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Three Family Fritzsch and Stech Models with Minimal and Two-Doublet Higgs Structures

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Abstract

The invariant function approach is applied to the Fritzsch and Stech quark mass matrices to derive explicit analytic formulas for the measurable KM matrix elements and commutator determinant associated with CP violation. Imposition of the ARGUS $B-\bar{B}$ mixing results greatly restricts the allowed top quark mass range. The Fritzsch model with minimal Higgs structure is marginally viable with $95 \lesssim m_t \lesssim 105$ GeV, but the top mass range can be easily extended downward to 70 GeV in the two-doublet Higgs version. The Stech model, on the other hand, restricts the maximum top mass to ~ 50 GeV and is ruled out except in the two-doublet Higgs version with somewhat unnatural parameters.

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I. INTRODUCTION

One of the outstanding issues in particle physics concerns the pattern of quark masses and mixings.

In the minimal version of the Standard Electroweak Model[1] (EWM) there is only a single mass scale $v \sim 250$ GeV, the vacuum expectation value of the Higgs field. Nevertheless, the quark (lepton[2]) masses are all arbitrary because the Higgs-fermion coupling constants are not fixed by the EWM. Furthermore, the EWM can not predict the individual elements of the quark mixing matrix; however, it does require that the mixing matrix be unitary[3]. In the extended versions of the EWM where two Higgs doublets are introduced, there are two fundamental mass scales, v and v', the vacuum expectation values of the two doublets. In order to avoid severe problems due to flavor-changing neutral currents, it is assumed[4] that one of the doublets is responsible for the masses of the up-type quarks and the other for the down-type quarks. Again, as in the case of the minimal version, the quark masses and mixings remain arbitrary.

The above unsatisfactory feature of the EWM has, in the past decade, generated two major approaches to the question of masses and mixings as follows:

i) Since the quark masses and mixings originate from the quark mass matrices, a number of authors have attacked the issue by assuming "reasonable" specific forms for the quark mass matrices. These mass matrices have either been inspired by gauge models or have been derived by imposing exterior principles, such as discrete symmetries, hierarchical structure, mixing of nearest neighbors, etc. Given the mass matrices, for the up-type and down-type quarks, one may compute the eigenvalues (quark masses) and mixings as functions of the parameters of the model. A desirable requirement is that the model in question be as predictive as possible, i.e., the number of parameters in the model should be as small as possible. In the minimal EWM, with

n families of quarks, the number of independent measurables (masses and mixings) is

$$2n + (n-1)^2 = n^2 + 1 \tag{1.1}$$

where $2n$ equals the number of masses and $(n-1)^2$ is the number of independent measurables of the quark mixing matrix. Furthermore, one should keep in mind that many of the measurables in (1.1) are not known. In fact, even the number of families is unknown. A great deal of work has been done within the framework i) described above. See, for example, refs. 5 - 6.

ii) A second mainstream of papers has addressed the question of how to parametrize the quark mixing matrix. This might seem strange, for afterall the general parametrization of a unitary $n \times n$ matrix in terms of rotation angles and phases has been discussed in great detail by Murnaghan[7]. In fact, all parametrizations proposed in the literature are simply special cases of the Murnaghan parametrizations and are, of course, equivalent to one another. The only justification for introducing different parametrizations is that the quark mixing matrix is already quite complicated for the case of three families. For larger numbers of families, this complexity increases dramatically; therefore, it may turn out that physics would look simpler in some parametrizations than in others. Some examples of parametrizations of the quark mixing matrix in terms of angles and phases are given in refs. 8 - 10. Among these papers, ref. 8 is the fundamental result by Kobayashi and Maskawa, who showed that for $n = 3$ the quark mixing matrix could be parametrized by three angles and a phase.

Since angles and phases are convention dependent, it is desirable to introduce combinations of them which are directly measurables and thus convention independent. The plaquettes in the quark mixing matrix provide such measurables. For $n = 3$, such measurables are given in refs. 11 - 13. The general structure of the quark mixing matrix in terms of its plaquettes and plaques, for arbitray n, has been studied by Bjorken and Dunietz[14]. Furthermore, for $n = 3$, Wolfenstein[15] has provided us with a useful and easy to remember empirical parametrization of the quark mixing matrix.

Finally, one of us (C.J.) has proposed[16] that, for $n = 3$, the simplest parametrization of the quark mixing matrix is obtained by forgetting all about the angles and the phase. Instead, one can use four (independent) moduli of the mixing matrix. The point is that the moduli are directly measurables, while the angles and phase are convention dependent and, furthermore, have to be extracted from measurables. So far, since the angles and the phase are not known to have a fundamental status, there is no need to bother about extracting them. This state of affairs could change in the future, if we would find that the angles and phase, in some convention, do actually have important physical "meaning".

In this paper, we wish to reinvestigate the above issues using the method of invariant functions of mass matrices[17] and the flavor projection operators[18]. As we shall demonstrate below, this method is convenient as follows: Given the two $n \times n$ mass matrices $\mathbf{M}$ and $\mathbf{M}'$ for the up- and down-type quarks, respectively, what one usually does is

a) find the eigenvalues (masses),

b) calculate the measurables of the quark mixing matrix,

c) compare them with data.

As we shall see below, the task b) is much simplified by using the projection technique, where one simply projects out the appropriate measurables in closed analytic forms. In general, the measurables are rather complicated functions of the parameters of the mass matrices which are not manageable by hand unless one is willing to make approximations. Therefore, it is useful to have exact analytic expressions which the computer can easily handle. The computer can, for example, easily check the unitarity of the resulting quark mixing matrix, etc.

In this paper, we extend our earlier work[19] and examine in some detail two popular models of quark mass matrices. Since the methods described in this paper are new, we shall present them in a pedagogical fashion, so that the interested model builder may easily apply them to the analysis of her/his favorite model. The plan of the paper is as follows: In Section II we present the technique of flavor projection operators in generality for hermitian, as well as nonhermitian, mass matrices. The commutator of the quark mass matrices plays a special role if CP is violated as illustrated for three families. The invariant function approach is applied in Section III to the Fritzsch and Stech mass matrices to extract closed analytic forms for the absolute squares of the KM mixing matrix elements and the determinants of the mass matrix commutators. In Section IV two determinations of the quark masses at 1 GeV are presented along with expressions needed to evolve the top quark mass up to its physical mass scale. A recent determination of the KM matrix elements is also presented. A numerical analysis is carried out in Section V in both the Fritzsch and Stech models with minimal Higgs and two-doublet Higgs structures to find the allowed range of top quark masses which best fits the KM matrix information as well as the $B - \bar{B}$ mixing information from the ARGUS collaboration. Our results are summarized in Section VI.

II. TECHNIQUE OF FLAVOR PROJECTION OPERATORS

In this Section we shall give a brief account of the invariant approach[17] and flavor projection operators[18]. Our purpose is to establish our notations and give explicit analytic formulas for measurables.

Assume that there are n families of quarks and denote the mass matrices for the up- and down-type quarks by $\mathbf{M}$ and $\mathbf{M}'$, respectively. Since in the EWM one may[20], without lack of generality, take $\mathbf{M}$ and $\mathbf{M}'$ to be hermitian, we shall first consider the hermitian case. The generalization to nonhermitian mass matrices is given later.

A. Hermitian Mass Matrices

The hermitian $n \times n$ mass matrices $\mathbf{M}$ and $\mathbf{M}'$ are, as usual, diagonalized via unitary rotations. We shall denote the eigenvalues of the mass matrices by λ_u, λ_d, etc., the point being that these eigenvalues are real but not necessarily positive. Thus $\lambda_u^2 = m_u^2$, etc., where m_u is the (positive) mass of the up quark, etc. We have

$$\begin{aligned} U\mathbf{M}U^\dagger &= diag(\lambda_u, \lambda_c, \lambda_t, ...) \\ U'\mathbf{M}'U'^\dagger &= diag(\lambda_d, \lambda_s, \lambda_b, ...) \end{aligned} \tag{2.1}$$

The projection operators for the up-type quarks are given by

$$P_\alpha(\mathbf{M}) = v_\alpha(\mathbf{M})/v \tag{2.2}$$

Here v is the Vandermonde determinant

$$v = v(\lambda_u, \lambda_c, \lambda_t, ...) = \prod_{\beta>\gamma} (\lambda_\beta - \lambda_\gamma) \tag{2.3}$$

where $\lambda_1 = \lambda_u, \lambda_2 = \lambda_c$, etc. For example, for $n = 3$ we have $v = (\lambda_t - \lambda_c)(\lambda_t - \lambda_u)(\lambda_c - \lambda_u)$. The numerator $v_\alpha(\mathbf{M})$, in Eq. (2.2) is obtained from v by replacing λ_α with the mass matrix $\mathbf{M}$ and multiplying all $\lambda_\beta, \beta \neq \alpha$, with the unit matrix. Thus v_u in the case of $n = 3$ is given by

$$v_u(\mathbf{M}) = (\lambda_t - \lambda_c)(\lambda_t \mathbf{1} - \mathbf{M})(\lambda_c \mathbf{1} - \mathbf{M}) \tag{2.4}$$

Here $\mathbf{1}$ is the unit $n \times n$ matrix. If there are degeneracies in the up sector, the construction above needs some modifications (for details see ref. 18). The construction of the projection operators for the down-type quarks follows exactly in the same way as for the up-type quarks.

Given any pair $\mathbf{M}$ and $\mathbf{M}'$, we may rotate both of them with the same arbitrary unitary matrix X. The measurable quantities remain invariant[17] under such a rotation. Thus all measurables must be invariant functions of mass matrices, where an invariant function $f(\mathbf{M},\mathbf{M}')$ is defined to be such that

$$f(\mathbf{M},\mathbf{M}') = f(X\mathbf{M}X^{\dagger}, X\mathbf{M}'X^{\dagger}) \tag{2.5}$$

The measurables of the quark mixing matrix are expressible in manifestly invariant terms with the help of projection operators. The square of the modulus of the matrix element $V_{\alpha j}$, where $\alpha = u, c, t, ...$ and $j = d, s, b, ...$ is given by[17,18]

$$|V_{\alpha j}|^2 = Tr(P_\alpha(\mathbf{M})P_j'(\mathbf{M}')) \tag{2.6}$$

where $P_\alpha(\mathbf{M})$ is defined in (2.2) and P_j' is the projection operator for the down quark, etc. The purpose of the prime is only to remind us that we are dealing with a down-type quark. Similarly, other measurables can be projected out. For example,

$$V_{\alpha_1 j_1}(V^{\dagger})_{j_1\alpha_2}...V_{\alpha_n j_n}(V^{\dagger})_{j_n\alpha_1} = Tr(P_{\alpha_1}P_{j_1}'P_{\alpha_2}P_{j_2}'...P_{\alpha_n}P_{j_n}') \tag{2.7}$$

Here the indices are not summed.

B. Some Results for $n = 3$

For the case of three families, which will be our main concern in this paper, the commutator of the quark mass matrices defined by

$$[\mathbf{M},\mathbf{M}'] = i\mathbf{C} \tag{2.8}$$

plays a central role. CP is violated if, and only if, *det* $\mathbf{C} \neq 0$. The determinant is given by[11]

$$det\ \mathbf{C} = -2vv'J \tag{2.9a}$$

where

$$\begin{aligned} v &= (\lambda_t - \lambda_c)(\lambda_t - \lambda_u)(\lambda_c - \lambda_u) \\ v' &= (\lambda_b - \lambda_s)(\lambda_b - \lambda_d)(\lambda_s - \lambda_d) \end{aligned} \tag{2.9b}$$

and J is an invariant of the quark mixing matrix. It is given by

$$Im(V_{\alpha j}V_{\beta k}V^*_{\alpha k}V^*_{\beta j}) = J\sum_{\gamma,l}\epsilon_{\alpha\beta\gamma}\epsilon_{jkl} \tag{2.10}$$

where the sum in the RHS equals either plus or minus one, depending on which rows (α,β) and columns (j,k) are chosen. The quantity J itself, up to an overall sign, can also be written[16] in terms of the moduli of the elements of the quark mixing matrix. One finds

$$4J^2 = -\lambda(|V_{\alpha j}|^2|V_{\alpha k}|^2, |V_{\beta j}|^2|V_{\beta k}|^2, |V_{\gamma j}|^2|V_{\gamma k}|^2) \tag{2.11}$$

where the rows α,β,γ are any permutation of $1,2,3$ and the columns j and k satisfy $j \neq k$. An equivalent way of writing J^2 is in the column formulation, where

$$4J^2 = -\lambda(|V_{\alpha j}|^2|V_{\beta j}|^2, |V_{\alpha k}|^2|V_{\beta k}|^2, |V_{\alpha l}|^2|V_{\beta l}|^2) \tag{2.12}$$

Here the columns j,k,l are any permutations of $1,2,3$ and the rows α and β satisfy the condition $\alpha \neq \beta$. The function λ is sometimes referred to as the Källen's λ function[21] or the triangular function[22], the reason being that

$$\lambda(x,y,z) = x^2 + y^2 + z^2 - 2xy - 2yz - 2zx \tag{2.13}$$

determines the area of the triangle with the sides $\sqrt{x}, \sqrt{y}$ and $\sqrt{z}$. Thus J^2 defines a "CP violation area". However, it would be unwise[23] to use this area to define a measure of CP violation because the condition for CP violation does not involve only J but rather det **C** of Eq. (2.9).

In applications, we shall use Eq. (2.9) to compute J as function of the parameters of models of mass matrices and will use Eqs. (2.11) and (2.12) to relate J^2 to the elements of the quark mixing matrix.

C. Nonhermitian Mass Matrices

If the mass matrices $\mathbf{M}$ and $\mathbf{M}'$ are not hermitian, we take their hermitian "squares" $\mathbf{S}$ and $\mathbf{S}'$ defined by

$$\mathbf{S} = \mathbf{M}\mathbf{M}^{\dagger}, \qquad \mathbf{S}' = \mathbf{M}'\mathbf{M}'^{\dagger} \tag{2.14}$$

The eigenvalues of these matrices are nonnegative, viz.,

$$\begin{aligned} U\mathbf{S}U^{\dagger} &= diag(x_1, x_2, ..., x_n) \\ U'\mathbf{S}'U'^{\dagger} &= diag(x'_1, x'_2, ..., x'_n) \end{aligned} \tag{2.15}$$

where

$$\begin{aligned} (x_1, x_2, ..., x_n) &= (m_u^2, m_c^2, ...) \\ (x'_1, x'_2, ..., x'_n) &= (m_d^2, m_s^2, ...) \end{aligned} \tag{2.16}$$

Now the projection operators are given by

$$P_\alpha(\mathbf{S}) = v_\alpha(\mathbf{S})/v, \qquad P'_j(\mathbf{S}') = v'_j(\mathbf{S}')/v', \qquad \alpha, j = 1, 2, ..., n \tag{2.17}$$

where

$$v = v(x_1, x_2, ..., x_n) = \prod_{\beta > \gamma} (x_\beta - x_\gamma) \tag{2.18}$$

and $v_\alpha(\mathbf{S})$ is obtained from v by replacements $x_\alpha \to \mathbf{S}$ and $x_\beta \to x_\beta \mathbf{1}$, $\beta \neq \alpha$. Here $\mathbf{1}$ is the unit matrix. Thus the construction follows exactly the same pattern as the hermitian case, cf. after Eq. (2.2). Furthermore, the invariant functions of $\mathbf{S}$ and $\mathbf{S}'$ may be defined as in Eq. (2.5) by simply replacing $(\mathbf{M}, \mathbf{M}')$ by $(\mathbf{S}, \mathbf{S}')$. Finally, the commutator of the mass matrices now involves $\mathbf{S}$ and $\mathbf{S}'$ instead of $\mathbf{M}$ and $\mathbf{M}'$, and its determinant has the form (2.9), where v and v' involve the squares of the masses, cf. Eq. (2.18), but J is exactly the same as before as defined by Eq. (2.10).

III. FRITZSCH AND STECH MODELS

A. Fritzsch Mass Matrices

In order to demonstrate the power of the invariant approach and the projection technique, we apply it to the well known case of the Fritzsch mass matrices[24], for $n = 3$, given by

$$\mathbf{M} = \begin{pmatrix} 0 & A & 0 \\ A^* & 0 & B \\ 0 & B^* & C \end{pmatrix}, \qquad \mathbf{M}' = \begin{pmatrix} 0 & A' & 0 \\ A'^* & 0 & B' \\ 0 & B'^* & C' \end{pmatrix} \tag{3.1}$$

where $\mathbf{M}$ and $\mathbf{M}'$ are hermitian mass matrices for the up- and down-type quarks, respectively. The entries in these matrices are complex numbers. Without lack of generality one may take C and C' to be positive. Furthermore, as noted before in Eq. (2.5), one may rotate $\mathbf{M}$ and $\mathbf{M}'$ with the same arbitary unitary matrix without changing the physics. By choosing X in Eq. (2.5) to be

$$X = diag\,(exp(-i\phi_A), 1, exp(i\phi_B))$$

where ϕ_A and ϕ_B are the phases of A and B, $\mathbf{M}$ is made real with positive entries. Thus the number of parameters in $\mathbf{M}$ and $\mathbf{M}'$ is eight. These are $A, B, C, |A'|, |B'|, C'$ and the phases $\phi_{A'}$ and $\phi_{B'}$. Since there are, in principle, ten measurables according to (1.1), there are two predictions; however, since m_t is not known, one of the predictions is replaced by a prediction of the value of the top quark mass.

The Fritzsch mass matrices[24] have enjoyed a great deal of popularity in the past decade and have been studied by many authors[25-27]. To quote one of the investigators[26], the analysis is "painstaking" if one wishes to avoid approximations. We shall now show that in the invariant approach the analysis becomes easy.

First one may determine $A, B, C, |A'|, |B'|$ and C' as functions of the eigenvalues of $\mathbf{M}$ and $\mathbf{M}'$. Let the eigenvalues of $\mathbf{M}$ and $\mathbf{M}'$ be denoted by $(\lambda_u, \lambda_c, \lambda_t)$ and

$(\lambda_d, \lambda_s, \lambda_b)$, respectively. *A priori*, the λ's which are real by hermiticity are not necessarily positive. Thus λ_u could be either $+m_u$ or $-m_u$, where m_u is the physical mass of the up quark, etc. We have

$$\begin{aligned} Tr\mathbf{M} &= \lambda_t + \lambda_c + \lambda_u &&= C \\ Tr\mathbf{M}^2 &= \lambda_t^2 + \lambda_c^2 + \lambda_u^2 &&= C^2 + 2(B^2 + A^2) \\ Det\mathbf{M} &= \lambda_u \lambda_c \lambda_t &&= -CA^2 \end{aligned} \tag{3.2}$$

Solving for A, B and C yields

$$C = \lambda_t + \lambda_c + \lambda_u \tag{3.3a}$$

$$CB^2 = -(\lambda_t + \lambda_c)(\lambda_t + \lambda_u)(\lambda_c + \lambda_u) \tag{3.3b}$$

$$CA^2 = -\lambda_t \lambda_c \lambda_u \tag{3.3c}$$

Since we are looking for a solution with a hierarchial structure $|\lambda_t| >> |\lambda_c| >> |\lambda_u|$, we see from (3.3a) that $\lambda_t = m_t$, C being positive. From (3.3b) it now follows that λ_c is negative, i.e., $\lambda_c = -m_c$. Finally (3.3c) yields $\lambda_u = m_u$. This result that the Fritzsch form requires the second eigenvalue λ_c be negative is well known in the literature. Thus

$$\begin{aligned} C &= m_t - m_c + m_u \\ B &= [(m_t - m_c)(m_t + m_u)(m_c - m_u)/C]^{1/2} \\ A &= [m_t m_c m_u / C]^{1/2} \end{aligned} \tag{3.4}$$

Similarly, $C', |B'|$ and $|A'|$ are obtained from (3.4) by replacing (m_u, m_c, m_t) by (m_d, m_s, m_b).

By using these relations the above parameters may be eliminated, i.e., replaced by measurable quantities. Thus we have two remaining parameters, $\phi_{A'}$ and $\phi_{B'}$, and four measurables of the quark mixing matrix. The measurables of the quark mixing matrix are

$$
\begin{aligned}
|V_{\alpha j}|^2 &= Tr\left[P_\alpha(\mathbf{M})P'_j(\mathbf{M}')\right] \\
&= \{[\lambda_\beta\lambda_\gamma + A^2 + B^2]\,[\lambda_k\lambda_l + |A'|^2 + |B'|^2] \\
&\quad + [\lambda_\beta\lambda_\gamma + A^2]\,[\lambda_k\lambda_l + |A'|^2] \\
&\quad + [(\lambda_\alpha + \lambda_\beta)(\lambda_\alpha + \lambda_\gamma) + B^2]\,[(\lambda_j + \lambda_k)(\lambda_j + \lambda_l) + |B'|^2] \\
&\quad + 2(\lambda_\beta + \lambda_\gamma)(\lambda_k + \lambda_l)A|A'|cos\phi_{A'} \\
&\quad + 2\lambda_\alpha\lambda_j B|B'|cos\phi_{B'} + 2BA|B'||A'|cos(\phi_{A'} + \phi_{B'})\} \\
&\quad \times [(\lambda_\alpha - \lambda_\beta)(\lambda_\alpha - \lambda_\gamma)(\lambda_j - \lambda_k)(\lambda_j - \lambda_l)]^{-1}
\end{aligned}
\tag{3.5}
$$

where (α, β, γ) is any permutation of (u, c, t) and (j, k, l) any permutation of (d, s, b). Furthermore, since for this Fritzsch case

$$
\begin{aligned}
(\lambda_u, \lambda_c, \lambda_t) &= (m_u, -m_c, m_t) \\
(\lambda_d, \lambda_s, \lambda_b) &= (m_d, -m_s, m_b)
\end{aligned}
\tag{3.6}
$$

Eq. (3.5) together with (3.4) then yields the nine $|V_{\alpha j}|^2$ as functions of the six masses and two angles $\phi_{A'}$ and $\phi_{B'}$, but by unitarity only four of the $|V_{\alpha j}|^2$ are independent. We may check on the computer whether, in the allowed range of phases $\phi_{A'}$ and $\phi_{B'}$, there is any solution with m_t treated as a free parameter. This was the procedure followed in ref. 19.

Another way of approaching the problem is as follows: We compute the four independent measurables of the quark mixing matrix from the four independent traces appearing in (3.5). We find

$$
\begin{aligned}
Tr(\mathbf{M}\mathbf{M}') &= \sum_{\alpha,j} \lambda_\alpha\lambda_j|V_{\alpha j}|^2 \\
&= CC' + 2B|B'|cos\phi_{B'} + 2A|A'|cos\phi_{A'}
\end{aligned}
\tag{3.7a}
$$

$$
\begin{aligned}
Tr(\mathbf{M}^2\mathbf{M}') &= \sum_{\alpha,j} \lambda_\alpha^2\lambda_j|V_{\alpha j}|^2 \\
&= C'(C^2 + B^2) + 2CB|B'|cos\phi_{B'}
\end{aligned}
\tag{3.7b}
$$

$$\begin{aligned} Tr(\mathbf{M}\mathbf{M}'^2) &= \sum_{\alpha,j} \lambda_\alpha \lambda_j^2 |V_{\alpha j}|^2 \\ &= C(C'^2 + |B'|^2) + 2BC'|B'|cos\phi_{B'} \end{aligned} \tag{3.7c}$$

$$\begin{aligned} Tr(\mathbf{M}^2\mathbf{M}'^2) &= \sum_{\alpha,j} \lambda_\alpha^2 \lambda_j^2 |V_{\alpha j}|^2 \\ &= (C^2 + B^2)(C'^2 + |B'|^2) + (A^2 + B^2)(|A'|^2 + |B'|^2) + A^2|A'|^2 \\ &\quad + 2BCC'|B'|cos\phi_{B'} + 2AB|A'||B'|cos(\phi_{A'} + \phi_{B'}) \end{aligned} \tag{3.7d}$$

With the magnitudes of the quantities A, B, C and their primed versions determined before as functions of the masses via (3.4), we may eliminate $cos\phi_{B'}$ by combining Eqs. (3.7b) and (3.7c) to get a prediction for the t-quark mass as function of the five other quark masses and the $|V_{\alpha j}|^2$. In this way the top quark mass is related to other measurables via

$$\sum_{\alpha j}(C'\lambda_\alpha - C\lambda_j)\lambda_\alpha\lambda_j|V_{\alpha j}|^2 - C'^2B^2 + C^2|B'|^2 = 0 \tag{3.8}$$

We may use Eqs. (3.7b) or (3.7c) to determine $cos\phi_{B'}$ and subsequently use (3.7a) to compute $cos\phi_{A'}$. Then Eq. (3.7d) will provide a further consistency relation. Since in that equation what enters is $cos(\phi_{A'} + \phi_{B'})$, we see that there is a two-fold ambiguity, i.e., if the fit requires $sin\phi_{A'}sin\phi_{B'} > 0$, we will not be able to distinguish between both sine functions being positive or both being negative. Similarly if the product of the two sines is negative, we will not be able to distinguish which one is negative and which one is positive. As discussed in Section IIB, the reason for the ambiguity is that the sign of J is not fixed by the moduli $|V_{\alpha j}|$. To determine the sign we must use the sign of one of the imaginary parts in Eq. (2.10).

The quantity J, including its sign, can be determined from the determinant of the commutator in Eq. (2.9). In the Fritzsch model we have

$$\begin{aligned} \tfrac{1}{2}Det\ \mathbf{C} &= (B|B'|sin\phi_{B'} - A|A'|sin\phi_{A'})\,[A^2|B'|^2 + B^2|A'|^2 \\ &\quad - 2AB|A'||B'|cos(\phi_{A'} + \phi_{B'})] \\ &\quad + A|A'|sin\phi_{A'}\,[C^2|B'|^2 + B^2C'^2 - 2CC'B|B'|cos\phi_{B'}] \\ &= -vv'J \end{aligned} \tag{3.9}$$

With the replacements (3.6) in Eq. (2.9b) for v and v', we then find that J in (3.9) above can be written entirely in terms of measurable quantities with the help of Eqs. (3.7).

B. Stech Mass Matrices

As a second example of the application of the invariant approach we consider the very popular model by Stech[28] which has been studied by many authors. The Stech mass matrices are given by

$$\mathbf{M} = \begin{pmatrix} \lambda_u & 0 & 0 \\ 0 & \lambda_c & 0 \\ 0 & 0 & \lambda_t \end{pmatrix}, \qquad \mathbf{M}' = p\mathbf{M} + i\mathbf{A} = p\mathbf{M} + i\begin{pmatrix} 0 & a & d \\ -a & 0 & b \\ -d & -b & 0 \end{pmatrix} \tag{3.10}$$

where the mass matrices are hermitian, the λ's are the eigenvalues and p is a constant. The entries a, b and d are real. Again by judicious choice of X in Eq. (2.5), we may take a and b to be positive. The Stech model, having seven parameters, gives three relations among the measurables. Since m_t is not known, in principle, one of these relations will fix m_t and there will be two further predictions among the measurables. In the invariant approach, the analysis goes as follows: First we have the Stech relations[28]

$$p = Tr\mathbf{M}'/Tr\mathbf{M} = (\lambda_d + \lambda_s + \lambda_b)/(\lambda_u + \lambda_c + \lambda_t) \tag{3.11}$$

Furthermore

$$TrM'^2 = p^2(\lambda_u^2 + \lambda_c^2 + \lambda_t^2) + 2(a^2 + b^2 + d^2) = \lambda_d^2 + \lambda_s^2 + \lambda_b^2 \tag{3.12}$$

$$DetM' = p(p^2\lambda_u\lambda_c\lambda_t - a^2\lambda_t - d^2\lambda_c - b^2\lambda_u) = \lambda_d\lambda_s\lambda_b \tag{3.13}$$

Using these equations we may express a and b in terms of the quark masses and find

$$a^2 = \left[-\lambda_u E_1 + E_2 - (\lambda_c - \lambda_u)d^2\right]/(\lambda_t - \lambda_u) \tag{3.14a}$$

$$b^2 = \left[\lambda_t E_1 - E_2 - (\lambda_t - \lambda_c)d^2\right]/(\lambda_t - \lambda_u) \tag{3.14b}$$

where

$$\begin{aligned} E_1 &= \tfrac{1}{2}\left[\lambda_d^2 + \lambda_s^2 + \lambda_b^2 - p^2(\lambda_u^2 + \lambda_c^2 + \lambda_t^2)\right] \\ E_2 &= \left[p^3\lambda_u\lambda_c\lambda_t - \lambda_d\lambda_s\lambda_b\right]/p \end{aligned} \tag{3.14c}$$

Therefore the only remaining parameter is d, while we have four measurables $|V_{\alpha j}|^2$ which we have not used yet. In contrast to Eq.(3.5) for the Fritzsch model, these can be written

$$\begin{aligned} |V_{\alpha j}|^2 &= Tr\left[P_\alpha(\mathbf{M})P_j'(\mathbf{M}')\right] \\ &= [(\lambda_k - p\lambda_\alpha)(\lambda_l - p\lambda_\alpha) + (a^2 + d^2)\delta_{\alpha u} + (a^2 + b^2)\delta_{\alpha c} \\ &\quad + (b^2 + d^2)\delta_{\alpha t}]\,[(\lambda_k - \lambda_j)(\lambda_l - \lambda_j)]^{-1} \end{aligned} \tag{3.15}$$

where again (j, k, l) is a permutation of (d, s, b).

Computing the four traces in $|V_{\alpha j}|^2$ gives

$$Tr(\mathbf{MM}') = \sum_{\alpha,j} \lambda_\alpha\lambda_j|V_{\alpha j}|^2 = p(\lambda_u^2 + \lambda_c^2 + \lambda_t^2) \tag{3.16a}$$

$$Tr(\mathbf{M}^2\mathbf{M}') = \sum_{\alpha,j} \lambda_\alpha^2\lambda_j|V_{\alpha j}|^2 = p(\lambda_u^3 + \lambda_c^3 + \lambda_t^3) \tag{3.16b}$$

$$\begin{aligned} Tr(\mathbf{MM}'^2) &= \sum_{\alpha,j} \lambda_\alpha\lambda_j^2|V_{\alpha j}|^2 \\ &= p^2(\lambda_u^3 + \lambda_c^3 + \lambda_t^3) + \lambda_u(a^2 + d^2) + \lambda_c(a^2 + b^2) + \lambda_t(b^2 + d^2) \end{aligned} \tag{3.16c}$$

$$
\begin{aligned}
Tr(\mathbf{M}^2\mathbf{M}'^2) &= \sum_{\alpha,j} \lambda_\alpha^2 \lambda_j^2 |V_{\alpha j}|^2 \\
&= p^2(\lambda_u^4 + \lambda_c^4 + \lambda_t^4) + \lambda_u^2(a^2+d^2) + \lambda_c^2(a^2+b^2) + \lambda_t^2(b^2+d^2)
\end{aligned}
\quad (3.16d)
$$

Relations (3.16a) and (3.16b) give two relations for the top quark mass in terms of $|V_{\alpha j}|^2$. Multiplying the first by λ_t and subtracting the second from it gives

$$
\begin{aligned}
p\left[\lambda_c^2(\lambda_t - \lambda_c) + \lambda_u^2(\lambda_t - \lambda_u)\right] &= \textstyle\sum_{\alpha,j} \lambda_\alpha \lambda_j (\lambda_t - \lambda_\alpha)|V_{\alpha j}|^2 \\
&= \textstyle\sum_j \lambda_c \lambda_j (\lambda_t - \lambda_c)|V_{cj}|^2 + \sum_j \lambda_u \lambda_j (\lambda_t - \lambda_u)|V_{uj}|^2
\end{aligned}
\quad (3.17)
$$

Using this equation and (3.16a), we have two second order equations for m_t in terms of other measurables, i.e.,

$$
(\lambda_t + \lambda_c + \lambda_u)\sum_{\alpha,j} \lambda_\alpha \lambda_j |V_{\alpha j}|^2 - (\lambda_b + \lambda_s + \lambda_d)(\lambda_u^2 + \lambda_c^2 + \lambda_t^2) = 0 \quad (3.16a')
$$

and

$$
\begin{aligned}
&(\lambda_t + \lambda_c + \lambda_u)\sum_j \lambda_j \left[\lambda_c(\lambda_t - \lambda_c)|V_{cj}|^2 + \lambda_u(\lambda_t - \lambda_u)|V_{uj}|^2\right] \\
&\qquad - (\lambda_b + \lambda_s + \lambda_d)\left[\lambda_c^2(\lambda_t - \lambda_c) + \lambda_u^2(\lambda_t - \lambda_u)\right] = 0
\end{aligned}
\quad (3.17')
$$

As before, the above equations do not determine the sign of d. The sign of d is determined from the determinant of the commutator, which is found to be

$$
Det\ \mathbf{C} = -2vv'J = -2vabd \quad (3.18a)
$$

Thus

$$
J = abd/v' \quad (3.18b)
$$

IV. QUARK MASSES and EXPERIMENTALLY DETERMINED KM MATRIX

We shall require the quark masses as input as well as experimental constraints on the KM matrix in order to limit the unknown parameters $m_t, \phi_{A'}$ and $\phi_{B'}$ in the Fritzsch model, two of which can be independently determined.

A. Quark Masses

The light quark masses m_u, m_d and m_s follow from QCD sum rules in the $\overline{MS}$ scheme, while their ratios are determined more accurately from current algebra. On the other hand, for the heavy quark masses m_c, m_b and m_t, each pole mass can be defined at the singularity of the quark propagator, or again alternatively from QCD sum rules. Moreover, the quark masses run with energy since their Yukawa couplings satisfy the renormalization group equations. In the approximation that the evolution equation is linear and involves just the gauge couplings, the solution up through two loops can be written as[29]

$$m_i(\mu) = \bar{m}_i \left(\frac{L}{2}\right)^{-2\gamma_0/\beta_0} \left\{1 - \frac{2\beta_1\gamma_0}{\beta_0^3}\frac{\ln L + 1}{L} + \frac{8\gamma_1}{\beta_0^2 L} + O\left[\frac{(\ln L)^2}{L^2}\right]\right\} \qquad (4.1a)$$

where $\bar{m}_i$ is the renormalization group invariant mass, and in terms of the QCD scale Λ,

$$\begin{aligned} &L = \ln \tfrac{\mu^2}{\Lambda^2} \\ &\beta_0 = 11 - \tfrac{2}{3}N_f, \qquad \gamma_0 = 2 \\ &\beta_1 = 102 - \tfrac{38}{3}N_f, \quad \gamma_1 = \tfrac{101}{12} - \tfrac{5}{18}N_f \end{aligned} \qquad (4.1b)$$

for the appropriate number of flavors N_f. The running QCD coupling strength is given by

$$\alpha_s(\mu) = \frac{4\pi}{\beta_0 L}\left\{1 - \frac{\beta_1}{\beta_0^2}\frac{\ln L}{L} + O\left(\frac{(\ln L)^2}{L^2}\right)\right\} \qquad (4.2)$$

up through terms involving one loop.

In order to implement the invariant function approach described in Section II, it is important to determine the matrix elements of $\mathbf{M}$ and $\mathbf{M}'$, and hence their mass eigenvalues, at the same energy scale, μ. This can be accomplished by defining $m_i(m_i) = m_i$ and evolving the quark masses to 1 GeV. In doing so, it is conventional to select $\Lambda_3 = 100$ MeV as the 3-flavor QCD scale. To insure that $\alpha_s(\mu)$ is continuous across the flavor thresholds then requires that one take $\Lambda_4 = 76$ MeV and $\Lambda_5 = 47$ MeV, respectively, for the 4- and 5-flavor scales with the pole values $m_c(m_c) = 1.28$ GeV and $m_b(m_b) = 4.26$ GeV.

The quark mass determination by Gasser and Leutwyler[29] remains a standard choice:

$$\begin{aligned} m_u &= 5.1 \pm 1.5 \ MeV, & m_d &= 8.9 \pm 2.6 \ MeV \\ m_c &= 1.35 \pm 0.05 \ GeV, & m_s &= 175 \pm 55 \ MeV \\ m_t &= ? & m_b &= 5.3 \pm 0.1 \ GeV \end{aligned} \tag{4.3}$$

with light quark mass ratios

$$\frac{m_d}{m_u} = 1.76 \pm 0.13, \qquad \frac{m_s}{m_d} = 19.6 \pm 1.6, \qquad \frac{m_s}{m_u} = 34.5 \pm 5.1 \tag{4.4}$$

A new determination of the heavy quark masses based on the propagator singularity has been carried out by Narison[30] who obtains

$$m_c(1GeV) = 1.36 \pm 0.02, \qquad m_b(1GeV) = 5.70 \pm 0.07 \tag{4.5a}$$

with

$$\frac{m_c - m_u}{m_s - m_u} = 9 \pm 2 \tag{4.5b}$$

The latter ratio together with (4.4) then yield

$$\begin{aligned} m_s(1GeV) &= 155 \pm 36 \ MeV \\ m_d(1GeV) &= 7.9 \pm 2.5 \ MeV \\ m_u(1GeV) &= 4.5 \pm 1.7 \ MeV \end{aligned} \tag{4.5c}$$

for the light quarks.

Finally we note that once a value for $m_t(1GeV)$ is determined by the invariant approach, it is necessary to evolve $m_t(\mu)$ upward to $m_t(m_t) = m_t$ to find the appropriate mass scale for the top quark. In doing so, we use Λ_4 from 1 GeV to $m_b(m_b)$ and then Λ_5 from $m_b(m_b)$ to $m_t(m_t)$. Thus m_t is determined from the product ratio

$$\left.\frac{m_t}{m_t(m_b)}\right|_{N_f=5} \cdot \left.\frac{m_t(m_b)}{m_t(1GeV)}\right|_{N_f=4} \tag{4.6}$$

in terms of $m_t(1GeV)$. The mass of the dressed quark is then computed from the running mass with the first order QCD correction

$$m_t^{phys} = m_t(m_t)\left\{1 + \frac{4}{3\pi}\alpha_s + O(\alpha_s^2)\right\} \tag{4.7}$$

B. KM Matrix

For the KM matrix we use a recent determination by Schubert[31] as presented at the 1987 EPS meeting in Uppsala:

$$V = \begin{pmatrix} 0.9754 \pm 0.0004 & 0.2206 \pm 0.0018 & 0 \pm 0.0087 \\ -0.2203 \pm 0.0019 & 0.9743 \pm 0.0005 & 0.0460 \pm 0.0060 \\ 0.0101 \pm 0.0086 & -0.0449 \pm 0.0062 & 0.9989 \pm 0.0003 \end{pmatrix}$$
$$+i \begin{pmatrix} 0 & 0 & 0 \pm 0.0087 \\ 0 \pm 0.0004 & 0 \pm 0.0001 & 0 \\ 0 \pm 0.0085 & 0 \pm 0.0019 & 0 \end{pmatrix} \tag{4.8}$$

Here V_{ud} is determined from superallowed beta decays and μ decay, V_{us} from K_{e3} decay and hyperon decays with $SU(3)$ breaking, V_{cd} from neutrino production of charm and V_{cs} from D decay lifetimes. The less accurately known V_{cb} follows from the B decay lifetime, V_{ub} from the ratio of $(b \to u)/(b \to c)$, and V_{td} and V_{ts} from the ARGUS $B^o - \bar{B}^o$ mixing data[32]. Finally, unitarity of the 3-family KM matrix is imposed to determine V_{tb} and to reduce the errors on the other entries.

V. NUMERICAL ANALYSIS

We now use the previously derived relations in Section III to test the predictions of the Fritzsch and Stech mass matrices with the known experimental data. The latter involves information on the allowed range for the top quark mass, the KM matrix elements, the J value for CP violation, $B\bar{B}$ mixing results for $\Delta m/\Gamma$, the bag parameter B_K, etc. The new information on $B\bar{B}$ mixing has important consequences for both mass matrix models and the appropriate Higgs sector – standard model or extended – as we shall see below.

A. KM Matrix Elements Squared: $|V_{\alpha j}|^2$

Our procedure for comparing the predictions of the Fritzsch mass matrices with the experimental limits given in Section IV is as follows:

a) Select masses $m(1GeV)$ for the five lightest quarks u, d, s, c and b which lie within the bounds and ratio limits stated in Section IV for the Gasser - Leutwyler[29] or Narison[30] determinations.

b) Pick a top mass $m_t(1GeV)$ in the range 25 - 200 GeV and run through the complete range of phase angles $\phi_{A'}$ and $\phi_{B'}$. The upper limit of 200 GeV selected is imposed by radiative corrections on the neutral current neutrino scattering data[33].

c) Plot the allowed range of $\phi_{B'}$ for given $m_t(1GeV)$ and selected $\phi_{A'}$ for which all calculated $|V_{\alpha j}|^2$ from Eq. (3.5) lie within one standard deviation of the KM matrix evaluation of Schubert[31] given in Eq. (4.8).

With this prescription, we find that the allowed support region is an annular ring in the $\phi_{B'}$ *vs.* $m_t(1GeV)$ plots for rather tightly constrained $\phi_{A'}$, typically $\pm 2°$. We plot several examples in Fig. 1. Varying the input quark masses changes the size of the ring; notably the size is most sensitive to the strange quark mass $m_s(1GeV)$. Lowering the value down to 120 MeV increases the size of the annulus and the allowed range for $m_t(1GeV)$. Along the bottom horizontal axis we have plotted m_t^{phys} by scaling it relative to $m_t(1GeV)$ through Eqs. (4.6) and (4.7).

In Fig. 2 we plot $m_t(1GeV)$ and m_t^{phys} *vs.* $m_s(1GeV)$ to show the allowed range of m_t obtained with the exact formula (3.5) for the KM matrix elements, $|V_{\alpha j}|^2$. On the same Figure we also plot the upper bound on m_t obtained by the usual approximate method of first diagonalizing the Fritzsch mass matrices[34]:

$$m_t \lesssim \frac{m_c}{(\sqrt{\frac{m_s}{m_b}} - |V_{cb}|)^2} \tag{5.1}$$

which follows from the estimate

$$|V_{cb}| \sim \left|\sqrt{\frac{m_s}{m_b}} - exp\{-i\phi_2\}\sqrt{\frac{m_c}{m_t}}\right|$$

The cases illustrated in Fig. 2 correspond to $|V_{cb}|_{max} = 0.052$ and $m_c = 1.35$ GeV, $m_b = 5.3$ for the Gasser-Leutwyler masses and $m_c = 1.36$ GeV, $m_b = 5.7$ GeV for the Narison masses. Comparison of our exact procedure with the approximate bound (5.1) reveals roughly a 10% discrepancy. Whereas the approximate formula leads to $(m_t^{phys})_{max} \simeq$ 88 GeV and 97 GeV in the two cases above with $m_s(1GeV) = 120$ GeV, the exact upper limits are 97 and 107 GeV, respectively.

Let us now turn our attention to the four independent trace equations (3.7a-d). We select the central values of $|V_{\alpha j}|^2$ given by Schubert in (4.8) along with the five lightest quark masses $m(1GeV)$ for u, d, s, c and b. Either Eq. (3.7b) or (3.7c) can be used to calculate $cos\phi_{B'}$ as a function of $m_t(1GeV)$, and the results typically agree with each other to one part in 10^4, as well as with the values of $\phi_{B'}$ allowed in Fig. 1. Equation (3.7a) or (3.7d) can then be used to find $\phi_{A'}$, again with fair agreement with each other and the previous results in Fig. 1. When Eqs. (3.7b) and (3.7c) are combined to eliminate $cos\phi_{B'}$, one finds the cubic equation for m_t given in (3.8). Although this equation can, in principle, determine m_t for a given set of five quark masses and a set of $|V_{\alpha j}|^2$'s, it is so sensitive to the values of the

KM matrix elements used that the procedure is unreliable. We prefer instead to use the method described earlier in this Section to fit the KM matrix elements to one standard deviation uncertainty.

For the Stech model, our procedure for comparing the predictions of the KM matrix elements with those given in Section IV is as follows:

a′) Select masses $m(1GeV)$ for the five lightest quarks as in a) above for the Fritzsch model.

b′) Pick a top mass $m_t(1GeV)$ in the range 25 - 200 GeV and run through values for d lying in the range $0 \leq |d| \lesssim a$ as determined from (3.14a).

c′) Require that the $|V_{\alpha j}|^2$ calculated from (3.15) lie within one standard deviation of the KM matrix element evaluation of Schubert given in (4.8).

Acceptable predictions are obtained only for the identification (3.6) of the λ_α and λ_j with m_α and m_j as in the Fritzsch case and then *only* for the restricted band of m_t *vs.* m_s indicated in Fig. 2. A maximum m_t^{phys} of 48 - 51 GeV (depending upon which parametrization is used) is obtained for $m_s(1GeV) = 120$ MeV, well below the maximum value permitted in the Fritzsch model. No acceptable solution is found for $m_s \gtrsim 170$ MeV. The actual choice of m_u and m_d made within the allowed ratios of (4.4) limits the range of the matrix parameter d.

Alternatively, we can select the central values for the $|V_{\alpha j}|^2$, a set of the five lightest quark masses and find m_t from the four trace relations given in (3.16) and (3.17). This method yields a value for $m_t(1GeV)$ consistent with Fig. 2 and the procedure described above.

B. CP Violation Parameter J and Phase Angle δ

The invariant J parameter associated with CP violation can be determined from Eq. (3.9) for the Fritzsch model for any point in the annular rings appearing in Fig. 1.

For the Stech model, J is determined from Eq. (3.18b) for any point in the allowed region of d *vs.* m_t in (5.2) above with the aid of Eqs. (3.14). In the Fritzsch model the J values lie in the range $0.1 \times 10^{-4} \lesssim |J| \lesssim 0.45 \times 10^{-4}$, while in the Stech model this range is even greater. In a recent analysis of Donaghue, Nakada, Paschos and Wyler[35], the preferred value is close to 0.3×10^{-4}, well within the allowed range of either model.

It is customary to parametrize the 3-family KM matrix in terms of three angles and one phase[8]. If we adopt the scheme favored by Chau and Leung[9], Harari and Leurer[9], and Fritzsch[10], one rotates the s and b quarks first by the angle θ_{23}, then the first and third family of down quarks by an angle θ_{13} and phase ϕ, and finally the first and second familes by an angle θ_{12}. This procedure corresponds to the complete breaking of the chiral $U(3)_L \otimes U(3)_R$ symmetry in stages such that only nearest neighbors mix to yield the observed spectrum. In this scheme the KM matrix is then given by

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -c_{23}s_{12} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - c_{23}s_{12}s_{13}e^{i\delta} & c_{13}c_{23} \end{pmatrix} \tag{5.2}$$

Using Eq. (2.10) we then observe that J can be written as

$$\begin{aligned} J &= Im(V_{12}V_{23}V_{13}^*V_{22}^*) \\ &= c_{12}c_{13}^2c_{23}s_{12}s_{13}s_{23}sin\delta \\ &\simeq |V_{12}||V_{23}||V_{13}||V_{22}|sin\delta \end{aligned} \tag{5.3}$$

from which $sin\delta$ can be determined. Near the preferred value of $J \simeq 0.3 \times 10^{-4}$, we find $sin\delta \simeq 1.0$ in both models. Specific cases of interest are listed in Table I.

C. $B-\bar{B}$ Mixing in One and Two Doublet Higgs Models

We now turn to the restrictions imposed on the Fritzsch and Stech models by new combined information on the top quark mass and V_{td} and V_{tb} matrix elements. This arises from the recent ARGUS experimental results[32] on the $B_d^o-\bar{B}_d^o$ system. In Section IV we have noted that Schubert[31] used this information to restrict the KM matrix elements in a model-independent fashion. Here we can obtain even stronger, model-dependent bounds.

The ratio r_d of the decay widths for the mixing mode relative to the direct mode is measured to be

$$r_d \equiv \frac{\Gamma(B_d^o \to \bar{B}_d^o \to \ X')}{\Gamma(B_d^o \to \ X)} = 0.21 \pm 0.08 \tag{5.4a}$$

and can be well approximated by

$$r_d \simeq \frac{x_d^2}{2+x_d^2} \tag{5.4b}$$

in terms of the ratio of the mass difference to the average decay width, from which

$$x_d \equiv \Delta m_{B_{dL}-B_{dS}}/\Gamma_B \simeq 0.73 \pm 0.18 \tag{5.4c}$$

The mixing parameter, x_d, can be approximated by the one loop box diagrams involving just W boson exchange in the minimal Higgs model with one doublet but also charged Higgs exchange in the two doublet Higgs model. In these two cases, we can write[36,37]

$$x_d = \frac{G_F^2}{6\pi^2} m_B \tau_B B_B f_B^2 \eta_t m_t^2 |V_{tb}V_{td}^*|^2 R(z_t, z_\eta, v_2/v_1) \tag{5.5a}$$

and estimate this by using $m_B \simeq 5.3$ GeV for the mass of the B meson, $\tau_B \simeq 1.18 \times 10^{-12}$ sec for its lifetime , $B_B f_B^2 \simeq (0.140 \pm 0.040\ GeV)^2$ for the product of the bag parameter and square of the B meson decay constant, and $\eta_t \sim 0.85$ for a QCD correction[36] to obtain:

$$x_d \simeq 0.37 \, \frac{B_B f_B^2}{(0.140)^2} \, m_t^2 |V_{tb} V_{td}^*|^2 R(z_t, z_\eta, v_2/v_1) \tag{5.5b}$$

where[37]

$$\begin{aligned} R(z_t, z_\eta, v_2/v_1) \; = \; & \Big\{ \Big[\tfrac{1}{4} + \tfrac{9}{4} \tfrac{1}{(1-z_t)} - \tfrac{3}{2} \tfrac{1}{(1-z_t)^2} - \tfrac{3}{2} \tfrac{z_t^2}{(1-z_t)^3} ln \; z_t \Big] \\ & + 2 \left(\tfrac{v_2}{v_1}\right)^2 z_t \Big[\tfrac{1}{(z_t - z_\eta)(1-z_t)} + \tfrac{z_\eta}{(z_t - z_\eta)^2 (1-z_\eta)} ln \; z_\eta \\ & + \tfrac{z_t^2 - z_\eta}{(z_t - z_\eta)^2 (1-z_t)^2} ln \; z_t \Big] - \tfrac{1}{2} \left(\tfrac{v_2}{v_1}\right)^2 z_t \Big[\tfrac{z_t}{(z_t - z_\eta)(1-z_t)} \\ & + \tfrac{z_\eta^2}{(z_t - z_\eta)^2 (1-z_\eta)} ln \; z_\eta + \tfrac{z_t^2 + z_t^2 z_\eta - 2 z_t z_\eta}{(z_t - z_\eta)^2 (1-z_t)^2} ln \; z_t \Big] \\ & + \tfrac{1}{4} \left(\tfrac{v_2}{v_1}\right)^4 z_t \Big[\tfrac{z_t + z_\eta}{(z_t - z_\eta)^2} - \tfrac{2 z_t z_\eta}{(z_t - z_\eta)^3} ln \; \left(\tfrac{z_t}{z_\eta}\right) \Big] \Big\} \end{aligned} \tag{5.5c}$$

in which v_1 and v_2 are the vacuum expectation values of the Higgs giving masses to the up-type quarks and down-type quarks, respectively, and $z_t = (m_t/M_W)^2$ and $z_\eta = (m_\eta/M_W)^2$ are the squared ratios of the top quark and charged Higgs mass to the W mass. In these formulas m_t is the running top quark mass. The first term in (5.6c) is the W box contribution to the $B_L - B_S$ mass difference which is the only term present in the standard minimal Higgs model.

In Fig. 3 we plot R *vs.* m_t for $m_\eta = 30, 50$ and 70 GeV and the ratio $v_2/v_1 = 0, 0.5$ and 1. From Eqs. (5.4c) and (5.5b) we can then write

$$m_t^2 |V_{tb} V_{td}^*|^2 R(z_t, z_\eta, v_2/v_1) \simeq (2.0 \pm 0.5) \, \frac{(0.140)^2}{B_B f_B^2} \tag{5.6}$$

and can plot the bands corresponding to this for fixed v_2/v_1 for both the Fritzsch and Stech models. In Figs. 4 and 5 we superpose these $B - \bar{B}$ mixing bands with the KM allowed regions for the Fritzsch and Stech models, respectively, where we have used the central value of $(0.140 \; GeV)^2$ for $B_B f_B^2$ in the plots, so the bands cover the range 2.0 ± 0.5. The right-most band in each figure applies for the minimal Higgs model, while the double-hatched band corresponds to the two-doublet Higgs model with $v_2/v_1 = 1.0$ and charged Higgs mass $m_\eta = 50$ GeV in the Fritzsch case and $m_\eta = 30$

GeV in the Stech case. It is apparent that these bands for the Fritzsch model with minimal Higgs structure do not overlap the allowed KM region, while overlap does occur with $m_s = 120$ MeV in the two Higgs doublet version if $v_2/v_1 \gtrsim 1$. However, the Fritzsch model with just one Higgs doublet can not be ruled out[38] on the basis of the present ARGUS data due to the uncertainty in $B_B f_B^2$ indicated above. For example, by selecting $B_B f_B^2 = (0.160\ GeV)^2$ instead, the bands cover the range 1.5 ± 0.4 and nearly intersect the KM matrix-allowed annuli when the strange quark mass is taken to be 120 MeV in the Gasser-Leutwyler or Narison mass determinations. Slightly less overlap of the double-hatched band with the KM ring occurs in the Fritzsch case, if we raise m_η to 70 GeV with $v_2/v_1 = 1.0$. The Stech model can be ruled out in the minimal Higgs case. With two-doublet Higgs structure, and $m_\eta = 30$ GeV and $v_2/v_1 = 2.0$, the Stech model survives, but this case is very marginal, since the charged Higgs mass is just beyond present observation and the VEV ratio is anomalously large for the down quark sector relative to the up quark sector.

D. Other Parameters of Interest

We conclude our analysis by addressing several other parameters which are predicted by the Fritzsch and Stech models and are of interest here.

The predicted values for $|V_{ts}|^2/|V_{td}|^2$ for both models suggest a $B_s^o - \bar{B}_s^o$ mixing parameter $x_s \gtrsim 10x_d$ corresponding to $r_s \gtrsim 0.95$ for the physically interesting cases, nearly maximal mixing in this channel compared to r_d in (5.4a). The square of the amplitude ratio $|V_{ub}|/|V_{cb}|$ enters the prediction for the partial decay rates $\Gamma(b \to ux)/\Gamma(b \to cx')$. The ARGUS group has recently observed[39] the non-charmed decay modes $B^+ \to p\bar{p}\pi^+$ and $B^o \to p\bar{p}\pi^+\pi^-$ from which they have determined

$$0.07 \lesssim \frac{b \to u}{b \to c} \simeq \frac{|V_{ub}|}{|V_{cb}|} \lesssim 0.23 \tag{5.7}$$

The Fritzsch model predictions are slightly below the lower bound for this ratio of

KM matrix elements as seen from Table I.

Finally we note that the bag parameter B_K can be determined from the ϵ parameter in K decay. For this purpose we use the expressions of ref. 40 to write

$$\begin{aligned} |\epsilon| &= \tfrac{G_F^2}{6\pi^2\sqrt{2}} M_W^2 \tfrac{m_K B_K f_K^2}{\Delta m_K} \, |V_{ub} V_{cb} sin\delta| \\ &\times \left\{ [\eta_3 f_3(z_t) - \eta_1] \, z_c V_{us} + \eta_2 z_t f_2(z_t) |V_{cb}|^2 \left(V_{us} - \tfrac{|V_{ub}|}{|V_{cb}|} cos\delta \right) \right\} \end{aligned} \quad (5.8a)$$

in terms of the bag parameter B_K in K decay, $z_c = (m_c/M_W)^2$ and z_t given earlier in part C,

$$f_2(z_t) = 1 - \frac{3}{4} \frac{z_t(1+z_t)}{(1-z_t)^2} \left[1 + \frac{2z_t}{1-z_t^2} ln \; z_t \right] \quad (5.8b)$$

$$f_3(z_t) = ln \frac{z_t}{z_c} - \frac{3}{4} \frac{z_t}{1-z_t} \left[1 + \frac{z_t}{1-z_t} ln \; z_t \right] \quad (5.8c)$$

and $\eta_1 = 0.7, \eta_2 = 0.6, \eta_3 = 0.4$. Using standard values for the constants in (5.8), we find the values tabulated in Table I for the cases illustrated in Figs. 4 and 5. The values of B_K obtained with $m_s = 120$ MeV and the top quark mass yielding the best fit for the KM matrix and $B - \bar{B}$ mixing data are in fair agreement with the predictions of Bardeen, Buras and Gérard[41] in the Fritzsch model.

Neither a heavy top mass nor a heavy charged Higgs boson significantly modifies the standard u and c quark and W contributions to the ϵ parameter given in (5.8) above. The same is true for the ϵ'/ϵ ratio recently determined experimentally[42] which is well explained by the standard EW model.

VI. SUMMARY

In this paper we have investigated the Fritzsch and Stech models of the quark mass matrices, using the invariant function approach developed by one of us (CJ). With this approach we are able to present explicit analytic formulas for the measurables, i.e., the squares of the KM mixing matrix elements and the J-value associated with CP violation. It is then a simple matter to find the support regions in the top mass, m_t, and remaining free-parameter space where the predicted KM matrix elements lie within the experimentally determined bounds. For the Fritzsch model, this corresponds to an annular region in the $\phi_{B'}$ phase *vs.* m_t plane, while for the Stech model it is a very narrow elliptical region in the d *vs.* m_t plane.

The recent ARGUS data on $B_d^o - \bar{B}_d^o$ mixing further restricts the range of the combination $m_t^2 \, |V_{td}V_{tb}^*|^2 \, R$, with R defined in Eq. (5.6c), corresponding to bands in the $\phi_{B'}$ *vs.* m_t or d *vs.* m_t planes. We have considered the box diagram contributions to the mixing parameter for both the standard model with minimum Higgs structure and the two-doublet Higgs model.

Our analysis indicates that the Fritzsch model with minimal Higgs structure is somewhat marginal. The strange quark mass at 1 GeV must be set equal to $m_s(1GeV) \simeq 120$ MeV and the mixing parameter x_d must be taken on the low side of the experimentally determined value while the product of the bag parameter and decay constant squared, $B_B f_B^2$, must be taken on the high side. The top mass, m_t^{phys}, is then predicted to lie in the $95 \lesssim m_t^{phys} \lesssim 107$ GeV range, depending somewhat upon the light quark mass determination employed in the analysis, i.e., that by Gasser and Leutwyler or that by Narison. Note that this conclusion is somewhat more pessimistic than that arrived at in our earlier letter because here we make use of the exact expression for the box contribution to $B - \bar{B}$ mixing, and we have taken a slightly lower upper bound for $|V_{cb}|$ than previously for the range allowed by the latest Schubert data analysis.

As expected, if one expands the standard model to include a two-doublet Higgs structure, the Fritzsch model is less tightly constrained and more viable. We have considered a charged Higgs mass of 50 GeV and set the ratio of the two vacuum expectation values equal to unity. In this case, the allowed top quark mass range is $85 \simeq m_t^{phys} \lesssim 107$ GeV if we take the value $B_B|f_B|^2 = (140 \; MeV)^2$, while it can be lowered to $(m_t^{phys})_{min} \simeq 70$ GeV with the choice $B_B|f_B|^2 = (160\ MeV)^2$. However, a bag parameter B_K closer to 0.7 than 1.0, as suggested by the analysis of Buras *et al.*, favors the higher top quark masses. In either the minimal Higgs or two-doublet Higgs version of the Fritzsch model, we find the ratio of $|V_{ub}/V_{cb}| \simeq 0.055$, corresponding to a $(b \to u)/(b \to c)$ transition ratio slightly on the low side compared to recent ARGUS data.

In contrast, our exact treatment of the Stech model shows that the maximum top quark mass allowed by the present KM data lies in the range $m_t^{phys} \sim 48 - 51$ GeV. The standard model version with minimal Higgs structure is ruled out on the basis of the $B - \bar{B}$ mixing data, while the two-doublet Higgs version is still marginally viable, if we set the charged Higgs mass equal to 30 GeV and the vacuum expectation ratio to $v_2/v_1 \simeq 2$. The latter choice of parameters is somewhat unnatural, however, and the bag parameter in K decay is much too large.

ACKNOWLEDGEMENTS

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References

[1] S. L. Glashow, Nucl. Phys. **22**, 579 (1961); S. Weinberg, Phys. Rev. Lett. **19**, 1264 (1967); A. Salam, Proceedings of the 8th Nobel Symposium, ed. N. Svartholm (Almqvist and Wiksell, Stockholm, 1968).

[2] In this paper we shall not discuss models of the lepton mass matrices.

[3] For a review see, for example, C. Quigg, Gauge Theories of the Strong, Weak and Electromagnetic Interactions , (Benjamin/Cummings, Reading, MA, 1983).

[4] S. Weinberg, Phys. Rev. Lett. **57**, 657 (1976); S. L. Glashow and S. Weinberg, Phys. Rev. D**15**, 1958 (1977); E. A. Paschos, ibid. **15**, 1966 (1977).

[5] H. Fritzsch, Phys. Lett. **70B**, 436 (1977); **73B**, 317 (1978); **166B**, 423 (1986); T. Kitazoe and K. Tanaka, Phys Rev. **18**, 3476 (1978); H. Georgi and D. V. Nanopoulos, Phys. Lett. **82B**, 392 (1979) and Nucl. Phys. **B155**, 52 (1979); M. Shin, Phys. Lett. **145B**, 285 (1984).

[6] B. Stech, Phys. Lett. **130B**, 189 (1983); A. A. Davidson, Phys. Lett. **122B**, 412 (1983); B. Bijnens and C. Wetterich, Phys. Lett. **199B**, 525 (1987).

[7] F. D. Murnagham, in The Unitary and Rotation Groups, (Spartan Books, Washington, D.C., 1962).

[8] M. Kobayashi and T. Maskawa, Prog. Theor. Phys. **49**, 652 (1973).

[9] L. Maiani, in *Proc. of 1977 Intl. Symp. on Lepton and Photon Interactions at High Energies* (DESY, Hamburg, 1977); L. L. Chau and W. Y. Keung, Phys. Rev. Lett. **53**, 1802 (1984); H. Harari and M. Leurer, Phys. Lett. **181B**, 123 (1986).

[10] H. Fritzsch, Phys. Lett. **184B**, 391 (1987); **189B** 191 (1987).

[11] C. Jarlskog, Phys. Rev. Lett. **55**, 1039 (1985); Zeit. Phys. **C29**, 491 (1985).

[12] D.-d. Wu, Phys. Rev. **D33** 860 (1986).

[13] O. W. Greenberg, Phys. Rev. **D32**, 1841 (1985).

[14] J. D. Bjorken and I. Dunietz, Phys. Rev. D**36**, 2109 (1987).

[15] L. Wolfenstein, Phys. Rev. Lett. **51**, 1945 (1983).

[16] C. Jarlskog, in Proceedings of the 4th LEAR Workshop , (Villars-sur-Ollon, 1987); in Proceedings of the International Symposium on the Production and Decay of Heavy Flavors (Stanford University, Stanford, 1987).

[17] C. Jarlskog, Phys. Rev. D**35**, 1685 (1987); C. Jarlskog and A. Kleppe, Nucl. Phys. **B286**,245 (1987).

[18] C. Jarlskog, Phys. Rev. **D36**, 2138 (1987).

[19] C. H. Albright, C. Jarlskog and B.-Å. Lindholm, Phys. Lett. **199B**, 553 (1987).

[20] P. H. Frampton and C. Jarlskog, Phys. Lett. **154B**, 421 (1985).

[21] G. Källén, Elementary Particle Physics (Addison-Wesley, Reading, MA, 1985).

[22] See, for example, H. Pilkuhn, The Interaction of Hadrons (North-Holland Publishing Company, 1967, p. 6).

[23] See ref. 18, Section VII.

[24] H. Fritzsch, ref. 5.

[25] H. Georgi and D. V. Nanopoulos, ref. 5.

[26] M. Shin, in Proceedings of the Twenty-First Rencontre de Moriond, "Progress in Electroweak Interactions," Ed. J. Tran Thanh Van (Editions Frontieres, 1986).

[27] J.-M. Gérard, B. Grzadkowski and M. Lindner, Phys. Lett. **189B**, 453 (1987); L. Wolfenstein, Phys. Rev. **D34**, 897 (1986); L. F. Li, Phys. Lett. **84B**, 461 (1979).

[28] B. Stech, Phys. Lett. **130B**, 189 (1983).

[29] J. Gasser and H. Leutwyler, Phys. Rep. **87**, 77 (1982).

[30] S. Narison, Phys. Lett. **197B**, 405 (1987).

[31] K. R. Schubert, in Proceedings of the 1987 EPS Conference (Uppsala, 1987).

[32] ARGUS Collaboration (H. Albrecht, et al.), Phys. Lett. 192B (1987) 245; and in Proc. of 1987 Intl. Symp. on Lepton and Photon Interactions at High Energies (DESY, Hamburg, 1987).

[33] See eg., G. Altarelli, in Proceedings of the 1987 EPS Conference (Uppsala, 1987).

[34] H. Harari and Y. Nir, Phys. Lett. **195B**, 586 (1987); Y. Nir, SLAC preprint, SLAC-PUB-4368 (1987), to be published.

[35] J. F. Donoghue, T. Nakada, E. A. Paschos and D. Wyler, SIN preprint, SIN-PR-87-05, to be published.

[36] G. Altarelli and P. J. Franzini, Z. Phys. **C37**, 271 (1988).

[37] S. L. Glashow and E. E. Jenkins, Phys. Lett. **196B**, 233 (1987).

[38] In our previous letter of ref. 19, the Fritzsch model with minimal Higgs structure appeared in better shape because there we had approximated $R \simeq 1$ while here we have taken into account the exact expression quoted in Eq. (5.6).

[39] ARGUS Collaboration, in Proc. of 1987 Intl. Symp., cited in ref. 4.

[40] T. Inami and C. S. Lim, Prog. Theor. Phys. 65 (1981) 297; (E) 65 (1981) 1772; F. J. Gilman and M. B. Wise, Phys. Rev. D27 (1983) 1128.

[41] A. J. Buras and J.-M. Gérard, Nucl. Phys. B264 (1986) 371; W. A. Bardeen, A. J. Buras and J.-M. Gérard, Max Planck Institute preprint, MPI-PAE/PTh 22/87, to be published.

[42] I. Mannelli, in Proc. of 1987 Intl. Symp., cited in ref. 4; M. Woods *et al.*, U. of Chicago preprint, EFI-88-03.

Figure Caption

Figure 1: Phase angle $\phi_{B'}$ vs. $m_t(1GeV)$ and m_t^{phys} plots in the Fritzsch model showing the physically allowed annular regions for the KM matrix elements based on the one standard deviation results of Schubert[31] given in Eq. (4.8). The sets of quark masses for the four plots illustrated, two with the Gasser-Leutwyler and two with the Narison determinations, respectively, are

(a) $m_u = 5.1$ MeV, $m_d = 8.9$ MeV, $m_s = 175$ MeV, $m_c = 1.35$ GeV and $m_b = 5.3$ GeV;

(b) $m_u = 3.5$ MeV, $m_d = 6.1$ MeV, $m_s = 120$ MeV, $m_c = 1.35$ GeV and $m_b = 5.3$ GeV;

(c) $m_u = 4.5$ MeV, $m_d = 7.9$ MeV, $m_s = 155$ MeV, $m_c = 1.36$ GeV and $m_b = 5.7$ GeV;

(d) $m_u = 3.5$ MeV, $m_d = 6.1$ MeV, $m_s = 120$ MeV, $m_c = 1.36$ GeV and $m_b = 5.7$ GeV.

Figure 2: Allowed range of $m_t(1GeV)$ and m_t^{phys} *vs.* m_s in the Fritzsch model obtained by fitting the squares of the KM matrix elements $|V_{\alpha j}|^2$ to one standard deviation with the exact formula given in Eq. (3.5). The cases illustrated correspond to $|V_{cb}|_{max} = 0.052$ with $m_c = 1.35$ GeV and $m_b = 5.3$ GeV for the Gasser-Leutwyler masses (solid lines) and $m_c = 1.36$ GeV and $m_b = 5.7$ GeV for the Narison masses (broken lines). For comparison, the approximate upper bounds from (5.1) are plotted as dashed lines. The corresponding allowed range in the Stech model is indicated by the two narrow bands.

Figure 3: Plot of $R(z_t, z_\eta, v_2/v_1)$ defined in Eq. (5.6c) *vs.* m_t for the standard model with one Higgs doublet and the two Higgs doublet model. The

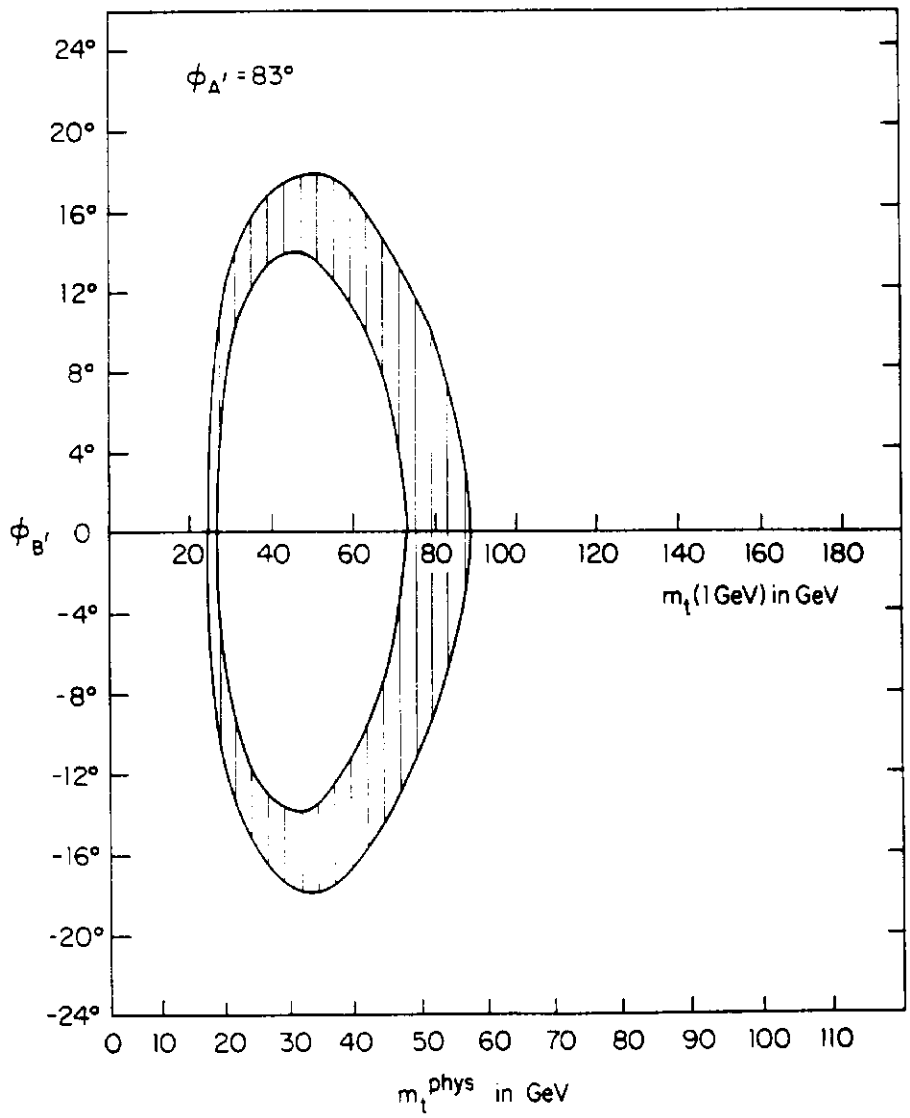
standard model result is represented by the lower solid curve, while the two- doublet model results are given by the two sets of three curves with the upper set referring to the vacuum expectation value ratio $v_2/v_1 = 1.0$ and the lower set to $v_2/v_1 = 0.5$. In each set, the curves refer to a charged Higgs mass of 30, 50 and 70 GeV, respectively.

Figure 4: Annular regions of Fig. 1 for the Fritzsch model allowed by the KM matrix elements and bounds following from Eq. (5.6) as determined by the $B_d - \bar{B}_d$ mixing results of the ARGUS collaboration. The single-hatched bands apply for the minimal Higgs standard model, while the double- hatched bands apply for the two Higgs doublet model with a charged Higgs mass of 50 GeV and VEV ratio $v_2/v_1 = 1.0$. The sets of quark masses are identical to those for Fig. 1.

Figure 5: Plot of d $vs.$ $m_t(1GeV)$ and m_t^{phy} for the Stech model with light quark masses $m_u = 3.6$ MeV, $m_d = 6.4$ MeV and $m_s = 120$ MeV for the Gasser - Leutwyler determination. The narrow elliptical region is that allowed by the KM matrix elements, while the single-hatched and double-hatched bands correspond to the standard model and two-doublet Higgs model results for $B_d - \bar{B}_d$ mixing as in Fig. 4.

Table 1: Values of additional parameters associated with selected points in the plots of Figs. 4 and 5.

Fig.	$m_s(1GeV)$ (MeV)	$m_t(1GeV)$ (GeV)	m_t^{phys} (GeV)	$\phi_{A'}$	$\phi_{B'}$	d	J $\times 10^4$	δ	$m_t^2\|V_{td}\|^2 R$	$\|V_{ts}\|^2/\|V_{td}\|^2$	r_s	$\|V_{ub}\|/\|V_{cb}\|$	B_K
	Fritzsch Model with Minimal Higgs Structure												
4(a)	175	85	56	83°	0°		0.35	105°	0.36	15.6	0.68	0.068	1.5
4(b)	120	150	94	85°	0°		0.30	99°	0.85	17.2	0.94	0.053	0.9
4(c)	155	115	74	83°	0°		0.34	101°	0.58	16.5	0.86	0.061	1.1
4(d)	120	170	106	85°	0°		0.30	97°	1.03	17.5	0.96	0.053	0.8
	Fritzsch Model with Two-Doublet Higgs Structure and $m_\eta = 50$ GeV, $v_2/v_1 = 1.0$												
4(a)	175	85	56	83°	0°		0.35	105°	0.67	15.6	0.88	0.068	1.5
4(b)	120	140	87	85°	9°		0.32	98°	1.65	17.3	0.98	0.058	0.9
4(b)	120	150	94	85°	0°		0.30	99°	1.86	17.2	0.99	0.053	0.9
4(c)	155	115	74	83°	0°		0.34	101°	1.17	16.5	0.96	0.061	1.1
4(d)	120	140	87	85°	14°		0.32	94°	1.60	17.6	0.98	0.058	0.9
4(d)	120	150	94	85°	8°		0.28	96°	1.65	17.4	0.98	0.056	1.1
4(d)	120	160	100	85°	4°		0.28	97°	1.93	17.4	0.99	0.055	1.0
4(d)	120	170	106	85°	0°		0.30	97°	2.33	17.5	0.99	0.053	0.8
	Stech Model with Minimal Higgs Structure												
5	120	70	46			0.000	0.00	180°	0.19	19.1	0.47	0.005	
5	120	70	46			0.017	0.32	93°	0.20	17.7	0.47	0.068	2.1
5	120	70	46			0.024	0.45	91°	0.22	16.5	0.46	0.096	1.5
	Stech Model with Two-Doublet Higgs Structure and $m_\eta = 30$ GeV, $v_2/v_1 = 2.0$												
5	120	70	46			0.000	0.00	180°	1.33	19.1	0.98	0.005	
5	120	70	46			0.017	0.32	93°	1.43	17.7	0.98	0.068	2.1
5	120	70	46			0.024	0.45	91°	1.53	16.5	0.98	0.096	1.5



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16°
12°
8°
4°
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-16°
-20°
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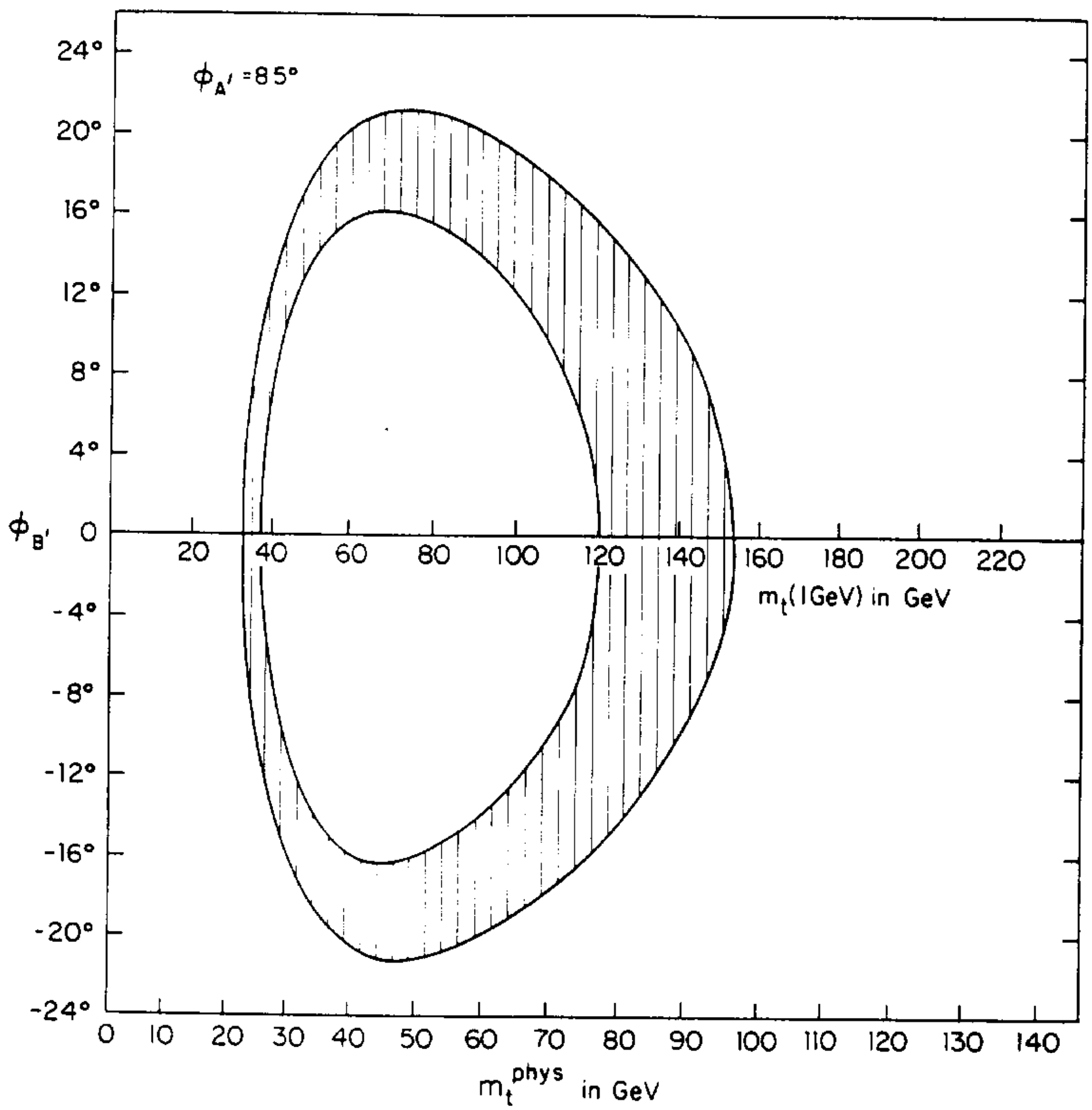
m_t^{phys} in GeV

Fig. 1a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-24°
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80
100
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200
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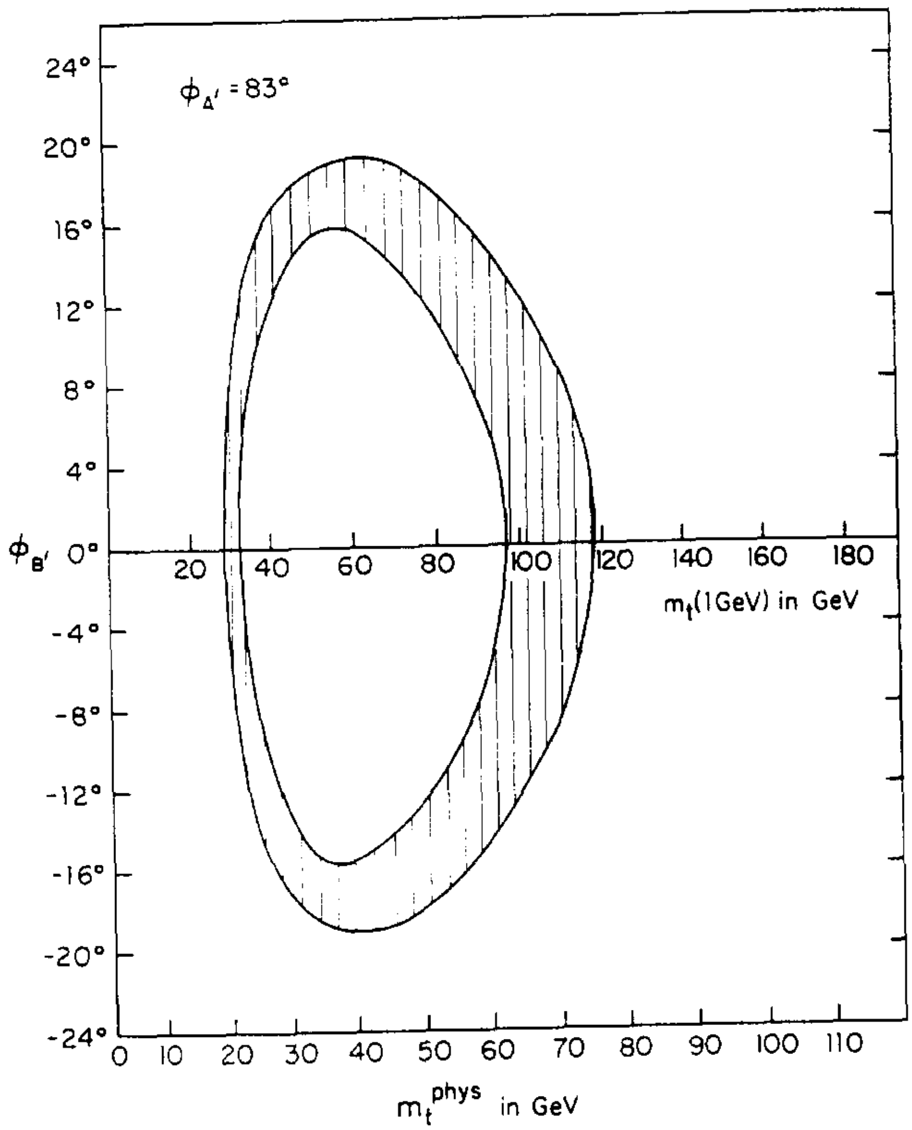
m_t^{phys} in GeV

Fig. 1b



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$\phi_{B'}$ 0°
-4°
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m_t(1GeV) in GeV
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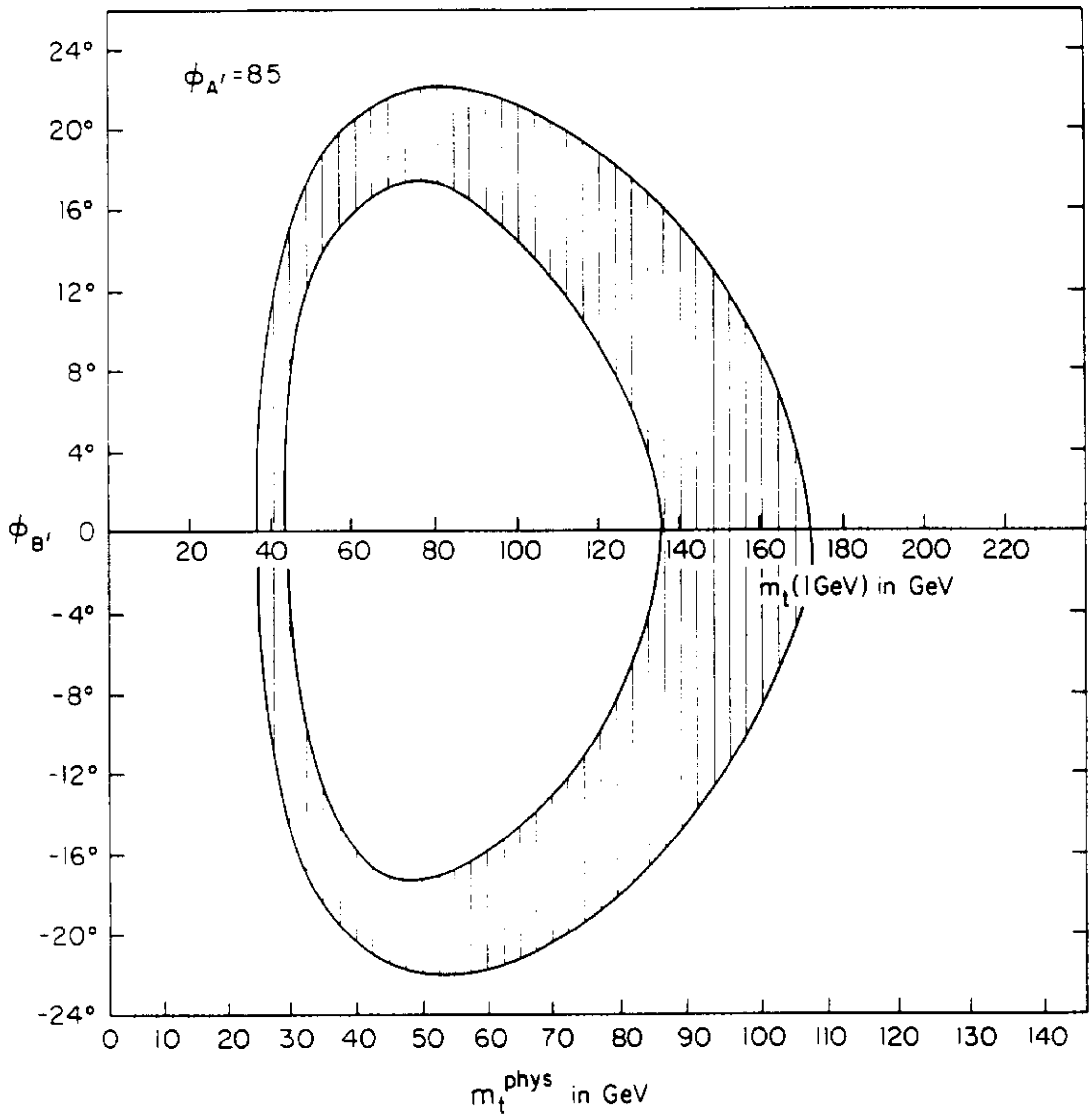
m_t^{phys} in GeV

Fig. 1c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-20°
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m_t(1GeV) in GeV
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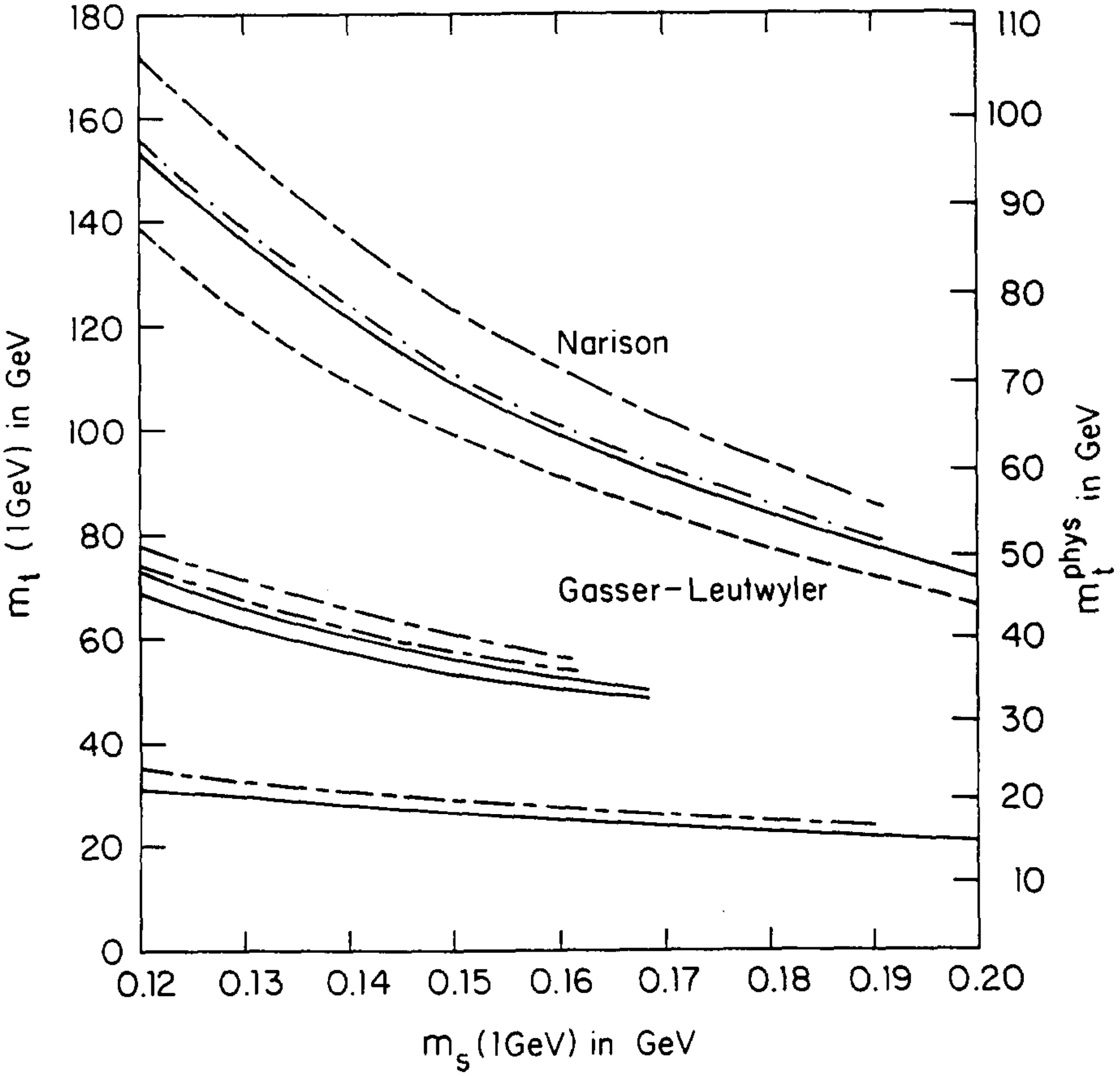
m_t^{phys} in GeV

Fig. 1d



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10
Narison
Gasser-Leutwyler
m_t (1GeV) in GeV
m_t^{phys} in GeV
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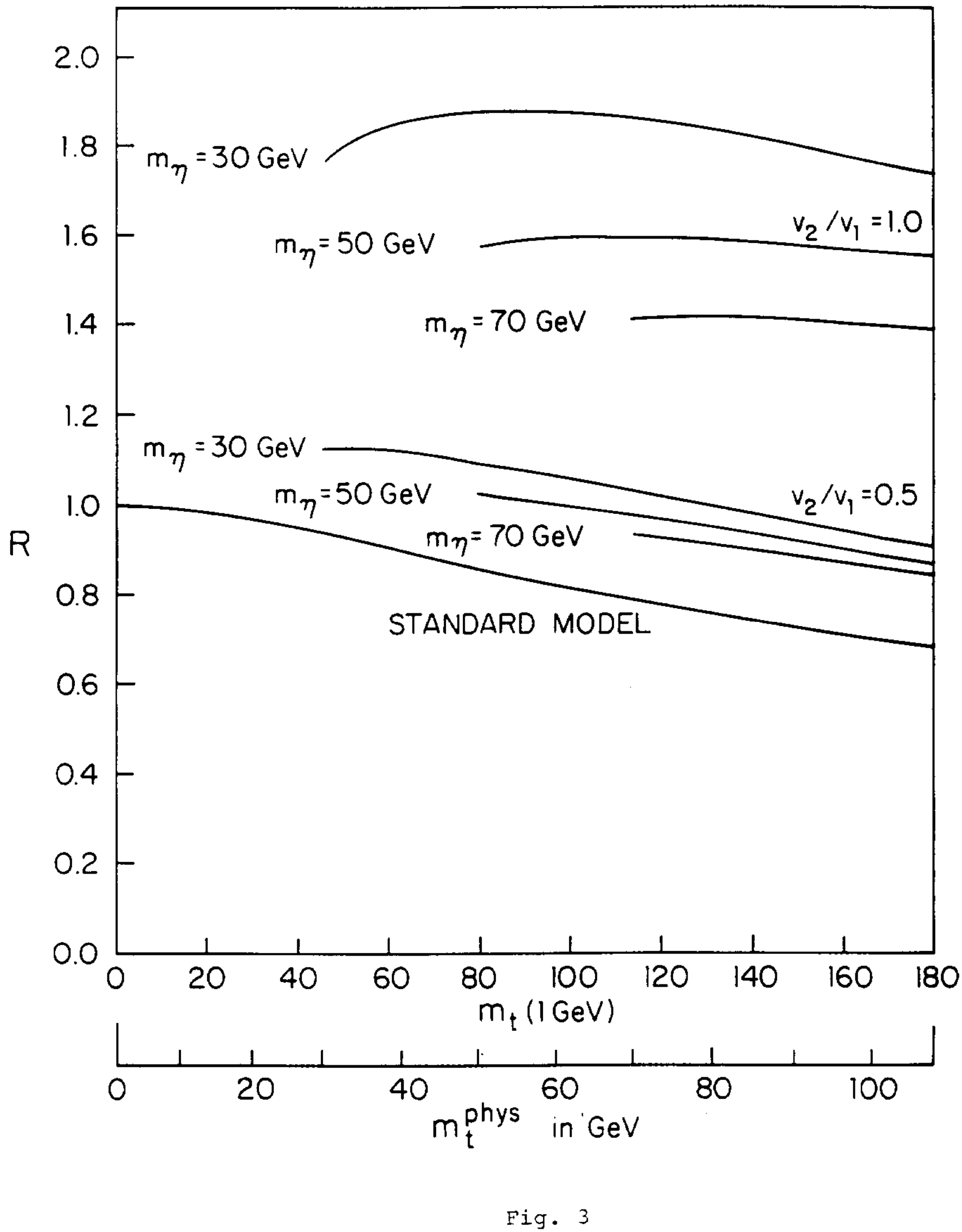
m_s(1GeV) in GeV

Fig. 2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R
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m$_\eta$ = 50 GeV
m$_\eta$ = 70 GeV
v_2/v_1 = 1.0
m$_\eta$ = 30 GeV
m$_\eta$ = 50 GeV
m$_\eta$ = 70 GeV
v_2/v_1 = 0.5
STANDARD MODEL
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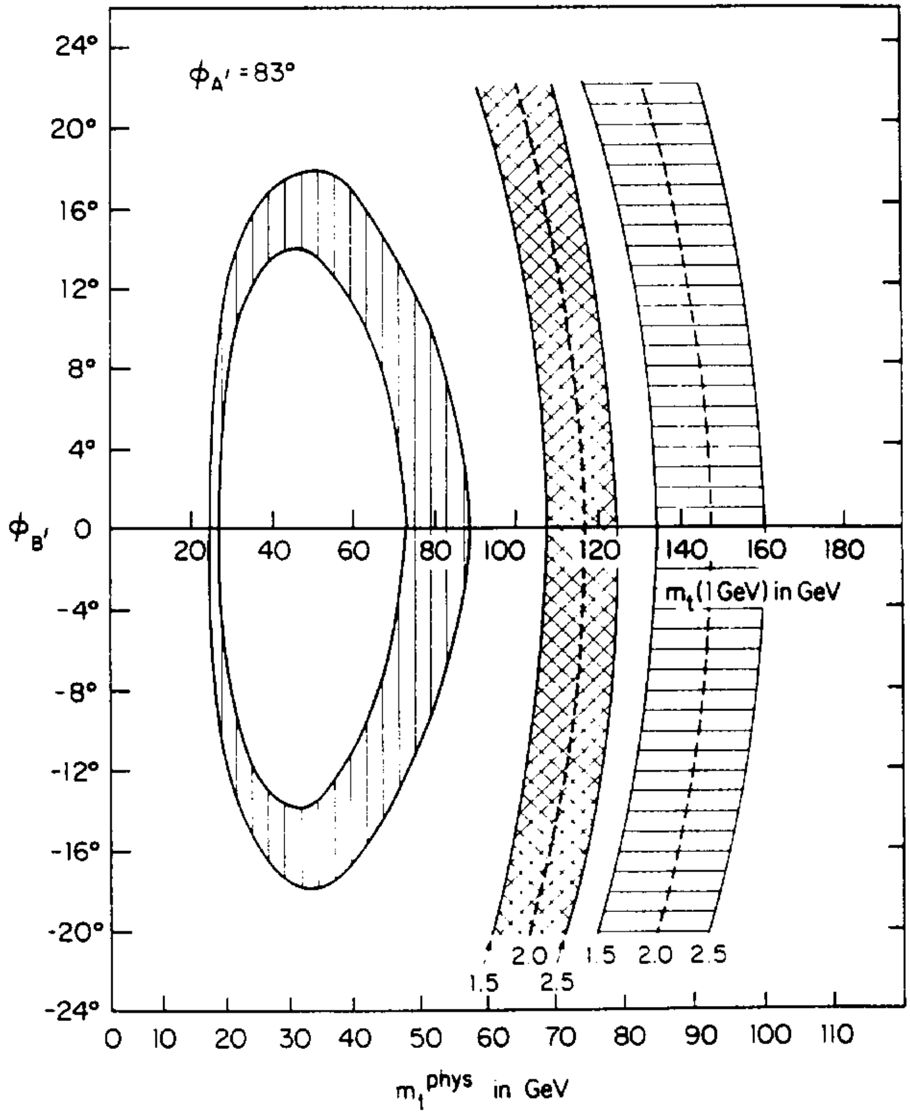
m_t^{phys} in GeV

Fig. 3



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8°
4°
$\phi_{B'}$ 0
-4°
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-12°
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-24°
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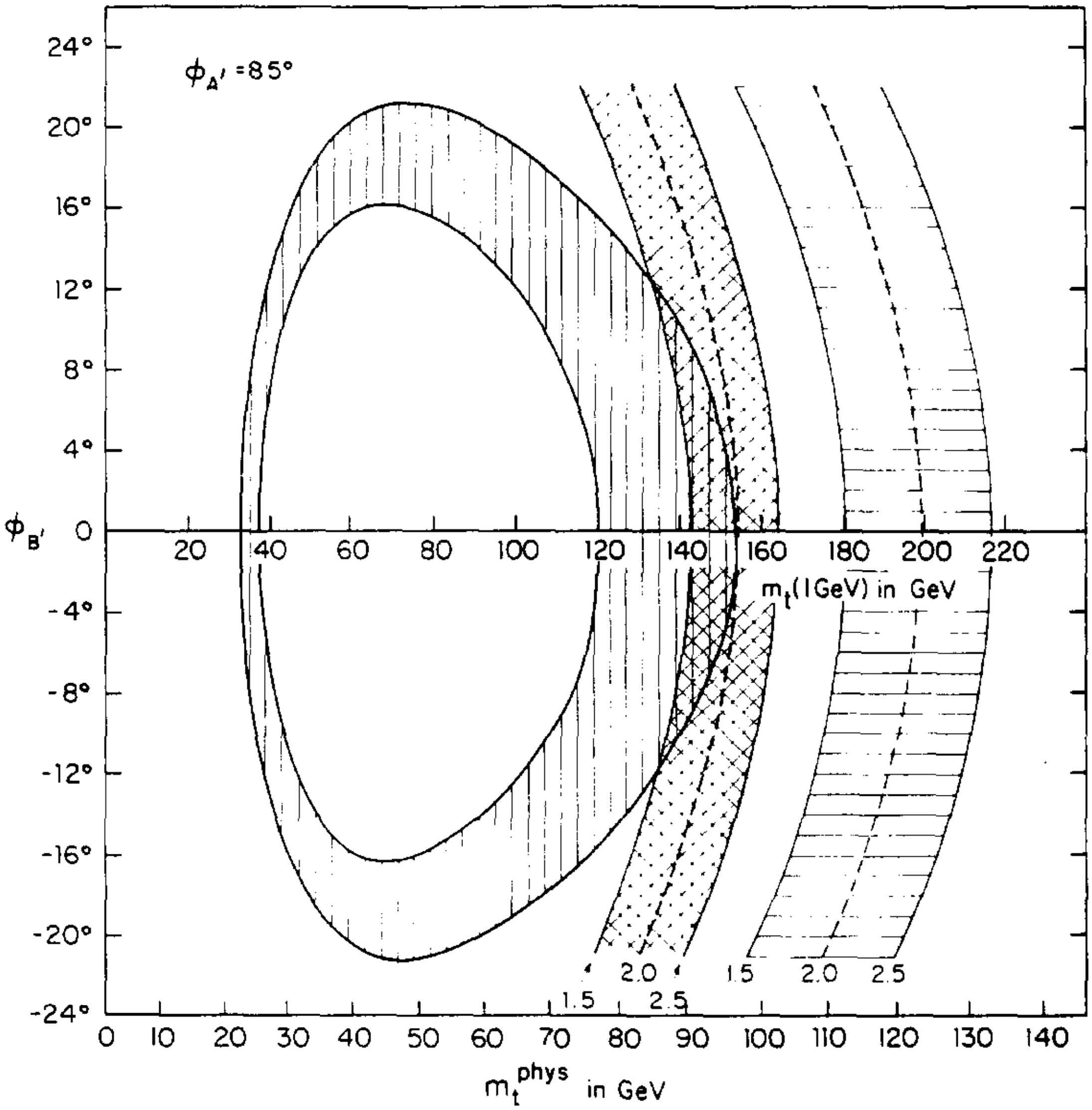
m_t^{phys} in GeV

Fig. 4a



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8°
4°
$\phi_{B'}$ 0
-4°
-8°
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-20°
-24°
20 40 60 80 100 120 140 160 180 200 220
m_t(1GeV) in GeV
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1.5 2.0 2.5
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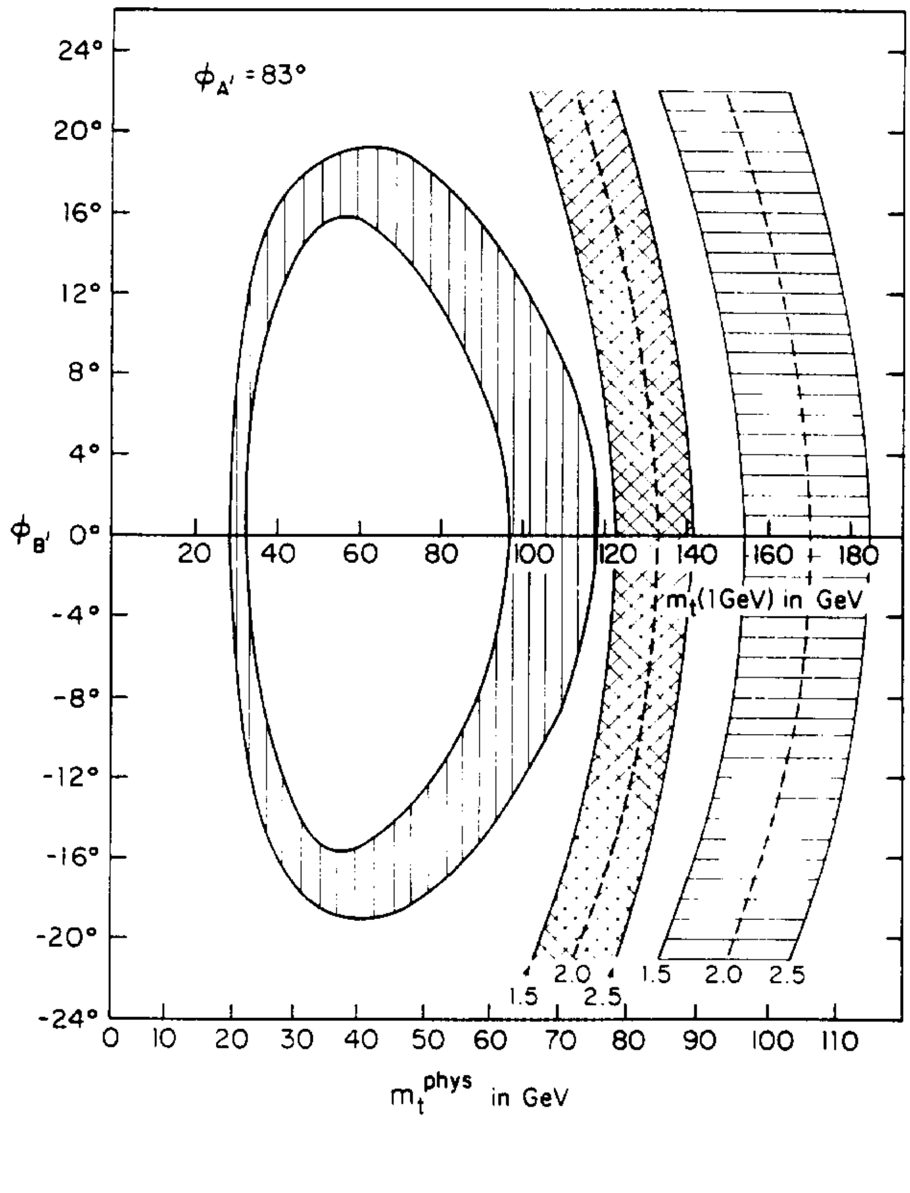
m_t^{phys} in GeV

Fig. 4b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20
40
60
80
100
120
140
160
180
m_t(1GeV) in GeV
1.5
2.0
2.5
1.5
2.0
2.5
0
10
20
30
40
50
60
70
80
90
100
110
m_t^{phys} in GeV

Fig. 4c

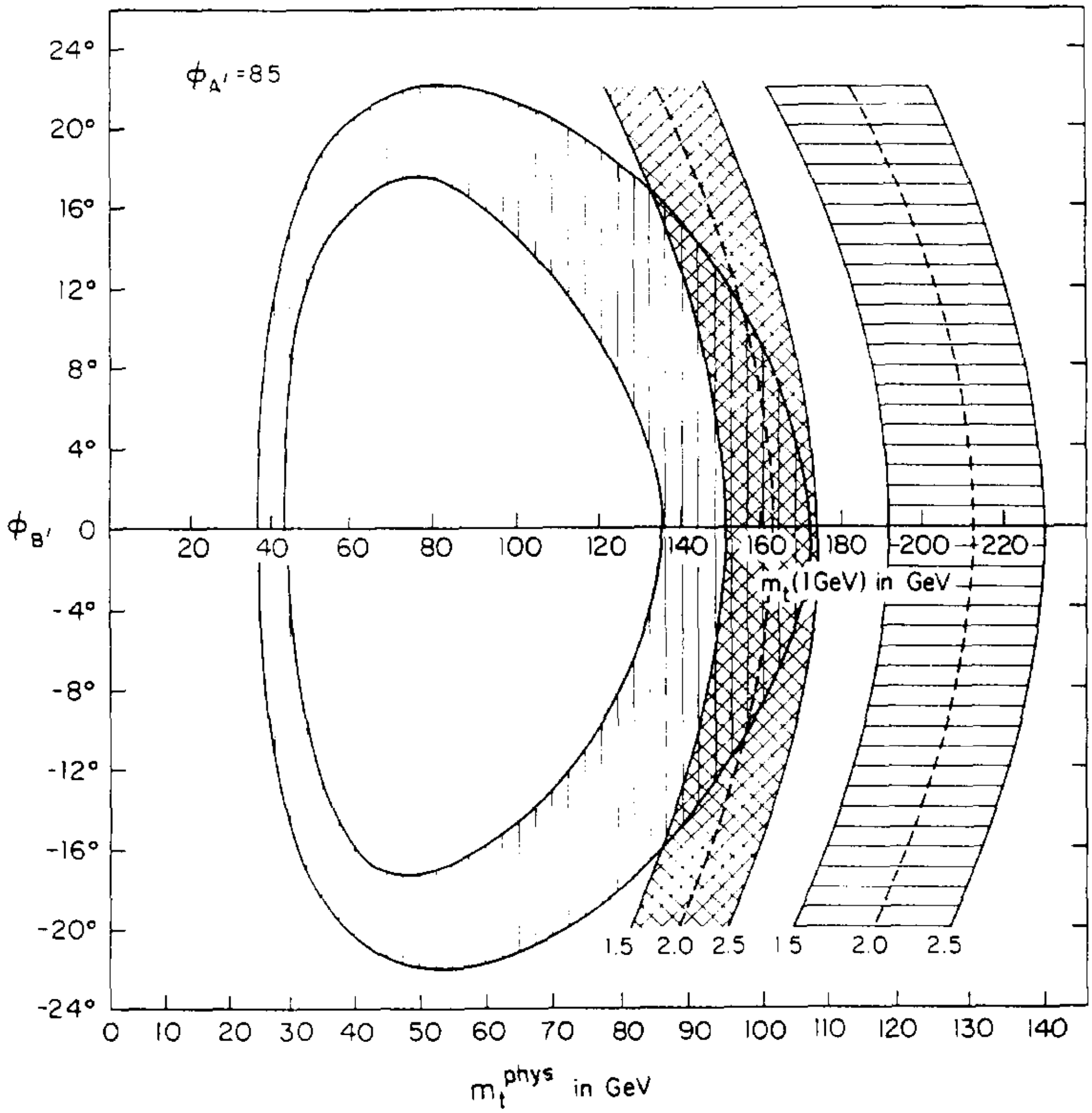
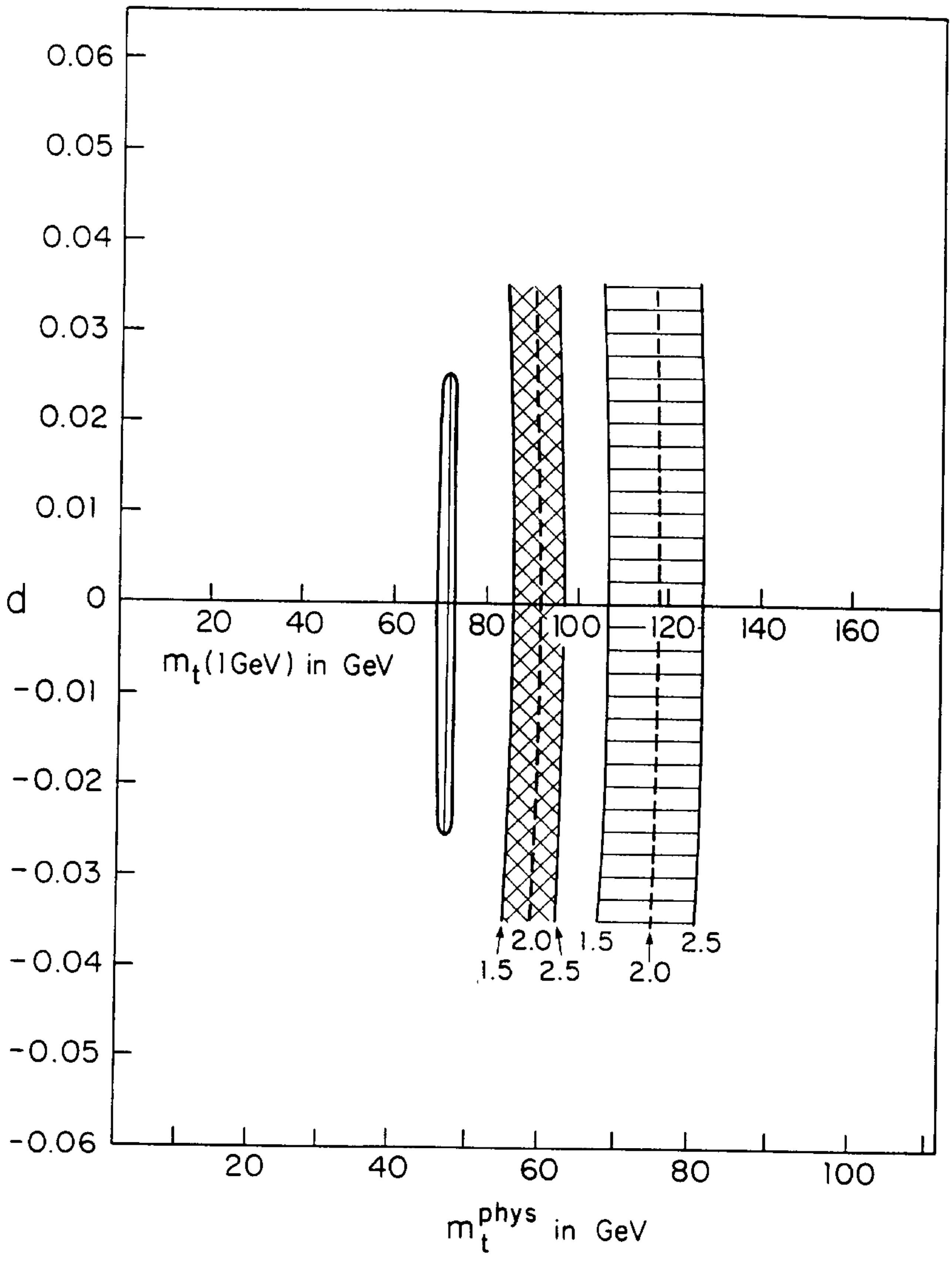


Fig. 4d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-0.06
20
40
60
80
100
120
140
160
m_t(1GeV) in GeV
1.5
2.0
2.5
1.5
2.0
2.5
20
40
60
80
100
m_t^{phys} in GeV

Fig. 5