidence. It is possible, if unlikely, that neutron stars will not convert to strange stars, even if the strange matter hypothesis is correct.

However, once there is a seed of strange matter inside a neutron star it is possible to calculate the rate of growth.⁹ The strange matter front absorbs neutrons, liberating u and d quarks into the strange matter. Weak equilibrium is then reestablished by the diffusion of strange quarks and by the weak interactions. The rate of progress of this front has a strong inverse temperature dependence.

If this conversion happens just after the supernova explosion one expects a neutrino signature of 10^{52} erg over a period between minutes and hours. Neutrino astronomy will be able to detect neutrinos from nearby supernova and this signature can be tested.

This conversion can happen in later stages of neutron star evolution. If it happens in an active pulsar, a macrogitch will be observed because of the change in moment of inertia. An old defunct pulsar will convert even faster, and a gamma-ray burst will be its signature.

4.2.5 The Relationship between Strange Stars and White Dwarf Stars

Since there is a clear and well-known relationship between neutron stars and white dwarf stars, it is of interest to discuss the relationship between strange stars and white dwarf stars. In the standard model, there is a unique zero temperature equation of state $P(\rho)$ with the property $\rho \rightarrow 0$ as $P \rightarrow 0$. A sequence of solutions of the Oppenheimer-Volkoff equation may be obtained using this equation of state, with the central density ρ_c as a parameter. These solutions are dynamically stable if and only if $dM/d\rho_c > 0$. There are two stable portions of the sequence, a low- ρ_c portion (the white dwarfs) and a high- ρ_c portion (the neutron stars). These two stable portions are separated by a series of unstable models bounded by the Chandrasekhar limit for white dwarfs and the minimum mass neutron star.

There is no direct analogue of this relationship in the strange star picture. This is because $\rho \rightarrow 4B$ as $P \rightarrow 0$, and the strange star sequence extends essentially to zero mass with $M \simeq R^3$, as described above. However, there is a connection between white dwarf stars and strange stars with crusts, which arises because the equation of state for the crust is also the equation of state for white dwarfs.

For each strange star, there is a sequence of possible crusts ranging from zero crust up to some maximum crust mass. The maximum crust mass $M_{c,max}$ increases as the mass of the strange core, M_* , decreases. A point is reached in this sequence where $M_{c,max} = M_*$, and for lower values of M_* the mass of the object is mostly in the crust. It now is more sensible to regard the object as a white dwarf with a small strange core. This sequence terminates as $M_* \rightarrow 0$, where $M_{c,max}$ equals the Chandrasekhar mass for white dwarfs.

This sequence has been explored in part by Romanelli³³. However, an adequate study

of the dynamical stability of the models with $M_{c,max} > M_*$ was not performed. These models may have important astrophysical application since they lie in a region of the mass-radius diagram that is otherwise devoid of stable models. Further study is needed.

4.3 Strange Matter in the Sun

There may be small numbers of low mass strangelets in the universe, produced by some process for which we have only a poor understanding. One such process might be collisions between strange stars; such collisions will certainly occur at the end of gravitational evolution of binary pulsars such as PSR 1913 + 16.¹

Takahashi and Boyd³⁴ examined the possible consequences of a small abundance of very low-mass strangelets in the sun. They found a possible catalytic burning cycle involving these low-mass strangelets, which operates analogous to a CNO cycle, and which results in hydrogen burning at as lightly lower temperature than in the standard solar models, possibly resolving the solar neutrino problem.³⁵

The burning cycle arises via a series of proton capture reactions on strangelets, with weak decays to keep at the optimum configuration. In order for a cycle to operate, after four proton captures the resulting strangelet must emit an alpha particle. This will not happen if the masses of strangelets with low baryon number are those given by Farhi & Jaffe;² in their case, unbounded growth of strangelets will occur. Takahashi & Boyd introduced "shell corrections" by direct analogy with nuclear shell models and found that, in the vicinity of special "magic number" configurations, a cycle would occur.

4.4 Model for Special Events

In the astronomical literature one finds many examples of phenomena that are both puzzling and extremely unusual or unique. Their interest derives in large from the fact that they do not belong to any known class. Strange matter has been involved in models for three such "special events."

4.4.1 The Centauro Cosmic-Ray Events

The Centauro cosmic-ray events are cosmic rays in which the primary particle appears to fragment almost exclusively into large numbers of baryons.³⁶ It is important to stress that this phenomenology is extremely unusual.

It has been suggested^{37,38} that the primary particle in these events is a glob of quark matter with baryon number $\sim 10^3$ and kinetic energy per baryon between 10^3 and $10^4 GeV$. This notion becomes much more plausible if the glob of quark matter is stable, which it would be under the strange matter hypothesis.

There remains the question of where such small lumps of strange matter originate. Since the only candidate location we know of is strange stars, Witten suggested that collisions of strange matter stars would be the likely origin. This is certainly a plausible hypothesis, since such collisions must occur with reasonable frequency in our galaxy. Of the $\sim 10^3$ known pulsars, three are in close binary systems with another compact object. At least one of these, PSR 1913 + 16, has an orbit that will decay in less than 10^9 years. The resulting collision will be violent, and some material may be expelled. The expulsion velocities will typically be $\sim 0.1c$, so any large lump will leave the galaxy immediately.

Smaller strangelets will be arrested by the galactic magnetic field, and may ultimately become Centauro events. The efficiency of this process may be very low and still account for the observed flux at Earth.

4.4.2 Exotic Hadrons from Cygnus X-3

A model was proposed to account for some of the bizarre phenomena of Cygnus X-3 in terms of the special properties of strange stars.²⁸ In particular, very small strangelets are produced at the exposed strange surface, and accelerated electrostatically to high energies. Spallation reactions in the atmosphere of the companion create some neutral strangelets (perhaps the $Z = 0, A = 2$ H particle) that propagate to the Earth. Collisions in the atmosphere produce the neutrinos that in turn produce the deep underground muons seen in proton detectors.³⁹

It was suggested²⁹ that the accelerated strangelets would produce a very characteristic high-energy neutrino spectrum, and that a test of the strange star hypothesis might be possible.

Both of these papers ignore the astrophysics of Cygnus X-3. It is an accreting compact star, as revealed by the X-ray emission. The strange star will certainly have a crust, and there will not be an exposed quark surface.

4.4.3 Very High-Luminosity Gamma-Ray Bursts

An extraordinary event was recorded by the interplanetary γ -ray sensor network on 5 March 1979.⁴⁰ This event exceeded by more than an order of magnitude the peak flux recorded from any other γ -ray event. The rise time of the γ -ray flux was $\leq 250\mu s$, more than 100 times faster than typical for γ -ray events. The rapid rise was followed by an intense phase of $\sim 0.15s$ duration. This in turn was followed by a much lower intensity phase, which was observed for about three minutes, during which the flux decayed exponentially with a characteristic time $\sim 50s$ and was periodically modulated with period $\sim 8s$. Precise determination of the arrival times of the burst photons at each of the nine spacecraft in the network allowed the source to be located on the sky in an "error box" $1' \times 2'$ in size. A young supernova remnant, N49 in the Large Magellanic Cloud, is found in the error box.⁴¹

The Large Magellanic Cloud is 55 kpc away. At this distance, the energetics of the event prove to be so extraordinary that the identification with N49 is customarily rejected.⁴² In particular, the inferred luminosity is $\sim 10^6$ times the Eddington limit for a solar mass compact object, and the rise time is very much smaller than the time needed to drop $\sim 10^{26}g$ of "normal" material onto a neutron star.

These considerations motivated a model⁴³ involving the particular properties of strange matter. A lump of strange matter of mass $\sim 10^{-8}M_{\odot}$ fell into a strange star. Since the density of strange matter is so high, there was little tidal distortion of the lump by the gravitational field of the strange star, and the duration of the impact was very short, $\sim 1\mu s$; this accounts for the rapid onset of the γ -ray flash. The surface of the strange star was heated by the impact and radiated γ -rays with very high luminosity for $\sim 0.15s$. Since the strange matter surface is held together by the strong force, there is no problem

with the Eddington limit, as there would be if the compact object was a neutron star. The lower intensity radiation that followed the original flash is attributed in this model to resettling of the crust, and the 8s modulation is attributed to the rotation of the compact star.

This model accounts (in "broad-brush" fashion) for the peculiarities of the event. The consequences of the idea have not yet been completely worked out. In particular, the spectrum of photons has not been computed. Since the observed spectrum is well studied, this could be a useful diagnostic of the model.

5. *Cooling of Neutron/Strange stars*

The cooling properties of neutron stars are very sensitive to their composition. Neutron stars cool primarily via neutrino emission, which is more effective than photon emission for about 10^4 years. During this time a small amount of heat is lost by photon emission at the surface; this flux is determined entirely by the temperature of the core and by the transport properties of the crust of the star.

Since neutrons do not interact with the crust, the thermal structure of the crust and the core evolve essentially independently. After a few hundred years the core becomes approximately isothermal, while the crust acts as a thin insulating envelope containing almost all of the temperature gradient. The temperature gradient occurs where electrons become nondegenerate, which corresponds to the outermost layer of a neutron star. The temperature drops between two and three orders of magnitude in this small region.

In the standard model the primary neutrino emission reactions are $n + n \rightarrow n + p + e^- + \bar{\nu}_e$ and $n + p + e^- \rightarrow n + n + \bar{\nu}_e$. The "spectator neutron" is necessary to satisfy four-momentum conservation at the top of the Fermi sea. The matrix elements for these processes are small; for a review see Shapiro & Teukolsky.²⁴

Neutrino emission may be greatly enhanced by the presence of meson condensates. The possibility of pion condensation in sufficiently dense matter was first pointed out by Migdal⁴⁴ and, independently, by Sawyer⁴⁵ and Sawyer & Scalapino.⁴⁶ More recently Nelson & Kaplan³² showed that kaons can also form a condensate. Meson condensates can be formed because as the density in nuclear matter increases the electron chemical potential increases, and may exceed the effective mass of pions or kaons. The effective mass of pions and kaons in dense matter can be significantly lower than their mass in vacuum because of attractive nuclear interactions. If the meson effective mass lies below the chemical potential, the meson field develops a classical expectation value and forms a condensate.

Whether or not pions condensate at neutron star densities is still a controversial issue. Early calculations of the critical baryon number density for pion condensation indicate that it was higher than densities inside nuclei, but lower than densities reached inside the core of massive neutron stars. Negative results of experimental searches for evidence of pion condensation in atomic nuclei indicate that the critical density is higher than nuclear densities. More recent studies have pushed up the critical density, and made pion condensation in neutron stars less likely.

Nelson and Kaplan showed that it is possible for kaons to condensate at lower densities than pions, in spite of kaons being so much heavier. Pions have attractive axial vector but repulsive vector interactions with nucleons, while kaons have attractive vector and axial vector interactions. The attractive nuclear interactions may compensate for the mass difference between pions and kaons and make kaon condensation possible at lower densities.

In either case, the condensate will soften substantially the equation of state of dense matter. Cooling rates for neutron stars are also strongly affected by the formation of a condensate. Pion condensates cool via $n + \text{“}\pi^- \text{”} \rightarrow n + e^- + \bar{\nu}_e$ (and its inverse reaction) where “ π^- ” represents the pion condensate built in the quasiparticle states of the neutron. Maxwell et al⁴⁷ showed that even a small amount of pion condensate will produce a dramatic enhancement of the neutrino emissivity. The condensate brings an additional four-momentum making it easier to satisfy energy-momentum conservation. Pion condensation is driven by derivative interactions, so it occurs for nonzero wave numbers (p wave). A neutron on the top of its Fermi sea does not have to change into a low-momentum neutron, which greatly enhances the rate for this reaction.

Kaon condensates cool via the analogous reaction $n + \text{“}K^- \text{”} \rightarrow n + e^- + \bar{\nu}_e$. Kaon interactions do not involve derivatives, and the condensate occurs zero wave number (s wave). The additional four-momentum is not as large as in the pion case, hence kaon condensates are not as effective in speeding the neutrino emissivity. Brown et al⁴⁸ find that the cooling of a neutron star with kaon condensate in the core is the same as the strange matter cooling curves.

Strange stars cool via neutrino emission as a result of the following reactions: $u + e^- \rightarrow d + \nu_e + e^-$, $u + e^- \rightarrow s + \nu_e$, $d \rightarrow u + e^- + \bar{\nu}_e$, and $s \rightarrow u + e^- + \bar{\nu}_e$. The emissivity for these processes is proportional to the electron fraction in strange matter. In turn, the density of electrons depends on the density of up and down quarks being higher than that of strange quarks. The emissivity is, therefore, sensitive to the choices of m_s and α_c . Fig. 4 shows how this dependence affects the cooling curves for some typical choices of these parameters.

If the core of a neutron star is made of strange quark matter (in which case it is stable only at high pressure), it has the same neutrino emissivity calculated for strange stars. The luminosity will be somewhat smaller since only a fraction of the total volume of the star has this higher emissivity. The cooling curves (a), (b), (c), and (d) should be shifted to the right accordingly.

Observation of x-ray thermal emission from known supernovae remnants places upper limits on the surface temperatures of the inferred neutron stars. Standard nuclear matter cooling curves lie above a few of these upper limits, making the more exotic alternatives somewhat appealing.

While the core cooling for strange stars and neutron stars with quark matter or kaon condensate cores are very similar, the surface temperature evolves very differently. Neutron stars have layers of normal matter separating the exotic inner core from the surface. The signature of faster cooling will take some thermal diffusion time scale to affect the surface. Brown et al⁴⁸ et al. estimated that it would take between 50 and 100 yrs for

kaon condensation to manifest itself at the surface. The same time scale is appropriate for quark matter cores. Strange stars are strange matter almost up to the surface. The diffusion time scale is much shorter for strange stars, and the faster cooling should be promptly manifest. The very large range for strange stars occurs because the mass of the outer crust may vary from zero to $M_{c,max}$. These possibilities may be explored by observing the young neutron star that may exist in SNR1987A.

6. Conclusion

Perhaps the most useful conclusion to be drawn from this survey of the literature is that low-energy QCD remains a fertile area of research for particle physics and astrophysics. In part this is the case only because it is so difficult to perform accurate calculations. It is certainly an extreme unsatisfactory state of affairs that the ground state for the strong interaction remains unknown.

Given the state of the theory, one should turn to experiment. There are some tantalizing possibilities of experimental verification of the strange matter hypothesis. Unfortunately, none of the experiments described above contain a clear possibility of contradicting the hypothesis.

The astrophysical consequences of strange matter are very interesting and will remain a most active area of research in the next few years. There is in the astrophysics of neutron stars and strange stars the possibility of distinguishing the two models observationally. A convincing distinction will require a deeper understanding of the dynamics of strange stars.

In summary, this field is just reaching the stage where the elementary questions have (sometimes equivocal) answers. Much more work is needed in order to answer the central question: What is the ground state in QCD?

Figure Caption

Fig. 1.- Phase diagram for QCD in the standard model, in the temperature (T)-chemical potential (μ) plane. The heavy line indicates a first-order phase transition that separates confined quarks (lower left) from unconfined quarks. The trajectory followed by the universe in the first 100 seconds is shown.

Fig. 2.- As for Fig. 1 but with the strange matter hypothesis the first-order phase transition terminates at a critical point. The dashed line separates bulk strange matter from the gas of strangelets; it is not a phase transition. The dotted line separates the dilute hadron gas from the dilute strangelet gas.

Fig. 3.- Mass (M/M_{\odot}) versus radius (R) relation for strange stars (dotted line) and for a representative sample of neutron stars (solid lines). The labels on the solid curves refer to the equations of state discussed in the text.

Fig. 4.- Core temperature (T_c) as a function of time (t) for different cooling mechanisms: (1) photon emission, (2) modified Urca, (3) crust bremsstrahlung, (4) pion condensate, and strange matter with strange quark mass, $m_s = 100$ MeV, and strong coupling $\alpha_c = 0.1$ (a), and $\alpha_c = 0.6$ (b), and $m_s = 300$ MeV, $\alpha_c = 0.1$ (c), and $\alpha_c = 0.6$ (d).

Fig. 5.- Surface temperature (T_s) versus time (t) ranges for different core compositions: standard nuclear equations of state, quark matter core, and strange stars.

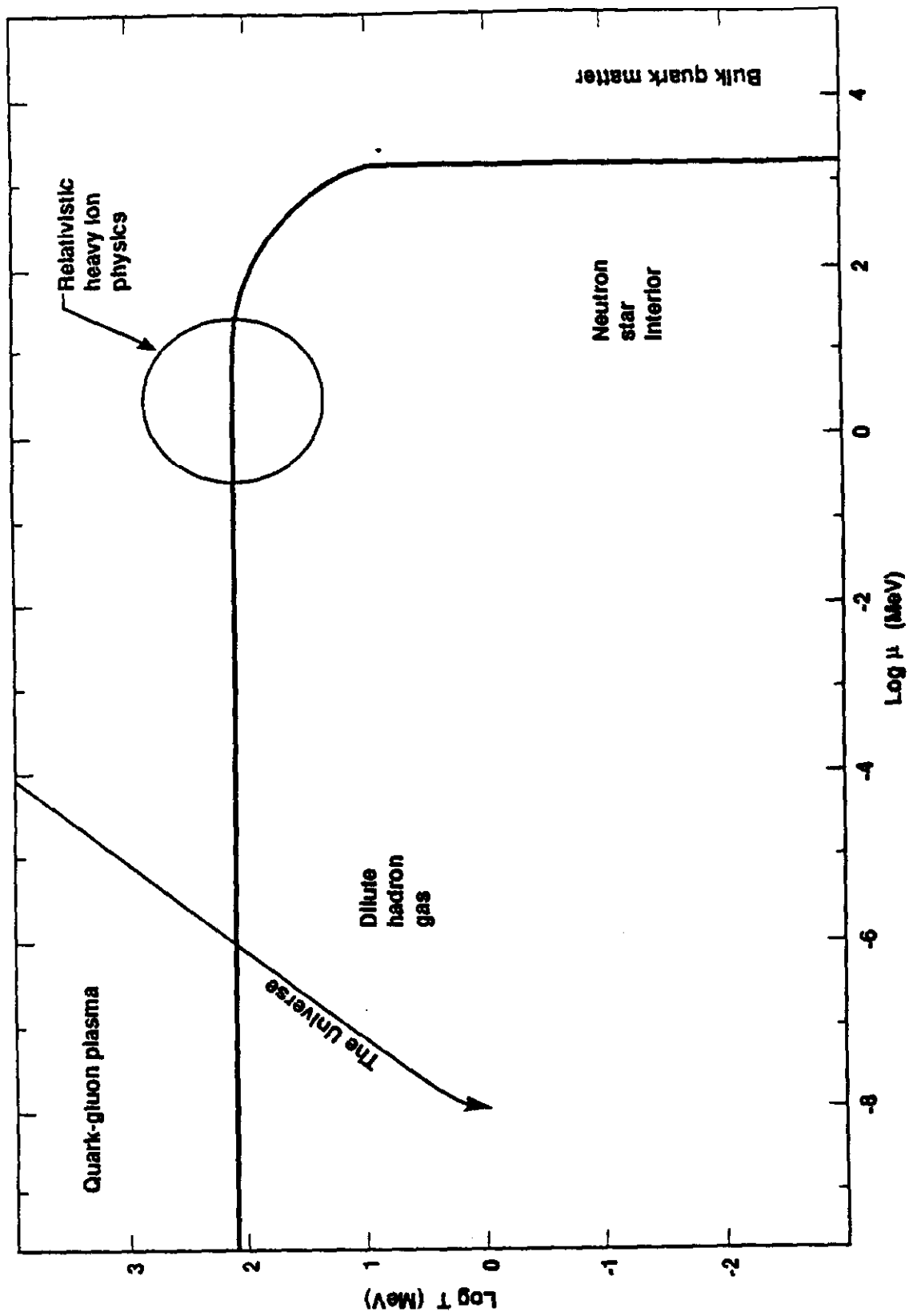


FIGURE 1

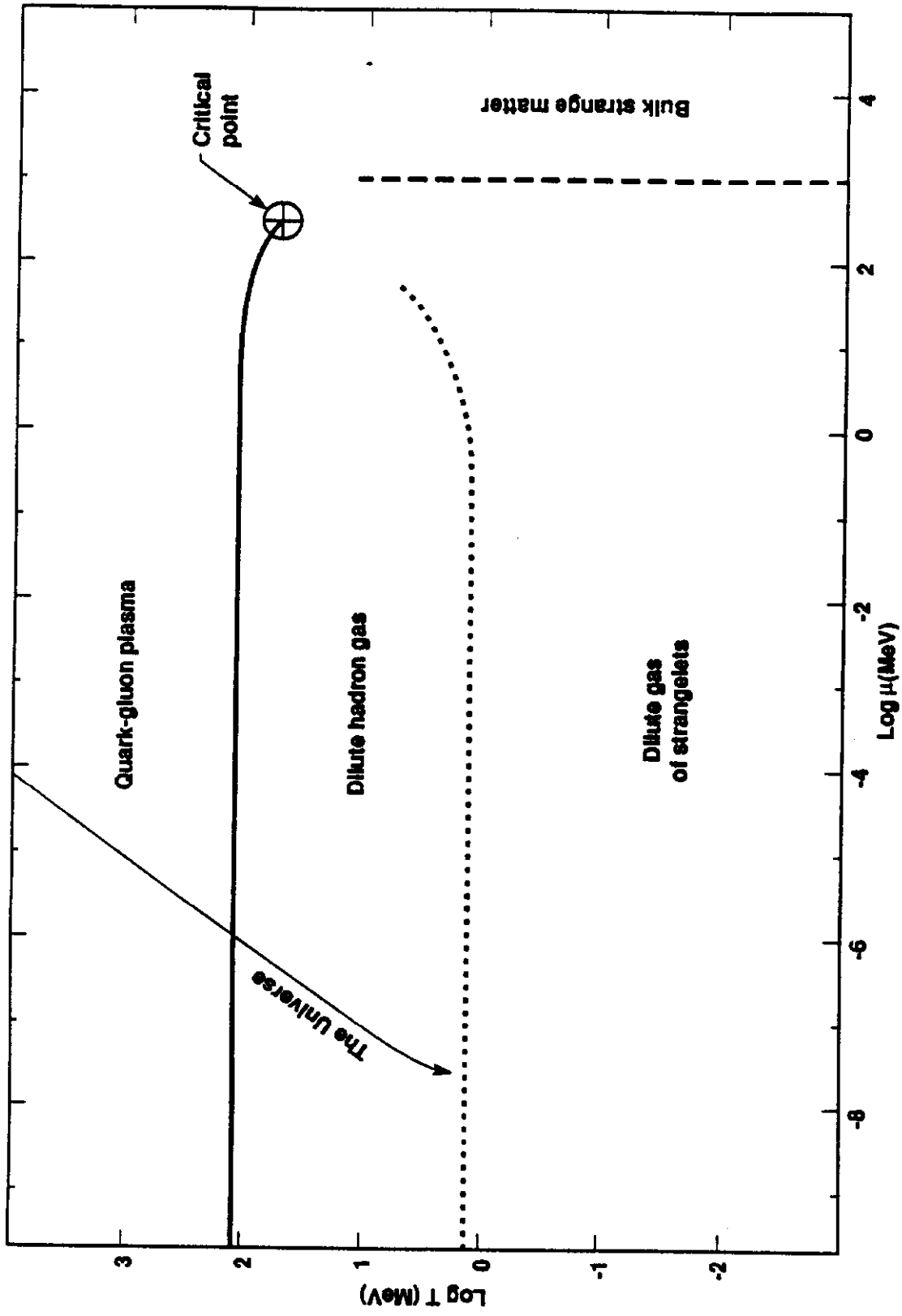


FIGURE 2

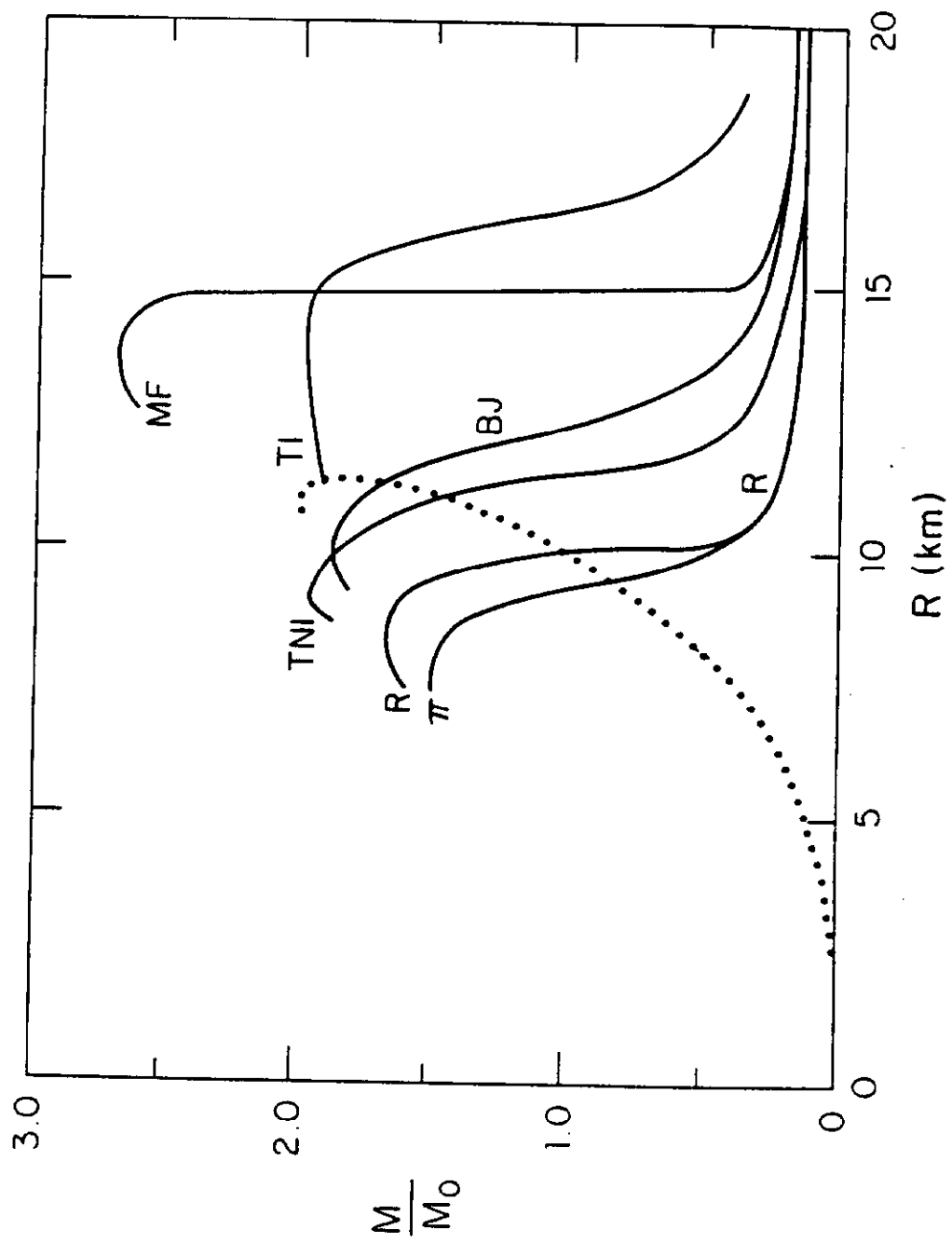


FIGURE 3

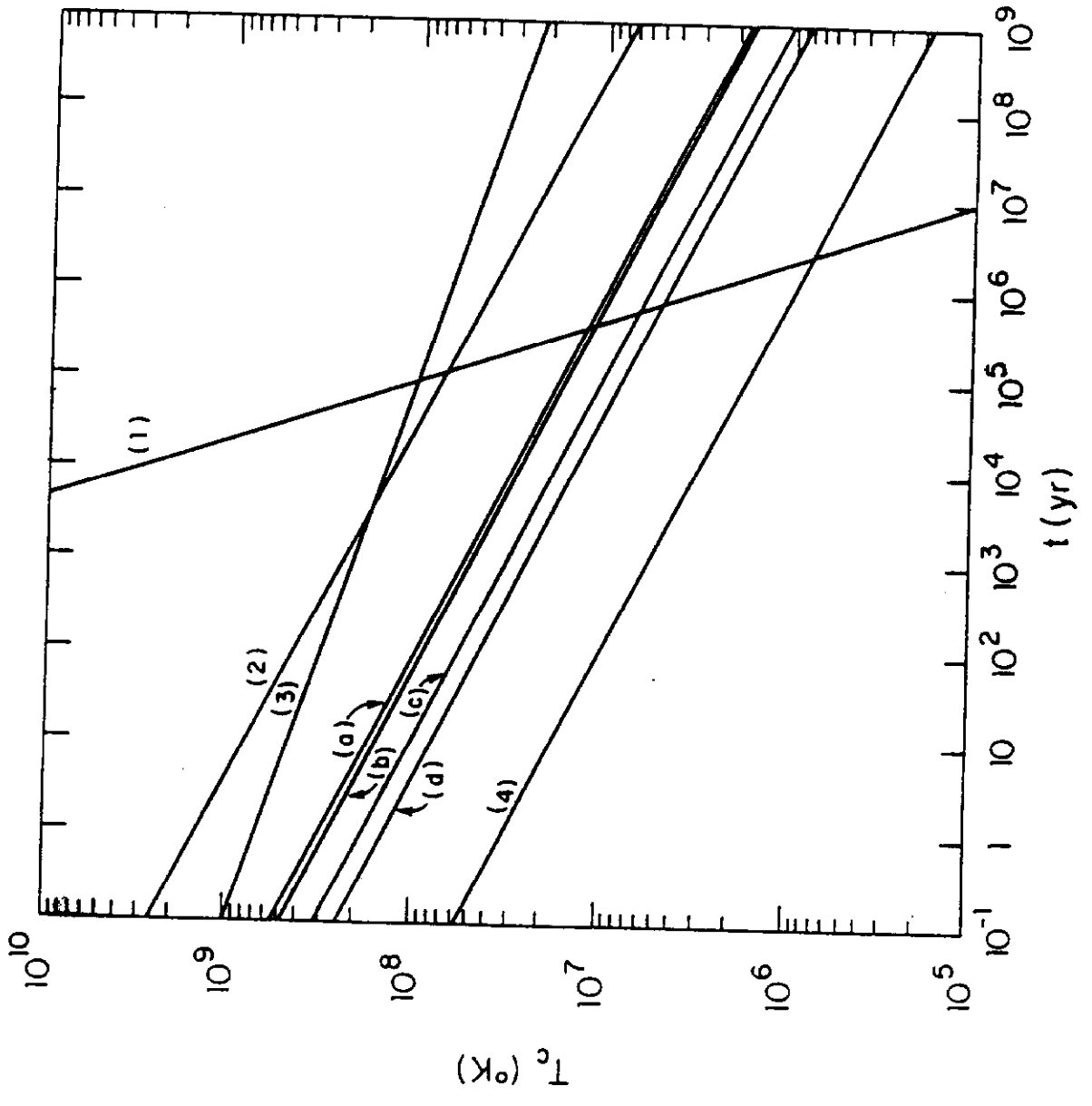


FIGURE 4

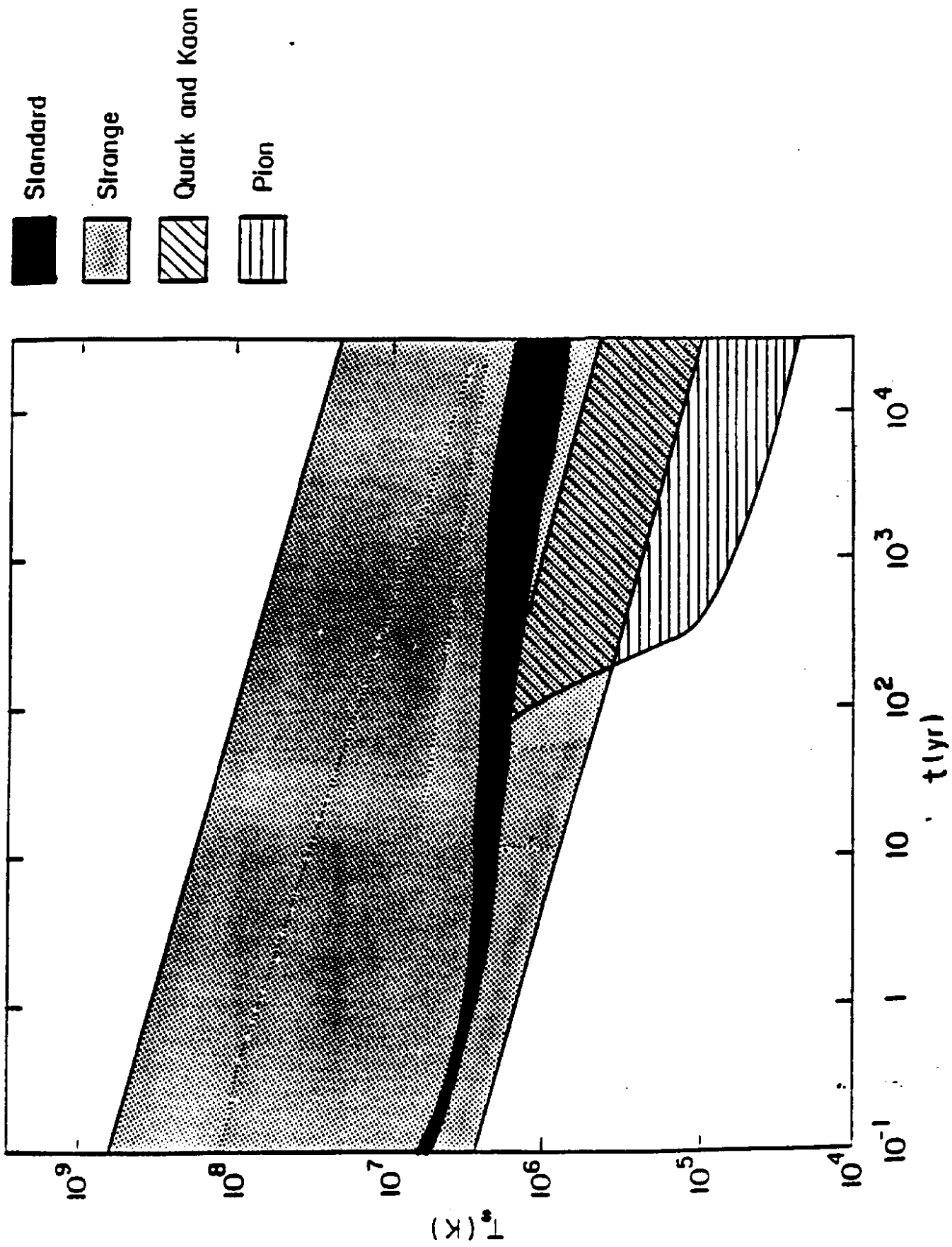


FIGURE 6

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