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The Status of Perturbative QCD*

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Abstract

The advances in perturbative QCD are reviewed. The status of determinations of the coupling constant α_S and the parton distribution functions is presented. New theoretical results on the spin dependent structure functions of the proton are also reviewed. The theoretical description of the production of vector bosons, jets and heavy quarks is outlined with special emphasis on new results. Expected rates for top quark production at hadronic colliders are presented.

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1. QCD and e^+e^- annihilation

1.1 The e^+e^- total cross-section

One of the theoretically cleanest predictions of QCD is $R^{e^+e^-}$, the ratio of the total e^+e^- hadronic cross-section to the muon pair production cross-section. An interesting new result for this quantity has been obtained by Gorishny *et al.*[1] who have calculated the third order coefficient in the perturbative expansion of $R^{e^+e^-}$. Ignoring for the moment weak interaction effects, the expansion for $R^{e^+e^-}$ is found to be,

$$\begin{aligned} R^{e^+e^-} &= 3\sum_f Q_f^2\left\{1+\left(\frac{\alpha_S}{\pi}\right)+\left(1.986-0.115f\right)\left(\frac{\alpha_S}{\pi}\right)^2\right. \\ &+ \left.\left(70.986-1.200f-0.005f^2\right)\left(\frac{\alpha_S}{\pi}\right)^3\right\}-\left(\sum_f Q_f\right)^2 1.679\left(\frac{\alpha_S}{\pi}\right)^3 \quad (1.1) \end{aligned}$$

The scale for the strong coupling has been chosen to be $\mu = \sqrt{S}$ and the renormalisation is performed in the $\overline{MS}$ scheme. Specialising to the case of $f = 5$, Eq. (1.1) becomes,

$$R^{e^+e^-} = 3\sum_f Q_f^2\left\{1+\left(\frac{\alpha_S}{\pi}\right)+1.411\left(\frac{\alpha_S}{\pi}\right)^2+64.810\left(\frac{\alpha_S}{\pi}\right)^3\right\}. \quad (1.2)$$

The third order term is very large, and perhaps larger than one would have expected based on the second order term alone. Two groups[2,3] have performed global fits to all data in e^+e^- annihilation using this result. Of course at the higher energies the effects of Z boson exchange cannot be ignored. Leaving $\sin^2\theta_W$ free, both groups find values consistent with the world average value[4]. The values of α_S obtained in ref. [2] fixing $\sin^2\theta_W$ at the world average are given in Table 1. The inclusion of the third order leads to a 10% change in the value of α_S at $\mu^2 = 1000$ GeV^2. Note that within the experimental errors the values of α_S derived using second and third order perturbation theory agree.

1.2 The QCD perturbation series

Given a value of α_S, one can extract a value of the QCD parameter Λ. The relationship between the measured coupling constant and Λ is unfortunately not unambiguous.

Expression for R	$\alpha_S(\mu^2 = 1000 \text{ GeV}^2)$	$\sin^2\theta_W$	$\Lambda_{\overline{MS}}$ (MeV)
2 loops, $7 < \sqrt{S} < 56$	0.158 ± 0.020	0.23	420^{+300}_{-230}
3 loops, $7 < \sqrt{S} < 56$	0.143 ± 0.016	0.23	250^{+160}_{-130}

Table 1: Results of global fit to e^+e^- data on total cross-section

f	b_0	b_1
3	.7162	.5659
4	.6631	.4902
5	.6101	.4013

Table 2: Corresponding values of b_0 and b_1 with f active flavours.

In order to make it completely clear one must also specify, a) which renormalisation scheme was used, b) how the heavy flavour thresholds were treated, c) which renormalisation scale was used, d) which functional relation between α_S and Λ was used. Some of these ambiguities can be eliminated simply by specifying α_S at a particular scale. The running coupling is a solution of the renormalisation group equation, $d\alpha_S(\mu)/d\mu^2 = -b_0\alpha_S^2(1 + b_1\alpha_S + b_2\alpha_S^2 \ldots)$. By convention Λ is defined by,

$$\alpha_S(\mu) = \frac{1}{b_0 \ln(\mu^2/\Lambda^2)}\left[1 - \frac{b_1}{b_0}\frac{\ln\ln(\mu^2/\Lambda^2)}{\ln(\mu^2/\Lambda^2)} + \ldots\right]. \tag{1.3}$$

Λ depends on the number of active flavours. Values of Λ for different numbers of flavours are defined by imposing the continuity of α_S at the scale $\mu = m$, where m is the mass of the heavy quark. In table 2 the values of the coefficients b_0 and b_1 for 3, 4 and 5 active light flavours are shown.

The coefficients in the perturbation expansion, Eq. (1.1) are given for the special choice of the renormalisation scale ($\mu = \sqrt{S}$). In general the coefficients of all QCD perturbative expansions depend on the choice made for the renormalisation scale μ. Thus, for example, for an arbitrary choice of the scale μ, Eq. (1.1) can be written,

$$R^{(j)} = R^{QPM}\left\{1 + \sum_{i=1}^{j} r_i(\mu)\left(\frac{\alpha_S(\mu)}{\pi}\right)^i\right\}, \quad r_1 = 1. \tag{1.4}$$

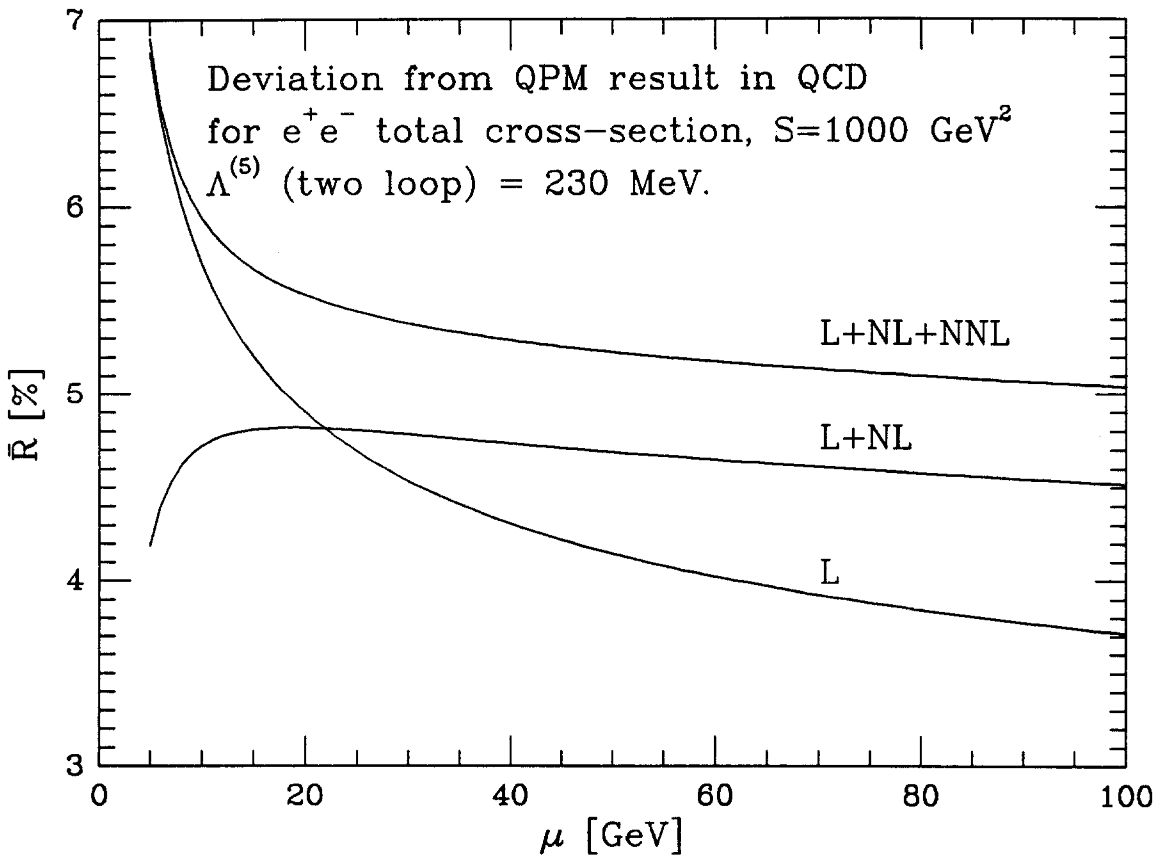

Figure 1: The quantity $\overline{R} = [R^{(j)}/R^{\rm QPM} - 1]$ as a function of the scale μ.

The dependence on the scale μ retaining only the first, second or third correction terms is shown in Fig. 1. In the literature, it has often been advocated that one should make specific choices for the scale μ such that,

$$\begin{aligned} R^{(1)}(\mu) &= R^{(2)}(\mu), \qquad \text{FAC} \\ \mu \frac{d}{d\mu} R^{(2)}(\mu) &= 0, \qquad \text{PMS.} \end{aligned} \tag{1.5}$$

After the inclusion of the recently calculated third order term we see that neither of these guesses do much better than any other choice for the scale μ (see also ref. [5]). Note that the third order correction cannot be defined away. The value of the renormalisation scheme invariant quantity $r_3 - r_2^2 - \pi b_1 r_2 + \pi^2 b_2$ is also large. Proponents of schemes in Eq. (1.5) are of course free to make these choices for μ. The error on

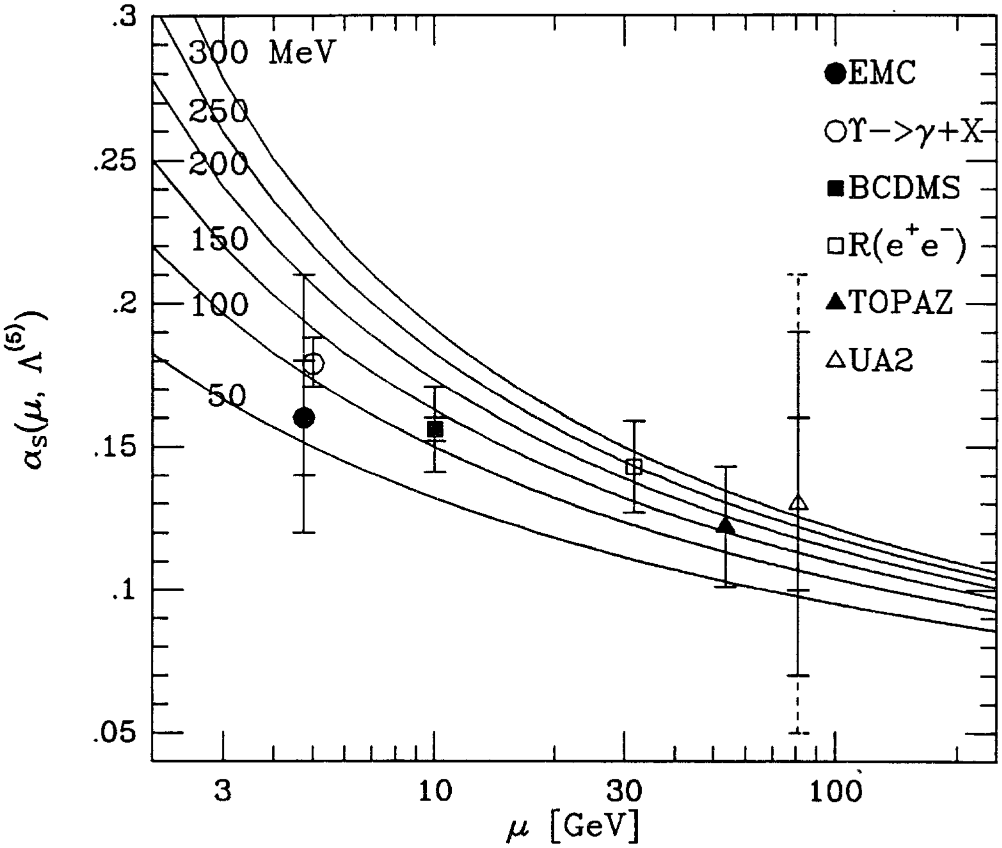

Figure 2: α_S measurements compared with theory for various $\Lambda^{(5)}_{\overline{MS}}$.

a physical prediction which has been calculated to $O(\alpha_S^n)$ remains $O(\alpha_S^{n+1})$. Another question which is raised by the large coefficient of the third order term is whether the perturbation series has begun to manifest the behaviour expected of a divergent, asymptotic series. Without further terms in the series this question cannot be definitively answered.

1.3 Other determinations of α_S

In Fig. 2 a partial compilation of α_S measurements is given. The value ascribed in Fig. 2 to the decay $\Upsilon \to \gamma + X$ is obtained from measurement of the branching ratio, $\Gamma(\Upsilon \to \gamma gg)/\Gamma(\Upsilon \to ggg)$. The value [6] shown in Fig. 2 includes only the statistical error which is very small. The extraction of the branching ratio from the data depends on a fit to a model of the photon spectrum which is based on a Monte Carlo program[7] rather than on an exact perturbative QCD evaluation. It is therefore subject to an additional theoretical uncertainty.

The values denoted by EMC[8] and BCDMS[9] are obtained from fits to deep inelastic scattering off Hydrogen. Since the data for these two experiments are in disagreement, at least one of these determinations of α_S must be suspect.

The UA2 value[10] is determined from the $W+$ jet events. The dashed line indicates an estimate of the theoretical error due to the fact that the calculation performed in ref. [11] is only partial. Although the errors are still large it is the highest energy measurement of $\alpha_S^{\overline{MS}}$. The measurement from TOPAZ[12] is obtained from energy-energy correlations.

Analysis of multijet final states in e^+e^- annihilation[13] yields a 3-jet rate consistent with a logarithmically decreasing coupling constant, but a value of α_S fixed with energy cannot be ruled out because of the limited statistics and energy range of the data. From Fig. 2 we can conclude that there is still not any convincing evidence for the running of $\alpha_S^{\overline{MS}}$ and that the value of α_S is still subject to a considerable uncertainty. This uncertainty in the value of α_S is reflected directly in the uncertainty in QCD production cross-sections.

2. Parton distribution functions

2.1 Quark distribution functions

Fig. 3 shows the ratio of the proton structure functions F_2^p as measured by the EMC [8] and BCDMS[9] collaborations. The experimental situation has been reviewed in refs. [14,15]. The discrepancy between the two experiments, although only a 15% effect, has a considerable impact on the form of the quark distribution functions extracted from the two data sets.

Another important measurement is the ratio F_2^n/F_2^p shown in Fig. 4 which controls the ratio of the up and down valence quarks in the proton. The two experiments are in agreement taking into account the systematic and statistical errors. However the measured value of F_2^n/F_2^p is still subject to about 20% uncertainty which translates directly into an error on the predicted ratio of W and Z cross-sections. The dashed

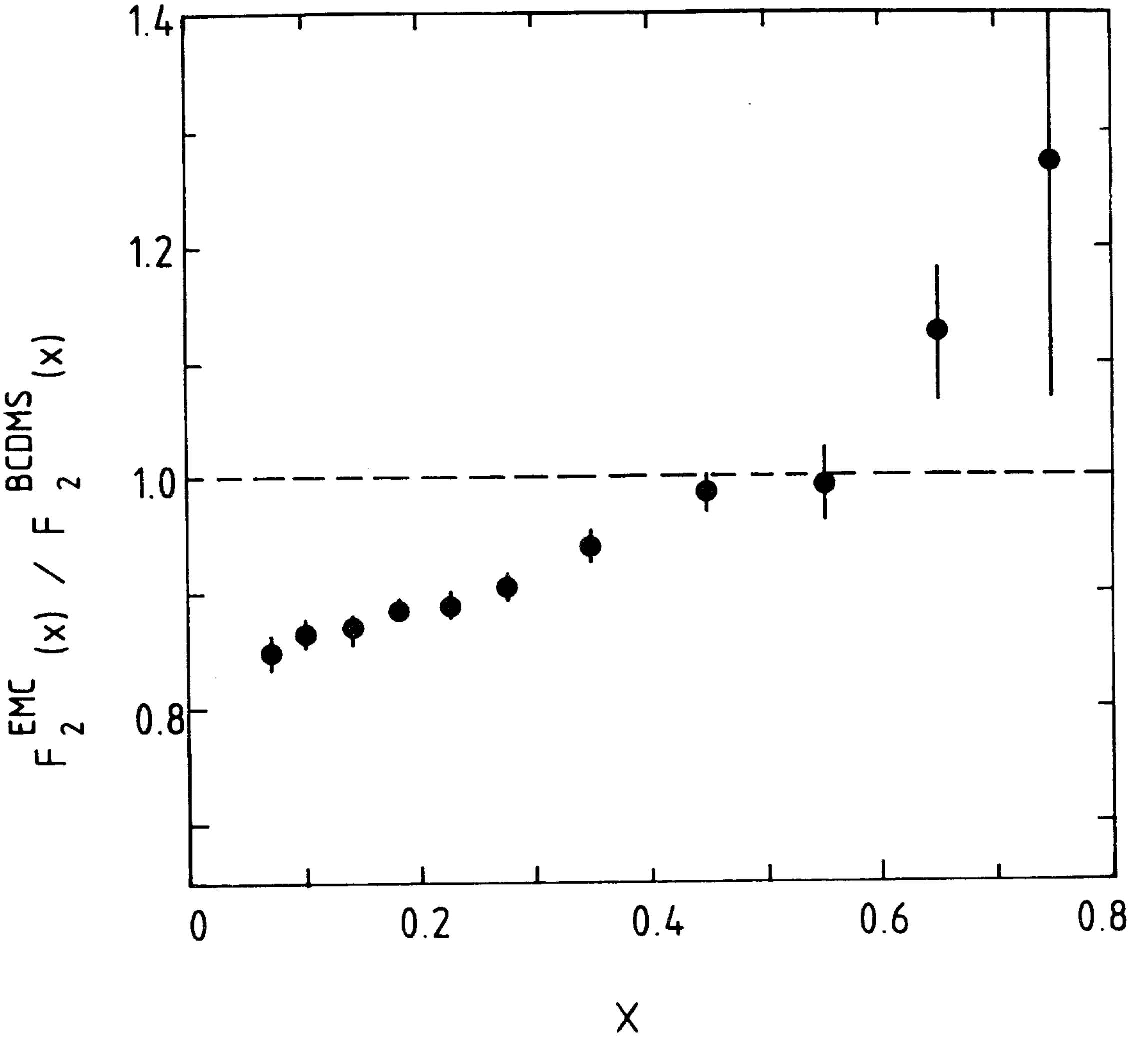

Figure 3: The ratio of the proton structure functions $F_2^{\mathrm{EMC}}(x)/F_2^{\mathrm{BCDMS}}(x)$.

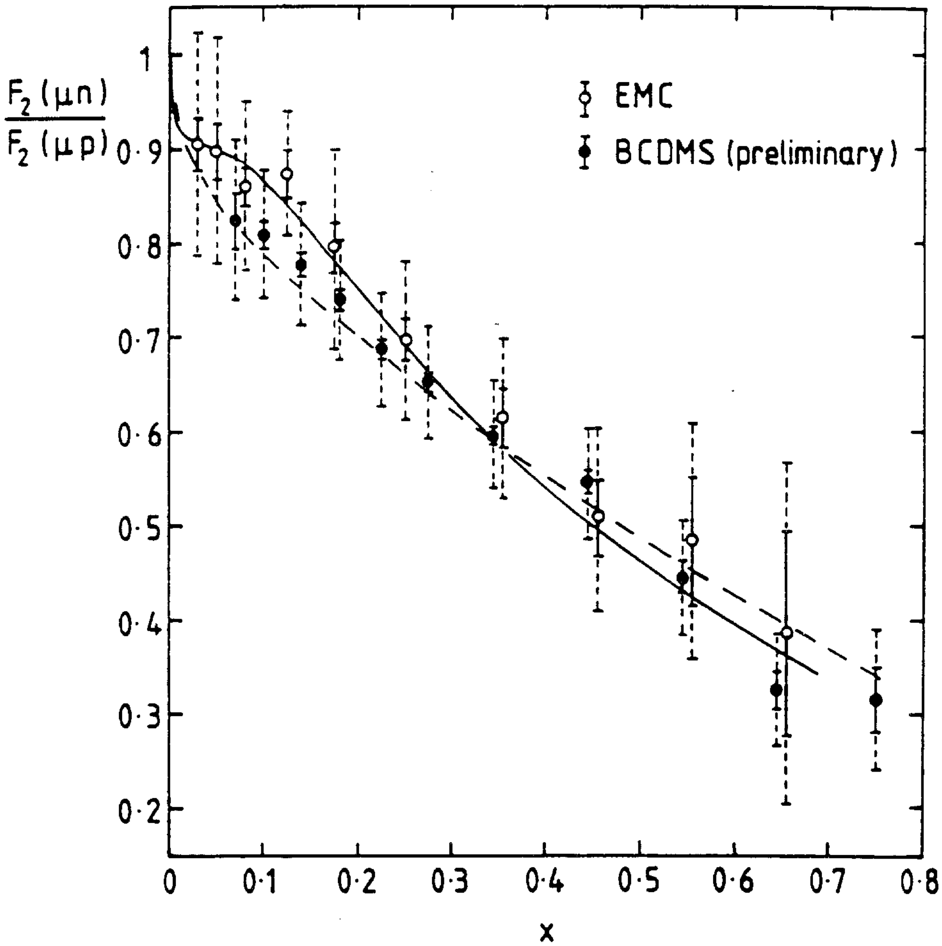

Figure 4: F_2^n/F_2^p as measured by EMC and BCDMS.

(solid) curve is the result of a global fit[16] to the BCDMS (EMC) μN data together with deep inelastic neutrino data. In the discussion of W and Z cross-sections I shall refer to these fits as the B and E-fits respectively.

2.2 Gluon distribution functions

A limited amount of experimental information on the gluon distribution function can be obtained from Deep Inelastic Scattering. The total momentum carried by the gluons is deduced from the momentum fraction carried by the quarks. Further indirect information on the gluons is obtained from the derivative of the measured structure functions $D = d\ln F_2/d\ln Q^2$. This is a difficult procedure. The gluons contribute only a small fraction of the measured D in a limited x range, ($x < 0.35$). In addition other effects, such as the opening of the charm threshold, give contributions of the same order of magnitude to D[17].

An accurate determination of the gluon distribution function becomes increasingly important for the calculation of hadronic processes at high energy, because the gluon distribution grows rapidly at small x. The Altarelli-Parisi (AP) equation [18] can be used to extract an asymptotic form for the gluon distribution function at small x and large Q^2. We denote the momentum distribution of the gluons by $G(x,t) = xg(x,t)$. At small x we obtain from the AP equation that,

$$\frac{dG(x,t)}{dt} = \frac{3\alpha(t)}{\pi} \int_x^1 \frac{dz}{z} G(z,t), \quad t = \ln\left(\frac{Q^2}{\Lambda^2}\right), \quad \alpha(t) = \frac{1}{b_0 t} \tag{2.1}$$

With the boundary condition $G(x,t_0)$ = constant, the asymptotic solution to this equation is[19],

$$G(x,t) \sim \exp\sqrt{\frac{12}{\pi b_0} \ln \frac{\ln Q^2/\Lambda^2}{\ln Q_0^2/\Lambda^2} \ln \frac{1}{x}}. \tag{2.2}$$

For a treatment of the solutions of Eq. (2.1) using more realistic boundary conditions see ref. [20]. Strictly speaking the AP equation is not valid in the small x region. It sums all the leading logarithms of Q/Λ but not all the leading logarithms of x which were included by Balitsky and Lipatov in ref. [21]. For values of $x > 10^{-4}$ the difference between solution to the BL equation and the full AP equation has been found numerically [22] to be less than 10%. So arguments based on the AP equation are qualitatively correct in the present energy regime and for most purposes at the SSC.

Note that Eq. (2.2) shows that $G(x)$ grows at small x violating both the input conditions and simple Pomeron dominance. Even if a constant behaviour is assumed at one value of Q^2 it is rapidly modified by AP evolution to give a steeper growth at small x. In the limited range of x available at present colliders it is therefore more appropriate to assume the following small x behaviour [23],

$$G(x) = xg(x) \sim \frac{1}{x^\delta}, \quad \delta \sim 0.5 \tag{2.3}$$

Such a form of the gluon distribution predicts that mini-jet cross sections should grow approximately like

$$\frac{d\sigma}{dp_T^2} \sim \frac{s^\delta}{(q_T^2)^{2+\delta}}. \tag{2.4}$$

This can be interpreted as the effect of the QCD perturbative Pomeron which predicts the growth of the minijet cross-section with energy. The present data[24] seem to favour a $1/x$ behaviour, but higher beam energies are needed to settle the question.

A method to disentangle the QCD Pomeron from the effects of the gluon distribution function is described in ref. [25]. The method measures the change in energy of a two jet inclusive cross-section in which the longitudinal momentum fractions x_1, x_2 at which the parton distributions are probed are held fixed. At fixed x_i the effects of the gluon distributions cancel. By increasing the energy at fixed x_1 and x_2 the rapidity gap between the two jets grows. When the rapidity gap $Y = \ln(x_1 x_2 S/M^2)$ between the two jets becomes large, the perturbation series becomes a series in $\alpha_s Y$. The two jet cross section is predicted to grow with energy as $\sigma \sim \sqrt{S}$.

Such power law growth cannot continue indefinitely since it would violate the Froissart bound. In the infinite momentum frame the Lorentz contracted proton is a disk of transverse radius r. The gluon distribution $G(x, \ln(Q^2))$ gives the number of gluons per unit of rapidity with a transverse size less than $1/Q$. At small x the number of gluons grows large. When they start to overlap, new non-perturbative effects come into play to curb the growth of the gluon distribution. A crude estimate of when this saturation begins to happen is provided by,

$$G(x, \ln(Q^2)) = \frac{\text{Area of hadron}}{\text{Area of parton}} \sim Q^2 r^2 \sim 25 \ Q^2 \ \text{GeV}^{-2}, \tag{2.5}$$

where $r \sim 1/m_\pi$ is the radius of the hadron. At presently attainable values of x the value of $G(x, \ln(Q^2))$ does not exceed 2 or 3, so the saturation limit is beyond the range of the present colliders. The modification of the Altarelli-Parisi equations as the saturation limit is approached is investigated in ref. [26].

3. The spin-dependent structure functions of the proton

The EMC collaboration have presented results[27] on the polarised structure functions of the proton, extending previous measurements made at SLAC[28] down to a smaller value of x. The primary measurement made by the EMC collaboration is the ratio

$$A = (d\sigma^{\uparrow\downarrow} - d\sigma^{\uparrow\uparrow})/(d\sigma^{\uparrow\downarrow} + d\sigma^{\uparrow\uparrow}) \tag{3.1}$$

where the arrows indicate the direction of longitudinal polarisation of the incoming muon and the struck proton. By including the information on the unpolarised structure function F_2 and $R = \sigma_L/\sigma_T$, they are able to deduce the polarised structure function g_1^p. Extrapolating from the measured x range ($0.01 < x < 0.7$) they obtain an estimate for the first moment of g_1^p,

$$\int_0^1 dx g_1^p(x) = 0.114 \pm 0.012 \pm 0.026, \quad \left\langle Q^2 \right\rangle \sim 10.7 \text{ GeV}^2. \tag{3.2}$$

Since they have no information on the polarised scattering off neutrons, they are unable to obtain a result for the Bjorken sum rule,

$$\int_0^1 dx \Big(g_1^p(x) - g_1^n(x) \Big) = \frac{1}{6} \frac{g_A}{g_V} \Big(1 - \frac{\alpha_S}{\pi} \Big). \tag{3.3}$$

Further approximations are therefore necessary to compare their data with theory. In the naive parton model the first moment of g_1^p may be written as,

$$\int_0^1 dx g_1^p(x) = \frac{1}{2} \Big(\frac{4}{9} \Delta u + \frac{1}{9} \Delta d + \frac{1}{9} \Delta s \Big) \tag{3.4}$$

where $\Delta q = \int_0^1 dx(q^\uparrow + \bar{q}^\uparrow - q^\downarrow - \bar{q}^\downarrow)$. In the following we shall see that Eq. (3.4) is incorrect in QCD due to the axial anomaly. Decomposing Eq. (3.4) into SU(3) octet and singlet pieces we obtain,

$$\int_0^1 dx g_1^p(x) = \frac{1}{12} I_3 + \frac{1}{36} I_8 + \frac{1}{9} I_0 \tag{3.5}$$

where $I_3 = \Delta u - \Delta d, I_8 = \Delta u + \Delta d - 2\Delta s, I_0 = \Delta u + \Delta d + \Delta s$.

Using the measured value of g_A/g_V and F/D ratios from semileptonic hyperon decays, the EMC collaboration obtain,

$$\Delta u = 0.75 \pm 0.038 \pm 0.078, \quad \Delta d = -0.51 \pm 0.038 \pm 0.078, \quad \Delta s = -0.23 \pm 0.038 \pm 0.078. \tag{3.6}$$

The estimated value of the total spin carried by the quarks in the infinite momentum frame is $I_0 = (1 \pm 12 \pm 24)\%$ in apparent contradiction with intuition from the non relativistic quark model. Note however that the determination of I_0 using Eq. (3.5) is subject to a large error [29], because it requires the subtraction of two numbers of about the same order of magnitude. Because of this sensitivity a small change, such as a $1/Q^2$ effect[30], can make a significant difference to the estimate of I_0.

Experimental information on the approach to the $Q^2 = 0$ limit is given in ref. [31]. In the Skyrme model both I_0 and $\Delta g = \int_0^1 dx(g^\uparrow - g^\downarrow)$ are equal to zero[32].

The operator product expansion treatment of the spin dependent structure function is contained in ref. [33]. The partonic interpretation of these results is complicated because there is only one spin–1, gauge-invariant local operator in the singlet sector, namely the axial current j^5_α. The axial current is not conserved because of the anomaly,

$$j^5_\alpha = \sum_{i=1}^{f} \overline{q}_i \gamma_\alpha \gamma_5 q_i, \quad \partial^\alpha j^5_\alpha = f \partial^\alpha k_\alpha, \quad k_\alpha = \frac{\alpha_S}{2\pi} \epsilon_{\alpha\nu\lambda\sigma} Tr \left[A^\nu \left(F^{\lambda\sigma} - \frac{2}{3} A^\lambda A^\sigma \right) \right] \quad (3.7)$$

In ref. [34] it was pointed out that the anomaly would alter the interpretation of the EMC results. The problem has been further studied in refs. [35,36,37]. The basic point is that the operator j_5^α which appears in the operator product expansion, does not measure $\sum \Delta q$ alone, but rather the combination $\sum_{i=1}^{f} (\Delta q_i - \Delta\Gamma)$ [36,37] where $\Delta\Gamma = \frac{\alpha_S}{2\pi} \Delta g$. This can be motivated by a physical argument. All the basic vertices of the theory conserve quark helicity. Hence Δq should not change with Q^2[38]. Since j_5 is not conserved it has an anomalous dimension. Matrix elements of j_5 should not be identified with Δq which does not change with Q^2 because of helicity conservation. Instead we have

$$\langle p | j_5^\alpha | p \rangle = 2m \sum_{i=1}^{f} (\Delta q_i - \Delta\Gamma) s^\alpha(p) \quad (3.8)$$

where s^α is the usual axial four vector describing the polarisation of a spin $\frac{1}{2}$ particle. The quantity $\Delta\Gamma$ is not necessarily small since at leading log level we have that,

$$\frac{d}{d \ln \mu^2} \left[\Delta\Gamma(\mu^2) \right] = 0 \quad (3.9)$$

Hence Δg is potentially very large and $\Delta\Gamma$ is of order $\alpha_S(Q_0^2)$ rather than $\alpha_S(Q^2)$. The values given Eq. (3.6) should be equated with $\Delta q - \Delta\Gamma$. If we assume that the strange quarks carry no spin, Eq. (3.6) implies that,

$$\Delta g \sim 8, \quad \left\langle Q^2 \right\rangle \sim 10.7 \text{ GeV}^2. \quad (3.10)$$

This estimate of Δg is still subject to the uncertainties pointed out in ref. [29]. The large value of Δg suggests that observable effects will be seen in polarised experiments

Experiment	$\sqrt{S}$ (GeV)	x_T range
WA70[41]	22.9	$0.35 < x_T < 0.56$
UA6[42]	24.3	$0.25 < x_T < 0.58$
E706[43]	31.5	$0.25 < x_T < 0.51$
UA2[44]	630	$0.04 < x_T < 0.22$
UA1[45]	630	$0.06 < x_T < 0.28$
CDF[46]	1800	$0.02 < x_T < 0.03$

Table 3: Reported x_T ranges for experiments giving information on the gluon distribution function in the proton.

involving gluons. Their discovery would give a nice confirmation of this interpretation of the EMC results.

4. Vector boson production

4.1 Direct photon production

The production of direct photons at large p_T proceeds at lowest order via the two parton sub-processes,

$$q + \overline{q} \to \gamma + g, \quad g + q(\overline{q}) \to \gamma + q(\overline{q}). \tag{4.1}$$

The observation of the direct photon p_T spectrum therefore provides an alternative way to measure the gluon distribution function. The advantage of this method over deep inelastic scattering is that the gluon distribution function contributes directly in the Born approximation. In addition the produced photon is observed directly, allowing a relatively simple kinematic reconstruction of the event. In practice it is often necessary to introduce an isolation cut in order to remove the background from hadronic decays. This isolation cut must be taken into account when making the comparison with theory. A complete next to leading order calculation $O(\alpha\alpha_S^2)$ is available[39] so that in principle both the value of $\Lambda_{\overline{MS}}$ and the gluon distribution function can be determined from these experiments.

A large number of experiments have been analysed[39] using the structure functions of Duke and Owens[40], which exist in two versions, a soft gluon version (DO1) with $\Lambda = 0.2$ GeV and a hard gluon version with $\Lambda = 0.4$ GeV. The experiments favour the soft gluon fit (DO1) corresponding to the smaller value of Λ. First results have also been presented on a combined second order fit to deep inelastic scattering and direct photon data. They are found to provide complementary information on $\Lambda_{\overline{MS}}$ and the form of the gluon distribution[47].

The separation of the direct photon signal from the background is only possible for a limited range of $x_T = 2p_T/\sqrt{S}$. The values of x_T to which the the experiments are sensitive is shown in Table 3. The range of x_T also determines the range in x in which the gluon distribution can be measured. The minimum value of x to which an experiment is sensitive is given by $x_{\min} = x_T/(2 - x_T)$. Therefore it is more appropriate to consider that these experiments determine the value of $\alpha_S(\mu^2)\ G(x,\mu^2)$ for $\mu \sim p_T, x \sim x_T$ rather than the shape of the whole gluon distribution function.

The observation of quasi-real photons which materialise into lepton pairs may allow the extension of direct photon studies down to lower values of p_T, yielding valuable information about the low x behaviour of the gluon distribution function[48]. Experimental results using this technique have been presented in ref. [49].

4.2 W and Z production

The production and decay properties of the W and Z bosons observed at hadron colliders test many features of the standard model. The branching ratio of the W into leptons depends on the mass of the top quark and/or the mass of a possible heavy lepton. The branching ratio of the Z also depends on the number of massless neutrino species. The production cross sections of the W and Z are calculable in perturbative QCD using the parton distribution functions measured in Deep Inelastic Scattering. The simplest quantity to analyse is the total cross-section. The total cross section for the production of a boson of mass M is determined by the convolution of the parton distribution functions F with the short distance cross-section Δ,

$$\sigma = \sum_{i,j} \int dx_1 dx_2 dz\ \delta(x_1 x_2 z - \tau)\ F_i(x_1,\mu) F_j(x_2,\mu) \Delta_{ij}(z) \tag{4.2}$$

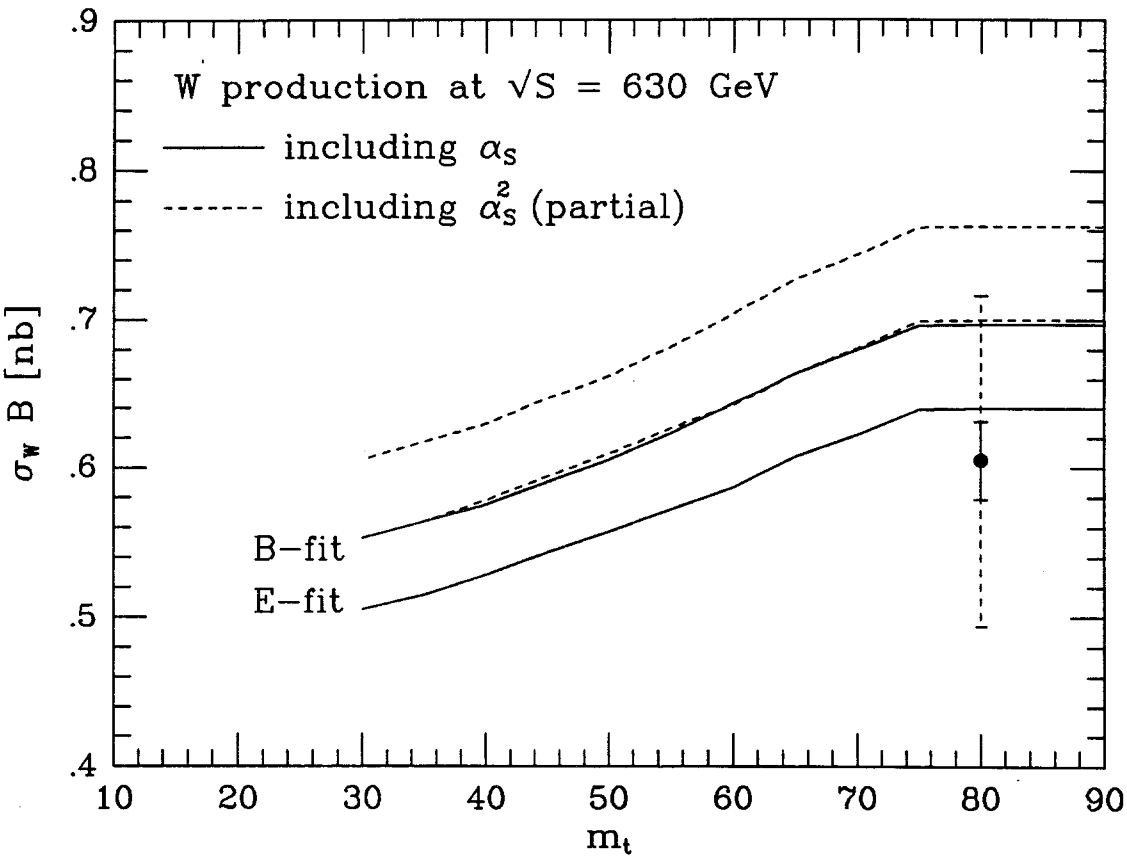

Figure 5: $\sigma_W B$ as a function of the top quark mass compared with data.

where $\Delta_{ij}(z) = \Delta_{ij}^{(0)}(z) + \alpha_S \Delta_{ij}^{(1)}(z) + \alpha_S^2 \Delta_{ij}^{(2)}(z) + \ldots$ and $\tau = M^2/S$. In lowest order only quark antiquark annihilation contributes, $\Delta_{q\bar{q}}^{(0)}(z) \sim \delta(1-z)$, whereas in higher orders initial states containing gluons give non-zero contributions. The full result for $\Delta_{ij}^{(1)}(z)$ is known[50]. In the case of W production at $\sqrt{S} = 0.63$ TeV, the inclusion of the term $\Delta^{(1)}$ increases the estimate based on $\Delta^{(0)}$ alone by about 30%.

A partial result has been presented in ref. [51] for $\Delta_{q\bar{q}}^{(2)}(z)$. All terms associated with soft and virtual gluons are included, but the effects of the emission of hard partons are neglected. Applying the same approximation to $\Delta^{(1)}(z)$ which is fully known one can test the validity of the soft and virtual approximation. It is found that approximate form deviates from the full result for $\Delta^{(1)}(z)$ by about 20% in the calculation of W production at CERN energies[52].

In Fig. 5 a comparison of the theoretical prediction for $\sigma_W B(W \to e\nu)$ with the experimental results of UA1[53] and UA2[54] is shown. The statistical errors of the two experiments have been combined in quadrature and the resultant error added linearly to the average of systematic errors of the two experiments. The theoretical curves correspond to the BCDMS like fit and the EMC like fit of ref. [56]. They are shown as a function of the top quark mass before and after the inclusion of the partial $O(\alpha_S^2)$ result. With the present errors no information on the top quark mass can be obtained from σB. Note however that agreement with the experimental results relies on the inclusion of the $O(\alpha_S)$ contribution. The quoted result for the W cross section at $\sqrt{S} = 1.8$ TeV[55] is $\sigma B(W \to e\nu) = 2.6 \pm 0.6 \pm 0.5$ nb which is consistent with the QCD prediction for all values of the top quark mass.

A potentially more sensitive method of obtaining a limit on the top quark mass is given by the ratio of the W and Z production cross sections, since experimental and theoretical uncertainties will tend to cancel,

$$R = \frac{\sigma_W B(W \to e\nu)}{\sigma_Z B(Z \to e^+e^-)} = R_\sigma R_B. \tag{4.3}$$

On the assumption that the masses of all charged objects to which the W and Z decay are known, R can provide a measurement of $B(Z \to e^+e^-)$ and hence of the number of massless neutrino species. If the W (and perhaps Z) boson can decay a top quark whose mass is not known, R_B will also depend on its mass. It is clear that this method relies on an accurate estimate for the theoretical ratio R_σ and its errors. The implications of the BCDMS measurements for this ratio and particularly the measurement of F_2^n/F_2^p shown in Fig. 4 are investigated in refs. [56,57]. The results of this analysis are shown in Fig. 6. The shaded bands correspond to the BCDMS and EMC like fits of ref. [16] and the other fits (A, C and F) correspond to further variations of the fits consistent with the experimental errors. In view of the experimental uncertainties in the structure functions it is not appropriate to draw a definite conclusion from Fig. 6. It is clear that one cannot exclude the possibility that there is no limit on the top quark mass from this method. If $m_t > M_W - m_b$ one can exclude a fourth massless neutrino. Also shown in Fig. 6 is the prediction for $\sqrt{S} =$ 1.8 TeV, where vector bosons are predominantly produced by the annihilation of sea partons which are isospin symmetric by assumption. The dashed lines correspond to the ± 0.04 variation in R_σ arising from the ± 0.005 uncertainty in the world average

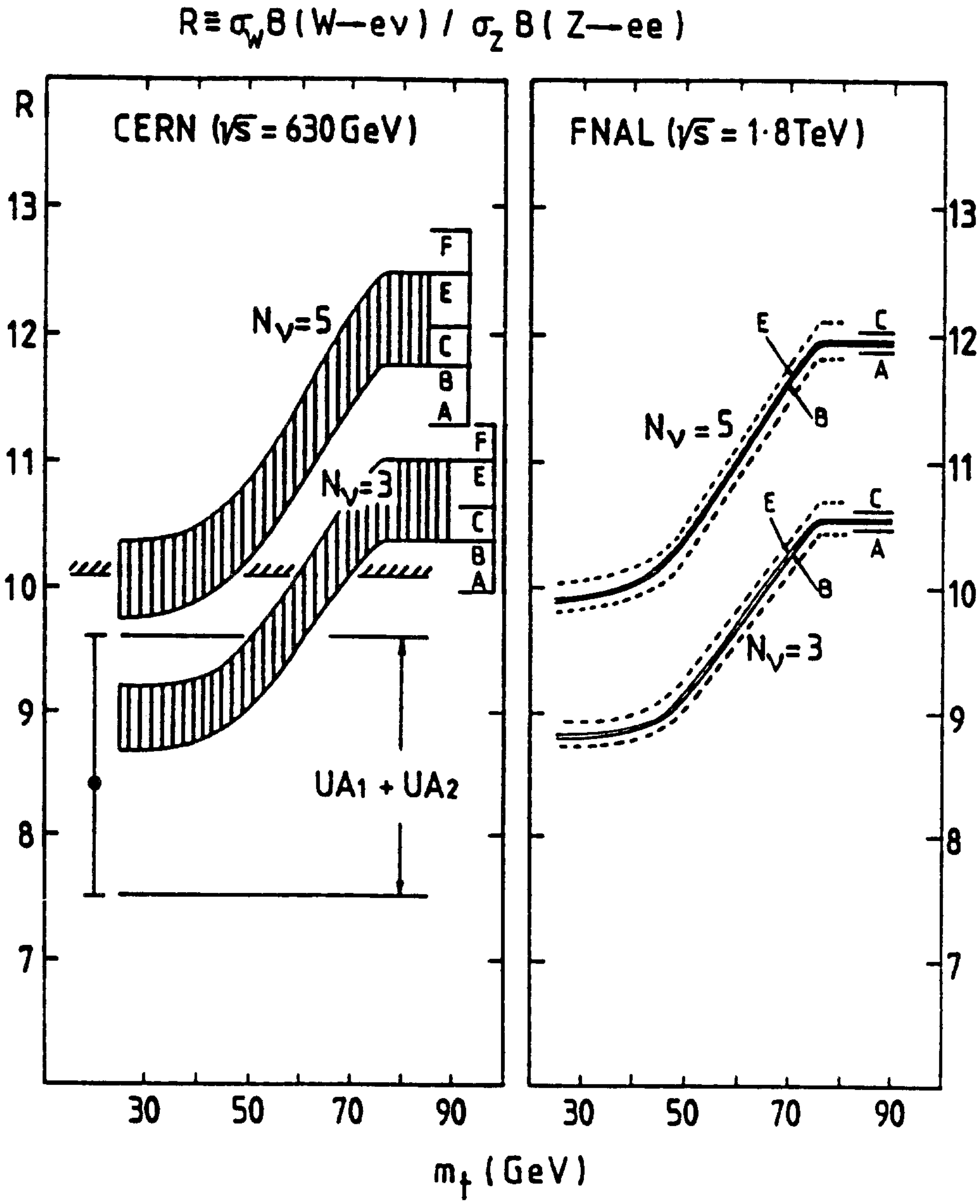

Figure 6: Predicted values of ratio R as a function of the top quark mass.

value of $\sin^2\theta_W$.

5. Jet physics

5.1 One jet and one particle inclusive cross-sections

New theoretical results have been presented at this conference on one jet inclusive cross-sections [59], and on one particle inclusive cross-sections [60]. Both calculations include the effects of radiative corrections using the explicit results for the higher order matrix elements given in ref. [61]. At present the results include only a limited number of parton sub-processes, although in principle the extension to all sub-processes presents no difficulty. The jets in the calculation of ref. [59] are defined as the amount of energy deposited in a certain solid angle $\Delta R = \sqrt{[\Delta y^2 + \Delta\phi^2]}$ where Δy and $\Delta\phi$ specify the extent of the cone ΔR in rapidity and azimuth. This allows one to investigate both theoretically and experimentally the dependence on the cone size ΔR. As expected the sensitivity of the physical prediction to the choice of the factorisation and renormalisation scale μ is reduced after the inclusion of the higher order corrections.

The results for the one parton inclusive cross-section in ref. [60] give complementary information and will be useful for the calculation of one hadron inclusive cross-sections. They can also be used to estimate the contribution of light quark fragmentation to the production of direct photons or heavy quarks at large p_T.

5.2 Theory of multijets

The theoretical description of events containing many jets can conveniently be illustrated by considering in detail the case in which there are four jets produced. There are two mechanisms which give rise to four jet events. The first of these is the Bremsstrahlung mechanism, which produces four jets because of the radiation of additional partons from a process involving a single hard scattering of two incoming partons. The second mechanism is double parton scattering which produces four jets by the independent scatterings of two pairs of partons at two distinct values of the impact parameter. These two mechanisms can be distinguished because they have quite different kinematic structures. The four jet events produced by bremsstrahlung are expected to have a typical bremstrahlung shape, (*i.e.* predominantly collinear and soft emission), whereas the double parton scattering events are expected to be pair-wise balanced in transverse momentum with no azimuthal correlation between the planes of the two pairs of jets.

The theoretical description of the four jet bremsstrahlung process can be treated within the context of the QCD parton model by calculating the matrix elements for the processes

$$p_1 + p_2 \rightarrow p_3 + p_4 + p_5 + p_6 \tag{5.1}$$

where p_i denotes a parton of type i. The transition probabilities for all possible six parton processes of this type have been calculated at tree graph level. The results for the six gluon process is given in refs. [62,63]. Processes involving four external gluons and two external quarks are studied in refs. [63,64]. Processes involving two external gluons and four external quarks are given in refs. [65]. Finally refs. [66] give the results for six quark processes. Numerical estimates of the rates of production of four or more jets are given in ref. [67].

The information on Bremsstrahlung processes with larger numbers of radiated partons is less complete. Exact results for higher bremssstrahlung amplitudes are in general not available even at tree graph level. For one particular helicity amplitude of the n gluon process the exact result is known[68] for all n. Also a first result for the seven gluon process was presented at this conference[69]. Recursive formulae relating processes with $n+1$ emitted gluons to processes with n emitted gluons are given in ref. [70]. Despite these advances it is probable that one will have to resort to approximate calculations because of the large numbers of Feynman diagrams which occur in processes involving more than six partons.

The approximation methods suggested in the literature depend on several techniques. Some approximations drop terms which are suppressed by $1/N_c^2$ where N_c is the number of colours. Other approximations add extra partons onto already calculated n-parton processes in such a way that known results are recovered when the momentum of the additional parton is soft or collinear with respect to other momenta in the graph[71,72]. Note that these approximations may well be sufficient if they are accurate to within a factor of two. For tree graph calculations the uncertainty associated with the choice of renormalisation scale may well lead to a larger error.

Multijet processes can also be modelled using a Monte Carlo program[73, 74].

These programs contain the matrix elements appropriate for collinear and soft emission of partons, expressed as a Markov process. They can therefore be used to produce an arbitrarily high number of jets, but they are not expected to give a reliable description when the emitted partons are hard or at wide angle. Discussion of the advances in Monte Carlo programs is given below.

Because of the rapid growth of the number of partons at low x inside the hadron, at some energy it will become favourable to have more than one hard scattering per event[75]. As explained above such double parton scatterings will have a distinctive kinematic structure. An exact description of events in which there is more than one active parton per hadron requires a knowledge of the correlations between the first active parton with longitudinal momentum fraction x_1 and the second active parton with longitudinal momentum fraction x_2. These parton correlation functions are not known. In order to give a crude interpretation of the information learnt from the double parton scattering we may write,

$$\sigma_{4-\mathrm{jet}} = \frac{(\sigma_{2-\mathrm{jet}})^2}{\pi R_{\mathrm{eff}}^2} \tag{5.2}$$

$\sigma_{2-\mathrm{jet}}$ is the measured value of the two jet cross-section evaluated at the p_T cut-off which is used to define the jets. The radius R_{eff} measures the correlations between partons within the proton. The UA2 collaboration[76] reports a limit of $R_{\mathrm{eff}} > 0.6$ fm at 90 % confidence level, whereas the AFS collaboration[77] using ISR data reports a value of $R_{\mathrm{eff}} = 0.3$ fm. The latter result is at a larger value of x and hence involves principally quarks rather than gluons. These two results can be explained by assuming that valence quarks are more tightly clustered than gluons, but a definite conclusion will have to await further experimental results.

5.3 Colour flow and Monte Carlo methods

Few high energy collider experiments can be analysed without the aid of Monte Carlo programs. Indeed Monte Carlo programs are now often used for physics comparisons, rather than as a method to model the response of a detector. It is therefore important that such programs are as faithful representations as possible of the physics following from QCD perturbation theory. Modern Monte Carlo programs attempt to describe all of the features of the hard scattering using perturbative QCD and resort to phenomenological models only to describe the underlying soft event and the fragmentation of the partons into hadrons. The model of the fragmentation process is assumed to satisfy a local parton hadron duality condition[78]. Because of the property of preconfinement[79], hadronisation involves only small momentum transfers so that the flows of energy and momentum of the observed hadrons are approximately determined by the corresponding quantities at the parton level.

The perturbative QCD stage of these Monte Carlo programs, in addition to the QCD matrix element for the basic hard scattering includes the dominant logarithmic effects due to the emission of collinear partons, both in the initial and final state. These effects are included by a parton shower. A second improvement is the inclusion of soft gluon radiation[80]. It is important to emphasize that soft in this context means soft with respect to the hard scale of the interaction. In the production of a jet of 1 TeV, a 50 GeV gluon could reliably be treated as soft. The coherence of the soft radiation is included in an approximate way by including dynamically restricting the phase space for gluon emission[81].

For the case of three jet events in e^+e^- annihilation the coherence of the radiation from the hard partons leads to the string effect[82,83]. In the language of perturbative QCD, the string effect is a result of constructive and destructive interference. Of course, it is entirely unremarkable that such interference effects should be observed in quantum field theory. However, it is interesting to note that the experimental evidence indicates that such interference effects survive the hadronisation process.

At sufficiently high energy, the colour structure of a hadron-hadron collision in which a hard scattering occurs will also determine the pattern of associated radiation. Because the distribution of this radiation is not significantly altered by hadronisation the observed pattern of the hadrons which lie between the jets will depend on the colour of the partons participating in the hard scattering. Possible method of observing such coherence effects at hadron colliders have been proposed in ref. [84]. The method involves the ratio of the transverse radiation in different parts of the calorimeter. Such measurements are also important because they allow one to distinguish the transverse energy associated with a hard scattering event from the transverse energy present in a normal minimum bias event[85]. It is important to try and find an experimental measurement which distinguishes the string picture of the radiation associated with a hard scattering from the QCD coherence picture[86]. This may be difficult experimentally, because the differences are expected to be suppressed by $1/N_c^2$.

6. Heavy quark production

The photoproduction and hadroproduction of hadrons containing heavy quarks has been extensively studied during the last two years. It is believed[87], (even though no all orders proof exists), that the total cross-section for the inclusive production of a heavy quark pair is described by a QCD improved parton model formula,

$$\sigma(S) = \sum_{i,j} \int dx_1 dx_2 \; \hat{\sigma}_{ij}(x_1 x_2 S, m^2, \mu^2) \; F_i^A(x_1, \mu) F_j^B(x_2, \mu) \tag{6.1}$$

where S is the square of the centre of mass energy of the colliding hadrons (A and B) and F are the number distributions of partons. The sum on i and j runs over the light quarks and gluons, but not over the heavy quarks. Processes involving heavy quark constituents in the incoming hadrons are suppressed by powers of m, the heavy quark mass[88]. Interactions of the produced heavy quarks with spectator partons are also suppressed by powers of m. A model of spectator interactions involving a limited class of graphs treated in a non-relativistic approximation has been presented in ref. [89]. When integrated outside the range of validity of the approximation the model suggests that these power corrections may be of order Λ/m, rather than Λ^2/m^2 as originally expected. In view of the potential phenomenological importance of these terms for charm production, it is important to check whether the result of ref. [89] survives in a complete analysis.

The short distance cross-section $\hat{\sigma}$ given in Eq. 6.1 is calculable as a perturbation series in the running coupling $\alpha_S(\mu)$ where μ is the renormalisation and factorisation scale. Early work on QCD corrections to heavy quark production is given in refs. [90, 91, 92, 93]. In ref. [94] the first radiative corrections to the heavy quark hadroproduction cross-section were presented, taking into account all sub-processes and both real and virtual corrections. The resultant form of the short distance cross-section is,

$$\hat{\sigma}_{ij}(s, m^2, \mu^2) = \frac{\alpha_S^2(\mu^2)}{m^2} \left\{ f_{ij}^{(0)}(\rho) + 4\pi\alpha_S(\mu^2) \left[f_{ij}^{(1)}(\rho) + \overline{f}_{ij}^{(1)}(\rho) \ln\left(\frac{\mu^2}{m^2}\right) \right] + O(\alpha_S^2). \right\} \tag{6.2}$$

with $\rho = 4m^2/s$, and s the square of the partonic centre of mass energy. In ref. [94] a complete description of the functions f_{ij} including the first non-leading correction was presented. These may be used to calculate heavy quark production at any energy and heavy quark mass. The form of the functions f_{gg} have been also calculated in ref. [95].

The important properties of the functions f are as follows. The functions $f_{gg}^{(1)}$ and $f_{gq}^{(1)}$ (as well as $\overline{f}_{gg}^{(1)}$ and $\overline{f}_{gq}^{(1)}$) tend to calculated constants[96] at high energy, because the higher order corrections involve the exchange of a spin one gluon in the t-channel. The lowest order terms, $f_{q\bar{q}}^{(0)}$ and $f_{gg}^{(0)}$ involve at most t-channel quark exchange and therefore fall off at large s. Near threshold the higher order terms $f_{q\bar{q}}^{(1)}$ and $f_{gg}^{(1)}$ display a very rapid $\ln^2(\beta^2)$ growth, ($\beta = \sqrt{1 - 4m^2/s}$). The origin of these correction terms which are numerically important is explained in ref. [94]. For attempts to resum these terms of this form in Drell-Yan processes we refer the reader to ref. [97].

Complete theoretical results for the photoproduction of heavy quarks in $O(\alpha_S^2\alpha)$ have been presented in ref. [98]. The real $O(\alpha_S^2\alpha)$ matrix elements are given in ref. [99].

6.1 Photoproduction of heavy flavours

New results have been presented on the photoproduction of charmed particles by E691[100] and NA14'[101]. These experiments have a large number of fully reconstructed charmed particles ($10^3 - 10^4$) and good acceptance in the forward region where the bulk of the photoproduction cross-section is expected to be produced. Since the experiments are performed on light nuclei the extraction of the cross-section per nucleon does not introduce a major ambiguity. Fig. 7 shows a comparison of the new data with the lowest acceptable theoretical predictions [98], taking into account the uncertainties associated with the choice of input parameters. The major uncertainty associated with the value of the heavy quark mass is shown explicitly. Values of the heavy quark mass $m_c < 1.5$ GeV are excluded. Substantial agreement is found between the data points in Fig. 7 and earlier values[102].

6.2 Hadroproduction of heavy flavours

The experimental situation for the hadroproduction of heavy quarks has been reviewed in refs. [103,104]. Before comparison can be made with total cross-section predictions per nucleon, one must extrapolate the measured rate to the whole of

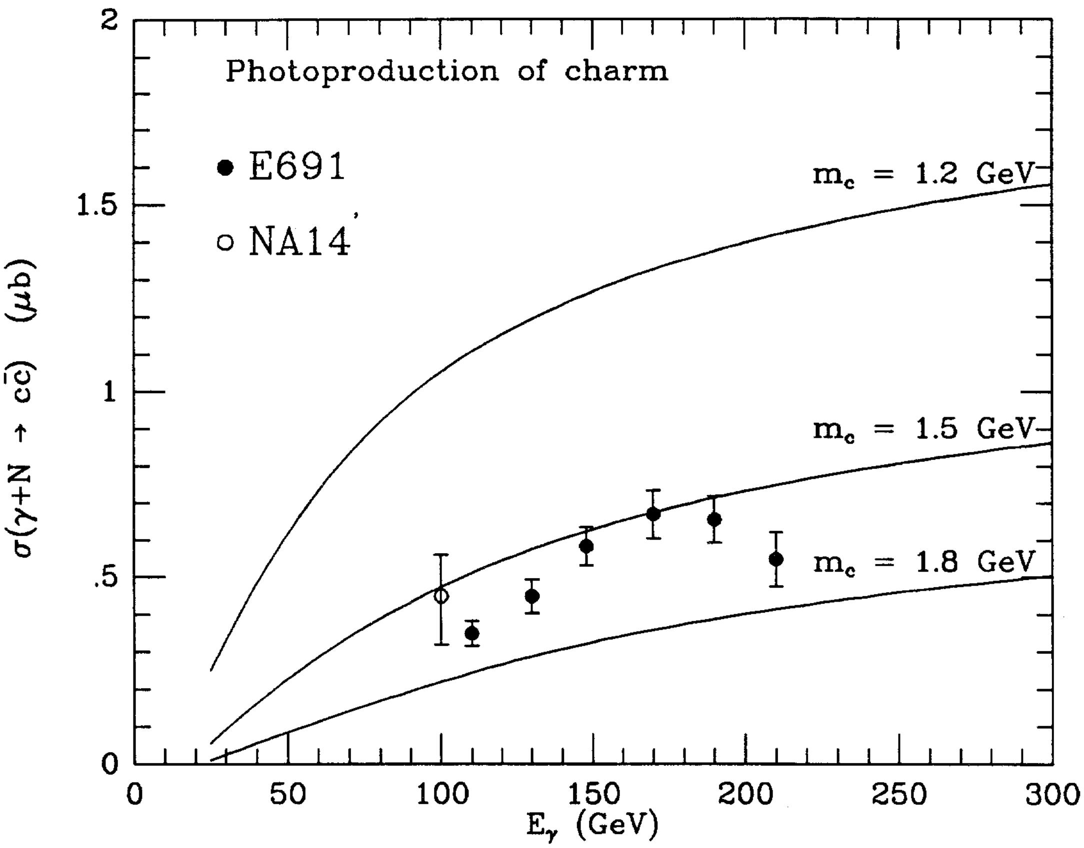

Figure 7: The measured $\gamma N \to c\bar{c} + X$ cross-section compared with theoretical lower bounds for various values of the mass.

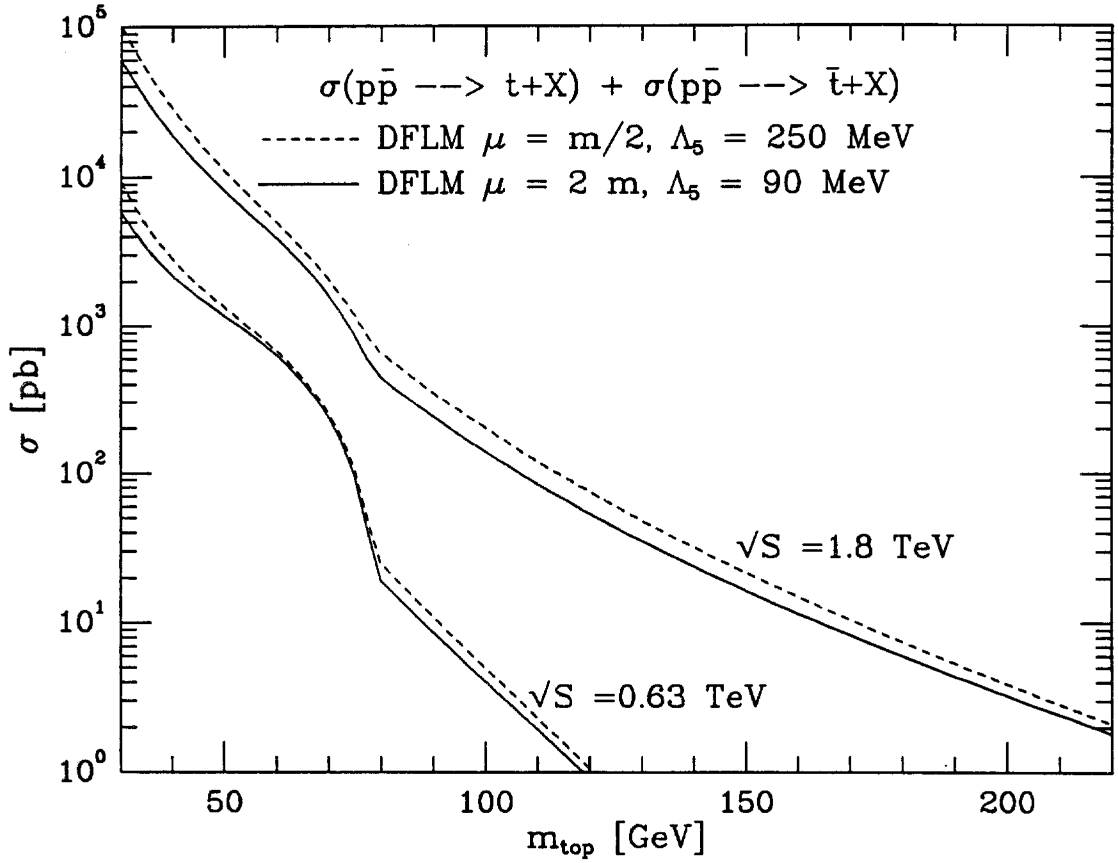

Figure 8: Top quark production as a function of the mass.

phase space, correct for the branching ratio to the observed decay mode and, if necessary, reduce the nuclear cross-section to a nucleon cross-section.

The new experimental results for charm production can be summarised as follows. There is now considerable evidence that the nuclear dependence of the cross-sections is given by A^{α} with $\alpha \neq 1$[105]. There are a large number of experiments for which agreement can be found with the QCD parton model predictions[106] including the higher order corrections, with a charm quark mass $m_c \sim 1.5$ GeV. However there are still some experiments which apparently after extrapolation lead to much larger total cross-sections than predicted by the QCD parton model as given in Eq. 6.1.

Experimental rates for bottom production have been given in refs. [107,108]. Because of the heavier quark mass the predictions for bottom production are expected to be more reliable. This is true at fixed target energies but at collider energies they become less certain because of the presence of two scales $S \gg m^2$. The agreement of the measurements with theoretical predictions is fair[109].

The expected rates for top quark production are shown in Fig. 8 based on the calculations of ref. [94]. The parton distributions of Diemoz *et al.*[110] were used and the top quark production from W decay was included using the measured cross-sections. As a result of the α_S^3 calculations the UA1 limit[111] has been revised[106] and is now $m_t > 41$ GeV. From this limit and Fig. 8 one can estimate that about 1000 t or $\bar{t}$ events need to be produced in order to set a limit. Based on this number one can extrapolate to the likely discovery limit on the top quark in the upcoming runs at CERN and FNAL. With 1 pb^{-1} at $\sqrt{S} = 1.8$ TeV or 10 pb^{-1} at $\sqrt{S} = 0.63$ TeV one should be able to discover a top quark with a mass less than 80 GeV.

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